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Analysis and Simulation of DoA Estimation Techniques

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Abstract

Detecting direction of arrival (DoA) using array antenna is considered as common challenge in different typeS of applications, such as sonar, radar, wireless communications. DoA estimation plays an important role in direction finding applications. Further, effective DoA estimation technique is the primary requirements for smart antenna system to realize beam forming technique. Multiple signal classification (MUSIC) and Estimation of Signal Parameters via Rotational Invariance (ESPRIT) algorithms have been widely studied, and they are considered as fundamental for the majority of the DoA methods in literature. In this paper, the principal of DoA estimations is presented. The main focus of the current paper is on the classical methods (Bartletts and MVDR), MUSIC and ESPRIT algorithms. Review of various efforts that have been made to enhance the performance of these algorithms was acheived. Then compared their performances using simulation results. Further, applying MUSIC algorithm to estimate the frequency of the multiple incident signals. The results show that MUSIC algorithm has higher resolution compared to other presented techniques. It is also shown that the accuracy of ESPRIT method is mainly rely on the number of antenna elements. Finally, the principle of Eigenvalues-based method was proposed. It is shown that this method has good resolution with very low complexity as it does not need decomposition of covariance matrix.

Keyword: DoA, MUSIC, Radar, ESPRIT, Bartletts, MVDR

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1. Introduction

Direction of arrival (DoA) estimation is defined as estimating the direction from which the propagated signals arrive to the receiver [1]. It plays an essential role in many crucial applications, for example sonar, medical diagnostics, radar, and wireless communications. Therefore, DoA estimation has gained significant attentions from research communities over the past decades. Various approaches are available to find DoA of the desired signals coming from multiple sources by using antenna array technology. Beamformer operates as a spatial filter since it is used to receive/send signals from/to a certain spatial direction in the presence of white noises [2]. In recent years, the number of DoA estimation techniques using array sensor has grown significantly, and different methods exist in literature that can be categorized based on the required-parameters to perform the estimation. The most widely used DoA techniques are multiple signal classification (MUSIC) and Estimation of Signal Parameters via Rotational Invariance (ESPRIT)**,** and they are the fundamental for majority of the developed methods in literature [3, 4]. The received signals from the output of the antenna array is the combination of noise and signal subspaces, and these subspaces are perpendicular (orthogonal) to each other. Most of the DoA algorithms operate based on the orthogonality between covariance-matrix- eigenvectors associated with the noise subspace and the array vector that corresponds to the true direction of the signals. This orthogonality can be realized since the array vector of the true DoA lies in the signal subspace. Practically, true covariance matrix does not exist, it has to be estimated from taking finite samples (snapshots) of the received signal and then averaged. This known as sample covariance matrix. The eigenvectors of this new covariance matrix are not the same as the those of true covariance matrix. This is due to taking finite number of snapshots during the estimation process. Thus, the array vector of the true DoA does not exactly lie in the signal subspace [5].

The objective of the present paper is threefold: (**i**) review the fundamentals of DoA estimation algorithms, (**ii**) compare the performance of classical methods (Bartletts and MVDR), MUSIC and ESPRIT algorithms via MATLAB simulations, and (**iii**) present the low complex eigenvalue-based DoA estimation.

The present paper is organized as follows: In section two, principle of DoA estimation is explained. In section three, the literature of MUSIC and ESPRIT algorithms is reviewed. Section four reveals the

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research methodology of this paper. The obtained results are presented and discussed in sections five. Section six concludes the main points of the paper.

2. Principle of DoA estimation

Matrix manipulation, particularly, covariance matrix plays an important role in signal processing. As most of the DoA algorithms in literature rely on the concepts of eigenvalues and eigenvectors of covariance matrix. It is crucial to review these concepts before knowing the technical detail of antenna array and DoA estimation algorithms.

2.1. Eigenvalues and Eigenvectors

Eigenvalues and eigenvectors are the mathematical tools that have many important applications in both science and engineering [6]. For the purpose of understanding the concept of this mathematical tool, an example; suppose that our data matrix A is:

$$A = [4 \quad 1\,1 \quad 4\,] \qquad (1)$$

Now to find its Eigenvalue and corresponding Eigenvectors , multiply the matrix with three different vectors, $x1$ $x2$, and $x3$, where

$$x_1 = [1,0]^T, x_2 = [0,1]^T, x_3 = [1,1]^T \qquad (2)$$

It is easy to show that the results of their products with original matrix A as follows:

$$Ax_1 = [4,1]^T \qquad (3)$$

$$Ax_2 = [1,4]^T \qquad (4)$$

$$Ax_3 = [5,5]^T \qquad (5)$$

These mathematical equations are alternatively illustrated in figure 1.

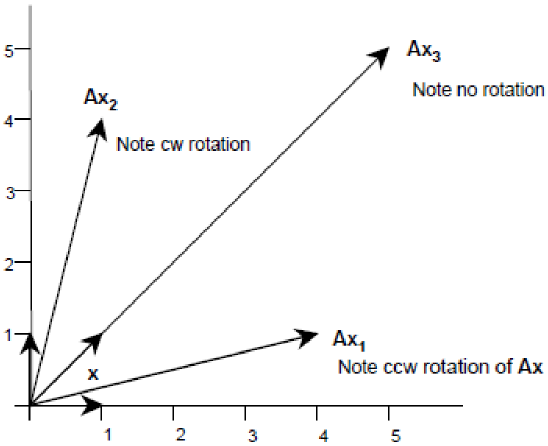

Figure 1. Vector - Matrix multiplications to illustrate eigenvectors [7]

It can be seen that, in addition to enlarged scaling, the directions of vectors $x1$ and $x2$ have been changed when multiplied by the matrix A. While, the direction of vector $x3$ remains the same. Thus, Ax3 is the scaled version of $x3$. Therefore, $x3$ is called Eigenvector of matrix A. In other words, x is called Eigenvectors of matrix A if Ax has the same direction of x. How much the vector is scaled is the Eigenvalues of A, in this case it is equal to five.

Thus, if Eigenvalues of the matrix are known, their corresponding Eigenvectors can be computed and vice versa, by using the following equation:

$$Ax = \lambda x \qquad (6)$$

Where A is data matrix, x is the Eigenvectors of matrix A, and λ is the corresponding Eigenvalues. It has been proven [7] that Eigenvalues and Eigenvectors have the following characteristics:

- If data matrix A is symmetric (i.e., A = A^T) and having unique eigenvalues, their corresponding eigenvectors are orthogonal.
- Symmetric matrix has *real eigenvalues*
- If A be a square matrix: summation of all eigenvalues $\sum_{k=1}^{k=m} \quad \lambda k \;=\; Trace\ of\ (A)$

2.2. Antenna Array Concepts

Before going into the details of the algorithms, it is important to review some fundamental concepts of antenna arrays. Antenna array refers to the multiple number of antennas (or sensors) that arranged in a particular fashion, for example linear antenna arrays and circular antenna arrays [8]. Wireless

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communications, sonar and radar are some of the application areas for array signal processing. In wireless communication, multiple copies of the transmitted signals are received by array antennas, then these signals go into the signal processors to obtain some useful information including their DoA estimation [9].

One of the most widely used array antenna configuration is uniform leaner antenna (ULA). ULA consisting of N antenna elements illustrated in figure 2.

