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Cox model for estimating the Piecewise Constant hazard function based on time segmentation of patients with renal failure.

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Summary

In some observations, especially medical observations (data), observations occur over a continuous timescale. Continuity is such that it is not used in estimating hazard functions. In this case, hazard functions can be estimated into Piecewise hazard functions, because time segmentation allows for flexibility in its application to segment hazard functions into Piecewise Constant hazard functions. This allows for the detection or identification of breakpoints in observations when using specific estimation methods. To address prediction and estimation of the Piecewise Constant hazard function and to improve the accuracy of prediction and estimation for dialysis patients, we used a Cox model with a Breslow estimator, which combines the Lasso and Lasso combined techniques. This allows for identifying breakpoints (jumps) in the Piecewise Constant hazard function. This model was applied to dialysis patients for the period (1/1/2023 to 1/7/2024). The Breslow estimator provides a good advantage in segmenting observation times, which improves the performance of the Cox model. The Piecewise Constant hazard function was estimated for dialysis patients using nine covariates. The results in this research show that the Cox model has high prediction accuracy for deterministic stationary hazard functions .

Keywords: Cox model, time discretization, Piecewise Constant hazard function, combined Breslow-Lasso estimator.

1- Introduction

Many statistical studies, especially those related to the medical field, suffer from data continuity, which makes risk prediction complex. Continuous data are processed using time discretization, which results in loss of access to continuous data.

Prediction and estimation are extremely important for researchers whose studies are medical [16]. Since the goal of the study is to estimate a deterministic fixed hazard function, it must be noted that estimating a Piecewise Constant function poses a significant challenge, making predictions about what will happen to the patient or observer a problem that must be addressed, especially in diseases whose treatment is staged [12].

Since medical, economic, and other data (observations) vary over time, time discretization has gained great importance in these observations, especially in many fields such as statistics and mathematics, because it allows for the extraction of accurate information and estimates for the time period [4] .

Time discretization is used to improve the accuracy of the estimation and prediction model. Using time segmentation, the piecewise constant hazard function is estimated into discrete time intervals. The time intervals selected are very useful for improving the estimation and prediction accuracy of the Cox model, making the data (observations) less difficult to compute. Estimation and prediction are then performed using the Cox model for each time interval, instead of applying continuous data (observations) directly to either the hazard function or the survival function [16] .

In this paper, we aim to estimate the piecewise constant hazard function using a time-slicing Cox model with Breslow's estimator using the Lasso and built-in Lasso techniques. The time-slicing cut-off points were determined using the built-in Lasso techniques, on which the piecewise constant hazard function is estimated for each period.

1.1 Literature Review

In recent years, a number of researchers have attempted to improve the accuracy of forecasting and estimation using various techniques. Allison (2008) proposed an estimation method that applies partial likelihood to discrete-time observational data to improve predictions of the survival and hazard functions. The results of the study showed that the proposed method is an effective method for forecasting and estimation [1] , significantly improving the accuracy of estimation and prediction. Bagdonavicius et al. (2013) proposed a non-parametric model consisting of dynamic regression and accelerated survival models, and applied it to business enterprises. The results of the study demonstrated the superiority of semi-parametric dynamic regression models over non-parametric models based on the results of predicting hazard functions for repayment companies [17]. Clements (2022) aimed to improve the performance of hazard function prediction in a series of missing data, after processing it with the multiple imputation (MI) method [7]. proposed a deep learning method by learning to jointly predict the timing and order of an event in the Cox partial log-likelihood model. This method yielded better results than previous state-of-the-art methods based on the C-comparison index [18]. Hutson (2021) proposed a discrete Weibull model for Poisson-distributed data, and survival and hazard functions were obtained in a discrete setting. Francisco (2018) proposed a method for calculating and estimating survival and hazard functions that combines the multitask logistic regression (MTLR) model and deep learning. When applied to lung cancer patients, the study results demonstrated, through comparison with several models (Coxph, MTLR, N-MTLR), that the deep learning method yielded better performance [10]. Sloma (2021) presented an approach for predicting survival and hazard functions based on partitioning time into a specific number of bins. The approach was applied to the FLCHAIN dataset, and the study demonstrated that the proposed time-binning approach performed better with the CoxpH model and had comparable features and accuracy with the Multi-Task Logistic Regression (MTLR) model [24]. Gensheimer (2019) applied neural networks to predict and estimate the survival and hazard functions based on discrete-time data. The discrete-time data were collected from the SUPPORT study dataset and used in this study. The results of the study demonstrated that the proposed neural network model, Nnet-survival, was the most suitable for prediction and estimation of the survival and hazard functions by comparing it with several models (Cox-nnet, Deepsurv, and the standard Cox proportional hazards model) [11]. Yaqoub and Ali (2020) compared the multiple Cox model with the multiple logistic model. The results of the study showed, through comparing the models and applying them to data sets of kidney failure patients obtained from the dialysis center in Basra Governorate, Iraq, that there is convergence between the two models based on comparison criteria [27].

