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Approximate solutions for nonlinear types of dispersive k (m, p. 1) equation using Adomian decomposition method

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Abstract

In this paper, nonlinear dispersive K(m, p. 1)-type equations were approximated using the Ado- mian decomposition method. This scheme is used to compute the explicit exact solution as a rapidly convergent series with easily computed components. Numerical results are obtained by using the calculated components of the decomposition series to illustrate the application of the method. It is discovered that the obtained results are extremely near to the precise solution.

Keywords: K(m, p. 1)-type equations, Soliton solutions, Adomain decomposition method, Nonlinear partial differential equations.

الحلول التقريبية للأنواع غير الخطية لمعادلة التشتت k (m, p.1) باستخدام طريقة التحلل أدوميان

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الملخص

هذه الورقة، تم تقريب معادلات تشتت غير خطية من النوع K(m, p. 1) باستخدام طريقة التحلل أدوميان. يتم استخدام هذا المخطط لحساب الحل الدقيق الصريح كسلسلة متقاربة بسرعة مع مكونات يمكن حسابها بسهولة. يتم الحصول على نتائج عددية باستخدام المكونات المحسوبة لسلسلة التحلل لتوضيح تطبيق الطريقة. تم اكتشاف أن النتائج التي تم الحصول عليها قريبة للغاية من الحل الدقيق .

الكلمات المفتاحية: معادلات من نوع K(m, p. 1)، حلول سوليتون، طريقة تحلل أدوميان، معادلات تفاضلية جزئية غير خطية

1. Introduction

Nonlinear partial differential equations are widely used to describe complex phenomena in var- ious fields of sciences, such as physics, chemistry and applied mathematics. Various methods have been devised to find the exact and approximate solutions of nonlinear partial differential equations in order to provide more information for understanding physical phenomena arising in numerous scientific and engineering fields. Nonlinear partial differential equations are widely used to describe complex phenomena in various fields of sciences, such as physics, chemistry and applied mathematics. Yonggui and Xiaoshan [1] presented a class of compactons of nonlinear K(m,n) equation as follows:

$$\boldsymbol{u_t + a\,(u^m)_x - (u^p)_{xxx} = 0} \qquad \textbf{(1.1)}$$

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If we take a = 1, then equation (1.1) is known as the focusing (+) branch. This (+) branch (1.1) contains compacton solutions [18]. If we take a = 1, the equation is referred to as the defocusing (-) branch. This defocusing (-) branch has a single pattern solution. Compactons are soliton solutions with finite wavelengths or that lack exponential wings. Unlike solitons, which narrow as amplitude increases, the width of a compacton is independent of amplitude. Compacton solutions have been used in many scientific applications, including super-deformed nuclei, phonon, photon. Various methods have been devised to find the exact and approximate solutions for nonlinear types of dispersive K(m, p. 1) equation, see [2, 3, 4, 5].

Over the last four decades, the Adomian decomposition method has been used to obtain formal solutions to a wide range of deterministic and stochastic partial differential equations. In recent years, the decomposition method has emerged as a viable alternative for solving a wide range of problems involving algebraic, differential, integral, integro-differential, higher-order ordi- nary differential equations, partial differential equations (PDEs), and systems [6, 7, 8]. The ADM was used by Adomian et al. [9, 10] to solve mathematical models of the dynamic interaction of immune response with a population of bacteria, viruses, antigens, or tumor cells as systems of nonlinear differential equations or delay-differential equations. In [11] authors used the decompo- sition method to construct approximate solutions for algebraic equations, time-fractional Riccati equations, time-fractional Kawahara equations and modified time-fractional Kawahara equation. [12] reviews the Adomian decomposition method (ADM) and its developments to handle singular and non-singular initial, boundary value problems in ordinary and partial differential equations that arise in the fields of science and engineering. The study [13] applies the spacetime generalized finite difference scheme to solve nonlinear dispersive shallow water waves described by the mod- ified Camassa Holm equation, the modified Degasperis Procesi equation, the Fornberg Whitham equation, and its modified form. Abbaoui and Cherruault [14] solved the cauchy problem using the decomposition method rather than the canonical form of Adomian. They also demonstrated convergence by employing a new formulation of the Adomian polynomials and comparing the ADM to the Picard method.