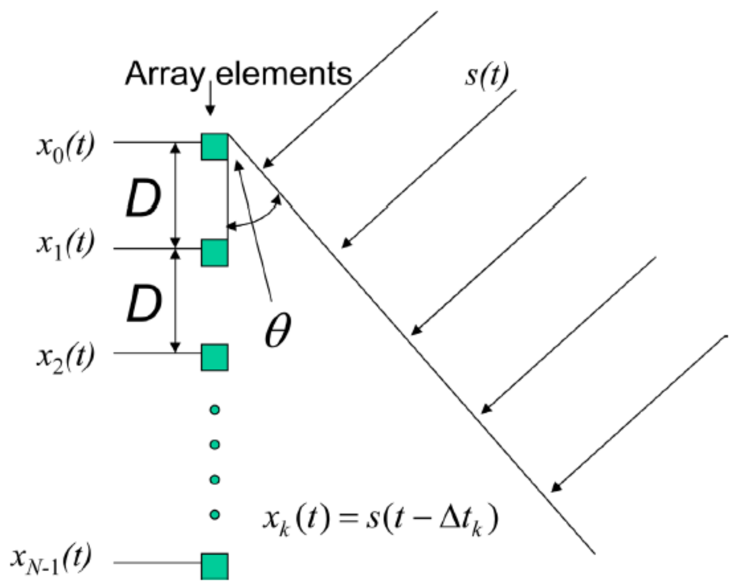

Figure 2. illustration of receiving signals by ULA [9]

Where D denotes the distance between array elements. In narrowband case, the delay between elements is modeled as phase shift between elements. It is assumed that the signal received from first element of the array (referred to the 0^{th} element) has zero phase shift, and the DoA of the signals received from other elements are calculated with respect to this signal. Therefore, arrival delay of the signal received from k^{th} element is expressed as:

$$\Delta t_k = \frac{D\ k\ sin\ \theta}{speed\ of\ light\ in\ spaec\ (c)} \qquad (7)$$

Then, received n^{th} sample (of the signal) at the k^{th} element of array can be expressed as:

$$x_k[n] = s[n]e^{-i2\pi f_c\,\Delta t_k} \qquad (8)$$

When the distance between the elements D is $d\ \lambda$ = ½ λ, and $c = f_c\ \lambda$; and the above equation can be simplified as follows:

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$$x_k[n] = s[n]e^{-i\,\pi\,k\,\sin\theta} \qquad (9)$$

$$x_k[n] = s[n]\,a_k(\theta) \qquad (10)$$

If there exist r sources, $x_k[n]$ is written as follows [7]:

$$x_k[n] = \sum_{j=0}^{r-1} S_j[n]a(\theta_j) \qquad (11)$$

Then, considering all the array elements, the received signals by the ULA can be expressed in the matrix form as follows:

$$[x_0[n]\; x_1[n] \ldots x_{N-1}[n]\,] = [a_0(\theta_0) \quad a_0(\theta_1) \;. \quad . \quad .\; a_0(\theta_{r-1}) \quad . \;\ldots\; a_{N-1}(\theta_0)\; a_{N-1}(\theta_1) \;.\;.\;.\; a_{N-1}(\theta_{r-1}) \quad]\; [S_0(n)\, S_1(n) \ldots S_{r-1}(n)\,] + [v_0\,(n)\, v_1\,(n) \ldots v_{N-1}\,(n)\,] \quad (12)$$

Equation (12) can be written in more compact form as:

$$x_n = [a\,(\theta_0)\;\; a\,(\theta_1) \ldots a\,(\theta_{r-1})]\, s_n + v_n \qquad (13)$$

$a\,(\theta_i)$ are the steering vectors of corresponding S_i ,where $i = 0,1,..r-1$

All the possible steering vectors can be represented in a compact-matrix known array manifold A. Thus equation (13) becomes:

$$x_n = A\,s_n + v_n \qquad (14)$$

Where x_n represents the received sample-signal vector, s_n and v_n represent signal and noise vectors, respectively.

2.3. Covariance Matrix

In the receiving side of communication systems, the received RF signals are converted to lower frequencies known intermediate frequency (IF) using downconverter. After that, the IF signal is digitized by using analog to digital convertor (ADC). In the case of antenna array, finite number of samples of the transmitted data will be collected from the output of the array to construct data matrix. In most of the applications, for example subspace DoA estimation technique which needs eigenvalue decommission (EVD), data matrix has to be transformed into a covariance matrix. This type of matrix is particularly important to investigate the

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correlations between received signals. Mathematically, covariance matrix of data set x[n] received from M antennas is written as follows [7]:

$$R_{xx} = E\,[x_i\, x_i^T]$$
$$= E[x_1\, x_1 \quad x_1 x_2 \ldots\ x_1 x_M\ x_2 x_1 \quad x_2 x_2 \ldots x_2 x_M \ldots x_M x_1 \quad x_M x_2 \ldots\ x_M x_M\,] \tag{15}$$

Where $E(x)$ represents the expected values of x

According to the statistics, $E(x_1\, x_1)$ = $E(x_1{}^2)$ which corresponds to $r(1,1)$ is the variance (σ^2) of $x1$. Thus, the main diagonal of R_{xx} represents the variance of the process. This is due to the special characteristics of stationary process that $r(1,1) = r(2,2) \ldots r(\,m,m)$. Further, the off-diagonal elements of R_{xx} are covariances such that $r(1,2) = r(2,1)$; $r(2,3) = r(3,2)$ and so on. Therefore, covariance matrix has *symmetric* property. However, true covariance matrix cannot be obtained as it needs infinite number of samples. Alternatively, covariance matric is estimated over finite number of samples:

$$R'xx = \frac{1}{N}\sum_{i=1}^{N} X_i X_i{}^H \tag{16}$$

Where $R'xx$ donates the estimated covariance matrix, N is the number of samples, X is the data matrix, and $(.)^H$ represents Hermitian transpose.

Covariance matrix has the following properties [7]:

- For stationary process, Rxx is *toeplits* matrix.
- Covariance matrix Rxx is symmetric, such that $r_{lk} = r_{kl}^*$, where * represents complex conjugate.
- Covariance matrix is *positive semi definite*. In other words, the eigenvalues of Rxx are always positive.
- The values of x are *correlated* if the off-diagonal elements of Rxx is significant compared to those on the main-diagonal.

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3. Literature review

In this section, a review of some for the works related to the DoA estimations. Particularly, special focus will be on reviewing three categories of DoA methods, namely classical techniques, MUSIC, and ESPRIT methods.

Classical DoA techniques, including Bartlett's method and MVDR, are mainly rely on beamforming. In these methods, the received powers are measured from all directions by using beam scanning. The direction from which the maximum power is received is considered as DoA.

Bartlett's method is one of the oldest DoA techniques introduced [10]. In this method, each antenna elements will be given an equal weight in order to form the steering vector. The output power of the Bartlett's method is represented as:

$$P_{Bartlett}(\theta) = E[y^H y] = E|w^H\, x_n|^2 = E|a(\theta)^H\, x_n|^2 = a(\theta)^H\, R_{xx}\, a(\theta) \qquad (17)$$

The output power (θ) reaches the peak when weight (w) is equal to the steering vector (θ) . The problem with this method is that its resolution mainly relies on the number of antenna elements. Increasing number of antennas, however, results higher cost and more complexity. Therefore, Bartlett's method has poor performance in terms of resolution.

MVDR-based DoA estimation is presented references in [11, 12]. This method is similar to the Bartlett's method since it calculates the strength of the signals in all directions. The direction from which the gain of the beamformer equals to one is selected as the direction of the signal. The weight of antenna based on this method is expressed as:

$$P_{MVDR}(\theta) = \frac{{R_{xx}}^{-1}\, a(\theta)}{a(\theta)^H(\theta){R_{xx}}^{-1}\, a(\theta)} \qquad (18)$$

Where ${R_{xx}}^{-1}$ is the inverse of covariance matrix. $a(\theta)$ is the steering vector corresponding to received signals.

Additionally, the Burg in [13] proposed Maximum Entropy (ME) method to estimate DOA. In this technique, the entropy of the signal is maximized by selecting the most appropriate extrapolation of the covariance matrix. ME finally finds the optimum weight by using Lagrange multiplier method. The

Pisarenko was the first researcher that used Eigenvectors of covariance matrix to estimate DoA, and introduced a method known as Pisanreko Harmonic Decomposition [14]. Pisarenko supposed that one column is the dimension of the noise without taking the number of the signals into account. This method has two limitations: it is applicable to ULA only, and sometimes it generates false peaks [15].

