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Comparative study on finite time tracking design based on PID for 1-degree of freedom manipulator

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HIGHLIGHTS

- A PID-based sliding surface ensures exponential convergence of the system's trajectory to zero.
- Combines PID precision with SMC robustness for enhanced stability against disturbances.
- Achieves finite-time stability, improving efficiency and responsiveness to disruptions.
- Integrates SMC with FTS for faster convergence, reduced errors, and smoother performance.
- Resists disturbances effectively, enabling rapid stabilization in unstable conditions.

Keywords:
Sliding mode controller (SMC); Conventional PID (C-PID); Finite time stability (FTS); 1-Degree of freedom (1-DOF).

ABSTRACT

In this paper, the application of Sliding Mode Control (SMC) and its integration with Finite-Time Stability (FTS) for controlling 1-degree of-freedom (DOF) system has been explored. By combining FTS with SMC, the convergence speed is significantly improved, enhancing the system's resistance to external disturbances. This paper demonstrates a detailed comparative study on the application of SMC based on FTS with a SMC, and with conventional-PID (C-PID). The SMC developed in this paper is based on a PID controller to ensure that the system's trajectory converges exponentially to zero. The results show that using SMC with FTS provides faster and more reliable system stability compared to the other two methods. The comparison also includes the utilization of different functions in the developed SMC, such as sign, tan, and saturation functions. The effects of disturbance and uncertainty have been considered as well. The results show that using SMC with FTS significantly outperforms SMC without FTS and the C-PID in all cases; the achieved Settling Time of SMC with FTS is approximately 1.5 seconds, compared to 4 seconds for the Conventional PID (C-PID) controller. This represents a 62.5% improvement in response speed. The reduction in Overshoot for the C-PID controller is around 15%, while using SMC with an FTS controller reduces the overshoot value to just 3%, indicating an 80% improvement in overshoot and demonstrating enhanced stability and performance.

1. Introduction

The sliding mode control (SMC) methodology is widely regarded as a robust control approach for complex, high-order nonlinear systems, especially in scenarios characterized by parametric uncertainties and external disturbances [1]. In the SMC structure, there are two stages (phases) of operation: the reaching stage and the sliding stage. During the reaching stage, the system states' trajectories are driven to the sliding surface, and then the states move toward the origin asymptotically [2]. Despite its early inception, variable structure systems did not initially gain significant traction in control engineering due to the implementation challenges, particularly issues related to chattering phenomena in sensors, actuators, and switching mechanisms. Nonetheless, SMC offers distinct advantages, notably its capacity to alter the dynamic behavior of a system by appropriately selecting a switching function, thereby rendering the closed-loop response insensitive to matched uncertainties. This inherent robustness has piqued the interest of researchers, driving advancements in SMC methodologies [3]. Recent progress in SMC design has centered on critical challenges such as the mitigation of chattering, compensation for unstructured dynamics, adaptability in the face of system uncertainties, and the enhancement of closed-loop dynamic performance [4]. A key advantage of SMC over traditional linear control strategies, such as Proportional-Integral-Derivative (PID) control, lies in its ability to ensure stability and robust performance in environments where PID control may fail, particularly under conditions of uncertainty [5]. The sliding mode controller (SMC) ensures system stability within a defined range by confining the state to an invariant set, preventing it from exceeding this boundary. This range is determined by initial conditions and design parameters, enabling precise control of system behavior and reducing steady-state errors [6]. Currently, sliding mode controllers have successfully been applied to a wide range of practical systems such as robot manipulators, aircraft, underwater vehicles, spacecraft, flexible space structures, power electronics, control of electric drives, doubly fed induction generators, robotics, and automotive engines

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[7], [8-10]. The rationale for employing SMC in these areas is grounded in its superior performance in managing nonlinear systems, its suitability for multiple-input multiple-output (MIMO) systems, and its applicability to discrete-time systems when appropriately designed [11]. SMC consistently delivers outstanding performance in the presence of bounded uncertainties, external disturbances, and unmodeled dynamics, outperforming established techniques such as robust adaptive control [12], H-infinity control [13], and backstepping control [14].

In the field of dynamic systems control, Finite-Time Stability (FTS) was developed in response to the need to enhance convergence speed and ensure system stability within a predefined time frame. While Sliding Mode Control (SMC) offers significant robustness and strong stability when dealing with complex nonlinear systems, one of its fundamental limitations is the potentially slow convergence toward the desired steady state. This issue is particularly pronounced in cases of asymptotic stability, where the convergence time may extend to infinity [15,16]. To overcome this limitation, the concept of FTS was introduced, ensuring that the system states reach the desired equilibrium within a specified time. This improvement not only accelerates the convergence process but also enhances the system's robustness against external disturbances and uncertainties. Compared to traditional asymptotic stability, FTS provides faster convergence and superior disturbance rejection, making it a highly desirable attribute in control systems [17,18]. By integrating FTS with SMC, researchers have been able to design hybrid control strategies that combine the robustness of SMC with the accelerated convergence offered by FTS [15]. This integration allows for the development of control systems that not only ensure strong stability but also guarantee that the system reaches the desired state within a predetermined time, even in the presence of external disturbances and system uncertainties. These enhancements have made SMC more effective in applications where time-critical performance is essential, such as aerospace, robotics, and power electronics [19-22]. The introduction of FTS into SMC has significantly improved convergence speed, resulting in faster and more reliable system responses. In scenarios involving actuator saturation or time delays, which are common in practical systems, the integration of FTS into SMC helps mitigate the potential negative impacts of these non-idealities, ensuring rapid and dependable system responses [15-22].

Disturbances and uncertainties destabilize conventional PID (C-PID) systems, causing significant oscillations. In contrast, SMC and SMC with FTS greatly enhance stability by effectively mitigating these effects. The primary challenges faced by a system lie in its ability to handle disturbances and uncertainties associated with nonlinear systems. Sliding Mode Control (SMC) is an effective strategy for this class of systems; however, it encounters significant obstacles, such as chattering phenomena caused by the rapid switching of controllers, which can negatively impact system performance. This challenge becomes more pronounced in applications that require fast and precise responses, such as robotics, power electronics, and aviation systems. To overcome these obstacles, a combined approach integrating Finite-Time Stability (FTS) with SMC has been developed, leading to significant improvements in the system's convergence speed and its ability to resist disturbances and uncertainties. This integration improves system stability and enhances its efficiency in managing challenging operational conditions [23,24].

This paper aims to design a controller based on SMC-integrated FTS for a 1-DOF system. This approach combines the robustness of SMC with the fast convergence properties of FTS, ensuring that the system reaches the desired state within a finite time despite the presence of uncertainties or external disturbances. The goal of this design is to enhance system stability and performance, especially in the presence of disturbances and uncertainties. It focuses on comparisons conducted between the performance of C-PID, SMC, and SMC with FTS under the influence of disturbances and uncertainties.

2. Modelling a 1-DOF manipulator

A 1-DOF manipulator, also known as a single-joint robotic arm, is a fundamental example in robotics, as shown in Figure 1. This section will provide a detailed explanation of the modeling process, including the derivation of the equations of motion and visual illustrations. The following parameters are considered in the modeling of a 1-DOF manipulator [25]:
m:Mass of the link (kg), l: Length of the link (m), I: Moment of inertia of the link about the pivot point (kg.m²), g: Acceleration due to gravity (m/s²), θ: Angular position of the link (rad), τ: Applied torque at the joint (Nm).

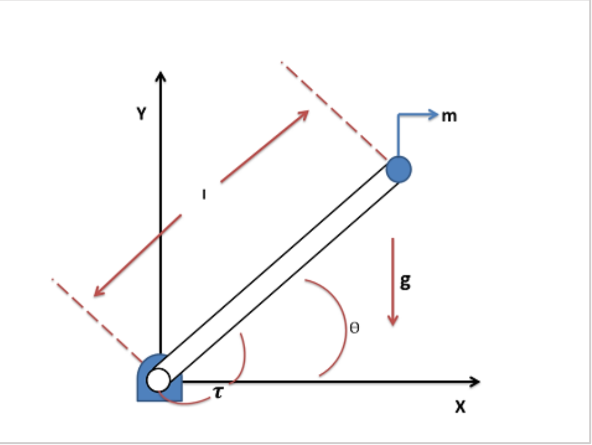


Figure 1: 1-DOF manipulator [25]

For a 1-DOF manipulator, the kinematic Equations (1-3) describe the relationship between the angular position θ angular velocity $\dot{\theta}$ and angular acceleration $\ddot{\theta}$ of the link [25]:

$$\theta = \theta(t) \tag{1}$$

$$\dot{\theta} = \frac{d\theta}{dt} \tag{2}$$

$$\ddot{\theta} = \frac{d^2\theta}{d^2t} \tag{3}$$

The dynamics of the manipulator shown in Equations below from (4) to (12) can be derived using the Lagrangian method, which involves calculating the system's kinetic and potential energies and then applying the Euler-Lagrange equation.

The center of mass of the link moves in a circular path, so its velocity (v) is given by [25]:

$$v = \frac{l}{2}\dot{\theta} \tag{4}$$

Thus, the translational kinetic energy (T_t) is:

$$T_t = \frac{l}{2}m\,(\frac{l}{2}\dot{\theta}\ )^2 = \frac{1}{8}m\,l^2\dot{\theta}^2 \tag{5}$$

The rotational kinetic energy (T_r) is:

$$T_r = \frac{l}{2}I\dot{\theta} \tag{6}$$

The total kinetic energy (T) is:

$$T = T_t + T_r = \frac{1}{8}m\,l^2\dot{\theta}^2 + \frac{l}{2}I\dot{\theta} \tag{7}$$

The potential energy (V) due to gravity is:

$$V = mg\,\frac{1}{2}cos(\theta) \tag{8}$$

The Lagrangian (L) is defined as the difference between the kinetic and potential energies:

$$L = T - V = \left(\frac{1}{8}m\,l^2 + \frac{l}{2}I\right)\dot{\theta}^2 - mg\,\frac{1}{2}cos(\theta) \tag{9}$$

The equation of motion is obtained using the Euler-Lagrange equation:

$$\frac{d}{dt}\left(\frac{\partial \mathrm{L}}{\partial \dot{\theta}}\right) - \frac{\partial \mathrm{L}}{\partial \theta} = \tau \tag{10}$$

Accordingly compute the partial derivatives:

$$\frac{\partial \mathrm{L}}{\partial \dot{\theta}} = (\frac{1}{4}m\,l^2 + I)\dot{\theta} \tag{11.a}$$

$$\frac{d}{dt}\left(\frac{\partial \mathrm{L}}{\partial \dot{\theta}}\right) = (\frac{1}{4}m\,l^2 + I)\ddot{\theta} \tag{11.b}$$

$$\frac{\partial \mathrm{L}}{\partial \theta} = mg\frac{1}{2}sin(\theta) \tag{11.c}$$

Then, by substituting in Equation (12), the final equation of motion for the 1-DOF manipulator is:

$$\left(\frac{1}{4}m\,l^2 + I\right)\ddot{\theta} + mg\frac{1}{2}sin(\theta) = \tau \tag{12}$$

This equation describes the relationship between the applied torque τ, the angular position θ, and its derivatives for the 1-DOF manipulator. It will be used for further analysis and control design.

3. Applied controller design

In this paper, the development of the sliding surface is based on PID control principle. This approach will ensure that the system's trajectory converges exponentially to zero by incorporating proportional, integral, and derivative components. This method enhances system robustness by improving response to disturbances and uncertainties.

3.1 Conventional PID (C-PID)

The general equation for a Conventional PID controller is given by:

$$u(t) = K_p e(t) + K_i \int e(t)dt + K_d \frac{de(t)}{dt} \quad (13)$$

where: u(t) is the control input, $e(t)$ is the error between the desired and actual system output, K_p, K_i, and K_d are the proportional, integral, and derivative gains, respectively.