The research problem is that time observations have difficulty in dealing with continuous times, which makes the process of prediction and estimation complicated. Despite the use of many distributions and models that deal with continuous times, among the methods and models that deal with continuous times, the Kaplan-Meier estimator, which deals with continuous measures and data and thus loses the ability to deal with discrete times. Therefore, the goal is to improve the accuracy of estimation and prediction by using the Breslau method with integrated Lasso techniques based on time discretization .

2. Materials and Methods

2.1 Data

This section uses data on kidney failure patients for the years 2023 and 2024. This data covers the period from January 1, 2023, to July 1, 2024. All observations were obtained from Baghdad Teaching Hospital .

2.2 Cutting times

In order to overcome the difficulty of dealing with continuous times, what is known as “time discretization” was developed, which has become important to do to estimate the deterministic stationary hazard function [16]. Time slicing relies on the use of time slicing methods to estimate a hazard function that is constant for each time period. Figure 1 illustrates the shape of the Piecewise Constant hazard function when time slicing. Although time slicing divides time into time intervals, it can also be referred to as a change point [9]. Therefore, time division can be defined as “obtaining information in real time about events, which is known as “time-event” (from time to event) by applying time division procedures in the form of time periods to better understand and estimate the mechanism of time before starting the study.

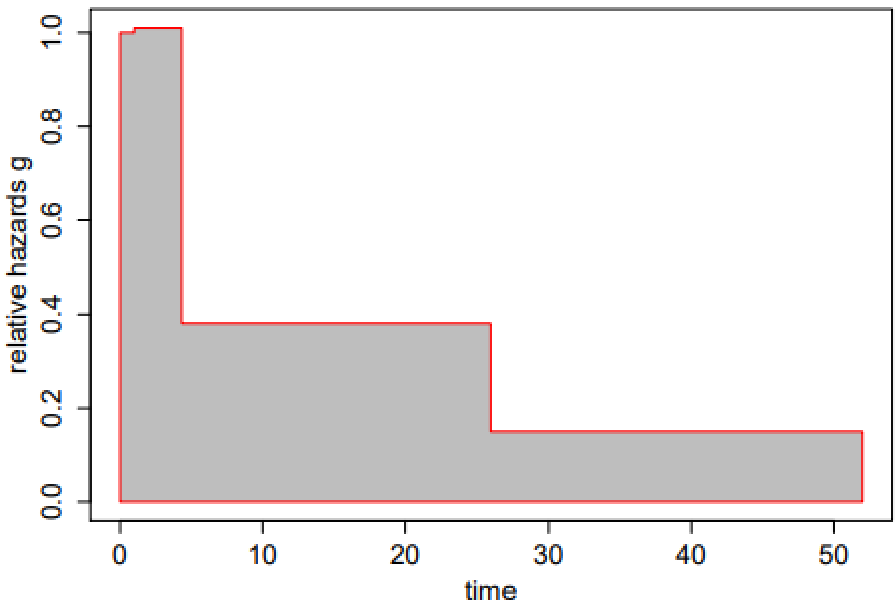

Figure 1: Time-discrete hazard function

There are three types of time segmentation: interval segmentation, percentile segmentation, and binning. The key distinction between the three is that they are comparable. We will use interval segmentation in this paper due to its increased flexibility and novelty [22]. The Cox model with a piecewise constant hazard function allows for segmenting time into eight intervals: $[0,t_1), [t_1, t_2), \ldots, [t_9, \infty)$. Each interval represents the time of the discrete random variable T and is also defined as T∈{1,…, t,…, q}. Intervals exhibit many of the features of survival times, including the presence of competing risks. According to the definition above, intervals play a role similar to the Kaplan-Meier estimator intervals, in that survival and hazard functions are estimated for each interval [25]. In time periods, it is (T=t), and the survival time for each event is $T = \min(T_1, \ldots, T_m)$ [28].