Due to its high resolutions, MUSIC algorithm has attracted a lot of attentions from the research communities. MUSIC algorithm technique introduced and considered [16] as the principle for all subspace methods and many efforts have been given to enhancing its performance. This method depends on the orthogonality between signal subspace and noise subspace. Steering vectors for the incoming signals are in the signal subspace part, thus, it is orthogonal to noise subspace. MUSIC algorithm can be applied not just to ULA, but also circular array antenna geometry. However, this method is computationally expensive as it has to search through all possible angles to find the steering vectors that are orthogonal to the noise subspace. Furthermore, when the incident signals are highly dependent/correlated, MUSIC's performance is degraded. This is because the signal covariance of the received signals loss its full-rank property, known as rank deficiency.

To address this problem, special smoothing (SS) technique is introduced [17, 18], to remove the dependency among the received signals. The principle of this technique shown in figure(3) is to divide the ULA into multiple overlapped subarrays, and then add some phase shifts between them.

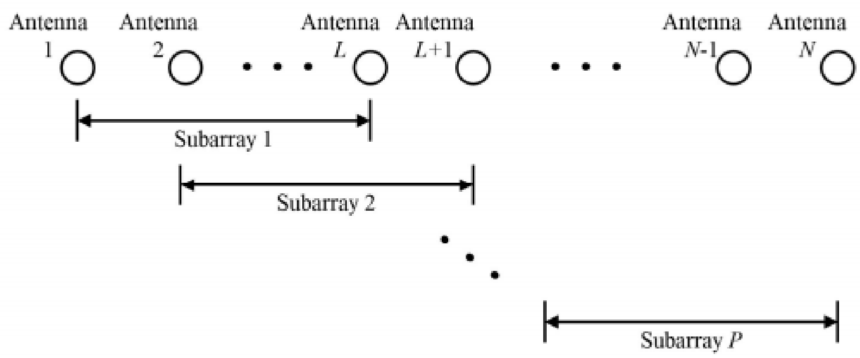

Figure 3. Constructed P subarrays from N antennas [18]

Jalali and Shakibaei [19] developed SS technique and they showed that SS can improve the resolution of the conventional-MUSIC algorithm for correlated sources significantly.

Various effort has been given to reducing the computational complexity of traditional MUSIC algorithm. Among those efforts, Root MUSIC technique is considered [20, 21]. The spectrum of root-MUSIC derived from the conventional MUSIC as follows:

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$$P_{MUSIC}(\theta) = \frac{1}{a^H(\theta)Q_n\, Q_n^H\, a(\theta)} = \frac{1}{a^H(\theta)C\, a(\theta)} \qquad (19)$$

Where **C** is a matrix from $Q_n\, Q_n^H$ and $(.)^H$ represents Hermitian transpose

Then,

$$D(z) = \sum_{l=N+1}^{N+1} C_l\, Z^{-1} \qquad (20)$$

Where C_l donates summation of the lth diagonal C, and D(z) is equal to the inverse of P_{MUSIC} assessed on the unite circle. Thus, k zeros of D(z) will be available on the unit circle. When k is the number of sources. In case of no noise, roots of polynomial D(z) will be exactly on the unit circle, however, when noise exist these roots will be near the unite circle. Therefore, roots of D(z) lie close to the unit circle corresponds to the peak in the MUSIC spectrum, thus it indicates direction of the incoming signals. Root MUSIC is computational less expensive than MUSIC algorithm. This is because root MUSIC technique does not perform searching through all array vectors to indicate DoA. Instead of that searches for the roots only that lie close to the unite circle which represent DoA. The researchers [22] perform a comparison between MUSIC and root MUSIC algorithm using ULA configuration. Their results indicate that root MUSIC algorithm has better estimation accuracy than that of conventional MUSIC technique. Although root MUSIC is faster and more accurate than MUSIC, it is applicable only to ULA configuration, and the roots of the polynomial D(Z) not always find the exact DoA [23].

As mentioned before, root MUSIC technique has a very poor performance if it is applied to an array configuration rather than ULA. This problem is solved [24, 25]. Jiang, Mao and Liu [24] applied modified root MUSIC algorithm to a unform circular array UCA. They showed that UCA-based root MUSIC can obtain higher accuracy and higher resolution with small number of snapshots compared to the unmodified root MUSIC. Coprime linear array is an important antenna configuration for DoA as it allows significant enhancement in parameter estimation. The researchers [25] co-prime antenna configuration is considered for root MUSC algorithm. They illustrated that their method reduces computational complexity and improves the resolution capability of the conventional root MUSIC.

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Another effort to minimize the computational complexity of conventional MUSIC algorithm is presented by Yan, Wang, Liu, Shen, and Jin [26]. They proposed Real-Valued MUSIC algorithm. Their idea comes from the fact that computational complex of multiplication between any two complex numbers as high as four times for two real numbers. In this method, only real values of the covariance matrix are considered for performing EVD and spectral search. Yan, Wang, Liu, Shen, and Jin showed that the proposed MUSIC technique provides approximately 75% reduction in the computational complexity without degrading its performance compared to the standard MUSIC method.

Cyclic MUSIC (C-MUSIC) algorithm is first introduced [27] for estimating DoA of the cyclo-stationary signals. By using the spectral correlation characteristics of the received signals, C-MUSIC method can separate desired and undesired signals and estimate the direction of the former one only due to signal selective property. Whereas, MUSIC detects DoA of the desired as well as interference sources. A new version of C-MUSIC is developed [28] called Extended C-MUSIC (EC-MUSIC). Charge, Wang and Saillard concluded that their proposed method gain two advantages over the standard C-MUSIC: the resolution of C-MUSIC is improved and EC-MUSIC can localize more sources than number of the antennas.

Blind DoA technique known as Multi-Invariance MUSIC (MI-MUSIC) is proposed [29]. Computational cost of MI-MUSIC is significantly lower than that of MUSIC. This comes from the fact that MI-MUSIC explores 1D-searching rather than 2D-searching done by MUSIC algorithm. To the maintain the high accuracy of the conventional MUSIC in the case of low SNR, improved MUSIC (I-MUSIC) is proposed [30]. The aim of this algorithm is to re-construct the MUSIC spectrum function in a such way that illuminates complexity introduced by the covariance steering vectors. Al-Tabatabaie stated that such covariance matrix can be replaced with a new matrix generated as the result of correlation between noise subspace and vector mode ($Q_n^H | \; 1 \; 0 \ldots M-1|$). Al-Tabatabaie found that I-MUSIC has lower complexity compared to original MUSIC algorithm and those presented [27-29].

ESPRIT algorithm for DoA estimation firstly introduced by Roy and Kailath [31] This technique operates based on two assumptions; first, the signal type must be narrowband, second the number of the sensors/antennas have to be greater than the number of the sources [23]. ESPRIT has gained a lot of interest from researchers [31-36].

Time and space ESPRIT (ST-ESPRIT) algorithm are introduced as an improved version of standard ESPRIT method [37]. The objective of ST-ESPRIT method is to eliminate errors introduced due to the effect of non-linearity and improve computation time. Firstly, (T-SPRIT) is applied to ULA, in this stage data matrix is

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divided into subspaces. This technique reduces the computational time and non-linearity effects since the only data enclosed by the subspaces are selected. After that T-SPRIT and S-SPRIT are combined to form the ST-SPRIT and applied to a non-uniform linear array geometry. T-SPRIT and S-SPRIT found that lower computational time and high estimation accuracy are obtained compared to the original ESPRIT method.