With only a few iterations, the decomposition method produces rapidly convergent series solutions for both linear and nonlinear deterministic and stochastic equations. The benefit of this method is that it provides a direct solution to the problem, eliminating the need for linearization, perturbation, massive computation, and transformation. Cherruault and colleagues investigated the convergence of this method. Cherruault proposed a new definition of the method in [14], and then insisted that it would be possible to

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prove the decomposition method's convergence. Cherruault and Adomian proposed a new convergence proof of Adomian's method based on convergent series properties in [10. 14). In this paper, we apply Adomian decomposition method (ADM) to solve nonlinear types of dispersive K(m, p. 1) equation. The study show that this methods is very efficient, convenient and can be applied effectively to these types of equations.

2. Construction of the Method

Adomian Decomposition Method(ADM) is a technique to find solutions for differential equations (partial, ordinary), linear and nonlinear, homogeneous and nonhomogeneous.

The aim of this paper is to extend the Adomian analysis method to derive the numerical and exact soliton solutions to the nonlinear dispersive K(m, p. 1) equation subject to the initial condition:

$$u_t(x,t) + (u^m(x,t))_x - (u^p(x,t))_{xxx} + u(x,t)_{5x} = 0. \quad (2.1)$$

subject to the initial condition

$$u(x,0) = f(x). \quad (2.2)$$

In this paper we consider equation (2.2) in a general nonlinear partial differential equations

$$Lu_t(x,t) = L_X u(x,t) + R\big(u(x,t)\big) + F\big(u(x,t)\big) + g(x,t) \quad (2.3)$$

where $\mathbf{L_x}$ is the highest order differential in x. L_x is the time operator, R(u(x, t)) contains the remaining linear terms of lower derivatives in x, F(u(x,t)) is an analytic nonlinear term, and g(x,t) is an inhomogeneous or forcing term. Apply the inverse operator $L_t^{-1}$to both sides of equation (2.3), we obtain

$$u(x,t) = u(x,0) + L_t^{-1}\left\{L_x u(x,t) + R\big(u(x,t)\big)\right) + F(u(x,t))) + g(x,t)\} \quad (2.4)$$

The ADM expresses the solution u(x,t)) of (2.4) by the decomposition series [9, 10]

$$u(x,t)) = \sum_{n=0}^{\infty} u_n\,(x,t) \quad (2.5)$$

and the nonlinear function F(u(x, t)) by an infinite sum of polynomials

$$F\big(u(x,t)\big) = \sum_{n=0}^{\infty} A_n(x,t) \tag{2.6}$$

where the components u_n (x, t) of the solution u(x, t) will be determined recurrently, and A_n are the so-called Adomian that can constructed for various classes of nonlinearity according to specific algorithms set by Adomian [9]. These polynomials can be constructed by using the general formula [15]

$$A_n \frac{1}{n!}\frac{d^n}{d \lambda^n}[\, F\,(\sum_{i=0}^{n} \lambda^i\, u_i)]_{\lambda=0}\,, n\ \geq 0$$

Substitution of (2.5) and (2.6) into (2.4) yields

$$\sum_{n=0}^{\infty} u_n(x,t) = u(x,0) + \tag{2.7}$$

$$L_t^{-1}\{L_x \sum_{n=0}^{\infty} u_n(x,t) + R(\sum_{n=0}^{\infty} u_n(x,t)) + (\sum_{n=0}^{\infty} A_n(x,t)) + g(x,x)\}$$

To determine the components $u_n(x, t)$ $n \geq 0$ we first identify the zeroth component u_0 (x, 0) by all terms that arise from the initial condition at t = 0 and the source term g(x, t). The remaining components u_n (x, t), $n \geq 1$ are then determined recursively using the components, in such a way that (2.7) is formally balanced. In other words, the method introduces the recursive relation

$$u_0(x,t) = u(x,0) +\ L_t^{-1}[g(x,t)],$$

$$u_{k+1}(x,t) =\ L_t^{-1}\big[L_x\big(u_k(x,t)\big) + \big(A_k(x,t)\big)\big],\ k \geq 0.$$

(2.8)

Because u_k depends heavily on the zeroth component u_0. it is computationally convenient to choose u_0 so as to contain the minimum number of terms. If the series converges in a suitable way, then we see that

$$u(x,t) = \lim_{M\to\infty} \sum_{n=0}^{M} u_n(x,t) \tag{2.9}$$

where M is the number of terms that we found.

