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The Reduction of Three Rows Weyl Module in the Skew-Shape (6, 4, 3)/(1, 0, 0)

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Abstract

The purpose of our research it is scanning an application for Lascoux resolution and the characteristic free resolution towards the partition (6, 4, 3)/(1, 0, 0), and we also survey the communication for the resolution of Weyl module in our case of every mode for the partition itself.

Keywords: Divided power algebra; lascoux resolution; polarization; boundary map; weyl module.

اختزال مقاس وايل لثلاث صفوف في الشكل المنحرف (6, 4, 3)/(1, 0, 0)

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الخلاصة

الغرض من بحثنا هو مراجعة تطبيق تحلل لاسكو وتطبيق المميزالحر بالنسبة للتجزئة(6, 4, 3)/(1, 0, 0) وايضا مراجعة العلاقة لتحلل مقاس وايل في حالتنا لكل وضع لنفس التجزئة.

الكلمات المفتاحية: جبر القوة المقسمة ؛ تحلل لاسكو؛ استقطاب ؛ دالة الحد ؛ مقاس وايل.

1. Introduction

Let F be a free module over a commutative ring with identity and D_nF be divided power algebra of degree n the foundation on which it is based the free module, [2] Z_{21}, Z_{32} and Z_{31} are the official polarization operators mentioned in [1]. Author in source [3] discuss the results to shape (8,6)/(2,0) in addition to (8,6)/(2,1). Source [4] and source [5] discuss results to shape (8,6,3)/(u,1) when the u=1 ; $u = 2$. In [6] it shows the relationship between the resolution of Weyl module (8, 6, 3)/(u,1) when the u=1; $u = 2$ on the shape itself.
In this work we studied the complicated of the resolution to partion (6, 4, 3) /(1, 0, 0) where is the number of triple overlaps with two triple overlaps is $(r - t_1 - t_2)$, in the first part we found the terms of characteristic free resolution, and in part two we provided conditions in addition to programs for Lascoux complex.

2. For the skew partion (6, 4, 3)/(1, 0, 0) the results are

We will present the formula is below to partition $[p, q, r, t_1, t_2]$ with two triple intersections to get conditions of the resolution to partition (6, 4, 3)/(1, 0, 0).

$\mathrm{Res}([p,q;t_1]) \otimes \mathrm{D}_r \oplus \sum_{l\geq 0} \underline{Z}_{32}^{(t_2+1)}\, y \odot \underline{Z}_{32}^{(l)}\, Res\,([p,q+t_2+1+l;t_1+t_2+1+l]) \otimes \mathrm{D}_{(r-(t_2+1+l)} \oplus \underline{Z}_{32}^{(l_2+1)} y \underline{Z}_{31}^{(t_1+1)} z\mathrm{Res}([p+t_1+1,q+l_2+1;t_2])$

$0 \longrightarrow Z_{ab} u Z_{ab} u \ldots Z_{ab} \longrightarrow \sum_{k_i\geq 1}\ ,\sum_{k_i=n}\ Z_{ab}^{(k_1)} u Z_{ab}^{(k_2)} u \ldots Z_{ab}^{(k_n-1)} \longrightarrow \cdots \longrightarrow Z_{ab}^{(n)} \longrightarrow 0$

So:

$\mathrm{Res}([5,4;1]) \otimes \mathrm{D}_3 \oplus \sum_{t\geq 0} \underline{Z}_{32}^{(1)}\, y \underline{Z}_{32}^{(l)} y\ \mathrm{Res}([5,5+l;2+l]) \otimes D_{2-l} \oplus \underline{Z}_{32}^{(1)} y \underline{Z}_{31}^{(2)} z\mathrm{Res}([7,5;0]) \otimes D_0$

Hence:

$\sum_{I\geq 0} Z_{32}^{(1)}\, y \odot \underline{Z}_{32}^{(I)}\, y\ Res([6,7+l;2+l]) \otimes D_{2-l} = Z_{32}^{(1)} y Res([5,5;2]) \otimes D_2 \oplus Z_{32}^{(1)} y Z_{32} y Res([5,6;3]) \otimes D_1$