Taym et al. [38] developed QR-TLS DoA estimation based on the principle of ESPRIT method, then they implemented the proposed technique on NI PXI platform. LU-based DoA estimation is proposed and implemented using FPGA in [38]. After factorizing the data matrix into lower (L) and upper (U) matrices, ESPRIT concepts are adapted to find direction of the incidence signals. They stated that this method has lower complexity, lower FPGA resource utilization, and high accuracy compared to its counterparts. Therefore, it is the best candidate for real-time implementation.

Oumar, Siyau and Sattar [39] performed a comparison between both MUSIC and ESPRIT algorithms. These algorithms are compared based on, number of snapshots, SNR level, and number of antennas. They presented that when the number of snapshots reduces (from 1024 to 64), accuracy of both algorithms is negatively affected since lower number of snapshots means the signals are more correlated. Overall, MUSIC algorithm outperforms ESPRIT algorithm in terms of estimation accuracy under different situations.

4. Research Methodology

In this section, we reveal the methodologies for each of the obtained result presented in the section five.

4.1 Detecting number of sources

In most of the DoA estimation techniques, it is assumed that the number of sources is previously known. This assumption, however, leads to fault estimation even with the very high-resolution algorithms such as multiple signal classification (MUSIC). Therefore, it is essential to correctly detect the number of the observed sources [40]. In this subsection was discussed the principle of some of the algorithms used for number of source detection.

4.1.1 Eigenvalue Grade Method (EGM)

EGM is the determines of the number of sources with the help of signal-subspace Eigenvalue and noise-subspace Eigenvalues. After forming the covariance matrix mentioned in section 2, it is required to apply EVD to determine the eigenvalues of both signal and noise subspaces. The eigenvalues then have to be

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sorted in descending order. Assume that **P** wavefronts arrive to the antenna array consisting **M** elements. Hence, the **M** x **M** covariance matrix has **M** eigenvalues sorted as follows:

$$\lambda_1 \geq \lambda_2 \ldots \geq \lambda_p \geq \lambda_{p+1} \geq \lambda_{p+2} \ldots \geq \lambda_M \qquad (21)$$

The basic principle of EGM comes from the fact that the difference between λ_p and λ_{p+1} is significant. Therefore, checking the differences between adjacent eigenvalues will help us to determine the number of sources P. The EGM procedures are concluded as follows[40]:

- Calculate the average grades $(\Delta\lambda')$**:**

$$\Delta\lambda' = (\lambda_1 - \lambda_M)/(M-1) \qquad (22)$$

- Determine the gradients $(\Delta\lambda_i)$ for the adjacent eigenvalues:

$$\Delta\lambda_i = (\lambda_i - \lambda_{i+1}) \qquad i = 1,2, \ldots M \qquad (23)$$

- Compare $\Delta\lambda'$ and $\Delta\lambda_i$, then store all the values of i that satisfy:

  $\Delta\lambda_i \leq \Delta\lambda'$ (24)

  Hence, $\{i_k\} = \{i \,|\Delta\lambda_i \leq \Delta\lambda'\}$

- The number of sources:

$$D = \{i_k\} - 1 \qquad (25)$$

4.1.2 AIC and MDL methods

For Akaike Information Criterion **(**AIC) and Minimum Description Length **(**MDL) methods are need to define a likelihood ratio statistic (ɣ) as follows:

$$ɣ\left(\lambda_{p+1,} \lambda_{p+2}, \ldots, \lambda_M\right) = ln\, ln \left[\frac{\left(\frac{1}{M-p} \quad \sum_{i=p+1}^{M} \quad \lambda_{i,}\right)^{m-p}}{\prod_{i=P+1}^{M} \quad \lambda_{i,}}\right]^N \qquad (26)$$

Where N denotes the number of samples, and the value of p is changing from 0 to M-1. For AIC, the number of sources is determined when the value of the following expression reaches its minimum[41]:

$$ɣ\left(\lambda_{p+1,} \lambda_{p+2}, \ldots, \lambda_M\right) + p(2M - p) \qquad (27)$$

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Whereas, determining the number of sources based on MDL method depends on minimizing the following expression:

$$\gamma\left(\lambda_{p+1,}\lambda_{p+2},\ \dots,\lambda_M\right)+\frac{p}{2}\ (2M-p)ln\ N \qquad (28)$$

The above expressions consist of two parts which are likelihood ratio and free adjusted parameters. It is easy to note that the value of the first part decreases and the value of the second part increases gradually as the value of p is changed from 0 to M-1.

4.1.3 General form of criterion (GFC)

GFC is another approach to identify the number of observed sources in the antenna array system and its formular is presented as follows[42]:

$$GFC = -K_1\ log\ [ML] + K_2\ [NFAP]\ K_3 \qquad (29)$$

Where:

$K_1 = -1;\quad K_2 = K_3 = 1;$

$ML = \left|\lambda_p - \lambda_{av}\right| \qquad p = 1,2\dots M$

$$\lambda_{av} = \frac{1}{M}\sum_{i=1}^{M}\ \lambda_i$$

$$Number\ of\ Free\ Adjustable\ Parameter\ (NFAP) = p\left(1+\frac{1}{M}\right) \qquad (30)$$

The number of **p** that minimizes GFC value is selected as the number of sources.

4.2 MUSIC algorithm

To obtain the spectral of MSUIC algorithm, covariance matrix (Rxx) explained in the section 2.3 need to be decomposed based on the principle of eigenvalue decomposition EVD as follows:

$$Rxx = V\ L\ V^H \qquad (31)$$

Where V is the matrix that its parameters are eigenvectors of Rxx and L represents a matrix that has eigenvalues at its main-diagonal and zeros elsewhere. Thus, equation (31) can be represented in matrix form as:

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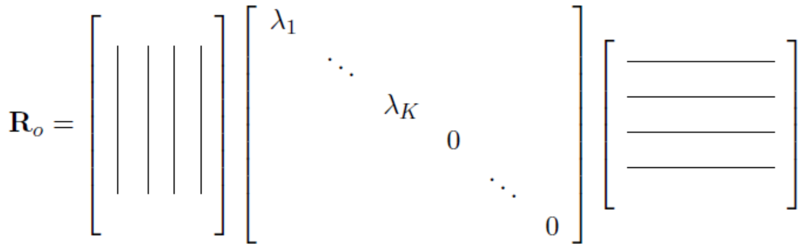

Figure 4. Matrix - illustration of EVD [39]

Or equivalent equation form:

$$R_{xx} = QDQ^H = [Q_s\ Q_n][D_s \quad 0\ 0 \quad \sigma^2 I\ ][Q_s\ Q_n]^H \tag{32}$$

In the case of K sources and M antennas, $(K < M)$. The dimension of R_{xx} is $M\ x\ M$, and it has K large eigenvalues corresponding to the energy of desired signals and $(M - K)$ zero- eigenvalues that correspond to the energy of the noise signals. Thus, eigenvectors corresponding to the large eigenvalue $(x_1 \ldots. x_k)$ form the *signal subspace* Q_s and eigenvectors corresponding to the small (or zero) eigenvalues $(x_{k+1} \ldots. x_M)$ form the *noise subspace* Q_n. D_s and $\sigma^2 I$ are the diagonal matrices with large eigenvalues and small eigenvalues at their entries, respectively. After separating signal subspaces (Q_s) and noise subspaces (Q_n), MUSIC algorithm needs to find steering vectors (θ) corresponding to the desired signals [39].

$$(\theta_i) = e^{-j\pi\ ksinsin\ \theta\ i} \tag{33}$$

(θ_i) represents the steering vector at k^{th} element of the array for the signal arrived at i angle. These vectors construct manifold array matrix A shown in (12). Thus, the peak value of the following spectral function indicates DoA in MUSIC algorithm:

$$P_{MUSIC}(\theta) = \frac{1}{a^H(\theta) Q_n\ Q_n^H\ a(\theta)} \tag{34}$$

In this method, searching technique is applied to find those steering vectors that when multiplied with the Q_n in (16) will generate peak value. Finally, MUSIC has to select those steering vectors that are perpendicular (orthogonal) to Q_n. Ideally, the peak of the MUSIC function would reach to infinity if

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steering vectors were completely orthogonal to Q_n . Practically, it is not the case due to occurring errors during Q_n estimation. The procedure of DoA detection using MUSIC algorithm is shown in the following flow chart.