Several researchers, including Cherruault and co-workers [14], among others (10), have previ ously proven the convergence of the Adomian decomposition series and the series of the Adomian polynomials. Cherruault

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and Adomian demonstrated the convergence of the decomposition se- ries without relying on the fixed point theorem, which is too restrictive for many physical and engineering applications. The Adomian decomposition series is a computationally advantageous rearrangement of the Banach-space analog of the Taylor expansion series around the initial soln- tion component function, allowing for recursion-based solution. The non-unique decomposition allows the analyst to design modified recursion schemes for easier computation in realistic sys- tems.

3. Analysis of ADM

Here we will present the method of solving the problem in this project using the usual Adomian's method with some details. We notice in the problem that there are two non-linear terms, where we need to calculate Adomian polynomials, but we will shorten this to finding only one term for the Adomian polynomials by inserting the derivative on the calculated term. In order to solve equation (2.4) using ADM, we rewrite it in an operator form as

$$L_t u = L_x\big(N(u)\big) + L_{3x}\big(N(u)\big) \qquad (3.1)$$

with an initial condition u(x, 0) = f(x), where L_t, L_x L_{3x} are linear operators defined as L_t (.)= $\frac{\partial}{\partial t}$ (.) , L_x (.) =$\frac{\partial}{\partial x}$ (.) , L_{3x} (.) $\frac{\partial^3}{\partial x^3}$ (.) While the term N(u) represents the non-linear terms either $u^m or\ u^p$. To start . we operate on both sides of equation (3.1) by the inverse of L_t , denoted by $L_t^{-1}(.) = \int_0^t .\,dt$ that yields to

$$u(x,t) = u(x,0) - L_t^{-1}\Big(-L_x\big(N(u)\big) + L_{3x}\big(N(u)\big)\Big) \qquad (3.2)$$

The ADM assumes that the unknown function u(x, t) can be expressed as a sum of components defined in a series of the form:

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t).$$

And the nonlinear operator N(u) can be written as

$$N(u) = \sum_{n=0}^{\infty} A_n(u_0\,, u_1\,, \dots, u_n).$$

Where A_n are called Adomian polynomials. The Adomian polynomials in our case can be found by the formula

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}[N\,(\sum_{i=0}^{n} \lambda^i\, u_i]_{\lambda=0}\,, n\ \geq 0.$$

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In which the first few Adomian polynomials, where we will consider two cases, when $N(u) = u^2$ and $N(u)= u^3$ are:

Case $N(u) = u^2$

$$A_0 = u_0^2, A_1 = 2u_0u_1, A_2 = 2u_0 + u_1^2, A_3 = 2u_0u_3 + 2u_1u_3, A_4 = 2u_0u_4 + 2u_1u_3 + u_2^2,$$

And ,

$$A_5 = 2u_0u_5 + 2u_1u_4 + 2u_2u_3.$$

Case $N(u) = u^3$

$$A_0 = u_0^3, A_1 = 3u_0^2u_1, A_2 = 3u_0^2u_2 + 3u_0u_1^2, A_3 = 3u_0^2u_3 + 6u_0u_1u_2 + u_1^3,$$

And

$$A_4 = 3u_0^2u_4 + 3u_0^2u_2 + 3u_0u_2^2 + 6u_0u_1u_3,$$

and so on. The rest of the polynomials can be constructed in a similar manner for the two cases. Now, Equation (3.2) becomes

$$\sum_{n=0}^{\infty} u_n(x,t) = u(x,0) - L_t^{-1}\left(-L_x\left(\sum_{n=0}^{\infty} A_n(u_0,u_1,\dots,u_n)\right) + L_{3x}\left(\sum_{n=0}^{\infty} A_n(u_0,u_1,\dots,u_n)\right)\right) \quad (3.3)$$

and we identify the zeroth component $u_0(x, t) = u(x, 0)$] by terms arising from initial condi- tions, and we obtain the subsequent components using the following recursive relation.

$$u_0(x,t) = u(x,0)$$

and the nth iterative term is given by

$$u_{n+1}(x,t) = -L_t^{-1}(A_n)_x + L_t^{-1}(A_n)_{3x}, \qquad n \geq 0$$

From which, all components of the decomposition are identified and calculated. Then we see that

$$u_A(x,t) = \lim_{m\to\infty} \sum_{n=0}^{M} u_n(x,t) \qquad (3.4)$$

Is our approximate solution, where M is the number of terms that we found. However, in many cases the exact solution in a closed form may be obtained.