3.2 SMC and SMC with FTS-based on PID controller

The integration of Sliding Mode Control (SMC) with PID combines the robustness of SMC against disturbances and uncertainties with the smooth response of PID, reducing chattering and improving system stability. This hybrid approach ensures faster convergence and better performance in dynamic systems [26]. Three types of nonlinear functions are used in the control law for illustrative comparison. The applied three functions will be Sign function, Sat function, and Tan function.

3.2.1 The applied Sliding Mode Control (SMC):

The fundamental elements of the developed SMC are outlined below, and the block diagram is shown in Figure 2.

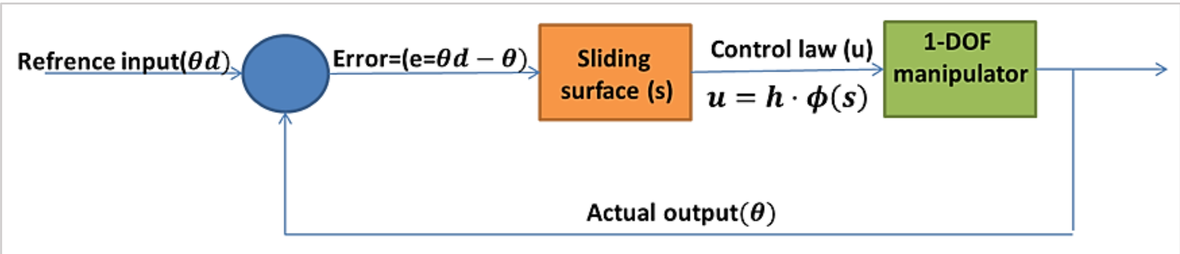


Figure 2: Block diagram of SMC Controller for 1-DOF manipulator

The Sliding Surface Design is mathematically expressed as [26]:

$$s = K_p \cdot e + K_i \cdot \int e\, dt + K_d \cdot \dot{e} \quad (14)$$

where $(e = \theta_d - \theta)$ is the error between the desired and actual system position, $(\dot{e} = \dot{\theta}_d - \dot{\theta})$ is the error in velocity and K_p, K_i and K_d are the control parameters representing proportional, integral, and derivative components, respectively. This formulation ensures that when the system trajectory lies on this surface, the error converges exponentially to zero. This sliding surface combines the advantages of PID control with the robustness of Sliding Mode Control (SMC). By incorporating the proportional, integral, and derivative terms, the sliding surface (s) not only helps minimize the error but also improves the system's response to disturbances and uncertainties. The proportional term $K_p \cdot$ e provides immediate response to errors, the integral term $K_i \cdot \int$ e dt eliminates steady-state errors, and the derivative term $K_d \cdot$ e reduces overshoot and oscillations, leading to a smoother and more stable system performance.

The control input u in SMC is typically designed to drive the system towards the sliding surface s=0. A general form of the SMC control law is:

$$u = h \cdot \phi(s) \quad (15)$$

where h the gain that controls the strength of the sliding mode control action. Three types of nonlinear functions are used in the control law for illustrative comparison. The Sign, Sat, and Tan functions are used in PID-based SMC to enforce the sliding condition, ensuring the system reaches the sliding surface quickly and maintains stability. The PID-based sliding ensures smooth convergence, while the nonlinear functions enhance robustness and reduce chattering. This combination improves system performance in the presence of disturbances and uncertainties. The applied three functions will be $\phi(s)$ The sliding mode function, which can be Sign function($\phi(s) = sign(s)$), Saturation function($\phi(s) = sat(s, \delta)$), and Tangent function ($\phi(s) = tan(T, s)$).

δ is the saturation level limits the control, which signals to prevent over-actuation. It enhances system stability by reducing oscillations. The saturation function limits the control input s to a level δ. T the scaling factor used to modify the value of (s) before applying the tangent function. It provides flexibility in control by offering a nonlinear response that can be adjusted to improve system performance. SMC Term $h \cdot \phi(s)$ Drives the system towards the sliding surface by applying a control force that adjusts based on the sliding surface (s). The choice of $h \cdot \phi(s)$ affects the behavior of the control action, where the sign function provides a robust but potentially chattering-prone response, the saturation function limits control efforts to reduce chattering, and the tangent function offers a smooth, nonlinear control response. The control input $\tau\backslash tau\tau$ is applied to the dynamic model in Equation (11).

3.2.2 Finite-time stability (FTS)

Finite-Time Stability (FTS) ensures that the system reaches the desired state s=0 within a finite time rather than asymptotically approaching it as in typical control systems. The control input u is modified to include a term that guarantees finite-time convergence:

$$u = \mu_s \tag{16}$$

where:

$$\mu_s = | s |^p \cdot sign(s) \tag{17}$$

μ_s term introduces a nonlinear control effort that increases as the sliding surface s moves away from zero, ensuring that the system converges to s=0 within a finite time. p: fractional exponent (0 < p < 10) that controls the rate of convergence. Smaller values of p lead to faster convergence but may also induce more aggressive control actions.

FTS Term μ_s drives the system to the sliding surface in a finite amount of time, providing robustness against disturbances and ensuring that the control system reacts promptly to errors. The finite-time convergence property is particularly useful in applications requiring quick stabilization.

3.2.3 Designing a controller based on sliding mode control with finite-time Stability (SMC with FTS) for a 1-DOF System

To design a controller with both SMC and FTS, as shown in Figure 3, the control law is designed to include both the SMC term and the FTS term:

$$u = h \cdot \phi(s) + \mu_s \tag{18}$$

This control law provides a robust and responsive control strategy that ensures finite-time convergence to the desired state, making it suitable for applications where both precision and speed are critical.

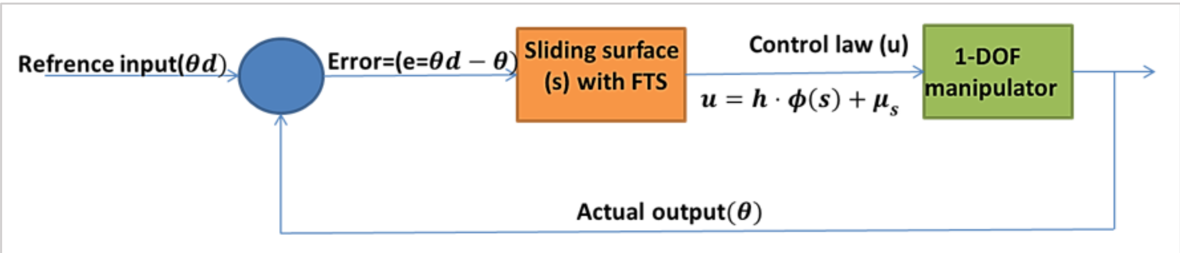


Figure 3: block diagram of SMC with FTS controller for 1-DOF manipulator

The system's stability under FTS can be analyzed using a Lyapunov function:

$$V(s) = \frac{1}{2}s^2 \tag{19}$$

where s is given in Equation (14) and $\dot{s}$ is:

$$\dot{s} = Kp \cdot \dot{e} + Ki \ \cdot e \ + Kd \cdot \ddot{e} \tag{20}$$

Substituting this into the derivative of the Lyapunov function:

$$\dot{V}(s) = s \ \cdot (\mathrm{Kp} \cdot \dot{e} + \mathrm{Ki} \ \cdot \mathrm{e} \ + \mathrm{Kd} \cdot \ddot{e} \,) \tag{21}$$

For stability, we need $\dot{V}(s)$ to be negative definite. This can be achieved if the control law is designed in such a way that $s \cdot s^{\cdot} < 0$, ensuring that the system energy decreases and the sliding surface reaches zero in finite time.

The negative definiteness of the Lyapunov function's derivative guarantees finite-time stability, ensuring that the system converges to the equilibrium point (i.e., s=0) within a finite amount of time. Therefore, the second derivative of the system must always be negative, leading to decreasing energy.

4. Simulation

The simulation is designed using MATLAB, and the reference trajectory is a low-frequency, low-amplitude sinusoidal signal. To investigate the performance of the three developed controllers, we started by applying the signal in Equation (22) using the parameters in Table 1 to the 1 DOF manipulator:

$$qt = 0.001\sin(0.05\pi t) \tag{22}$$

Table 1: The applied gains in the simulation

Gain	Value
h	0.5
Kp	400
Ki	0.0001
Kd	10
δ	0.02
T	0.5

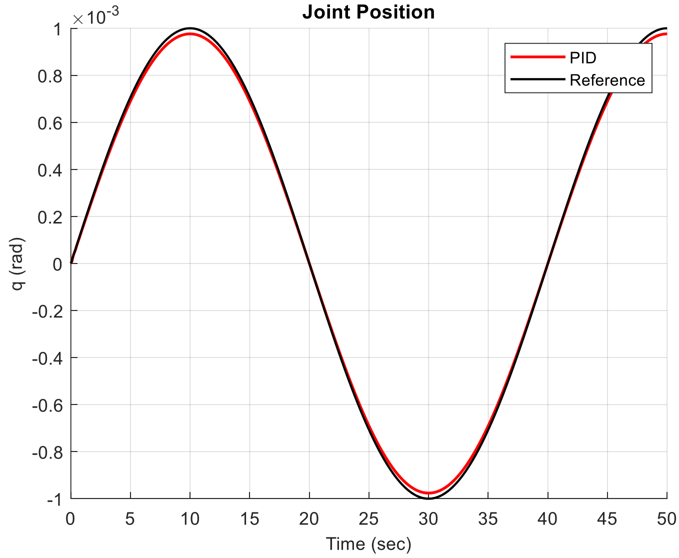
It can be noticed In Figure 4(a) that the performance of the controlled system using a C-PID controller is unsatisfactory. The system exhibits noticeable oscillations, a longer settling time, and potential overshoot. These characteristics indicate a slower response to deviations and less robust stability. In Figure 4(b), the implementation of Sliding Mode Control (SMC) demonstrates a significant improvement in system stability compared to the C-PID controller. SMC effectively reduces oscillations and accelerates the transition to a stable state, showcasing a more robust response to disturbances. The sliding mode mechanism ensures that the system adapts quickly and maintains stability even under varying conditions. Finally, in Figure 4(c), the combination of SMC with Finite-Time Stability (FTS) achieves the highest level of stability. The system reaches equilibrium rapidly without noticeable oscillations or overshoots. The integration of FTS into the SMC framework significantly enhances its performance, ensuring that the system achieves stability within a predefined time frame and efficiently mitigates the impact of disturbances.

In Figure 5(a), the control signals generated by C-PID may be oscillatory and unstable. This indicates that the C-PID controller exerts considerable effort to maintain system stability. Oscillations can be reduced by better tuning the parameters, but the challenge remains significant in more complex systems. In Figure 5(b), The control signals generated by SMC are more stable and less oscillatory than those produced by C-PID. This indicates that SMC can maintain system stability with less effort. SMC provides stable and smooth control signals, reducing oscillations and enhancing overall system stability. In Figure 5(c), the control signals generated by SMC with FTS are the most stable and smooth. The system shows a high capacity to respond efficiently with high stability. FTS works to enhance the stability of control signals further, ensuring the system operates smoothly and reduces the effort required to maintain stability.