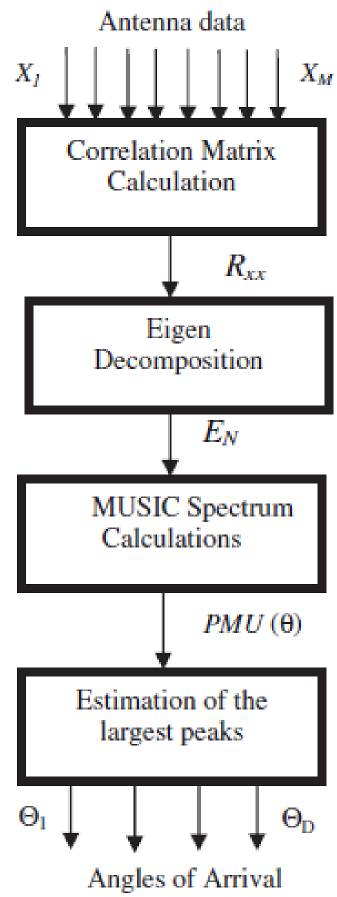

Figure 5. Flow chart representation of MUSIC algorithm procedure [40]

4.3 ESPRIT

The principle of this algorithm relies on the concept of rotational invariance of the desired signals. This concept can be realized with two subarrays that are translationally invariant.

Suppose there are K sources of signals and an ULA consists of M sensors/ antennas. This array is separated to construct two identical sub-arrays each with the dimension of M-1, and separation distance between each antenna element is d.

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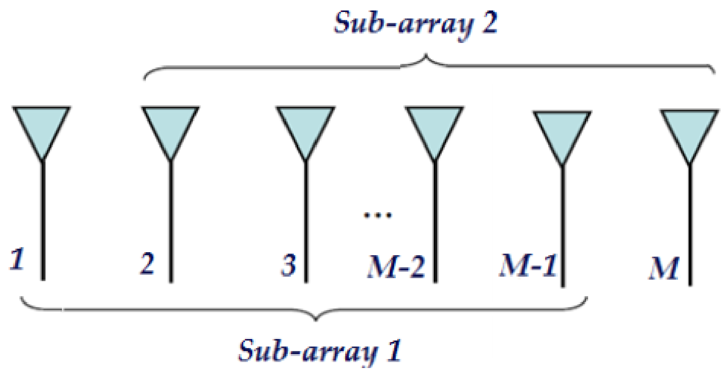

Figure 6 illustration of two subarrays derived from one ULA [32]

The output vector of sub-array1 and sub-array2 are expressed as $y_1(t)$ and $y_2(t)$ respectively.

$$y_1(t) = A(\theta)S(t) + n_1(t) \quad (35)$$

$$y_2(t) = A(\theta)\,\emptyset\, S(t) + n_2(t) \quad (36)$$

Where

$A(\theta)$ represents the manifold matrix

$S(t)$ is the baseband signal.

$n_1(t)$ and $n_2(t)$ are the noise vectors associated with sub-array1 and sub-array2, respectively.

$\emptyset$ donates is a k-by-k diagonal-matrix such as:

$$\emptyset = diag\left[e^{j\frac{2\pi}{\lambda}dsin(\theta_1)}, e^{j\frac{2\pi}{\lambda}dsin(\theta_2)}, \ldots, e^{j\frac{2\pi}{\lambda}dsin(\theta_k)}\right] \quad (37)$$

The overall output can be written as follows:

$$y(t) = [y_1(t)\; y_2(t)] = [A\; A.\emptyset].S_K(t) + n(t) \quad (38)$$

Let's $F1$ and $F2$ be two matrices that represent the signal subspaces of subarray 1 and subarray 2, respectively. These two data matrices can be written with non-singular transformation matrix ψ:

$$F2 = F1\,\psi \quad (39)$$

Furthermore, sub-arrays signal subspaces $F1$ and $F2$ are related to steering matrix A by transformation T such as:

$$F1 = AT \quad (40)$$

$$F2 = A\,\emptyset\, T \quad (41)$$

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By substituting (40) and (41) into (39), we get:

$$A\,\emptyset\,T = AT\,\psi \quad \text{or} \quad \psi = \; T^{-1}\,\emptyset\;T \qquad (42)$$

The eigenvectors of ψ are the columns of T and eigenvalues of ψ fills the diagonal elements of $\emptyset$. Thus, the eigenvalues of ψ need to be found to estimate DoA. Least square solution (LS) are applied to solve this problem and (39) will become:

$$\psi = \; [F1^{H}\,F1]^{-1}\,[F1\;F2] \qquad (43)$$

Finally, DoA is calculated by the following formular [38]:

$$\theta_l = \left[\frac{Arg(\gamma_l)}{2\pi d}\right] \quad l = 1 \dots K \qquad (44)$$

Where, θ_l is the DoA for the source number l, γ_l represents the eigenvalues of ψ .

5. Results and Discussion

In this section, MATLAB was used (R20017a) software to generate some results related to the topics explained in the previous sections.

Firstly, simulation results was provided for detecting number of sources based on the algorithms presented in methodology section. Thus, simulate a system consisting of an antenna array of 5 elements (M=5) and two observed sources (P=2). Then apply the procedure of EGM technique, and the obtained result is shown in the figure 7.

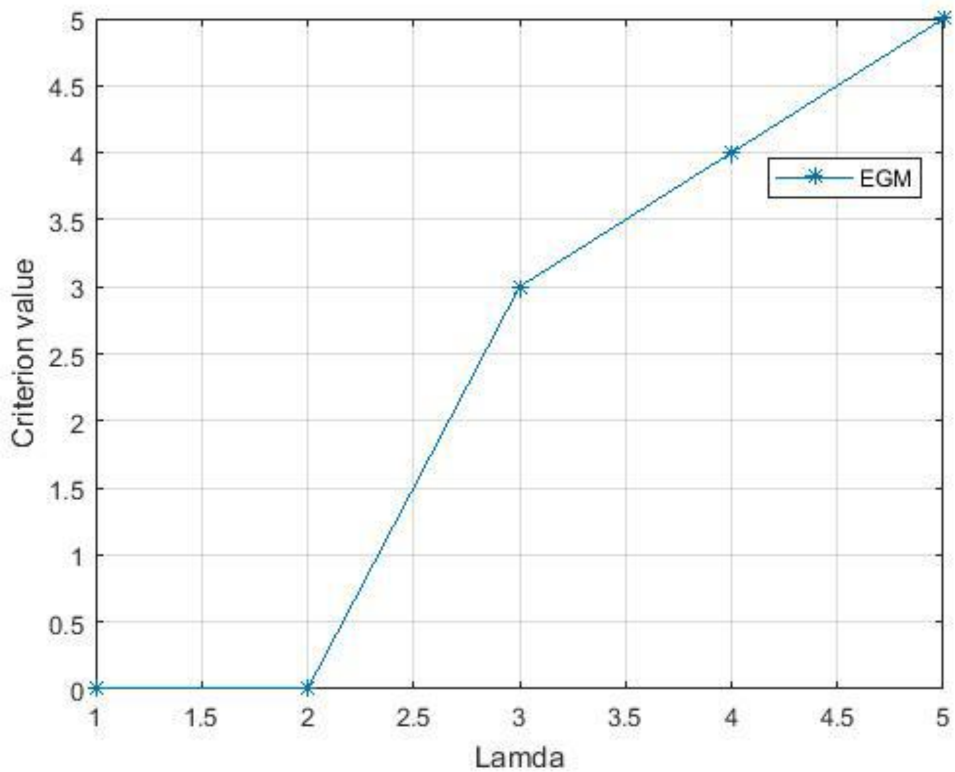

Figure 7. determining number of sources using EGM method

It can be seen that the difference between λ_2 and λ_3 is significant compared to others. Therefore, based on the EGM procedure, the number of sources (P = 2) is correctly detected.

