



A Study of Approximation Methods for Collision processes between Charged Particles and Atoms, Ions, or Molecules

Ahlam K. Yassir¹

Alaa A. Khalaf²

^{1,2}Department of Physics, College of Science, University of Basra

¹Corresponding author. E-mail ahlam.ashour.sci@uobasrah.edu.iq

²Corresponding author. E-mail alaa.abdulhasan@uobasrah.edu.iq

Abstract:

Atomic and molecular physics and chemistry are fundamentally dependent on the investigation of charged particle collisions. These collisions are crucial not only for industrial processes such as plasma technology, fusion energy, and radiation therapy, but also for understanding other natural phenomena, including ionization in Earth's atmosphere. This work aims to survey and evaluate various approximation methods used to analyses and understand collisions, emphasizing their practical applications across multiple scientific fields. This research investigated the subsequent methodologies: The Born Approximation is a first-order perturbation technique frequently utilized in weak potential scattering scenarios to calculate transition probabilities. It achieves this by considering the interaction as a negligible perturbation. The close-coupling approximation addresses complex collision scenarios by solving a system of coupled Schrödinger equations, accounting for the intense interactions between proximate particles.

Keywords: Collision processes, atoms, molecules, Born approximation, close-coupling approximation, quantum scattering.

دراسة طرق التقريب لعمليات التصادم بين الجسيمات المشحونة والذرات أو الأيونات أو الجزيئات

أحلام خضير ياسر¹ علاء عبد الحسن خلف²

^{1,2}قسم الفيزياء، كلية العلوم، جامعة البصرة

¹المؤلف المراسل. البريد الإلكتروني ahlam.ashour.sci@uobasrah.edu.iq

²المؤلف المراسل. البريد الإلكتروني alaa.abdulhasan@uobasrah.edu.iq

المخلص:

تعتمد الفيزياء والكيمياء الذرية والجزيئية بشكل أساسي على التحقيق في تصادمات الجسيمات المشحونة. هذه التصادمات مهمة ليس فقط للعمليات الصناعية مثل تكنولوجيا البلازما، و طاقة الاندماج، والعلاج الإشعاعي، ولكن أيضًا لفهم الظواهر الطبيعية الأخرى، بما في ذلك التأين في الغلاف الجوي للأرض. يهدف هذا العمل إلى مسح وتقييم طرق التقريب المختلفة المستخدمة لتحليل وفهم التصادمات، مع التركيز على تطبيقاتها العملية عبر مجالات علمية متعددة. بحث هذا البحث في المنهجيات التالية: تقريب بورن هو تقنية اضطراب من الدرجة الأولى تُستخدم كثيرًا في سيناريوهات التشتت المحتمل الضعيف لحساب احتمالات الانتقال. ويحقق ذلك من خلال النظر إلى التفاعل باعتباره اضطرابًا لا يُذكر. يعالج التقريب الوثيق سيناريوهات التصادم المعقدة من خلال حل نظام من معادلات شرودنجر المقترنة، مع مراعاة التفاعلات الشديدة بين الجسيمات القريبة.



الكلمات المفتاحية: عمليات التصادم، الذرات، الجزيئات، تقريب بورن، تقريب الاقتران الوثيق، التشتت الكومي.

Introduction

The study of charged particle, atomic, molecular, and ion collisions is fundamental to the understanding of many technological and natural processes and phenomena. A number of approximation techniques have been refined throughout the years to study and comprehend these crashes [1] . In an effort to present a thorough evaluation of these methods, this study will include a reference list that goes all the way back to the earliest studies in the subject and will focus on the work of important scholars who created these techniques [2] . When studying particle collisions, one of the first approximation approaches employed was the Born Approximation. Max Born proposed this approach in 1926[3], for systems with weak interactions; it determines transition probabilities by considering the interaction as a little perturbation. Its effectiveness and simplicity have made it useful in atomic and particle physics. Important studies that have used this approach include Tsitsi and Griffiths's (1999) and Kulik and Jacobsen's (1993) research. [4, 5].

To account for strong interactions between closely spaced particles, the Close-Coupling Approximation solves a system of coupled equations. Studying low-energy particle collisions, when interactions are substantial and cannot be ignored, requires this approach. Leading scholars in this area are James and Bolton (1990) and Williamson and Gallagher (1965) [6].

The Born approximation, the close-coupling approximation, and other analogous methods are commonly utilised to address inelastic processes in collision events involving charged particles interacting with atoms or molecules over a wide energy range [7, 8] .