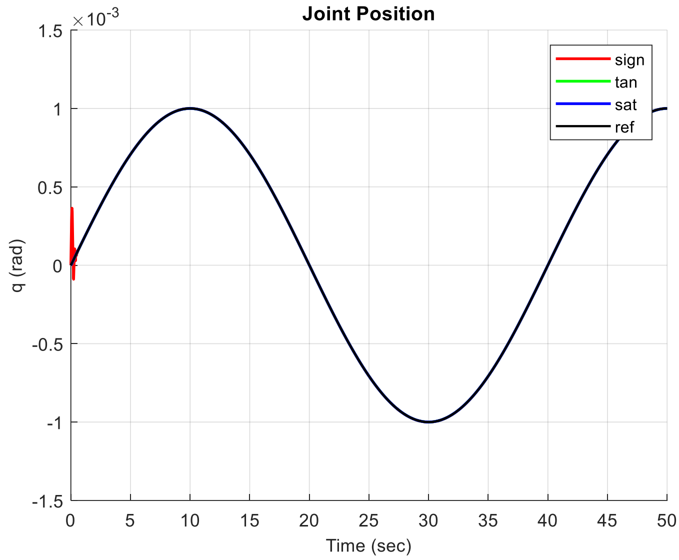
In Figure 6(a), the system suffers from noticeable oscillations in error, indicating that the system is not sufficiently stable under C-PID control. In Figure 6(b), SMC significantly reduces the error and shows higher stability. The system quickly reaches a stable error value without significant oscillations. SMC greatly improves system stability by reducing error oscillations and maintaining system stability. In Figure 6(c), SMC with FTS reduces the error to a minimum, reflecting high stability and precision in control. FTS enhances the performance of SMC by ensuring that the error is corrected quickly and that the system maintains precise stability.

Figures 7 (a, b, and c) shows the sliding surface s versus its derivative $\dot{s}$ over time for the sliding mode control (SMC) strategy using three functions for sign, tan, and sat respectively. The plot helps visualize the dynamics of the control system in the sliding mode. Using the sign-based SMC, the trajectory exhibits noticeable oscillations before settling down. The presence of oscillations is a result of the switching nature of the sign function, which causes chattering. Chattering is a common issue in sign-based SMC and can lead to wear and tear in practical applications. However, the plot shows that the system eventually stabilizes, suggesting that despite the initial oscillations, the control system achieves stability over time. Using the tan-based SMC provides a balance between control effort and stability, making it a suitable choice for applications where reduced chattering and smooth control are important. The saturation (sat) and tangent (tan) functions reduce chattering in Sliding Mode Control (SMC) by replacing the abrupt switching of the sign function with smooth transitions. The sat function limits control input within a threshold, while the tan function provides a gradual, nonlinear response, both minimizing high-frequency oscillations and improving system stability. This results in smoother, more reliable control in practical applications. Using the sat-based SMC can be considered the best choice based on the following reasons:

1) High Stability: The system exhibits high stability with very minimal oscillations.
2) Smooth Response: The control provides a smooth response without abrupt changes in the control input.
3) Reduced Vibrations: The saturation function reduces mechanical vibrations that could cause wear and tear in real systems.
4) Achieving Control Objectives: The system effectively achieves the control objective of driving the error to zero.



(a)



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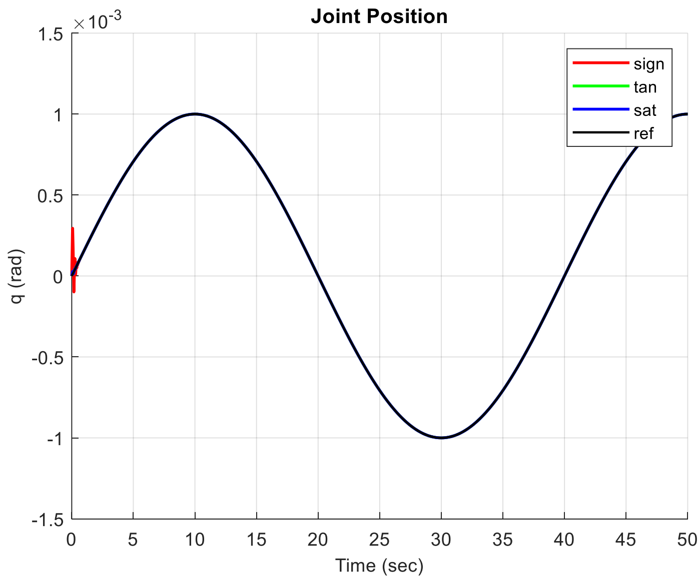
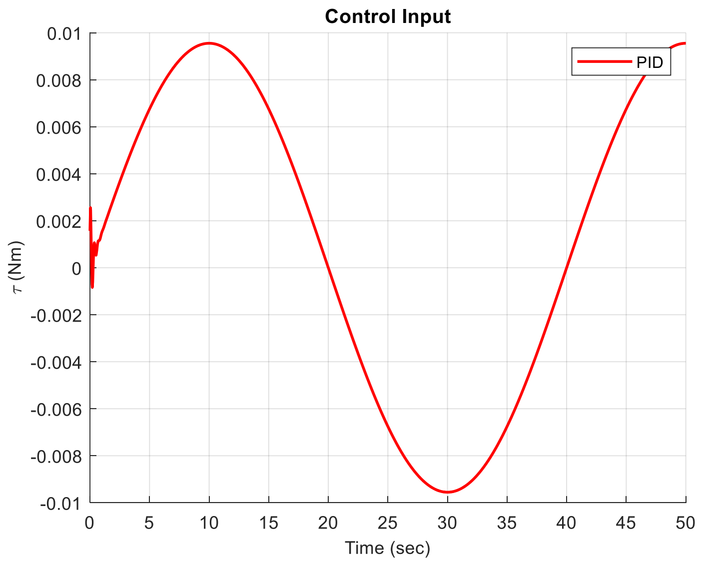
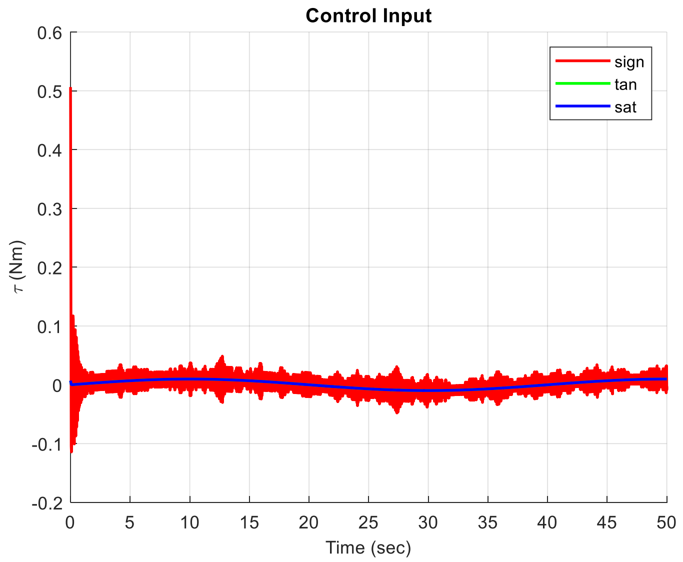


(c)

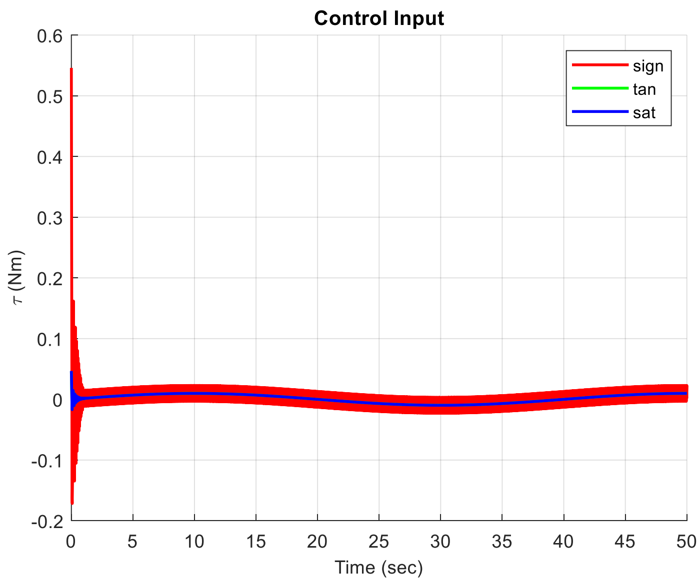
Figure 4: Joint position from a) C-PID b) SMC c) SMC with FTS controllers



(a)

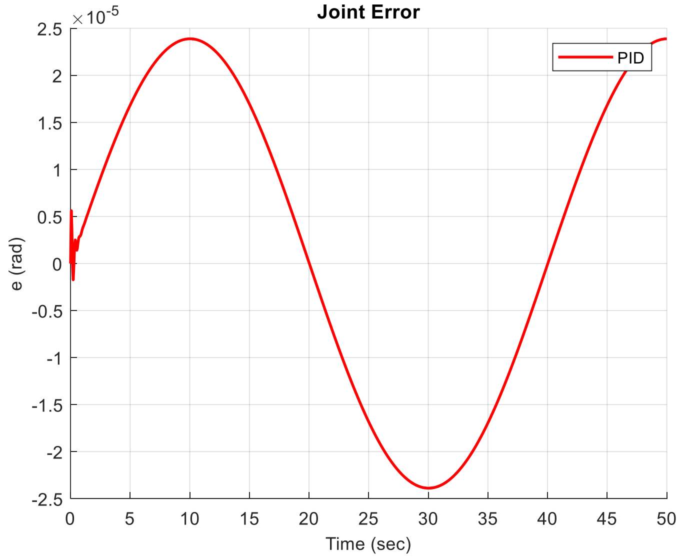


(b)

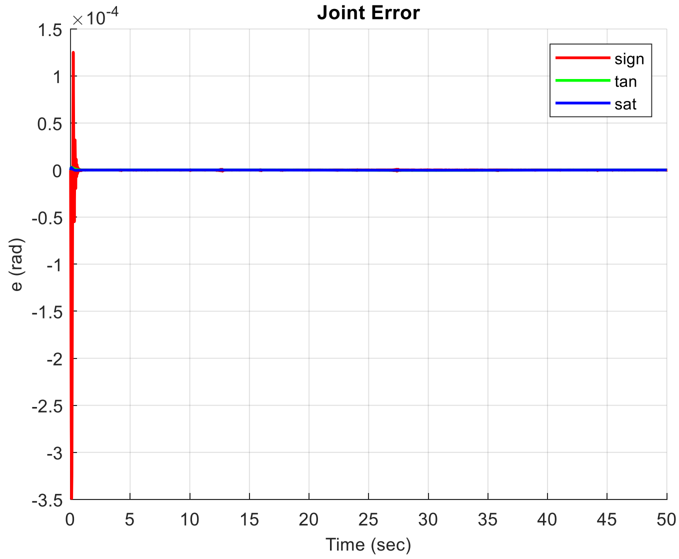


(c)

Figure 5: Control input from a) C-PID b) SMC c) SMC with FTS controllers



(a)