To evaluate the performance of AIC, MDL, and GFC**,** apply the procedure of these algorithms to an antenna array of 6 elements (M=6) with 3 observed sources (P=3). Firstly, plot the likelihood ratio (γ) as shown in figure 8. Then, found the number of sources based on these algorithm as shown in figures 9.

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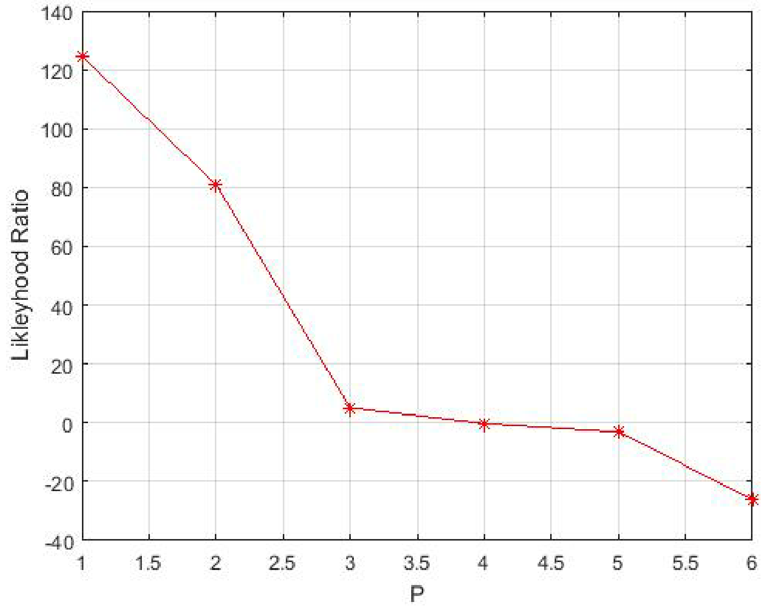

Figure 8. Likelihood ratio as the function of p.

It can be seen that the slope of likelihood ratio changes rapidly when **p** is equal to the real number of sources **P** (i.e., p=3)

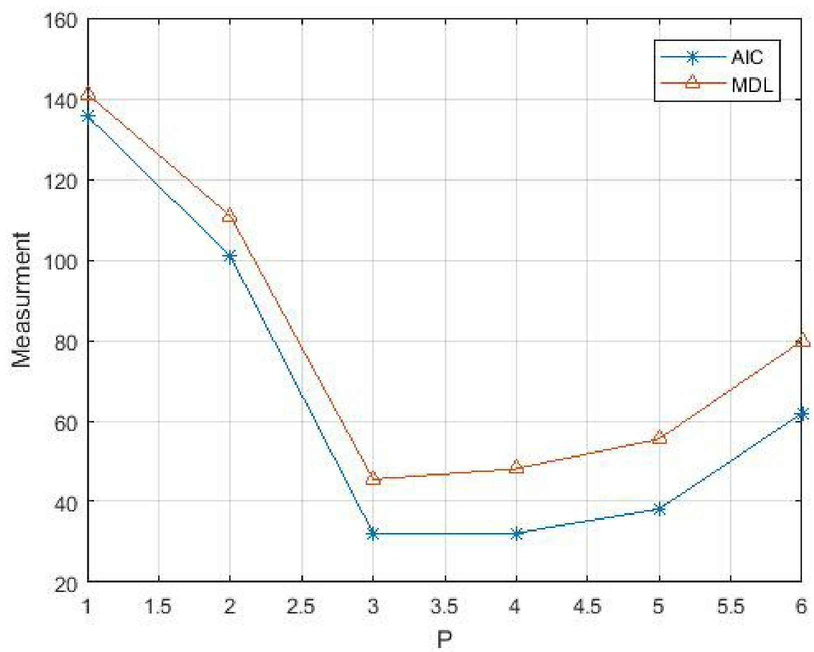

Figure 9. detecting number of sources based on AIC and MDL methods

It is observable that the rapid change in the likelihood ratio results minimum occurrence in AIC and MDL methods. Therefore, the value of both algorithms reaches minimum when **p** is equal to the number of source (**p=3**). Hence, both techniques have detected the number of sources successfully.

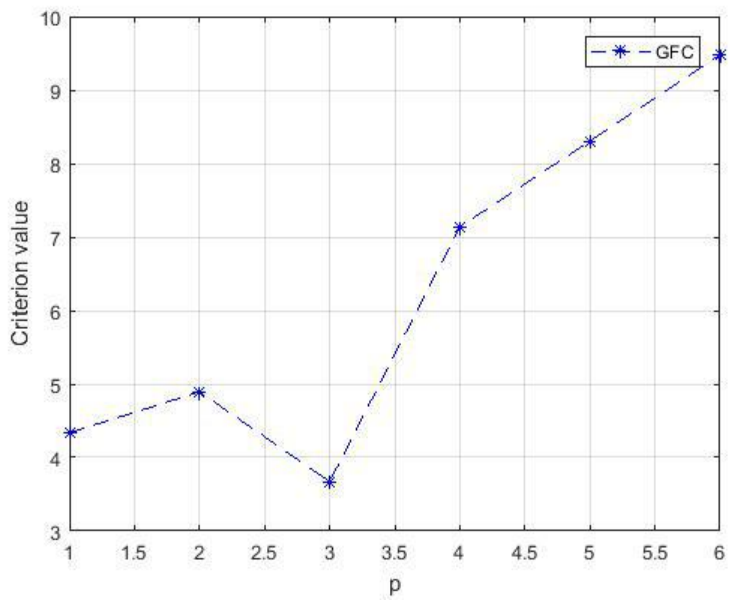

Figure 10. detecting number of sources based GFC method

Figure 10 clearly shows that the result of the GFC becomes minimum when the value of **p** equals to the number of sources (**p=3**). Hence, the number of sources has been identified correctly.

The performance was investigated for two classical DoA estimation techniques that has been explained in detail in previous sections, Bartletts and MVDR. Finally, provide a comparison between these algorithms in terms of resolution. Equation (17) and (18) used along with parameters listed in table 1.