$= Z_{32}^{(1)} y Res([5,5;2]) \otimes D_2 \oplus Z_{32}^{(1)} y Z_{32} y Res([5,6;3]) \otimes D_1$

Where $\underline{Z}_{32} y$

$$0 \longrightarrow Z_{32} y \xrightarrow{\partial_y} Z_{32} \longrightarrow 0$$

$\underline{Z}_{32}^{(2)} y$ is the bar complex

$$0 \longrightarrow Z_{32} y Z_{32} y \xrightarrow{\partial_y} Z_{32}^{(2)} y \xrightarrow{\partial_y} Z_{32}^{(2)} \longrightarrow 0$$

$\underline{Z}_{32}^{(3)} y$ is the bar complex

$$0 \longrightarrow Z_{32} y Z_{32} y Z_{32} y \xrightarrow{\partial_y} \begin{matrix} Z_{32}^{(2)} y Z_{32} y \\ \oplus \\ Z_{32} y Z_{32}^{(2)} y \end{matrix} \xrightarrow{\partial_y} Z_{32}^{(3)} y \xrightarrow{\partial_y} Z_{32}^{(3)} \longrightarrow 0$$

and $\underline{Z}_{31} z$ is the bar complex

$$0 \longrightarrow Z_{31} z \xrightarrow{\partial_z} Z_{31} \longrightarrow 0$$

The terms of the characteristic free resolution for partition (6, 4, 3)/(1, 0, 0) when the $b, b_1, b_2,\ b_3, b_4 \in Z^+$ are:

$(M_0) = D_5 \otimes D_4 \otimes D_3$

(M_1) equals the total :

•$Z_{21}^{(b)} \varkappa \mathrm{D}_{5+b} \otimes \mathrm{D}_{4-b} \otimes \mathrm{D}_3$; with $2 \leq b \leq 4$

•$Z_{32}^{(1)} y \mathrm{D}_5 \otimes \mathrm{D}_5 \otimes \mathrm{D}_2$

•$Z_{32}^{(2)} y \mathrm{D}_5 \otimes \mathrm{D}_6 \otimes \mathrm{D}_1$

(M_2) equals the total :

•$Z_{21}^{(b_1)}\varkappa Z_{21}^{(b_2)}\varkappa D_{5+|b|}\otimes D_{4-b}\otimes D_3$; with $3 \leq |b| = b_1 + b_2 \leq 4$

•$Z_{32}^{(1)}yZ_{21}^{(b)}\varkappa D_{5+b}\otimes D_{4-b}\otimes D_2$; with $3 \leq b \leq 5$

•$Z_{32}yZ_{32}\varkappa D_5\otimes D_6\otimes D_1$

•$Z_{32}^{(1)}yZ_{31}^{(2)}zD_7\otimes D_5\otimes D_0$

(M_3) equals the total :

•$Z_{21}^{(b_1)}\varkappa Z_{21}^{(b_2)}\varkappa Z_{21}^{(b_3)}\varkappa D_{5+|b|}\otimes D_{4-|b|}\otimes D_3$; with $4 \leq |b| = \sum_{i=1}^{3} b_i \leq 4$ and $b_1 \geq 2$

•$Z_{32}^{(1)}yZ_{21}^{(b_1)}\varkappa Z_{21}^{(b_2)}\varkappa D_{5+|b|}\otimes D_{4-|b|}\otimes D_2$; with $4 \leq |b| = b_1 + b_2 \leq 5$ and $b_1 \geq 3$

•$Z_{32}yZ_{32}yZ_{21}^{(b)}\varkappa D_{5+b}\otimes D_{5-b}\otimes D_1$; with $3 \leq b \leq 5$

•$Z_{32}yZ_{31}^{(2)}zZ_{21}^{(b)}\varkappa D_{7+b}\otimes D_{5-b}\otimes D_0$; with $1 \leq b \leq 5$

(M_4) equals the total :

•$Z_{32}yZ_{21}^{(b_1)}\varkappa Z_{21}^{(b_2)}\varkappa Z_{21}^{(b_3)}\varkappa D_{5+|b|}\otimes D_{5-|b|}\otimes D_2$;with $5 \leq |b| = \sum_{i=1}^{3} b_i \leq 5$ and $b_1 \geq 3$