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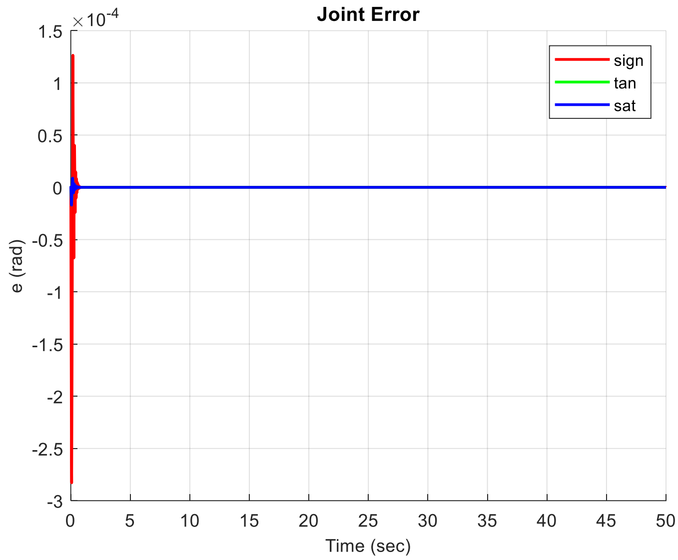
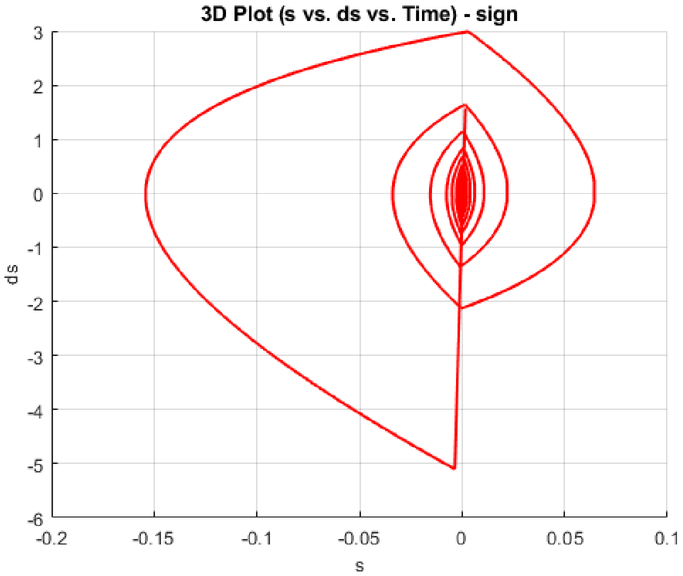
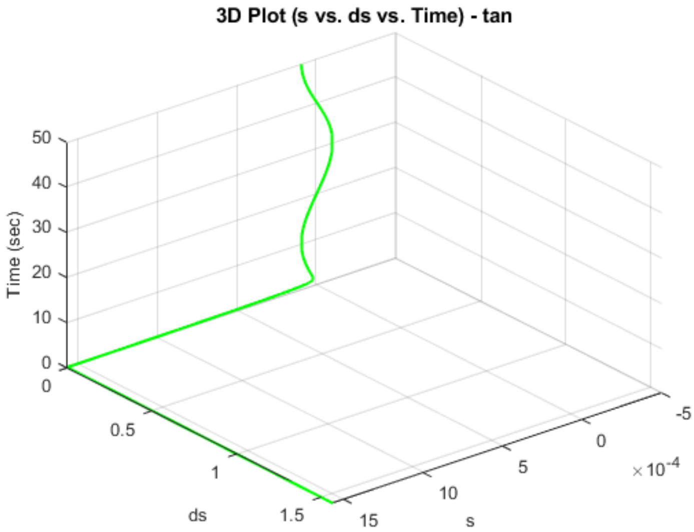


(c)

Figure 6: Joint error from a) C-PID b) SMC c) SMC with FTS controllers



(a)



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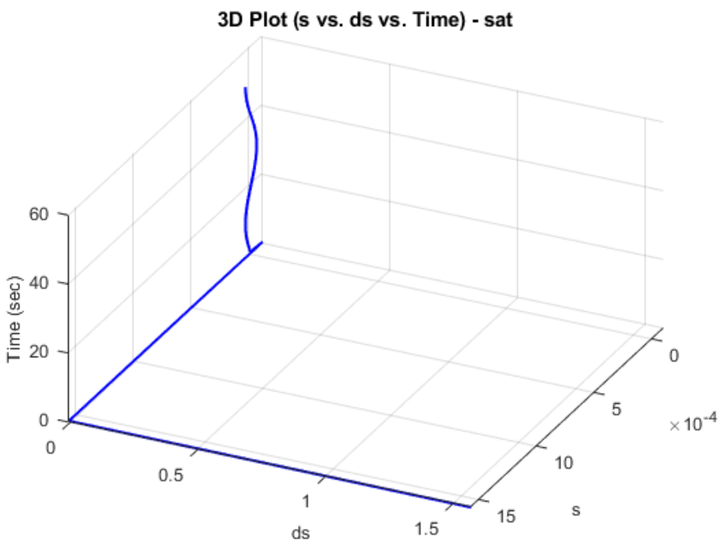
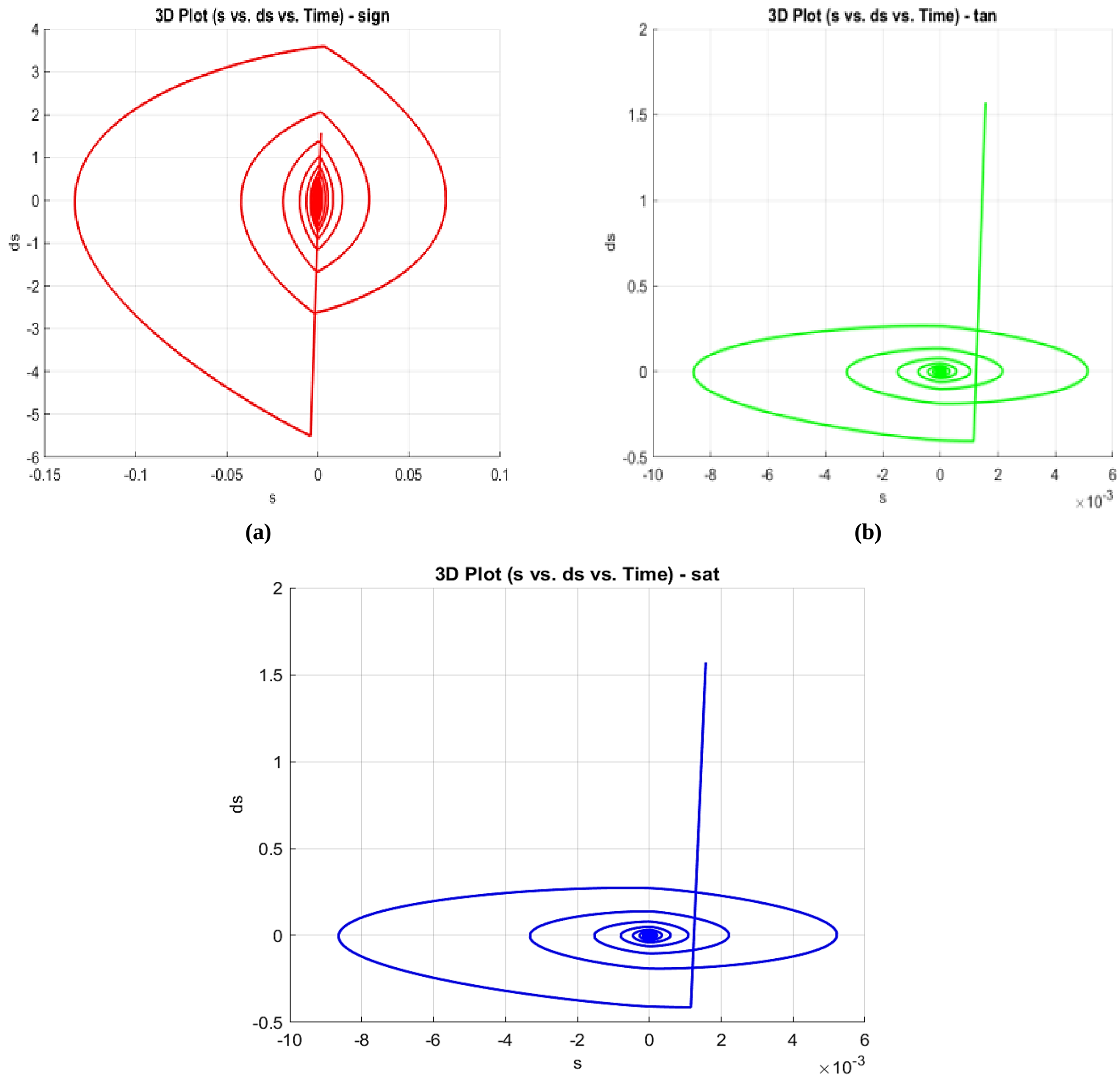


(c)

Figure 7: 3D plot for sliding surface from SMC controller with functions (a) sign, (b) tan, (c) sat

Compared to other control strategies like the sign or tan functions, the sat-based control provides an excellent balance between speed and stability, making it an ideal choice for many practical applications.

This 3D plots shown in Figures 8 (a, b, and c) for represent the sliding surface s versus its derivative s˙ over time for the sliding mode control (SMC) with FTS strategy using three functions for sign, tan, and sat, respectively. Using the sat provides controlled convergence but with limited flexibility due to the imposed control limits. Sign function ensures rapid convergence but at the cost of potential chattering. Tan function offers a smooth and effective control strategy that mitigates chattering while maintaining robust performance. Each control strategy has its trade-offs, but the integration of FTS generally ensures that the system achieves the desired state reliably and within a specified time frame, making the system more predictable and stable in complex applications.



(c)

Figure 8: 3D plot for sliding surface from SMC and FTS controller with functions (a) sign, (b) tan, (c) sat

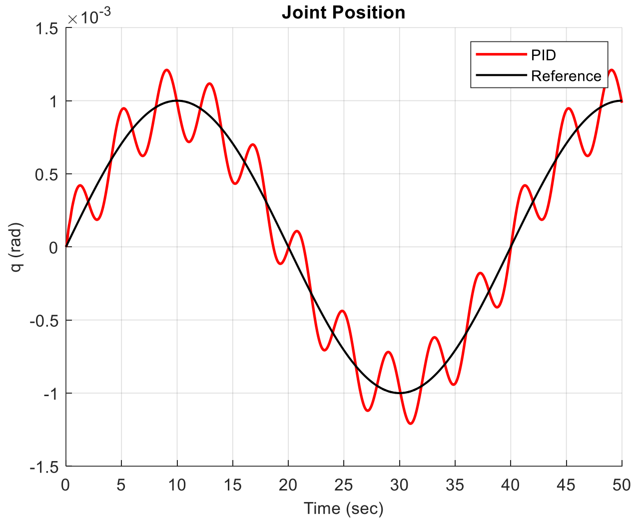
4.1 Investigation of the effects of disturbances

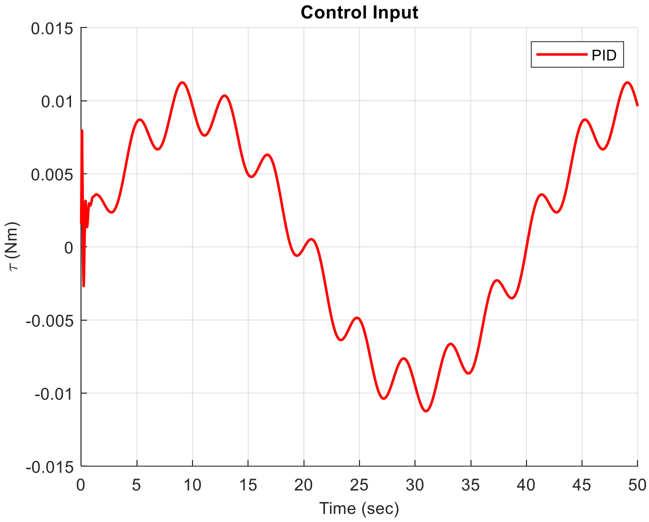
By adding disturbances, we have tested the system's ability to maintain stability in a realistic environment and achieved significant improvements in performance, especially with the integration of techniques like SMC with Finite-Time Stability (FTS). The External Disturbance is:

$$tau_{ex} = 0.1 * \sin(0.5 * pi * t) \tag{23}$$

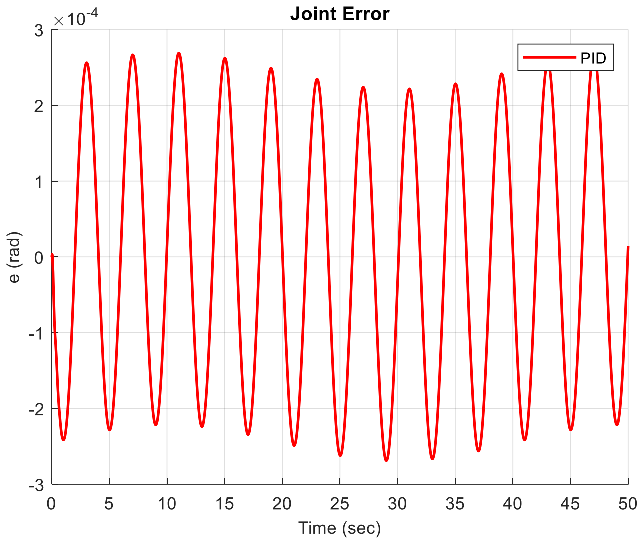
4.1.1 C-PID

Figure 9 (a joint position, b control input, c joint error) shows that the system is significantly affected by disturbance and shows large oscillations in position, control inputs, and error.