Our publications[9-11] . in this filed were used the Born and Born-Bethe and other approximation methods were used to calculate the interaction cross sections and other issues[12-14] .

In previous research, we concentrated on the interactions of electrons and positrons with atoms and molecules, specifically calculating the elastic impact cross sections, annihilation parameters, and ionisation cross sections [15-19] .

The paper further contrasts theoretical outcomes with actual data, illustrating the efficacy of these approximation methods in forecasting cross-sections for atomic and molecular targets in excited collision processes. The findings indicate that



these methods not only improve theoretical comprehension but also have practical applications in fields such as plasma physics, particle physics, nuclear processes, and medical physics. This study underscores the essential function of approximation approaches in enhancing our understanding of particle interactions and their wider scientific ramifications.

Theory

This discussion will concentrate on two methodologies and demonstrate their efficacy in computing the inelastic differential cross sections for interactions between charged particles and atomic or molecular targets.

1. Born approximation

In the Born approximation, the differential cross section for scattering can be expressed using the following equation:

$$\frac{d\sigma}{d\Omega} = \left(\frac{m}{2\pi\hbar^2}\right)^2 |f(q)|^2$$

Where :

- $\frac{d\sigma}{d\Omega}$ is the differential cross section
- m is the mass of the particle.
- $\hbar$ is the reduced plank's constant.
- $f(q)$ is the scattering amplitude
- q is the momentum transfer, given by $q=k-k'$, where k and k' are the initial and final wave vectors of the particle.

The Scattering amplitude $f(q)$ is given by the Fourier transform of the potential $V(r)$:

$$f(q) = \int e^{-iq \cdot r} V(r) d^3r$$

The formulation is applicable to weak potential scattering where the Born potential scattering where the Born approximation is valid.

2. Close-Coupling approximation

Quantum scattering theory describes the scattering process using the differential cross section in the close-coupling approximation. As part of this process, a system of coupled channels' Schrodinger equation must be solved [20].

The basic form is given by:

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2$$



Where the $f(\theta)$ is the scattering amplitude, which can be expressed in terms of partial waves and involves the solutions of the coupled –channel equations.

In the close- coupling approach, the scattering amplitude is calculated from

$$f(\theta) = \sum_{i=0}^{\infty} (2i + 1) e^{i\delta_i} \sin \delta_i P_i(\cos \theta)$$

Here, δ_i are the phase shifts obtained from solving the coupled equations , and $P_i(\cos \theta)$ are the Legendre polynomials.

The close- coupling method involves solving a set of coupled equations for the radial part of the wave function, which takes into account all relevant interactions and couplings between different channels.

Results

Here, we break down how to put the Born approach to work. Theoretical data for the rotating excitation transition from (0-0), (0-3) and (0-4) were compared with our DCS results at 5 eV in figure (1). Regarding Machado et al [19]. Brescansin et.al[21] , Jain et.al[22]. and the experimental data of Muller et.al[23].