The time intervals are obtained at the level (1, 2,…, m), where m is the total number of times. As mentioned above, the above time segmentation meets the following conditions [25]:

$t_0 = 0$

$t_j = \infty$

The Breslow estimator is used with the built-in Lasso techniques to segment the times, and this estimator is represented by the following mathematical model:

$$\hat{\alpha}_{\lambda} \epsilon \ argmin_{a \epsilon R^n} \left\{ \frac{1}{n} \sum_{j=1}^{n} \left(y_j - a_j \right)^2 + \lambda \sum_{j=2}^{n} \left| a_j - a_{j-1} \right| \right\} \ldots.. \quad (1)$$

While: y_j represents the amount of increase in Breslow's estimator. In more detail:

$$y_j = \frac{\hat{A}\ (\ t_j) - \hat{A}(t_{j-1})}{t_j - t_{j-1}} \quad \ldots.(2)$$

$$A^*(t) = \int_0^t \alpha^*(s) ds$$

$$\hat{A}(t) = \int_0^t \frac{J\left(s, \hat{\beta}\right)}{\bar{z}\left(s, \hat{\beta}\right)} d\bar{N}(s)$$

$$\int_0^t \frac{J\left(s, \hat{\beta}\right)}{\bar{z}\left(s, \hat{\beta}\right)} d\bar{N}(s) = \int_0^t \alpha^*(s) ds + \Delta^J(t) + \Delta^\beta(t) + \eta(t)$$

Thus, the exact points of the original distance t_j, which lie between the minimum value t_{min} and the maximum value t_{max}, are obtained as follows:

$t_j = t_{min} + j \backslash n$

The reason for using the Breslow estimator with built-in Lasso in our research is that PAM suffers from drawbacks, such as:

- The resulting solutions are not stingy, i.e. the parameters of the model are not very large.
- PAM is not constant in jumps, i.e. if the jump is from one number to another, there will be an unspecified and different number between the nodes [20].

•Many estimation methods, including MAP and AI methods, can be used to predict and estimate the Piecewise Constant hazard function, however, in this study, we will use the

Breslau estimator with built-in Lasso techniques, because we know that the slicing and identification of jumps for times will be this way [20].

2.3 Hazard Function Model

Hazard function modeling is essential for predicting observations from time to event. The hazard function model is generally proposed to predict the date of death or failure at time t [25]. However, if the hazard function is applied to continuous observations, it is preferable to convert the time observations to discrete times by segmenting them [16]. The hazard function is generally represented as follows [8] [13] :

$$\Pr\{t \leq T \leq t + \Delta t\} = R(t) - R(t + \Delta t) \ldots.(4)$$

Here, T denotes the failure rate in the time period from time t to time t + Δt, provided that the patient (individual) has survived until time t [8]. According to the above hazard function, the hazard function for discrete data is represented as follows in Equation:(5)

$$h(\tau_{j\backslash x}) = p(T^* = \tau_j \backslash T^* > \tau_{j-1}, x) = f(\tau_j \backslash x) \backslash S(\tau_{j-1} \backslash x) \quad \ldots(5)$$

where T^* is the conditional probability of failure in the time interval $[t_{j-1}, t_j)$, and the other part $T^* > \tau_{j-1}, x$ is the death rate at time τ_j provided that the person (patient) remains alive until time τ_{j-1} [25].

2.4 The piecewise constant hazard function (pc-hazard) model

A discrete hazard function model that aims to estimate hazard rates within time intervals, introduced by Hoem in 1987 [23]. One of the advantages of the pc-hazard function is its ability to estimate the hazard rate for each segment separately (i.e., for each time interval)[16][15][29] . The pc-hazard function model demonstrates the validation of the effects of covariates on the model used [16][15][29] . The pc-hazard function model reflects the stability properties over L time intervals, distinguishing it from the continuous-time hazard function model. The pc-hazard function model can be expressed as follows:

$$\lambda(t) = \lambda_1 \text{ for } 0 < t < t_1$$

λ_2 for $t_1 < t < t_2$

.

.

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λ_j for $t_{j-1} < t < \infty$

where λ_j represents the constant piecewise hazard rate of the pc-hazard function, t represents the time intervals, and t_j is the number of time periods. The parameters take the range from 0 to t_{j-1} and take infinity otherwise.

$$\lambda(t) = \begin{cases} \lambda_i & if \quad 0 < t < t_{j-1} \\ \infty & if \quad t_{j-1} < t \end{cases}$$

According to the above constant piecewise hazard rate, the Cox model with constancy is valid if and only if [14]:

h(t\x)= λ_i . e$^{\beta^t x}$

2.5 Breslow's fused lasso estimator:

Breslow's fused lasso estimator has attracted considerable attention for the PC-hazard function due to its excellent performance and good properties [20]. Fusible lasso techniques have attracted much attention and have been proposed as computational tools for estimating and predicting the constant piecewise hazard function. Breslow's estimators for estimating true hazard rates first appeared in the mid-1970s, created by Breslow and Crowley [6]. Many life tables have used Breslow's estimator extensively to estimate the survival distribution [6]. Because it operates as an aggregate function, it is known as the fusion lasso estimator [26]. For our study, Breslow's estimator combines the fused lasso techniques [20]. Due to its diverse performance, this method has developed into one of the most popular methods for estimating risk rates [26] [2].