(a) **(b)**



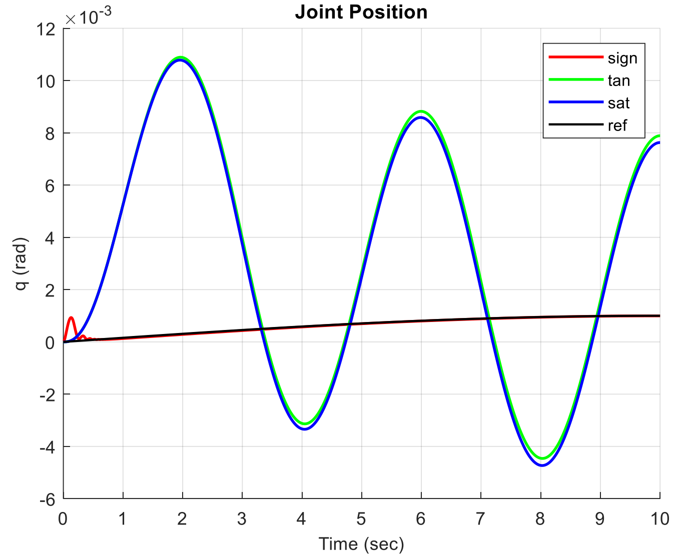
(c)

Figure 9: **(**C-PID**)** controller with disturbance a) joint position b) control input c) joint error

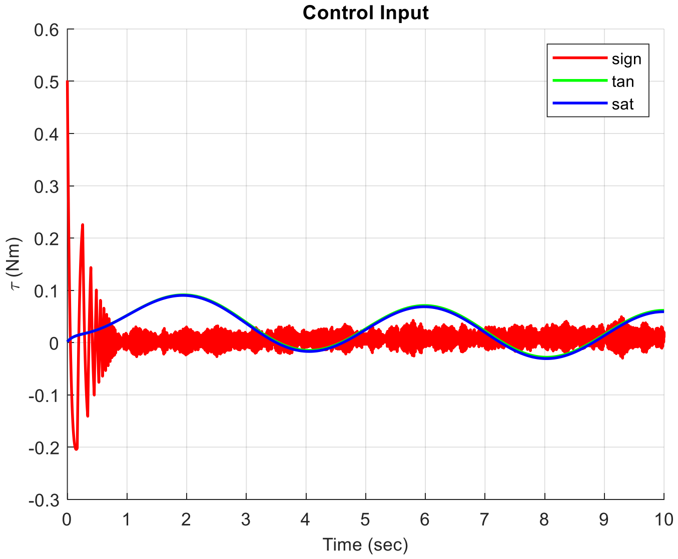
4.1.2 SMC

In Figure 10(a joint position, b control input, c joint error), SMC shows a better ability to resist disturbance, reduce oscillations, and improve system stability. SMC significantly enhances system stability even in the presence of disturbances by responding strongly and quickly to changes.

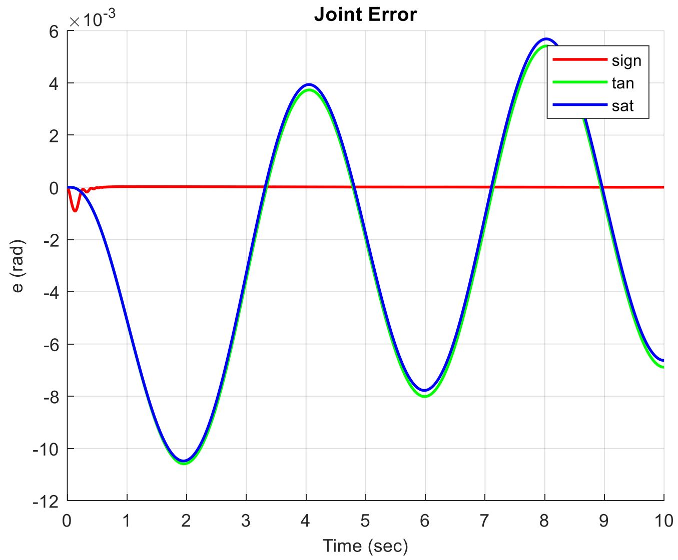
The 3D sliding surface plots in Figure 11 illustrate the impact of different functions within the SMC controller under disturbance. Figure 11a shows all functions, 11b–c depict the sign function, 11d–e are for the tan function, and 11f–g are for the sat function. These variations highlight differences in stability and performance. This 3D plot effectively demonstrates the robustness of the SMC controller in the presence of disturbances. The trajectory eventually stabilizes, showing the system's ability to resist disturbances and maintain control. The specific shape of the trajectory depends on the function used within the SMC strategy, with smoother functions (like sat or tan) generally leading to better performance in terms of reducing chattering and achieving quicker stabilization. Chattering and trajectory shape are influenced by the function type used in the SMC. The "sign" function would typically be caused by more pronounced oscillations or chattering, which can be seen as more erratic movements in the trajectory. Smoother Convergence: The use of the "sat" or "tan" functions typically leads to smoother control actions, which are reflected in the more controlled and less oscillatory paths in the 3D plot. Disturbance Compensation: The initial oscillations are due to the system's response to the disturbance. As the controller works to negate the disturbance, the trajectory smoothens, indicating that the system is successfully compensating for the external perturbations.



(a)



(b)



(c)

Figure 10: SMC controller with disturbance, a) joint position b) control input, c) joint error

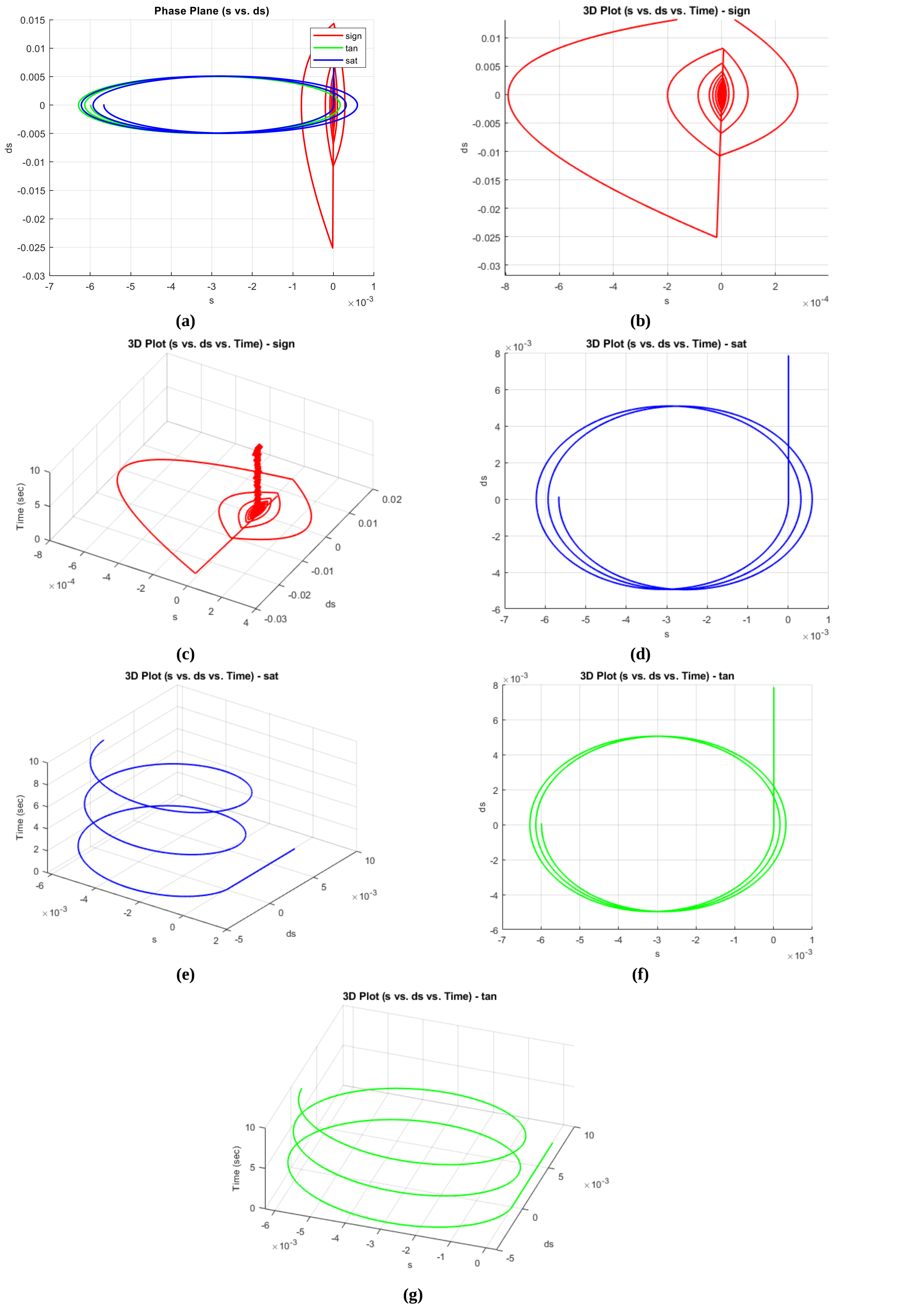


Figure 11: 3D plot for sliding surface from SMC controller with disturbance for (a, all functions) ,(b,c) sign (d,e) tan and (f,g) sat

4.1.3 SMC with FTS and disturbance

In Figure 12(a joint position b,c control input, and d joint error) SMC with FTS shows exceptional performance in handling disturbances, ensuring rapid and accurate system stability. FTS strengthens SMC's ability to deal with disturbances effectively, making the system stabilize quickly without significant overshoot or oscillations.

Figure 13 shows the controller's response and disturbance effects. It compares different functions: (a) all functions, (b, c) sign, (d, e) tan, and (f, g) sat, highlighting their impact on sliding dynamics. The system shows a much more controlled and stable trajectory compared to Figure 11. The 3D plot reveals that the sliding surface (s) and its derivative (ds) converge more smoothly and rapidly toward stability despite the presence of disturbances. The oscillations observed in the trajectory are significantly reduced compared to those in Figure 11, indicating that the Finite-Time Stability (FTS) controller enhances the SMC's ability to handle disturbances more effectively. The addition of FTS to the SMC results in a marked improvement in system stability. The system not only resists the disturbances but also rapidly converges to a stable state without significant overshoot or prolonged oscillations. The combination of SMC and FTS works to minimize chattering. The smoother trajectory observed in the 3D plot suggests that the control strategy effectively dampens high-frequency oscillations, which is a common issue in traditional SMC. The trajectory in Figure 13 is more stable and exhibits fewer oscillations than that in Figure 11. This is due to the FTS component, which ensures that the system reaches a stable state within a finite and predetermined time frame. As a result, the system responds more predictably and efficiently to disturbances. The smoother convergence towards the desired sliding surface indicates that the FTS controller provides a more robust control mechanism, effectively dealing with disturbances while maintaining system stability.