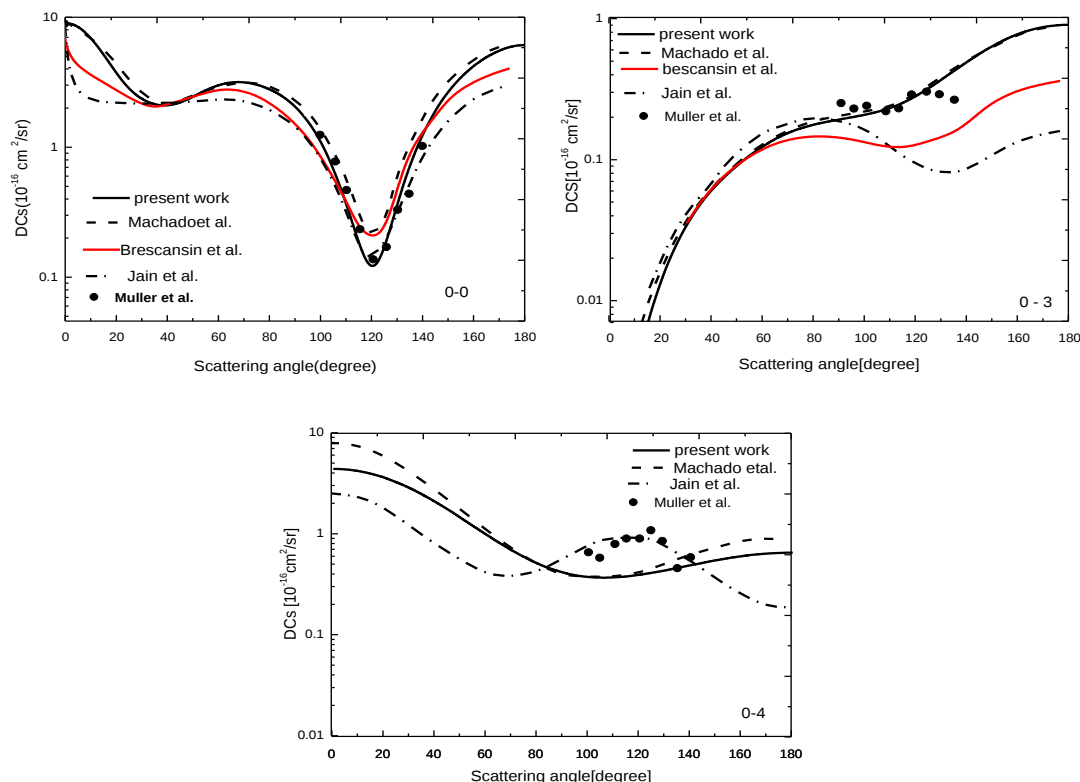




Fig.1: Rotational excitation ($J \rightarrow J'$) DCS for the e^-CH_4 scattering at 5eV. The full curves are the present results. Other theoretical calculations are due to Machado [19], Brescansin [21], Jain [22], And experimental Muller[23].

The application of close-coupling approach is explained in figure (2) [18], where it shows how the DCS acts for both elastic and inelastic interaction for electrons colliding with ionic Mg atom.

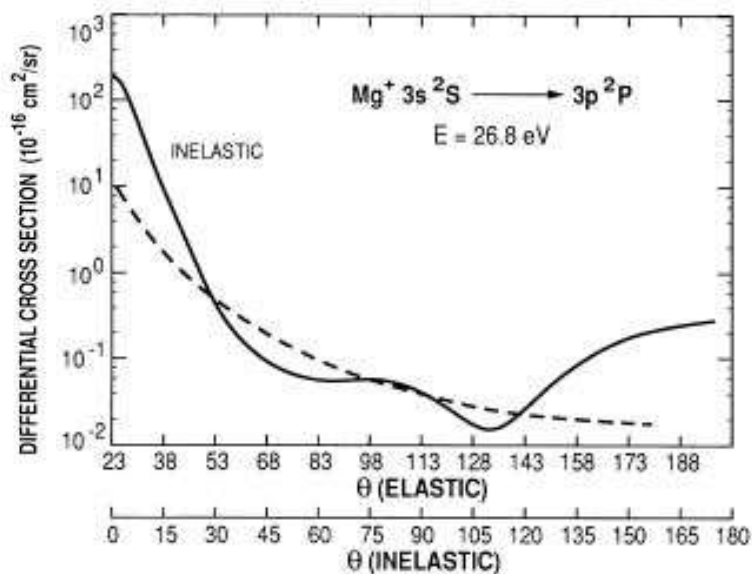


Fig. 2. Differential elastic and inelastic $3s \rightarrow 3p$ scattering cross sections for Mg^+ at energy of 26.8 eV are from 5CC (HF).[24]

Conclusi

From the above, we conclude that the approximate methods we adopted showed good behavior for the cross sections of each of the targets with which the electrons collided, whether atomic or molecular. By comparing the results with the experimental and theoretical data, we have shown the effectiveness of using these approximate methods in the case of excited collisions.

In conclusion, the approximate methods discussed are indispensable tools for understanding and studying collisions between charged particles, atoms, and ions. By simplifying complex equations and enabling accurate analysis of particle behavior, these methods significantly enhance our theoretical understanding and practical applications in various scientific and technological fields. This comprehensive review underscores the importance of these methods in advancing our knowledge and highlights their practical applications across different domains such as plasma physics, particle physics, analytical chemistry, nuclear reactions, and medical research. We hope this study will serve as a valuable resource for



researchers and contribute to further advancements in the field of collision analysis.