According to Figure 1, the basic structure of Breslow's estimator is a counting process model for n observations $N_i(t)$. Each counting process $N_i(t)$ is a doob-meyer decomposition for each observation i. The doob-meyer decomposition is consistent when it divides the random processes into two parts: a martingale and an increasing process (which increases over time) of the following form [5][20] :

$$N_i(t) = \Lambda_i\ (t) + M_i(t) \ldots . (3)$$

$\Lambda_i\ (t)$ can be any mathematical process that is increasing or constant over time. $\Lambda_i\ (t)$ associates the density function λ_i with the Doob-Meyer decomposition model $N_i(t)$, forming the multiplicative structure (i=1,2,…,j). Each series of mathematical processes determines the prediction from time to event [20]. $\lambda_i(t)$ is directly substituted into the Cox model to become:

$$\lambda_i(t) = 1(T_i \geq t) z_i(t, \beta) \alpha^*(t) \ldots . (4)$$

Typically, α^* is a positive (non-negative) function. Finally, the doob-meyer counting process generates the Breslow estimator. In other words, the Breslow estimator is generated from the counting process model $N_i(t)$ after multiplying both sides of equation (4) by $\frac{J(s,\beta)}{\bar{z}(s,\beta)}$.

$$\int_0^t \frac{J(s,\beta)}{\bar{z}(s,\beta)} \, \mathrm{d}\bar{N}(t) = \int_0^t J(s,\beta) \alpha^*(t) dt + \int_0^t \frac{J(s,\beta)}{\bar{z}(s,\beta)} \, d\bar{M}(t) \ldots .(5)$$

Where we can express the true cumulative hazard function of the model by $A^*(t) = \int_0^t \alpha^*(t) dt$, and $\Delta^J(t)$ is the bias term, and $\Delta^\beta(t)$ is the error term. As we see, in this study, the use of the standard theory for the term $\eta(t)$:

$$\eta(t) = \int_0^t \frac{J(s,\beta)}{\bar{z}(s,\beta)} \, d\bar{M}(t)$$

According to Breslow (1974), the left-hand part of equation (5) is represented by $\hat{A}(t)$, which represents Breslow's estimator. We use the following formula to calculate the increments of Breslow's estimator:

$$y_j = \frac{\hat{A}(t_j) - \hat{A}(t_{j-1})}{t_j - t_{j-1}} \ldots . (6)$$

Where the grid of time points in the time interval $[T_{min}, T_{max}]$ is t_j.

For the estimation of the cumulative hazard function in the Breslau estimator, it is required to derive a regression model.