Table 1. Simulation parameters

Parameters	**Value**
Number of antennae elements	10
Number of snapshoots	1000
SNR	2 dB
Antenna spacing	0.5 of wave length
Number of sources	2
Modulation scheme	BPSK
DoA	0° and 8°

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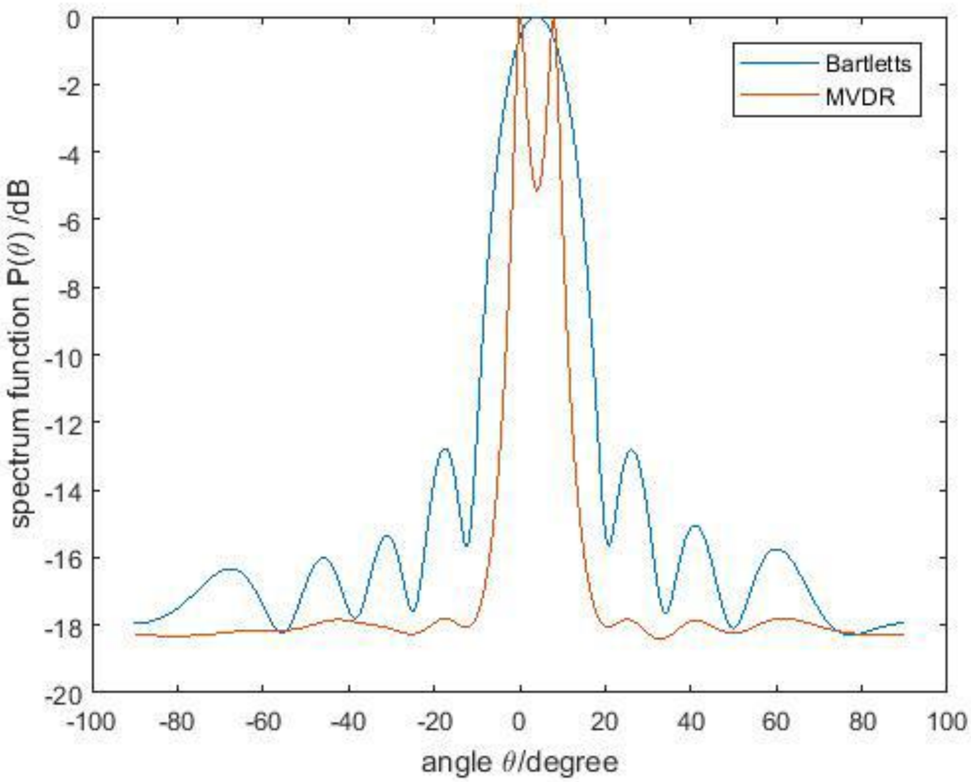

Figure 11. Comparison between the Bartlett and MVDR methods

From the figure 11, it can be observed that the MVDR can detect two closely separated sources (0° - 8°) clearly, but Bartlett's method fails to identify them. Therefore, MVDR has better performance compared to the Bartlett's method. This improvement, however, comes at the cost of more computational complexity as it has to perform inverse of covariance matrix.

Furthermore, simulation was carried for investigating the MUSIC algorithm performance. The parameters of table 1 and equation (34) are used to plot the MUSIC power spectrum.

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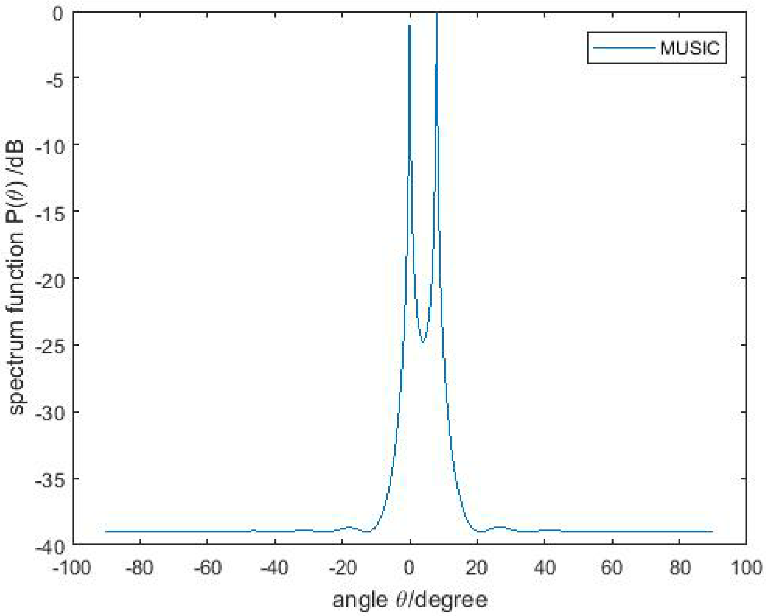

Figure 12. MUSIC algorithm

From the figure 12, it can be observed that the MUSIC algorithm has a sharp spectrum and it can identify the sources (0° - 8°) more precisely. This indicates its high resolution.

To confirm the performance of the MUSIC algorithm, it compared it with the two classical algorithms, Bartlett's and MVDR. The archived result is shown in figure 13

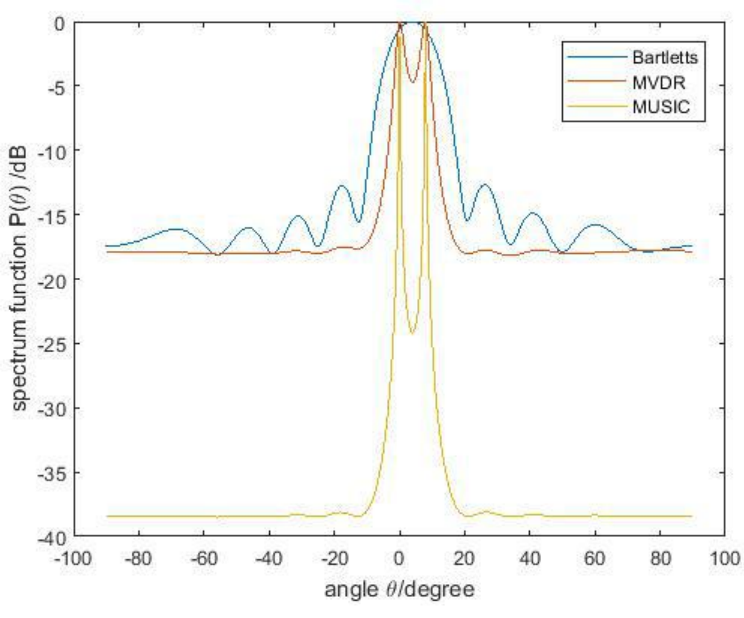

Figure 13. Comparison between MUSIC algorithm and classical techniques

It can be seen that MUSIC algorithm outperforms the classical techniques in terms of resolution. In other word, MUSIC technique can resolve to close-separated sources (0° and 8°) with a resolution that is much higher than those for Bartlett and MVDR techniques using the same parameters.

As explained above, I_MUSIC has been proposed to reduce the computational complexity of MUSIC techniques where its high resolution is maintained. To confirm that, plotting MUSIC and I-MUSIC based on the principle of [26], and the results is illustrated in figure 14.

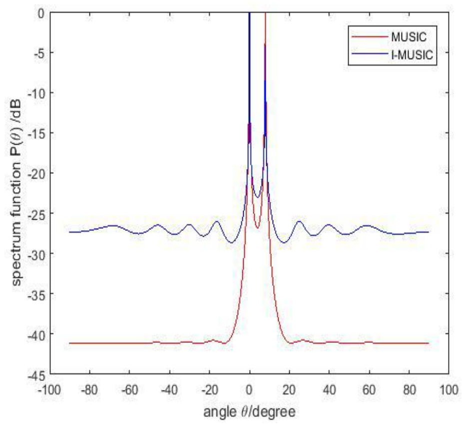

Figure 14. Comparison between MUSIC and I-MUSIC algorithms

Figure 14 shows that, in addition to the lower complexity of I_MUSIC, both MUSIC and I_MUSIC has very close resolution in detecting two sources located in (0° and 8°).

However, the resolution of MUSIC algorithm degrades when the SNR is low. As shown in the figure 15.

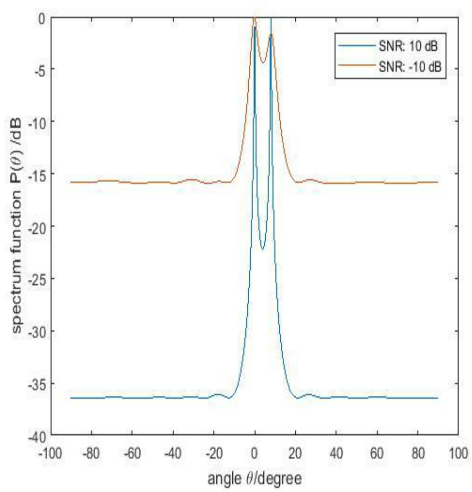

Figure 15. Performance of MUSIC algorithm with different SNR levels

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MUSIC algorithm can be used not only to DoA estimation, but also frequency estimation. To estimate the frequency of multiple signals, the steering vector have to be edited as follows.

$$a(w) = \begin{bmatrix}1 & e^{-i\,w} & e^{-i\,2\,w} & . \; . \; . & e^{-i\,(M-1)w}\end{bmatrix} \tag{45}$$

Where $w = 2\,\pi\,f$.

