Al-Qadisiyah Journal of Pure Science

Manuscript 1229

Single Step Block Linear Approach for Solving Initial Value Problems Second Order Ordinary Differential Equation

Ayinde Muhammed Abdullahi

John Sabo

Taiye Oyedepo

Chukkol Buba Yusuf

Salihu Ibrahim

ARTICLE

Single Step Block Linear Approach for Solving Initial Value Problems Second Order Ordinary Differential Equation

Ayinde M. Abdullahi [a,*], John Sabo [b], Taiye Oyedepo [c], Chukkol B. Yusuf [d], Salihu Ibrahim [e]

[a] University of Abuja, FCT, Gwagwalada, Abuja, Nigeria
[b] Adamawa State University, Mubi, Nigeria
[c] Federal College of Dental Technology and Therapy, Enugu, Nigeria
[d] Modibbo Adama University, Yola, Nigeria
[e] University of Al Qadisiyah, College of Science, Iraq

Abstract

The analysis of single step block hybrid method using the linear block approach for solving second order oscillatory initial value problems of ordinary differential equation is developed in this paper. The numerical properties of this block method are established, demonstrating its consistency, convergence, and zero-stability. To demonstrate the superiority of the block method, it was tested on three mathematical problems involving second-order initial value problems. The results obtained from these tests outperformed the existing approaches found in literature, showcasing the enhanced performance and accuracy of the proposed block method.

Keywords: Block hybrid method, Consistency, Convergence, Zero-stability, Numerical properties, Single-step

1. Introduction

Differential Equations are widely recognized as fundamental mathematical tools extensively employed in various disciplines such as engineering, mathematics, physics, aeronautics, elasticity, astronomy, dynamics, biology, chemistry, medicine, environmental sciences, econometrics, social sciences, banking, and numerous other fields [1].

In the aforementioned areas, mathematical models are commonly utilized to comprehend and analyze physical phenomena. These models invariably lead to the formulation of differential equations. Ross [2] identified a range of problem domains that extensively rely on differential equations, including:

i. the problem arising from determining the projectile motion, satellite, rocket or planet,
ii. the problem of how to determine the charge or current in an electric circuit,
iii. the study of chemical reactions and
iv. the study of decomposition rate of radioactive substance or population growth rate.

The aforementioned problems adhere to specific scientific laws that entail the consideration of rates of change of one or more quantities. Mathematically, these rates of change are represented by derivatives. Consequently, when these problems are mathematically formulated, they manifest as hybrid block differential equations.

The predictor-corrector method commonly employs an explicit technique for the predictor step and an implicit approach for the corrector step. Scholars, including [3–6] among others, have extensively investigated the development of the Linear Multistep Method (LMM) within the framework of the predictor-corrector mode.

Received 26 August 2023; accepted 11 April 2024.
Available online 20 August 2025

* Corresponding author.
E-mail address: ayinde.abdullahi@uniabuja.edu.ng (A.M. Abdullahi).

https://doi.org/10.29350/2411-3514.1229

In their work, Omar et al. [7] introduced a seventh-order Predictor-Corrector block method designed specifically for solving third-order ordinary differential equations. However, this method is associated with computational burden due to the evaluation of many functions per iteration in both predictor and corrector method. Awoyemi [3] stated the advantage of continuous linear multistep method over the discrete methods such that; it gives a simplified form of coefficients for additional analytical work at different points that guarantee easy approximation of solutions at all the interior points of the interval.

The block method was developed to correct some mistake in Predictor-Corrector [8]. According to Ref. [9], the block method was firstly proposed by Milne [10] who advocated the use of block as a means of getting a starting value for predictor-corrector algorithm and later adopted as a full method [11].

Authors who previously worked on block method for solving ODEs [12−15] to mention a few. They stated that numerical solutions on block method are produced with less computational efforts when compared with non-block method [16].