•$Z_{32}^{(1)}y\, Z_{32}yZ_{21}^{(b_1)}\varkappa Z_{21}^{(b_2)}\varkappa D_{5+|b|}\otimes D_{6-|b|}\otimes D_1$; with $5 \leq |b| = \sum_{i=1}^{2} b_i \leq 6$ and $b_1 \geq 4$

•$Z_{32}^{(1)}yZ_{31}^{(2)}zZ_{21}^{(b_1)}\varkappa Z_{21}^{(b_2)}\varkappa D_{7+|b|}\otimes D_{5-|b|}\otimes D_0$; with $2 \leq |b| = b_1 + b_2 \leq 5$ and $b_1 \geq 1$

(M_5) equals the total :

•$Z_{32}yZ_{32}yZ_{21}^{(b_1)}\varkappa Z_{21}^{(b_2)}\varkappa Z_{21}^{(b_3)}\varkappa D_{5+|b|}\otimes D_{6-|b|}\otimes D_1$; with $6 \leq |b| = \sum_{i=1}^{3} b_i \leq 6$ and $b_1 \geq 4$

•$Z_{32}yZ_{31}^{(2)}zZ_{21}^{(b_1)}\varkappa Z_{21}^{(b_2)}\varkappa Z_{21}^{(b_3)}\varkappa D_{7+|b|}\otimes D_{5-|b|}\otimes D_0$; with $3 \leq |b| = \sum_{i=1}^{3} b_i \leq 5$ and $b_1 \geq 1$

(M_6) equals the total :

$\bullet Z_{32} y Z_{31}^{(2)} z Z_{21}^{(b_1)} \varkappa Z_{21}^{(b_2)} \varkappa Z_{21}^{(b_3)} \varkappa Z_{21}^{(b_4)} \varkappa D_{7+|b|} \otimes D_{5-|b|} \otimes D_0$

; with $4 \leq |b| = \sum_{i=1}^{4} b_i \leq 5$ and $b_1 \geq 1$

(M_7) equals the total :

$\bullet Z_{32} y Z_{31}^{(2)} z Z_{21} x Z_{21} x Z_{21} x Z_{21} x Z_{21} x D_{12} \otimes D_{0|} \otimes D_0$

In the source [7] the Lascoux resolution of the Weyl module paired to partition (6, 4, 3)/(1, 0, 0) will be:

$$0 \to D_8F \otimes D_4F \otimes D_0F \to \begin{array}{c} D_8F \otimes D_2F \otimes D_2F \\ \oplus \\ D_7F \otimes D_5F \otimes D_0F \end{array} \to \begin{array}{c} D_5F \otimes D_5F \otimes D_2F \\ \oplus \\ D_7F \otimes D_2F \otimes D_3F \end{array} \to D_5F \otimes D_4F \otimes D_3F$$

Like in [4] the terms maybe appear as a follow-up

$M_0 = \mathcal{L}_0 = E_0$

$M_1 = \mathcal{L}_1 \oplus E_1$

$M_2 = \mathcal{L}_2 \oplus E_2$

$M_3 = \mathcal{L}_3 \oplus E_3$

$M_j = E_j$; for $j = 4, 5, \ldots, 7$.

Then we know $\sigma_1 : E_1 \to \mathcal{L}_1$ as follows

- $Z_{21}^{(3)} \varkappa(v) \longmapsto \frac{1}{3} Z_{21}^{(2)} \varkappa \partial_{21}(v)$;where $v \in D_8 \otimes D_1 \otimes D_3$
- $Z_{21}^{(4)} \varkappa(v) \longmapsto \frac{1}{6} Z_{21}^{(2)} \varkappa \partial_{21}^{(2)}(v)$;where $v \in D_9 \otimes D_0 \otimes D_3$
- $Z_{32}^{(2)} y(v) \longmapsto \frac{1}{2} Z_{32} y \partial_{32}(v)$;where $v \in D_5 \otimes D_6 \otimes D_1$

We should point out that the map σ_1 identity is achieved:

$\delta_{\mathcal{L}_2\mathcal{L}_1} \sigma_1 = \delta_{E_1 E_0}$ ... 1

$$\begin{tikzcd} \mathcal{L}_1 \arrow[r, "\delta\mathcal{L}_2 1\mathcal{L}_0"] & \mathcal{L}_0 = E_0 \\ & E_1 \arrow[ul, "\sigma_1"] \arrow[u] \end{tikzcd}$$

Where $\delta_{\mathcal{L}_1\mathcal{L}_0}$ boundary component of fat complex which transfer $\mathcal{L}_1$ to $\mathcal{L}_0$.
The notation we will use $\delta_{\mathcal{L}_{t+1}\mathcal{L}_t}, \delta_{\mathcal{L}_{t+1}}E_1 \ldots etc.$

We can also define $\partial_1: \mathcal{L}_1 \longrightarrow \mathcal{L}_0$ as $\partial_1 = \delta_{\mathcal{L}_1\mathcal{L}_0}$
Easily we can show that ∂_1 provision 1, we can give an example :

$$(\delta_{\mathcal{L}_1\mathcal{L}_0} \circ \partial_1)(\, Z_{21}^{(4)}\varkappa(v)) = \delta_{\mathcal{L}_1\mathcal{L}_0} \circ \partial_1(\tfrac{1}{6}\, Z_{21}^{(2)}\,\varkappa\partial_{21}^{(2)}(v))$$

$$= \frac{1}{6}(\partial_{21}^{(2)}\partial_{21}^{(3)}(v))$$

$$= \partial_{21}^{(4)}(v)$$

$$= \delta_{E_1E_0}(Z_{21}^{(4)}\varkappa(v))$$

As $\partial_2: \mathcal{L}_2 \longrightarrow \mathcal{L}_1$ as $\partial_2 = \delta_{\mathcal{L}_2\mathcal{L}_1} + \partial_1 \circ \delta_{\mathcal{L}_2E_1}$

Proposition 2.1: The installation $\partial_1 \circ \partial_2 = 0$.

Proof:

$$\partial_1 \circ \partial_2(g) = \delta_{\mathcal{L}_1\mathcal{L}_0} \circ (\delta_{\mathcal{L}_2\mathcal{L}_1}(g) + \partial_1 \circ \delta_{\mathcal{L}_2E_1}(g))$$

$$= \delta_{\mathcal{L}_1\mathcal{L}_0} \circ \delta_{\mathcal{L}_2\mathcal{L}_1}(g) + \delta_{\mathcal{L}_1\mathcal{L}_0} \circ \partial_1\delta_{\mathcal{L}_2E_1}(g)$$

$$= \delta_{\mathcal{L}_1\mathcal{L}_0} \circ \partial_1 = \delta_{E_1E_0}$$

we get

$$\partial_1 \circ \partial_2(g) = \delta_{\mathcal{L}_1\mathcal{L}_0} \circ \delta_{\mathcal{L}_2E_1}(g) + \delta_{E_1E_0}\delta_{\mathcal{L}_2E_1}(g) = 0$$

We can now to define $\partial_2: E_2 \longrightarrow \mathcal{L}_2$ this is the situation:

$$\delta_{E_2\mathcal{L}_1} + \partial_1 \circ \delta_{E_2E_1} = \delta_{\mathcal{L}_2\mathcal{L}_1} + \partial_1 \circ \delta_{\mathcal{L}_2E_1} \circ \partial_2 \ldots 2$$

We define

- $Z_{21}^{(2)}\varkappa Z_{21}\varkappa(v) \longmapsto 0$;where $v \in D_8 \otimes D_1 \otimes D_3$
- $Z_{21}^{(3)}\varkappa Z_{21}\varkappa(v) \longmapsto 0$;where $v \in D_9 \otimes D_0 \otimes D_3$
- $Z_{32}yZ_{21}^{(4)}\varkappa(v) \longmapsto \frac{1}{4}\, Z_{32}yZ_{21}^{(3)}\varkappa\partial_{21}(v)$;where $v \in D_9 \otimes D_1 \otimes D_2$
- $Z_{32}yZ_{21}^{(5)}\varkappa(v) \longmapsto \frac{1}{10}\, Z_{32}yZ_{21}^{(3)}\varkappa\partial_{21}^{(2)}(v)$;where $v \in D_{10} \otimes D_0 \otimes D_2$
- $Z_{32}yZ_{32}y(v) \longmapsto 0$;where $v \in D_5 \otimes D_6 \otimes D_1$