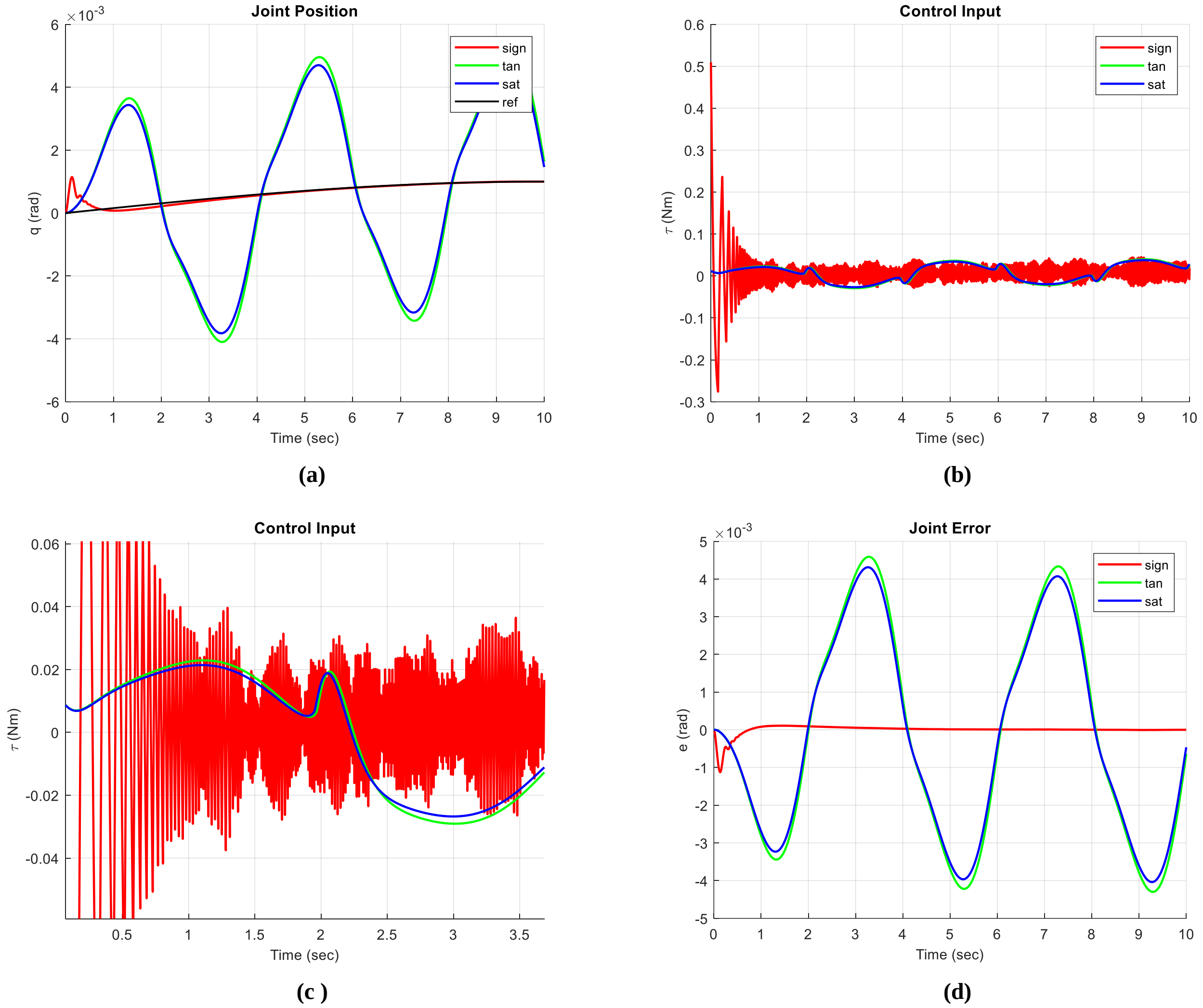
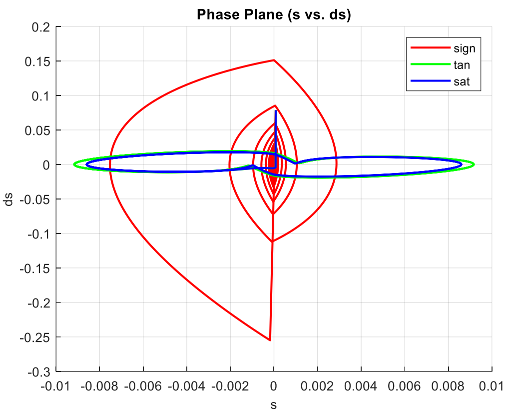
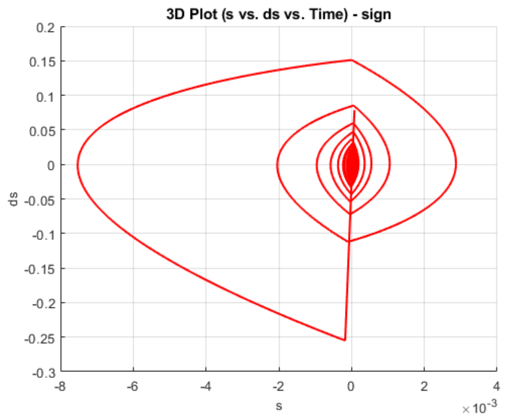


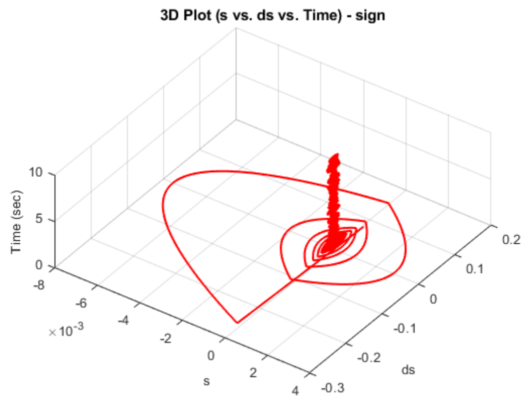
Figure 12: SMC with FTS controller with disturbance a) joint position b,c) control input d) joint error



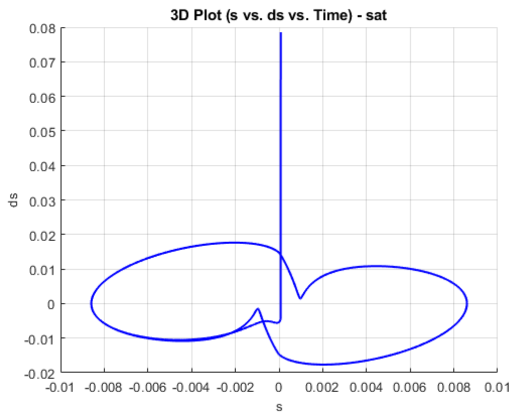
(a)



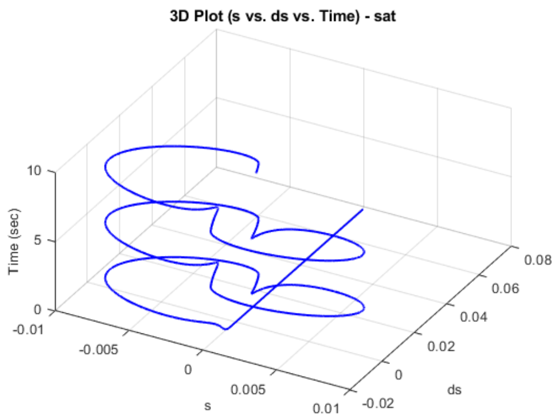
(b)



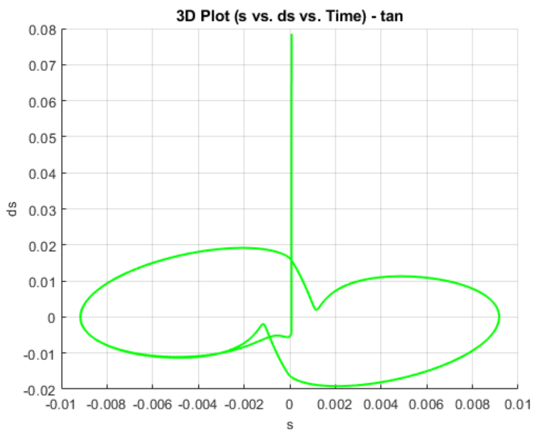
(c)



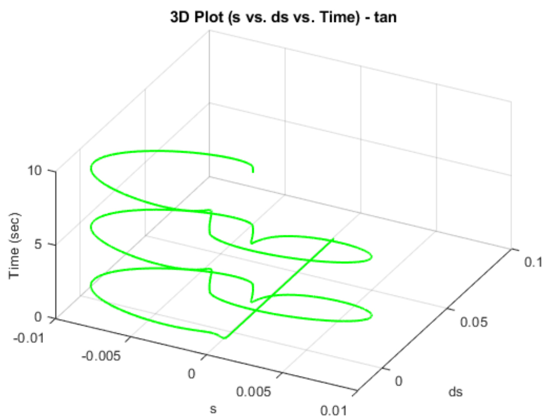
(d)



(e)



(f)



(g)

Figure 13: 3D plot for sliding surface from SMC with its controller and disturbance for (a, all functions) and (b,c) sign (d,e) tan, (f,g) sat

4.2 Considering the effects of both disturbance and uncertainty

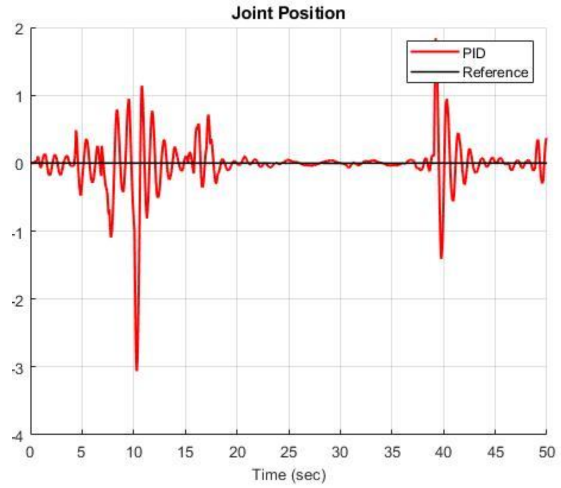
The addition of uncertainty to the system helps in understanding how well the control method can handle unpredictable conditions:

$$M = compute_{M_{1DOF(q)}} * (1 + 0.1 * randn);\ \%\ Adding\ 10\%\ uncertainty \tag{24}$$

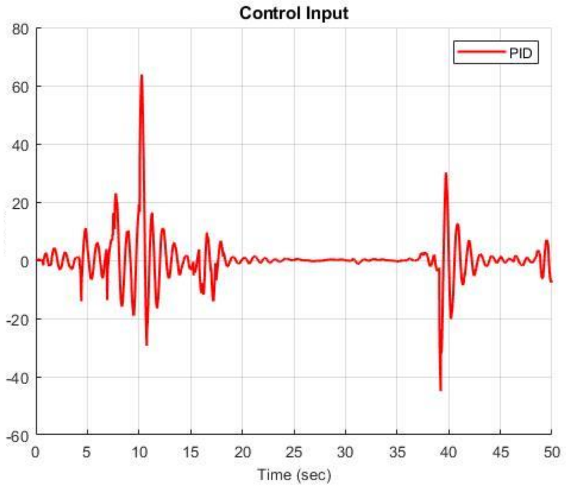
$$C = compute_{C_{1DOF(q,dq)}} * (1 + 0.1 * randn);\ \%\ Adding\ 10\%\ uncertainty \tag{25}$$

4.2.1 C-PID

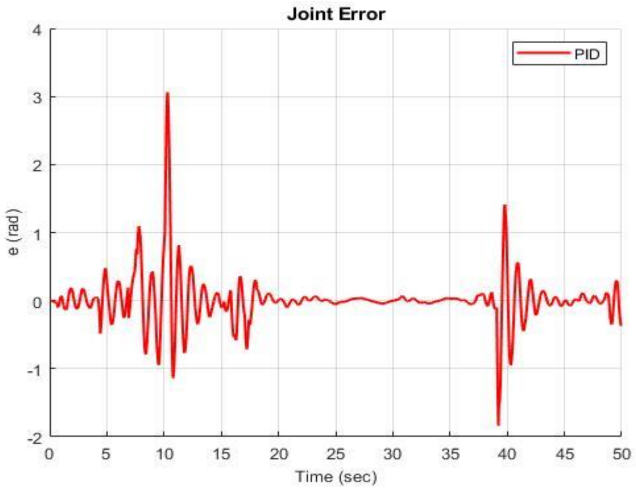
Figure 14 (a joint position, b control input, c joint error) shows that the system suffers greatly under the influence of disturbance and uncertainty, leading to significant oscillations and loss of stability.