2. Mathematical formulation of the method

This section describe a linear hybrid block for the formation of maximal-order second derivative block method. The following prepositions discuss the development of one-step implicit block method. Considering the general form of the block method while implementing it one-by-one to obtain the expected block method for solving second order ordinary differential equations.

Preposition 2.1. Obtain the block method from the given expression

$$y_{n+\xi} = \sum_{i=0}^{2} \frac{(\xi h)^i}{i!} y_n^{(i)} + \sum_{i=0}^{1} \phi_{i\xi} f_{n+i}, \xi = \frac{1}{8}, \frac{3}{8}, \frac{5}{8}, \frac{7}{8}, 1 \quad (1)$$

Preposition 2.2. Obtain the first and second derivative schemes of the block method from

$$y_{n+\xi}^{(a)} = \sum_{i=0}^{2-a} \frac{(\xi h)^i}{i!} y_n^{(i+a)} + \sum_{i=0}^{7} \omega_{\xi ia} f_{n+i}, a = 1_{\left(\frac{1}{8},\frac{3}{8},\frac{5}{8},\frac{7}{8},1\right)}, \quad a = 2_{\left(\frac{1}{8},\frac{3}{8},\frac{5}{8},\frac{7}{8},1\right)} \quad (2)$$

$\phi_{\xi i} = A^{-1}B$ and $\omega_{\xi ia} = A^{-1}D$

where

$$A = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & \frac{h}{8} & \frac{3h}{8} & \frac{5h}{8} & \frac{7h}{8} & h \\ 0 & \frac{\left(\frac{h}{8}\right)^2}{2!} & \frac{\left(\frac{3h}{8}\right)^2}{2!} & \frac{\left(\frac{5h}{8}\right)^2}{2!} & \frac{\left(\frac{7h}{8}\right)^2}{2!} & \frac{(h)^2}{2!} \\ 0 & \frac{\left(\frac{h}{8}\right)^3}{3!} & \frac{\left(\frac{3h}{8}\right)^3}{3!} & \frac{\left(\frac{5h}{8}\right)^3}{3!} & \frac{\left(\frac{7h}{8}\right)^3}{3!} & \frac{(h)^3}{3!} \\ 0 & \frac{\left(\frac{h}{8}\right)^4}{4!} & \frac{\left(\frac{3h}{8}\right)^4}{4!} & \frac{\left(\frac{5h}{8}\right)^4}{4!} & \frac{\left(\frac{7h}{8}\right)^4}{4!} & \frac{(h)^4}{4!} \\ 0 & \frac{\left(\frac{h}{8}\right)^5}{5!} & \frac{\left(\frac{3h}{8}\right)^5}{5!} & \frac{\left(\frac{5h}{8}\right)^5}{5!} & \frac{\left(\frac{7h}{8}\right)^5}{5!} & \frac{(h)^5}{5!} \end{pmatrix}, B = \begin{pmatrix} \frac{(\xi h)^2}{2!} \\ \frac{(\xi h)^3}{3!} \\ \frac{(\xi h)^4}{4!} \\ \frac{(\xi h)^5}{5!} \\ \frac{(\xi h)^6}{6!} \\ \frac{(\xi h)^7}{7!} \end{pmatrix}, D = \begin{pmatrix} \frac{(\xi h)^{2-a}}{(2-a)!} \\ \frac{(\xi h)^{3-a}}{(3-a)!} \\ \frac{(\xi h)^{4-a}}{(4-a)!} \\ \frac{(\xi h)^{5-a}}{(5-a)!} \\ \frac{(\xi h)^{6-a}}{(6-a)!} \\ \frac{(\xi h)^{7-a}}{(7-a)!} \end{pmatrix}$$

The implementation of Preposition 2.1 and equation (1), yield the following form

To get the unknown coefficients ϕ, it is defined that $\phi_{\xi i} = A^{-1}B$ were A and B are defined above.