Easy for her to show it σ_2 it is to recognize the implementation above 2, we count where $v \in D_{10} \otimes D_0 \otimes D_2$

$(\delta_{E_2\mathcal{L}_1} + \sigma_1 \circ \delta_{E_2E_1})(\,Z_{32}yZ_{21}^{(5)}\varkappa(v))$

$= \sigma_1(Z_{21}^{(5)}\varkappa\partial_{32}(v)) + Z_{21}^{(4)}\varkappa\partial_{31}(v)) - Z_{32}y\partial_{21}^{(5)}(v))$

$= \frac{1}{5}Z_{21}\varkappa\partial_{21}^{(4)}\partial_{32}(v) + \frac{1}{4}Z_{21}\varkappa\partial_{21}^{(3)}\partial_{31}(v) - Z_{32}y\partial_{21}^{(5)}(v))$

And

$(\delta_{\mathcal{L}_2\mathcal{L}_1} + \sigma_1 \circ \delta_{\mathcal{L}_2E_1})(\frac{1}{10}Z_{32}yZ_{21}^{(3)}\varkappa\partial_{21}^{(2)}(v))$

$= \sigma_1(\frac{1}{10}Z_{21}^{(3)}\varkappa\partial_{21}^{(2)}\partial_{32}(v) + \frac{1}{10}Z_{21}^{(3)}\varkappa\partial_{21}\partial_{31}(v) + \frac{1}{10}Z_{21}^{(2)}\varkappa\partial_{21}^{(2)}\partial_{31}(v)) - \frac{1}{10}Z_{32}y\partial_{21}^{(3)}\partial_{21}^{(2)}(v))$

$= \frac{1}{6}Z_{21}\varkappa\partial_{21}^{(5)}\partial_{32}(v) + \frac{1}{10}Z_{21}\varkappa\partial_{21}^{(4)}\partial_{31}(v) + \frac{1}{10}Z_{21}\varkappa\partial_{21}^{(4)}\partial_{31}(v) - Z_{32}y\partial_{21}^{(6)}(v)$

$= \frac{1}{5}Z_{21}\varkappa\partial_{21}^{(4)}\partial_{32}(v) + \frac{1}{4}Z_{21}\varkappa\partial_{21}^{(3)}\partial_{31}(v) - Z_{32}y\partial_{21}^{(5)}(v)$

Proposition (2.2): In our possession exactness at $\mathcal{L}_i$.

Proof**:** Be seen [8].

Using us σ_2 we can see $\partial_3:\mathcal{L}_3 \longrightarrow \mathcal{L}_2$ as $\partial_3 = \delta_{\mathcal{L}_3\mathcal{L}_2} + \sigma_2 \circ \delta_{\mathcal{L}_3E_2}$.

Proposition (2.3):

The installation $\partial_2\partial_3 = 0$.

Proof: Using such method employment in (2.1).

Observation (2.1):

The case we took $t_1 = 1$, $t_2 = 0$, being a lead to the inevitable problem of where polarization takes place y to 1 in form terms.

$Z_{32}^{(t_2+1)}yZ_{31}^{(t_1+1)}zRes([p_1 + t_1 + 1, p_2 + t_2 + 1; t_2])$

Because we do not have the conditions of the form $Z_{32}^{(1)}Z_{31}^{(2)}z$.

From observation (2.1) it became clear all terms in the third dimension immediately reduce to zero.