(a)



(b)



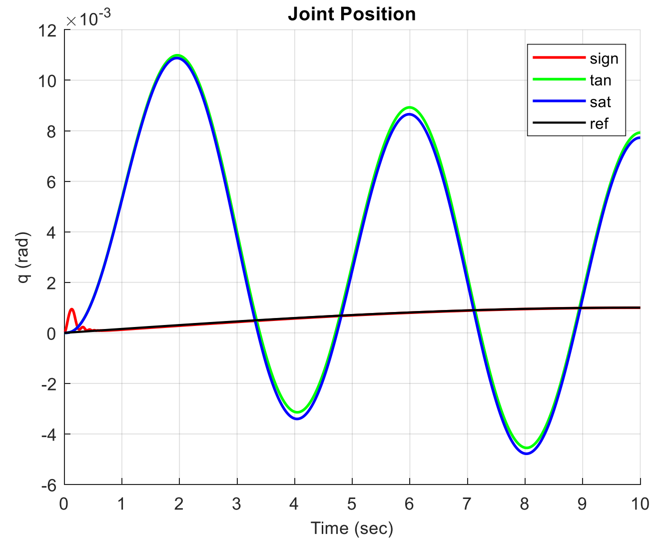
(c)

Figure 14: C-PID controller with (disturbance and uncertainty) a) joint position b) control input c) joint error

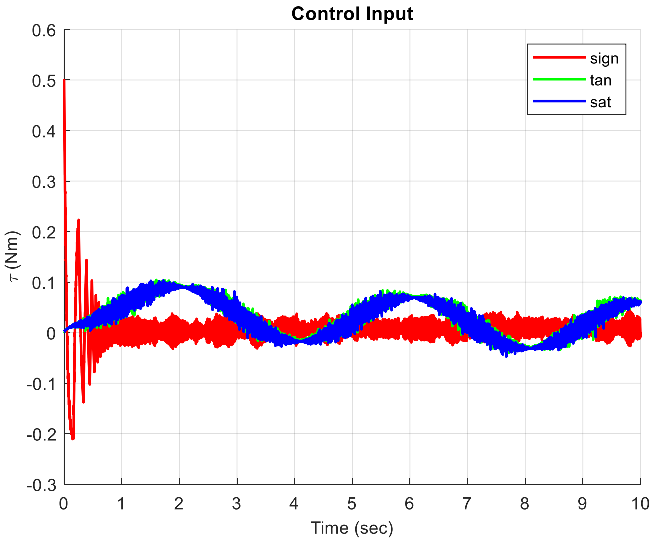
4.2.2 SMC

Figures 15(a joint position, b control input, and c joint error) show that SMC performs better than C-PID under these conditions, reducing the impact of disturbance and uncertainty on the system. Thanks to its effective response to changes and disturbances, SMC maintains stability even in uncertain conditions.

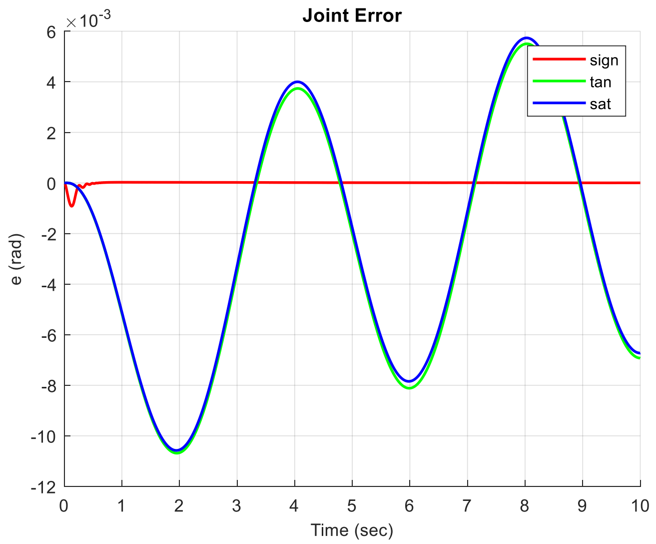
The pronounced oscillations and less smooth convergence are evident in Figure 16(a) for all functions, (b, c) for sign, (d, e) for tan, and (f, g) for sat. These effects are due to the combined effects of disturbance and uncertainty. The SMC controller is trying to compensate for these effects, but the additional uncertainty makes it harder to achieve a smooth trajectory. The SMC's robustness helps eventually stabilize the system, but the trajectory's initial instability highlights the challenge of controlling a system with both disturbances and uncertainties. Figure 16 demonstrates the effectiveness of the SMC controller in dealing with a system affected by both disturbance and uncertainty. While the system does achieve stability, the process is more turbulent compared to situations with fewer external factors. This figure emphasizes the challenges posed by combined disturbances and uncertainties, where achieving a smooth and stable trajectory becomes more difficult.



(a)



(b)



(c)

Figure 15: SMC controller with(disturbance and uncertainty) a) joint position b) control input c) joint error

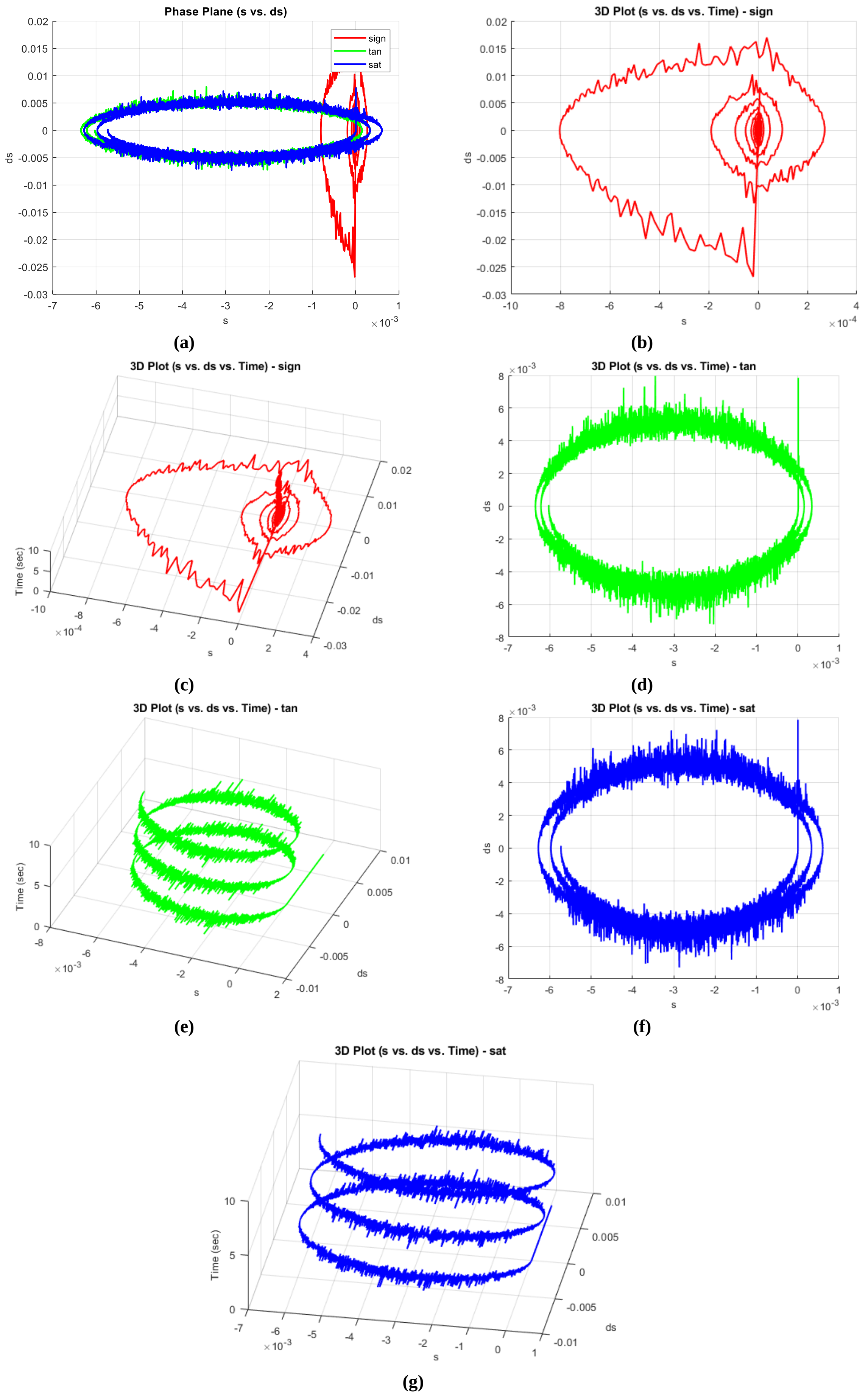
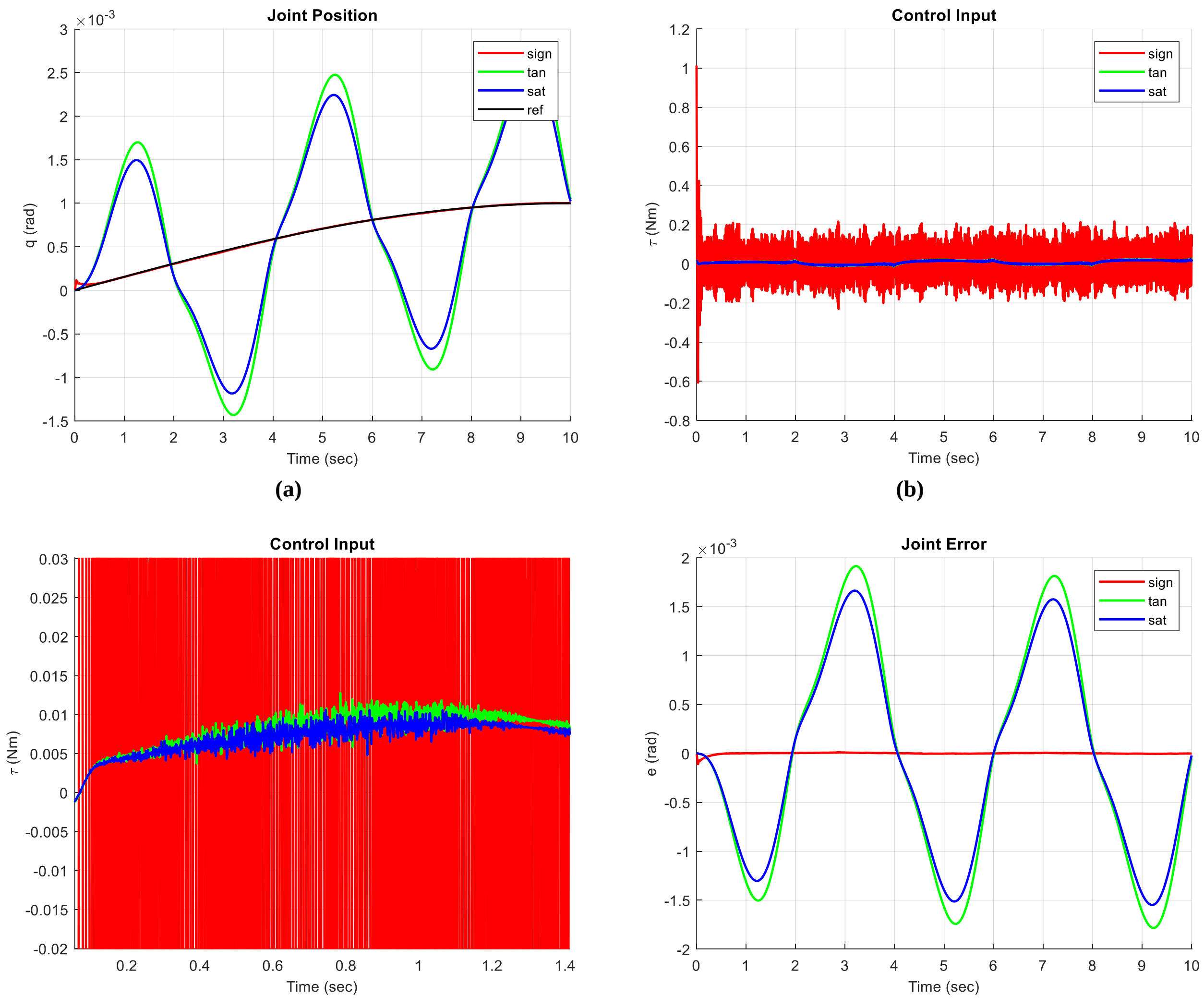