$$\left.\begin{aligned}
y_{n+\frac{1}{8}} &= y_n + \frac{h}{8}y'_n + \left[\phi_{10}f_n + \phi_{11}f_{n+\frac{1}{8}} + \phi_{12}f_{n+\frac{3}{8}} + \phi_{13}f_{n+\frac{5}{8}} + \phi_{14}f_{n+\frac{7}{8}} + \phi_{15}f_{n+1}\right]\\
y_{n+\frac{3}{8}} &= y_n + \frac{3h}{8}y'_n + \left[\phi_{20}f_n + \phi_{21}f_{n+\frac{1}{8}} + \phi_{22}f_{n+\frac{3}{8}} + \phi_{23}f_{n+\frac{5}{8}} + \phi_{24}f_{n+\frac{7}{8}} + \phi_{25}f_{n+1}\right]\\
y_{n+\frac{5}{8}} &= y_n + \frac{5h}{8}y'_n + \left[\phi_{30}f_n + \phi_{31}f_{n+\frac{1}{8}} + \phi_{32}f_{n+\frac{3}{8}} + \phi_{33}f_{n+\frac{5}{8}} + \phi_{34}f_{n+\frac{7}{8}} + \phi_{35}f_{n+1}\right]\\
y_{n+\frac{7}{8}} &= y_n + \frac{7h}{8}y'_n + \left[\phi_{40}f_n + \phi_{41}f_{n+\frac{1}{8}} + \phi_{42}f_{n+\frac{3}{8}} + \phi_{43}f_{n+\frac{5}{8}} + \phi_{44}f_{n+\frac{7}{8}} + \phi_{45}f_{n+1}\right]\\
y_{n+1} &= y_n + hy'_n + \left[\phi_{50}f_n + \phi_{51}f_{n+\frac{1}{8}} + \phi_{52}f_{n+\frac{3}{8}} + \phi_{53}f_{n+\frac{5}{8}} + \phi_{54}f_{n+\frac{7}{8}} + \phi_{55}f_{n+1}\right]
\end{aligned}\right\} \tag{3}$$

The implementation of Preposition 2.2 and equation (2), yield the following form

$$\left.\begin{aligned}
y'_{n+\frac{1}{8}} &= y'_n + \left[\omega_{101}f_n + \omega_{111}f_{n+\frac{1}{8}} + \omega_{121}f_{n+\frac{3}{8}} + \omega_{131}f_{n+\frac{5}{8}} + \omega_{141}f_{n+\frac{7}{8}} + \omega_{151}f_{n+1}\right]\\
y'_{n+\frac{3}{8}} &= y'_n + \left[\omega_{201}f_n + \omega_{211}f_{n+\frac{1}{8}} + \omega_{221}f_{n+\frac{3}{8}} + \omega_{231}f_{n+\frac{5}{8}} + \omega_{241}f_{n+\frac{7}{8}} + \omega_{251}f_{n+1}\right]\\
y'_{n+\frac{5}{8}} &= y'_n + \left[\omega_{301}f_n + \omega_{311}f_{n+\frac{1}{8}} + \omega_{321}f_{n+\frac{3}{8}} + \omega_{331}f_{n+\frac{5}{8}} + \omega_{341}f_{n+\frac{7}{8}} + \omega_{351}f_{n+1}\right]\\
y'_{n+\frac{7}{8}} &= y'_n + \left[\omega_{401}f_n + \omega_{411}f_{n+\frac{1}{8}} + \omega_{421}f_{n+\frac{3}{8}} + \omega_{431}f_{n+\frac{5}{8}} + \omega_{441}f_{n+\frac{7}{8}} + \omega_{451}f_{n+1}\right]\\
y'_{n+1} &= y'_n + \left[\omega_{501}f_n + \omega_{511}f_{n+\frac{1}{8}} + \omega_{521}f_{n+\frac{3}{8}} + \omega_{531}f_{n+\frac{5}{8}} + \omega_{541}f_{n+\frac{7}{8}} + \omega_{551}f_{n+1}\right]
\end{aligned}\right\} \tag{4}$$