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Moreover, the decomposition series solutions are generally converge very rapidly. The convergence of the decomposition series have investigated by several authors [14], [6]. To give a clear overview of the discussion presented above, the following examples will be investigated.

4. Numerical Results

Choosing examples with known solutions allows for a more complete error analysis. In order to assess the advantages of the proposed methods, in terms of accuracy and efficiency for solving the modified regularized long wave, we have apply our schemes to solve equation (2.2) with In this chapter, we will discuss the solution method used in this thesis by applying it to four different examples. In the first two examples, the numerical results and graphics show that the improved method gave excellent results, while the method gave exact solutions in the third and fourth examples, and this indicates that the improved method is effective and accurate.

5 Example 1

In this example, we solve equation (2.2) for values of m = p = 2, which has the form

$$u_t + u_x^2 - u_{xxx}^2 + u_{xxxxx} = 0 \tag{5.1}$$

which has exact solution

$$u(x,t) = u(x,0) = \frac{16c-1}{12} cosh^2\left(\frac{x-ct}{4}\right)$$

We verify our technique on the above model problem when c = 1 In order to be able to apply ADM first, we integrate equation (5.1) with respect to t, we get the Volterra integral equation

$$u(x,t) = u(x,0) + \int_0^t [-u^2(x,\tau) + u_{xxx}^2(x,\tau) - u_{xxxxx}(x,\tau]\, dr \tag{5.2}$$

Following the same steps in the previous chapter, which refer to substituting the Adomian infinite series on the left side of the above equation, and the series that represent nonlinear terms inside the kernel of the integral equation, and after the comparison, we get the general iterative Adomian solution as follows:

$$u_{k+1}(x,t) = \int_0^t \left[-\frac{\partial}{\partial x}A_k(x,t) + \frac{\partial^2}{\partial x^2}A_k(x,\tau)\right] d\tau, k = 0,1,2,\dots \tag{5.3}$$

$(\boldsymbol{x_i},\boldsymbol{t_i})$	$\lvert \boldsymbol{uE(x,t)} - \boldsymbol{u_A(x,t)}\rvert$
(1,0.1)	1.52449E-11

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(1.0.25)	3.70421E-09
(1,0.5)	2.35294E-07
(1,0.75)	2.661-40E-06
(1,1.0)	1.48563E-05
(3,0.1)	3.17018E-11
(3,0.25)	7.66593E-09
(3,0.5)	4.82967E-07
(3,0.75)	5.41707E-06
(1,1.0)	2.99794E-05
(5,0.1)	8.25926E-11
(5,0.25)	1.99541E-08
(5,0.5)	1,25522E-06
(5,0.75)	1.40566E-05
(5,1.0)	7.76649E-05

Table 1: The absolute error at different points (x, t) for Example 1

Where

$$u_0(x,t) = u(x,0) = \frac{5}{4}\, cosh^2\left(\frac{x}{4}\right),$$

and the nonlinear terms are represented by Adomian polynomials;

$$F\big(u(x,t)\big) = u^2 = \sum_{n=0}^{\infty} A_n(x,t).$$

Using Mathematica, and after some simplifications of the series solution, we obtain

$$u_A(x,t) = \frac{3840 + 10\,(394 + 48\,t^2 + t^2)\cosh(x/2) - t\,(1920 + 80t^2 + t^4)\sinh(x/2)}{6144}$$

Conclusions

We describe an approach that uses Adomian's method to obtain exact and numerical solitary pattern solutions to the nonlinear dispersive K(2, 2, 1) and K(3, 3, 1) equations with initial conditions. The approximate and exact solutions are contrasted in Table 1. The findings demonstrate that the present approach is a useful mathematical tool for finding additional solutions with solitary patterns to a variety of nonlinear dispersive equations with initial conditions. It is important to note that the solution in this case is provided in closed form and requires only the use of the initial condition, in contrast to traditional numerical techniques. One of the advantages of using the ADM is that it can tackle the problem in a straightforward manner without requiring the use of transformation

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formulae or boundary condition constraints. Ultimately, ADM circumvents the challenges and extensive computational work by identifying the analytical solutions. The efficiency of the Adomian scheme increases its range of applications.

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