Reference

- [1] I. Percival and D. Richards, "The theory of collisions between charged particles and highly excited atoms," in *Advances in Atomic and Molecular Physics*, vol. 11: Elsevier, 1976, pp. 1-82.
- [2] Z.-B. Chen, "Electron-impact excitation of atoms or ions with the screened Coulomb potential," *Physics of Plasmas*, vol. 30, no. 3, 2023.
- [3] M. Born, "Quantenmechanik der stoßvorgänge," *Zeitschrift für physik*, vol. 38, no. 11-12, pp. 803-827, 1926.
- [4] K. M. Stirk, "Identification and reactivity of distonic radical cations," Purdue University, 1993.
- [5] D. J. Griffiths and D. F. Schroeter, *Introduction to quantum mechanics*. Cambridge university press, 2018.
- [6] D. Yennie, F. Boos Jr, and D. Ravenhall, "Analytic distorted-wave approximation for high-energy electron scattering calculations," *Physical Review*, vol. 137, no. 4B, p. B882, 1965.
- [7] F. Gianturco and D. Thompson, "The scattering of slow electrons by polyatomic molecules. A model study for CH₄, H₂O and H₂S," *Journal of Physics B: Atomic and Molecular Physics*, vol. 13, no. 3, p. 613, 1980.
- [8] J. Mitroy, "Close coupling calculations of positron-hydrogen scattering at low energies," *Journal of Physics B: Atomic, Molecular and Optical Physics*, vol. 26, no. 24, p. 4861, 1993.
- [9] R. M. A. Hassan and A. A. Khalaf, "Atomic Lithium Excitation by Electron Impact," *Journal of Kufa-Physics*, vol. 11, no. 02, pp. 29-33, 2019.
- [10] R. M. A. Hassan and A. A. Khalaf, "Born-Bethe approximation to study the process of excitation atoms by electron impact," *Basrah Journal of Sciences*, vol. 37, no. 2, pp. 153-162, 2019.
- [11] A. K. Yassir and A. A. Khalaf, "Rotational excitation of methane molecule by electron impact," 2024.
- [12] A. K. Yassir and A. A. Khalaf, "Electron collision with Ammonia and phosphine at wide range of energies," *Journal of Kufa-Physics*, vol. 14, no. 02, pp. 59-74, 2022.
- [13] M. Augie, "Sarcouncil Journal of Applied Sciences," 2022.
- [14] A. A. Khalf, "Calculating the Atomic electron impact Ionization Cross Section for some elements," *Journal of Wasit for Science and Medicine*, vol. 1, no. 2, pp. 44-51, 2008.



- [15] M. Abdullah and A. Khalf, "Study of the scattering and Annihilation of low energy positron with noble atom," *Basrah Journal of Science (Bas J Sci)*, vol. 25, no. 1A english, 2007.
- [16] A. Khalf, "The Elastic Scattering of Electrons from Cadmium Atom with the use of Model-Potential Approach," *Basrah Journal of Science (Bas J Sci)*, vol. 26, no. 1A english, 2008.
- [17] A. A. Khalf, "Study of the electron impact ionization from molecules," *Al-Mustansiriya Journal of Science*, vol. 20, no. 3, 2009.
- [18] A. Khalaf and F. Ali, "Critical Points of Scattering Positrons from Neon, Krypton and Xenon atoms at low energy regions by using dipole polarization potential," *Basrah Journal of Science (Bas J Sci)*, vol. 25, no. 1A english, 2007.
- [19] L. Machado, L. Bescansin, and M.-T. Lee, "Elastic and rotational excitation cross sections for electron scattering by polyatomic molecules," *Brazilian Journal of Physics*, vol. 32, pp. 804-811, 2002.
- [20] P. McGuire and D. J. Kouri, "Quantum mechanical close coupling approach to molecular collisions. jz-conserving coupled states approximation," *The Journal of Chemical Physics*, vol. 60, no. 6, pp. 2488-2499, 1974.
- [21] L. M. Bescansin, M. A. Lima, and V. McKoy, "Cross sections for rotational excitation of CH₄ by 3–20-eV electrons," *Physical Review A*, vol. 40, no. 10, p. 5577, 1989.
- [22] A. Jain and D. Thompson, "Rotational excitation of CH₄ and H₂O by slow electron impact," *Journal of Physics B: Atomic and Molecular Physics*, vol. 16, no. 16, p. 3077, 1983.
- [23] R. Muller, K. Jung, K.-H. Kochem, W. Sohn, and H. Ehrhardt, "Rotational excitation of CH₄ by low-energy-electron collisions," *Journal of Physics B: Atomic and Molecular Physics*, vol. 18, no. 19, p. 3971, 1985.
- [24] S. J. Smith *et al.*, "Excitation cross sections for the ns 2 S → np 2 P resonance transitions in Mg⁺(n= 3) and Zn⁺(n= 4) using electron-energy-loss and merged-beams methods," *Physical Review A*, vol. 48, no. 1, p. 292, 1993.