$$y_j = \alpha_j^* + u_j$$

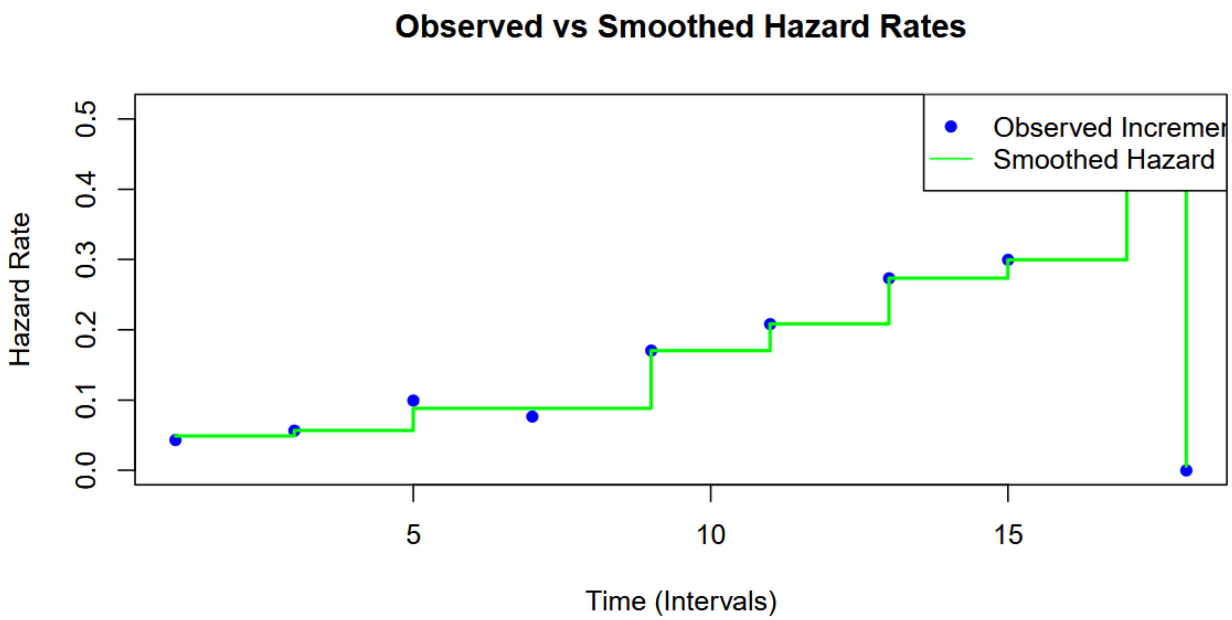

Figure 2: Structure of Breslow's piecewise constant hazard function

The hazard rate $\alpha_j^* = \alpha^*(t_j)$ is estimated from the Lasso technique, and the change location (jump location) is selected using $n_{k(j)}$ as a parameter of the distance between the time of the jump point and the smallest time in the time interval $n_{k(j)} = (T_k - T_{min}) * n$. This is because research has shown that after checking the jump location, the piecewise constant hazard rate can be better predicted. The piecewise constant hazard is best for each change point for a fine grid of time points as follows:

$$\hat{\alpha}_\lambda(t) = \begin{cases} \hat{\alpha}_{\lambda,1} & for\ t \in [t_0, t_1) \\ \hat{\alpha}_{\lambda,j} & for\ t \in [t_j, t_{j+1}) \\ \hat{\alpha}_{\lambda,n} & for\ t = t_n \end{cases} \quad \ldots.. \ (7)$$

As for the $r_{k(j)}$ measure, this measure accurately measures the length of the time period between jump indicators (i.e. the length of the time period between one change point and another) using the following equation:

$$r_{k(j)} = n_{k(j+1)} - n_{k(j)} \dots . (8)$$

where j=1,2,…,n

$n_{k(j)}$ represents the current (actual) jump index. $n_{k(j+1)}$ represents the next jump index.

2.7 Cox Model Design

With increasing prediction accuracy as the primary focus of research, a Cox model was developed based on the piecewise constant baseline hazard rate $\lambda_i(t)$ to increase prediction accuracy and estimate the piecewise constant hazard function (pc-hazard). The following are the components of the Cox model:

First: The Cox model relies on the parametric component, which consists of the covariates, taking into account the parameters and their number for each variable. The parametric component is a type of exponential function used in this model. Its function is $e^{\beta^t x}$, where t represents the time interval, β represents the model parameters, and x is the covariate [21].

Second: Among the components of the Cox model is the non-parametric part, which is called the baseline hazard rate $h_0(t)$. Therefore, the relationship between the parametric and non-parametric parts is expressed in the Cox model as follows: [3].

$$h(t \backslash x)= h_0(t) \, . \, e^{\beta^t x} \quad \dots\dots(16)$$

Cox model components can estimate the piecewise constant hazard function from data with great accuracy and efficiency[14] [19] .

The piecewise constant hazard function is estimated by $h_0(t) = \lambda_j$, which represents the piecewise constant baseline rate in the Cox model, while the parametric part is estimated by the combined Breslow-Lasso estimator, which contains the study-related data information.

The piecewise constant hazard function is obtained by another form of the Cox model:

$$h(t\backslash x)= h_0(t)\ \exp(\beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_p x_p \quad ....(17)$$

Where $\beta^t = [\beta_1, \beta_2, \dots, \beta_p]$: represents a vector of coefficients of the common variables x

X: represents a matrix of covariates

i=1,2,…,n Refers to the number of people (individuals)

j=1,2,…,J Indicates the number of time periods.

3 Discussion of Results

Since the goal of this research is to estimate a piecewise constant hazard function, in this article, the initial work will be carried out in several stages: In the first stage, the cumulative baseline hazard is calculated using the Doob-Meyer equation and Breslow's hazard rate increments, as shown in Figure 2.