Figure 16: 3D plot for sliding surface from SMC controller (disturbance and uncertainty) for (a, all functions) and (b,c) sign (d,e) tan, (f,g) sat

4.2.3 SMC with FTS

Figure 17(a joint position b,c control input, and d joint error) shows the highest level of stability under these conditions when using SMC with FTS. The system responds quickly and reaches a stable state without significant oscillations. FTS enhances SMC's ability to handle difficult conditions like disturbances and uncertainty, making the system more stable and efficient.

Figure 18 shows 3D plot for sliding surface from SMC with its controller, where 18(a) is for all functions, (b) and (c) are for sign, (d) and (e) are for tan, and in (f) and (g) are for sat. the trajectory indicates that the system maintains a more controlled and stable path compared to Figure 16. The Finite-Time Stability (FTS) controller integrated with the SMC helps in achieving rapid convergence and reduces the effects of both disturbance and uncertainty on the system's performance. The trajectory in Figure 18 is smoother and demonstrates fewer oscillations, showing the robustness of the SMC with FTS in managing these challenges. The combination of SMC with FTS significantly enhances the system's stability, even under adverse conditions of disturbance and uncertainty. The system quickly converges to a stable state, with minimal chattering and reduced oscillations. The FTS component ensures that the system not only stabilizes within a finite time but also minimizes high-frequency oscillations. This results in a more predictable and stable trajectory, as seen in Figure 18. The smoother trajectory in Figure 18 compared to Figure 16 can be attributed to the FTS component, which helps the system reach a stable state more quickly and efficiently, even in the presence of uncertainties and disturbances. The integration of FTS with SMC provides a robust control mechanism that enhances the system's ability to handle external perturbations and uncertainties, resulting in a more stable and controlled trajectory. Figure 18 demonstrates that SMC with FTS is more effective in stabilizing the system under the combined effects of disturbance and uncertainty compared to SMC alone, as shown in Figure 16. The FTS controller provides better control by ensuring rapid convergence, reducing oscillations, and achieving a smoother and more stable trajectory.



(c) (d)

Figure 17: SMC controller with FTS and (disturbance and uncertainty) a) joint position b,c) control input and d) joint error

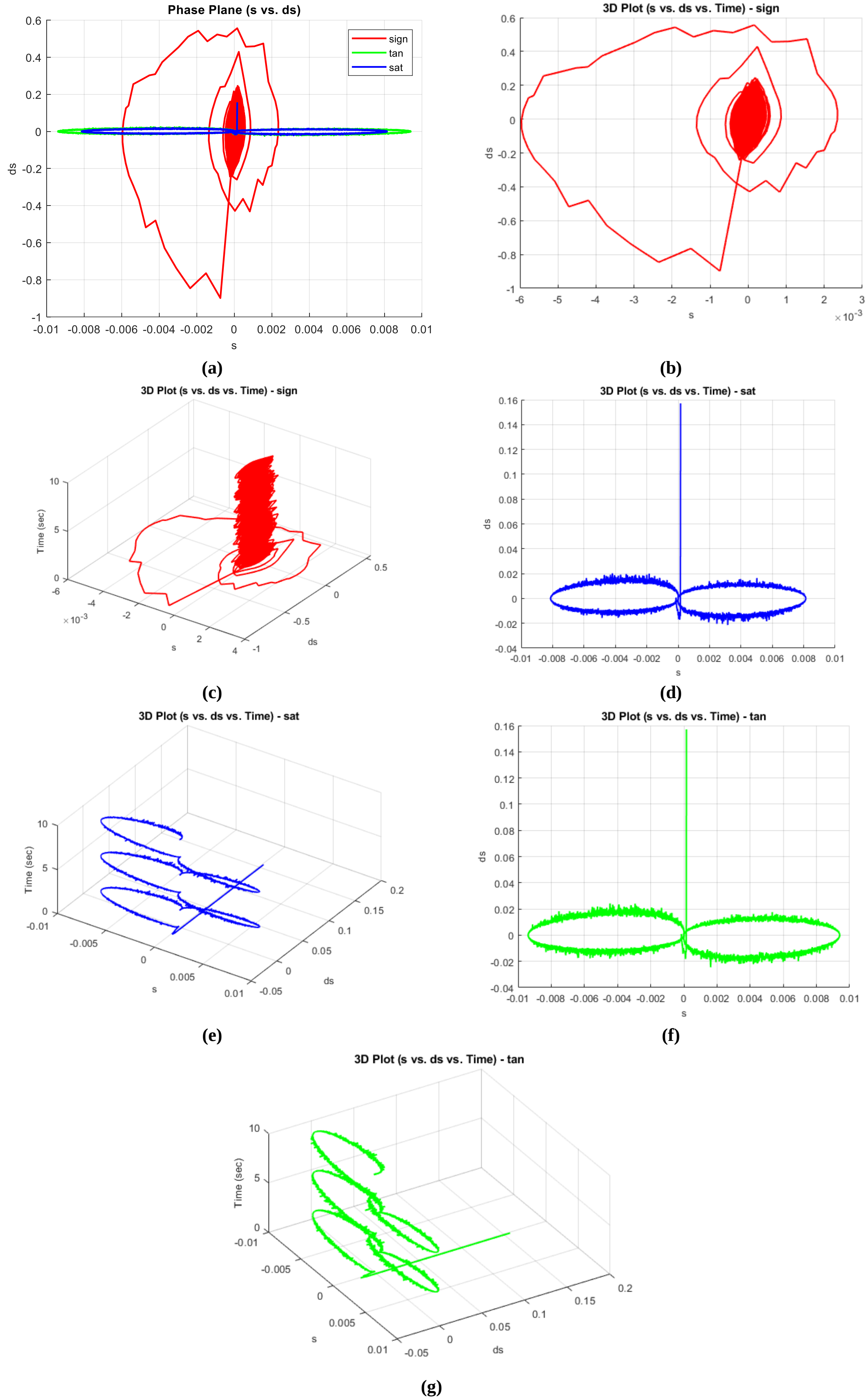


Figure 18: 3D plot for sliding surface from SMC with its controller (disturbance and uncertainty) for (a, all functions) and (b,c) sign (d,e) tan, (f,g) sat

Based on the Root Mean Square Error (RMSE) results, a metric was used to assess the differences between actual and predicted values, and the system's accuracy in tracking the desired trajectory was evaluated. A lower RMSE indicates better system performance, as it reflects reduced errors and improved stability. In this study, the Sliding Mode Control (SMC) with Finite-Time Stability (FTS) demonstrated the lowest RMSE, as shown in Table (2). This confirms its superior capability in managing disturbances and uncertainties compared to other methods. Subsequently, the performance of the different applied controllers can be summarized as follows:

- C-PID Controller: Performance declines significantly when disturbances are present, and while there's a slight improvement with added uncertainty, it remains less effective overall.

- SMC Controller: Shows better performance than C-PID, particularly under disturbances, with lower RMSE values. However, performance remains relatively stable when uncertainty is introduced.

SMC with FTS Controller: This delivers the best performance by significantly reducing RMSE in all conditions. The integration of FTS enhances the system's stability and effectiveness, even under disturbances and uncertainties, outperforming both C-PID and standard SMC.

Table 2: Summary of the Simulation Results, using Root Mean Square Error RMSE for the three controllers

Controller	(RMSE)
C-PID	1.6878e-05
SMC	sign: 1.5186e-05
	tan: 2.7280e-07
	sat: 1.6146e-07
SMC with FTS	sign: 1.5186e-05
	tan: 2.7280e-07
	sat: 1.6146e-07
C-PID - disturbance	0.9273
SMC - disturbance	sign: 9.6212e-05
	tan: 0.0054
	sat: 0.0053
SMC with FTS - disturbance	sign: 1.0085e-05
	tan: 0.0011
	sat: 9.9490e-04
C-PID- (disturbance & uncertainty)	0.3683
SMC - (disturbance & uncertainty)	sign: 9.7016e-05
	tan: 0.0054
	sat: 0.0054
SMC with FTS- (disturbance & uncertainty)	sign: 1.0085e-05
	tan: 0.0011
	sat: 9.9490e-04

5. Conclusion

In conclusion, this study shows the effectiveness of combining Sliding Mode Control (SMC) with Finite-Time Stability (FTS) for a one-degree-of-freedom system. The results demonstrate that SMC with FTS performs much better than traditional PID controllers and standard SMC, especially in environments with disturbances. The use of FTS helps the system reach stability faster and within a set time, reducing oscillations and improving performance. Simulations show that SMC with FTS provides smoother control, reduces errors, and stabilizes the system more effectively, making it an appealing choice for applications that need quick and stable responses.

Simulations confirm that SMC with FTS provides smoother control signals, reduced tracking errors, and faster stabilization compared to conventional methods. These improvements make it an ideal choice for applications requiring high precision and quick response times, such as robotics, aerospace, and power electronics.

In future work, this study will be extended to 2-DOF manipulators, where additional challenges, such as increased complexity and coupling effects, will be addressed. Furthermore, intelligent and adaptive control strategies will be explored further to enhance system performance in dynamic and uncertain environments.

Author Contribution

Conceptualization, **S. Al-dahlaky,** and **S. Raafat**; data curation, **S. Al-dahlaky**; formal analysis, **S. Al-dahlaky**.; investigation, **S. Al-dahlaky** and **S. Raafat.**; methodology, **S. Al-dahlaky** and **S. Raafat**.; project administration, **S. Raafat.**, resources, **S. Al-dahlaky,** and **S. Raafat**.; software **S. Al-dahlaky**.; supervision, **S. Raafat.**; validation, **S. Al-dahlaky,** and **S. Raafat**.; visualization, **S. Raafat.**; writing—original draft preparation, **S. Al-dahlaky**.; writing—review and editing, **S. Al-dahlaky,** and **S. Raafat**. All authors have read and agreed to the published version of the manuscript.

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Data availability statement

The data that support the findings of this study are available on request from the corresponding author.

Conflicts of interest

The authors declare that there is no conflict of interest.

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