The MUSIC spectrum for frequency estimation is written as follows.

$$P_{MUSIC}(w) = \frac{1}{a^H(w)Q_n\,Q_n^H\,a(w)} \tag{46}$$

Therefore, the MUSIC algorithm was applied to estimate the normalized frequency of source one (0.5) and source two (0.6). The archived result is shown in figure 16.

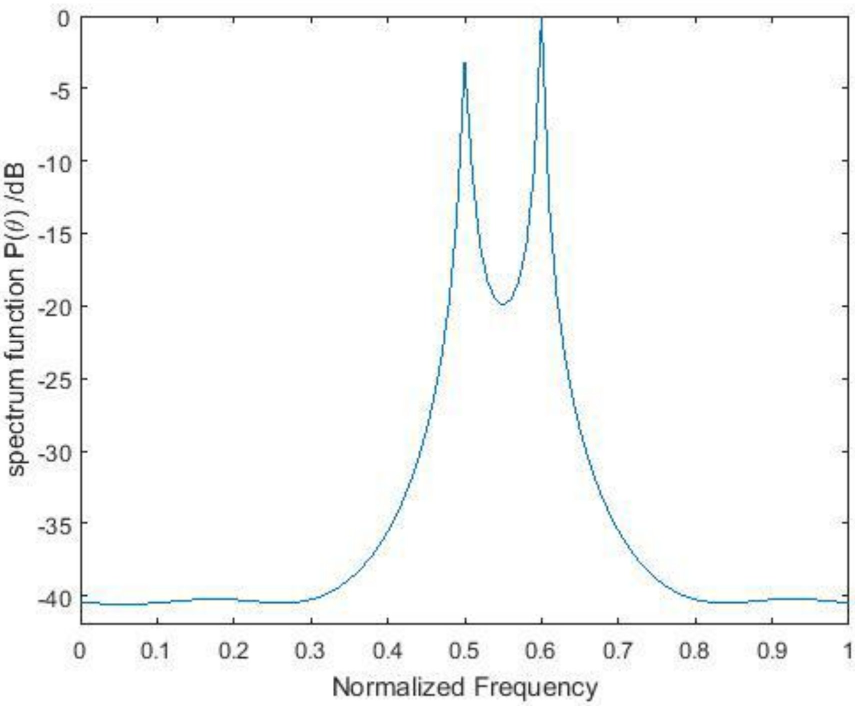

Figure 16. frequency estimation based on MUSIC algorithm

Figure 16 confirms that the principle of MUSIC algorithm (i.e., orthogonality between steering vector and noise subspace) can be used to estimate the frequency of multiple incident signals.

The second most widely used DoA estimation techniques is ESPRIT. The performance of this method observed through calculating its error rate. Finally, simulate the ESPRIT method using the parameters listed in table 2, with varying number of antenna elements. The error percentages of ESPRIT method for detecting two sources (10° and 20°) are shown in the table 3.

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Table 2. Simulation parameters

Parameter	**Value**
Number of sources	2
Number of snapshots	1000
Antenna spacing	0.5 of wavelength
SNR	2 dB
Antenna elements	**vary**
DoA	10° and 20°

Table 3. error percentage for ESPRIT algorithm with varying number of antennas

Number of antenna elements	**Actual DoA**	**Error Rate (%) (Difference between Actual and Estimated DoA**
3	10°	2.08 %
	20°	4.91 %
4	10°	1.46 %
	20°	1.99 %
5	10°	0.2 %
	20°	0.19 %
6	10°	0.1 %
	20°	0.56 %
7	10°	0.7 %
	20°	0.4 %
8	10°	0.04 %
	20°	0.01%

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The results presented in table 3 show that error rate for ESPRIT algorithm decreases as the number of antennas increase. However, higher number of antennas means higher cost and complexity.

As the Eigenvalues represent the energy of the signal, it is required to have maximum Eigenvalues corresponding to the DoA of the desired signals. Therefore, presenting a new method for estimating DoA of the coming signals known as Eigenvalues-based technique. The aim was to find the DoA of source one through calculating its eigenvalues. Finally, changed the value of θ_2, while the value of θ_1 is constant. Then, we observe the DoA of source one. The obtained result is illustrated in figure 17 using the parameters listed in table 4.

Table 4. simulation parameters

Parameter	Value
Number of antennas	8
Antenna spacing	0.5 of wave length
Number of samples	1000
SNR	5 dB
Number of sources	2
DoA of source 1 (θ_1)	0°
DoA of source 2 (θ_2)	Vary (0° - 360°)
Modulation type	BPSK

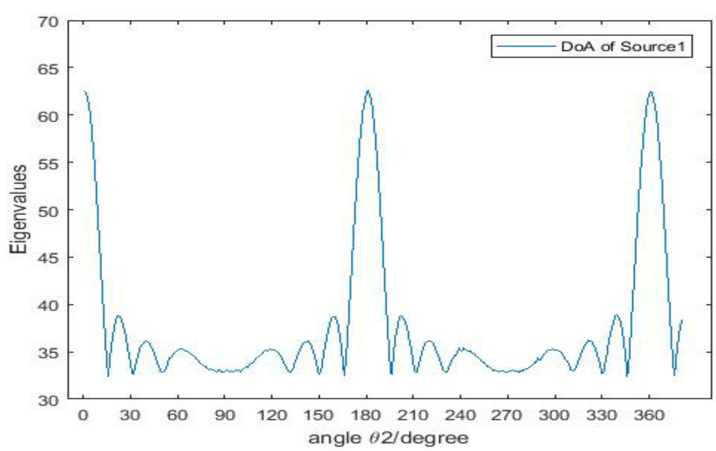

Figure 17. Eigenvalue-based DoA technique.

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It can be seen that the DoA is at the angle where source two are equal to the angle of source one, meaning that they are added constructively Therefore, maximum power at this angle (0°) is observed and this is repeated every π. Thus, this method has very low computational complexity compared to MUSIC algorithm. This is because it does not need EVD for making signal and noise subspaces.

6. Conclusion

Estimating direction of the incoming signals is a backbone for many critical applications including radar and sonar. This paper has presented the principle of DoA estimation. Particularly, the main focus of the paper was on classical methods, MUSIC and ESPRIT algorithms.

Eigenvalues and Eigenvectors are the mathematical tools that indicate about the energy and direction of the signals. The correlation between received signals can be analyzed with the help of covariance matrix. In addition, EVD is a significant approach to construct signal and noise subspaces. Subspace algorithms, MUSIC and ESPRIT, are introduced to overcome the poor- resolution problem of classical DoA methods. Bartlett and MVDR. MUSIC algorithm takes advantages from the orthogonality between steering vectors and noise subspace. Whereas, ESPRIT method depends on the special characteristics of the signal subspace known as rotational invariance.

The simulation result has shown that MUSIC algorithm has much better resolution compared to the classical techniques. However, its performance degrades with low SNR. The frequency of multiple signals can be estimated using MUSIC algorithm. The error rates of ESPRIT technique increase with small number of antenna elements. In this paper, the idea of Eigenvalue-based DoA estimation is presented. It is shown that this new method can detect the DoA with resolution very close to that for MUSIC with very low complexity as it does not need EVD. However, non- of the presented algorithms considered the broadband scenario. Therefore, these algorithms can be extended for wideband scenarios.

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