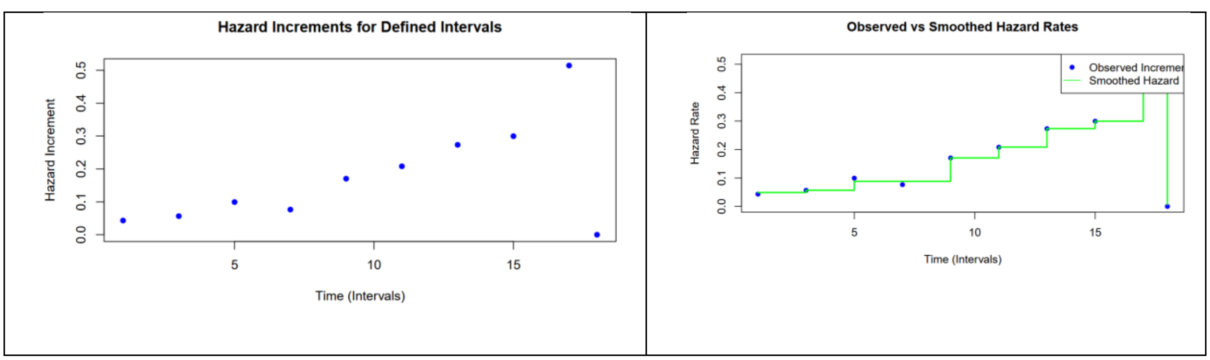

Figure (3) shows on the left the representation of the increases for Breslow's estimator and on the right the cumulative hazard rate.

In the second stage, the Likelihood ratio test is conducted on the study variables. After applying Breslow and Lasso estimators to each time period, they are compared to determine which of the variables has the highest significance. Then, the standard deviations (Std. error) are calculated for the study variables. The appropriate standard deviations (Std. error) should be selected from Table 1.

urea, sugar, pressure, age, and academic.

Table 1: Selection of standard error for Cox model variables

Variables	Std.error	Mini value
Urea	0.001772	*
Pct	1.046952	
Suger	0.003831	*
Pressure	0.007022	*
Age	0.008692	*
Academic	0.076169	*
Sex	0.197235	
Hcv	0.139018	

Hence, from the above table it will be shown that urea, sugar, pressure, age and academic are the most influential variables in the Cox model because Std.error is the smallest among the other variables. Table 2 also shows the parameters of the covariates and the p-values that were chosen as the best significance to represent the piecewise constant hazard function (pc-hazard) in the Cox model.

Table 2: Shows the coefficients of the Cox model variables.

Dependent variable: survival time.

Method: Breslow and Lasso-Gamma distribution.

Sample: January 1, 2023 - July 1, 2024.

$$h(t\backslash x) = \lambda_0(t)ex\,p(\beta_1 * academi\beta_2 * age + \beta_{-3} * cancer + \beta_4 * Hepatitis\ + \beta_5 * renal\ artery\ + \beta_6 pressure + \beta_7 * sex + \beta_8 * sugar + \beta_9 urea\ )$$

variable	Coefficient	p-value
urea	0.004319	0.01481
pct	1.430236	0.17191
sugar	-0.000529	0.89018
pressure	0.004237	0.54628
Age	0.026023	0.00275
Academic	0.047886	0.52956
Sex	0.308732	0.11751
hcv	0.308922	0.02627

The next step is to estimate the cumulative baseline risk using the Doob-Meyer equation (3). Breslow's hazard rate increments are applied to all time periods ([0, 2),[2, 4),[4, 6],[6, 8),[8, 10),[10, 12),[12, 14),[14, 16),[16, 18),[18, ∞). Hazard functions are predicted for each time period, and then the static, piecewise hazard functions for each time period are calculated in the Cox model. By identifying the Breslow change points (jumps) and verifying the validity and heteroscedasticity of the model, the results in Table (3) show that the Cox model is suitable for predicting and estimating the static, piecewise hazard function. Figure 3 illustrates the estimation results.

Table (3) Baseline hazard rate values using Breslow and Lasso

Period	True hazard rate	Breslow hazard estimate	Lasso hazard estimate
[0,2)	0.002013478	0.05915365	2.407205e-02
[2,4)	0.003420674	0.09973937	2.384281e-02
[4,6)	0.004435388	0.12840146	5.425937e-03
[6,8)	0.004969788	0.14337317	1.550870e-03
[8,10)	0.005388641	0.15440117	1.297496e-03
[10,12)	0.006719239	0.19014972	1.264352e-03
[12,14)	0.011475025	0.31772316	1.248516e-03
[14,16)	0.013869371	0.38039925	9.508867e-04
[16,18)	0.015194815	0.41449953	9.037269e-04
[18, ∞)	0.015542693	0.42347791	7.443293e-04

Table (4): Comparison criteria values for the True, Breslow, and Lasso hazard estimate for all time intervals using the comparison criteria Integrated mean squared Error (IMSE).