First we needed to define the map

$\partial_3: E_3 \longrightarrow \mathcal{L}_3$ s.t. $\delta_{E_3\mathcal{L}_2} + \sigma_2 \circ \delta_{E_3E_2} = (\delta_{\mathcal{L}_3\mathcal{L}_2} + \sigma_2 \circ \delta_{\mathcal{L}_3E_2})\circ \sigma_3$ (*)

As follows:

- $Z_{21}^{(2)}\varkappa Z_{21}\varkappa Z_{21}\varkappa(v) \longmapsto 0$;where $v \in D_9 \otimes D_0 \otimes D_3$

- $Ꝣ_{32}yꝢ_{21}^{(3)}\varkappa Ꝣ_{21}\varkappa(v) \longmapsto 0$;where $v \in Ɗ_9 \otimes Ɗ_1 \otimes Ɗ_2$
- $Ꝣ_{32}yꝢ_{21}^{(4)}\varkappa Ꝣ_{21}\varkappa(v) \longmapsto 0$;where $v \in Ɗ_{10} \otimes Ɗ_0 \otimes Ɗ_2$
- $Ꝣ_{32}yꝢ_{32}yꝢ_{21}^{(4)}\varkappa(v) \longmapsto 0$;where $v \in Ɗ_9 \otimes Ɗ_1 \otimes Ɗ_1$
- $Ꝣ_{32}yꝢ_{32}yꝢ_{21}^{(5)}\varkappa(v) \longmapsto 0$;where $v \in Ɗ_{10} \otimes Ɗ_0 \otimes Ɗ_1$
- $Ꝣ_{32}yꝢ_{31}^{(2)}\varkappa Ꝣ_{21}^{(2)}\varkappa(v) \longmapsto 0$;where $v \in Ɗ_9 \otimes Ɗ_3 \otimes Ɗ_0$
- $Ꝣ_{32}yꝢ_{31}^{(2)}\varkappa Ꝣ_{21}^{(3)}\varkappa(v) \longmapsto 0$;where $v \in Ɗ_{10} \otimes Ɗ_2 \otimes Ɗ_0$
- $Ꝣ_{32}yꝢ_{31}^{(2)}\varkappa Ꝣ_{21}^{(4)}\varkappa(v) \longmapsto 0$;where $v \in Ɗ_{11} \otimes Ɗ_1 \otimes Ɗ_0$
- $Ꝣ_{32}yꝢ_{31}^{(2)}\varkappa Ꝣ_{21}^{(5)}\varkappa(v) \longmapsto 0$;where $v \in Ɗ_{12} \otimes Ɗ_0 \otimes Ɗ_0$

It has become easier to display (*) is satisfy.

Now from observation (2.1) in X_3 all the terms reduced to zero.
Ultimately, in the next complex we know the boundary map:

$$0 \longrightarrow \mathcal{L}_3 \xrightarrow{\partial_3} \mathcal{L}_2 \xrightarrow{\partial_2} \mathcal{L}_1 \xrightarrow{\partial_1} \mathcal{L}_0 \;;$$

Where ∂_1 and ∂_2 find out as follows:

- $\partial_1(Ꝣ_{21}^{(2)}\varkappa\,(v)) = \partial_{21}^{(2)}(v)$; where $v \in Ɗ_7 \otimes Ɗ_2 \otimes Ɗ_3$
- $\partial_1(Ꝣ_{32}y\,(v)) = \partial_{32}(v)$; where $v \in Ɗ_5 \otimes Ɗ_5 \otimes Ɗ_2$
- $\partial_2(Ꝣ_{32}yꝢ_{21}^{(3)}\varkappa\,(v)) = \frac{1}{3}\,Ꝣ_{21}\varkappa\partial_{21}^{(2)}\partial_{32}(v) + \frac{1}{2}\,Ꝣ_{21}\varkappa\partial_{21}\partial_{31}(v) - Ꝣ_{32}y\partial_{21}(v)$

; where $v \in$
$Ɗ_8 \otimes Ɗ_2 \otimes Ɗ_2$

Proposition (2.4)

The complex

$$0 \longrightarrow \mathcal{L}_3 \xrightarrow{\partial_3} \mathcal{L}_2 \xrightarrow{\partial_2} \mathcal{L}_1 \xrightarrow{\partial_1} \mathcal{L}_0 \longrightarrow K_{(6,4,3)/(1,0)}$$

Is exact.
Proof: see [8] and [9].

3. Conclusions

By doing this ,the terms of the third row are reduced .

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