Period	hazard estimate	IMSE
[0,2)	Breslow	0.09
	Lasso	0.01

	Best	Lasso
[2,4)	Breslow	0.09
	Lasso	0.01
	Best	Lasso
[4,6)	Breslow	0.04
	Lasso	0.01
	Best	Lasso
[6,8)	Breslow	0.09
	Lasso	0.01
	Best	Lasso
[8,10)	Breslow	0.09
	Lasso	0.01
	Best	Lasso
[10,12)	Breslow	0.09
	Lasso	0.01
	Best	Lasso
[12,14)	Breslow	0.25
	Lasso	0.01
	Best	Lasso
[14,16)	Breslow	0.16

	Lasso	0.01
	Best	Lasso
[16,18)	Breslow	0.25
	Lasso	0.01
	Best	Lasso
[18, ∞)	Breslow	0.36
	Lasso	0.01
	Best	Lasso

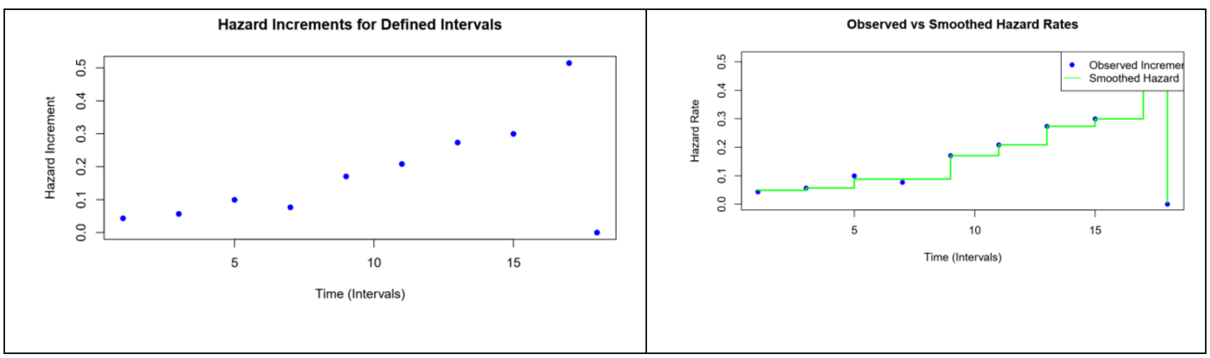

Figure (4) shows on the left the representation of the increases for Breslow's estimator and on the right the cumulative hazard rate.

Conclusion

In this study, we presented an estimation method based on the Cox model. This method is used to segment time into time periods (jumps). It is also used to estimate the deterministic fixed hazard function for each time period. The results demonstrated the ability of the Breslow method with Lasso techniques to segment time, project it into a time period, and predict and estimate deterministic fixed hazard functions. Breslow and Lasso, which use the Cox model, were proven to be superior and accurate in estimating and predicting hazard functions.

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An appendix explaining the factual data

NO.	Cancer Antigen	urea	Pct.	sugar	pressure	age	Academic achievement	sex	Hcv.	C:cen
1	9.1	125	0.11	1	2	29	3	1	2	0
2	8.8	102	0.12	2	2	60	4	1	1	0
3	8.9	104	0.13	2	2	36	2	1	1	1
4	10	93	0.17	1	2	53	1	1	3	1
5	8.1	124	0.17667	1	1	56	5	2	3	0
6	9	145	0.18333	1	2	72	2	1	1	0
7	8.3	112	0.19	1	1	68	1	2	3	1
8	7.4	93	0.135	1	2	63	2	1	1	0
9	8.9	91	0.08	1	2	36	2	2	1	1
10	9.6	109	0.16	2	2	56	1	1	2	1
11	8.9	111	0.12	1	2	48	4	2	1	0
12	8.5	133	0.08	1	2	77	4	1	3	1
13	7.85	131	0.115	2	2	56	3	2	1	1
14	7.2	86	0.15	2	2	42	4	1	1	1
15	9	85	0.8	1	2	45	5	1	1	1
16	7.4	146	0.17	1	2	48	1	1	3	0

17	8.4	147	0.14	1	2	48	3	1	1	0
18	8.3	99	0.135	1	2	48	4	2	3	0
19	9.7	111	0.13	2	2	70	2	2	1	1
20	6.7	142	0.135	1	2	49	2	2	1	1
21	9.1	123	0.14	2	2	57	5	1	2	1
22	8.2	155	0.12	2	2	66	1	1	1	1
23	5.9	98	0.11333	1	1	46	2	2	1	1
24	8.2	80	0.10667	2	2	78	2	1	3	1
25	8.1	89	0.1	2	2	60	1	1	1	1
26	9.5	65	0.12	1	2	64	1	2	1	1
27	11.5	105	0.15	1	2	44	6	1	2	1
28	7.3	140	0.15	1	2	30	2	1	3	1
29	9	100	0.08	1	2	32	5	2	1	0
30	9.2	465	0.095	1	2	34	2	2	1	1
31	7.2	112	0.11	2	2	65	1	1	1	1
32	8.8	132	0.11	2	2	69	1	2	1	1
33	8.6	103	0.11	2	2	52	4	2	2	1
34	8.9	117	0.15	1	2	54	4	1	3	1
35	7.9	154	0.1	2	2	54	2	2	1	1
36	11.8	155	0.1	2	2	55.5	4	1	1	1

37	8.9	209	0.1	2	2	57	3	1	1	1
38	6	152	0.1	2	2	68	1	1	1	1
39	8.9	119	0.12	1	2	66	5	2	2	0
40	8.2	225	0.14	1	2	70	4	2	3	1
41	8.4	214	0.16	1	2	62	4	1	3	1
42	10.3	173	0.12	1	2	24	1	1	3	1
43	9.3	119	0.11	2	2	39.5	2	1	1	1
44	9.3	96	0.1	2	2	55	1	2	1	1
45	7.6	149	0.11	1	2	45	3	2	1	1
46	7	114	0.1	1	2	64	5	1	2	1
47	8.1	133	0.11	1	2	33	1	2	1	1
48	10.9	128	0.06	1	2	55	5	2	1	1
49	8.7	83	0.08	1	2	60	5	1	1	0
50	8	190	0.06	1	2	57	1	1	3	0
51	7.6	192	0.24	2	2	33	1	2	1	1
52	7.4	134	0.20667	1	2	53	5	1	2	1
53	8.7	116	0.17333	2	2	30	2	1	1	1
54	7.5	96	0.14	1	2	52	3	2	1	0
55	8.4	136	0.13	1	2	32	2	1	1	0
56	8.4	116	0.126	2	2	57	2	1	3	1

57	9.3	178	0.122	2	2	53	2	2	1	1
58	7.7	126	0.118	1	1	68	2	1	1	1
59	8.4	141	0.114	1	2	38	2	2	1	1
60	9.1	88	0.11	2	2	30	3	1	2	0
61	7.8	115	0.105	1	2	54	5	1	1	1
62	7.2	121	0.1	2	2	56	5	1	1	1
63	10	137	0.11	2	2	57	2	2	3	1
64	9.6	112	0.11	1	1	37	2	2	1	1
65	8	82	0.12	1	2	58	1	2	1	1
66	8.2	144	0.21	1	2	53	4	1	1	1
67	8	121	0.15	1	2	58	1	2	1	0
68	8.5	86	0.09	1	1	50	5	2	2	1
69	8.8	146	0.16	2	2	78	4	1	1	1
70	8.7	271	0.13	1	2	55	1	2	1	0
71	9.2	83	0.11	1	2	51	3	2	1	1
72	9.1	85	0.09	2	2	65	3	2	2	1
73	9.9	99	0.15	1	1	56	2	2	2	1
74	10	54	0.14	2	2	56	3	1	1	0
75	9	128	0.04	1	2	67	5	1	3	1
76	7.8	197	0.16	1	2	53	4	2	1	1

77	9.3	112	0.12	1	2	26	3	1	1	1
78	8.5	70	0.08	1	2	43	2	1	2	1
79	10	101	0.08	1	2	51	3	2	2	1
80	8.5	168	0.08	1	2	52	3	2	2	1
81	8.8	315	0.08	1	2	63	4	1	1	1
82	8.6	149	0.11	1	1	61	4	2	1	0
83	6.4	106	0.11	2	2	72	2	1	1	1
84	9.4	153	0.11	1	2	69	4	1	3	1
85	6.7	76	0.11	1	2	38	1	1	1	1
86	9.1	94	0.14	1	2	78	5	2	1	1
87	8.8	224	0.13	1	2	53	2	1	2	0
88	7.9	114	0.05	1	2	45	2	2	1	1
89	9.1	96	0.13	1	2	33	3	2	1	1
90	9.1	144	0.13	1	2	33	2	2	1	1
91	9.3	126	0.09	2	2	57	2	1	2	1
92	7.8	99	0.13127	2	2	40	2	2	1	1
93	9.8	140	0.17	1	2	40	2	2	1	1
94	10	113	0.15	1	1	53	1	2	3	0
95	8.3	180	0.06	1	1	55	1	2	3	0
96	9.5	129	0.19	1	2	43	2	1	1	0

97	7.6	145	0.11	1	2	41	2	1	3	1
98	7.5	90	0.09	1	1	36	1	2	1	1
99	8.9	113	0.13127	2	2	40	2	1	1	0
100	9.4	120	0.13127	1	2	41	2	1	2	1
101	9.9	133	0.05	1	1	56	1	2	1	1
102	8.3	150	0.11	1	2	39	2	2	3	1