Therefore, using equation (3) and Preposition 2.1,

$$\left.\begin{aligned}
&(\phi_{10},\phi_{11},\phi_{12},\phi_{13},\phi_{14},\phi_{15})^T = \\
&\left(\frac{48281h^2}{11289600},\frac{1217h^2}{282240},-\frac{44051h^2}{3225600},\frac{1147h^2}{1612800},-\frac{1601h^2}{4515840},\frac{1391h^2}{11289600}\right) \\
&(\phi_{20},\phi_{21},\phi_{22},\phi_{23},\phi_{24},\phi_{25})^T = \\
&\left(\frac{17139h^2}{1254400},\frac{4905h^2}{100352},\frac{201h^2}{22400},-\frac{549h^2}{358400},\frac{117h^2}{250880},-\frac{171h^2}{1254400}\right) \\
&(\phi_{30},\phi_{31},\phi_{32},\phi_{33},\phi_{34},\phi_{35})^T = \\
&\left(\frac{9925h^2}{451584},\frac{45625h^2}{451584},\frac{8875h^2}{129024},\frac{25h^2}{8064},\frac{625h^2}{903168},-\frac{125h^2}{451584}\right) \\
&(\phi_{40},\phi_{41},\phi_{42},\phi_{43},\phi_{44},\phi_{45})^T = \\
&\left(\frac{7007h^2}{230400},\frac{14063h^2}{92160},\frac{31213h^2}{230400},\frac{26411h^2}{460800},\frac{49h^2}{5760},-\frac{343h^2}{230400}\right) \\
&(\phi_{50},\phi_{51},\phi_{52},\phi_{53},\phi_{54},\phi_{55})^T = \\
&\left(\frac{375h^2}{11025},\frac{79h^2}{441},\frac{263h^2}{1575},\frac{143h^2}{1575},\frac{67h^2}{2205},-\frac{37h^2}{2205}\right)
\end{aligned}\right\} \quad (5)$$

Similarly, to obtain the unknown coefficients ω, it is defined that $\phi_{\xi ia} = A^{-1}D$ were A and D are defined above. Therefore, using equation (5) and Preposition 2.2,

$$\left.\begin{aligned}
&(\omega_{101},\omega_{111},\omega_{121},\omega_{131},\omega_{141},\omega_{151})^T = \\
&\left(\frac{9679h}{201600},\frac{14339h}{161280},-\frac{2203h}{115200},\frac{409h}{38400},-\frac{851h}{161280},\frac{41h}{22400}\right) \\
&(\omega_{201},\omega_{211},\omega_{221},\omega_{231},\omega_{241},\omega_{251})^T = \\
&\left(\frac{663h}{22400},\frac{3963h}{17920},\frac{1869h}{12800},-\frac{381h}{12800},\frac{213h}{17920},-\frac{87h}{22400}\right) \\
&(\omega_{301},\omega_{311},\omega_{321},\omega_{331},\omega_{341},\omega_{351})^T = \\
&\left(\frac{295h}{8064},\frac{2125h}{10752},\frac{1325h}{4608},\frac{515h}{4608},-\frac{125h}{10752},\frac{25h}{8064}\right) \\
&(\omega_{401},\omega_{411},\omega_{421},\omega_{431},\omega_{441},\omega_{451})^T = \\
&\left(\frac{889h}{28800},\frac{4949h}{23040},\frac{28469h}{115200},\frac{10633h}{38400},\frac{2779h}{23040},-\frac{49h}{3200}\right) \\
&(\omega_{501},\omega_{511},\omega_{521},\omega_{531},\omega_{541},\omega_{551})^T = \\
&\left(\frac{103h}{3150},\frac{22h}{105},\frac{58h}{225},\frac{58h}{225},\frac{22h}{105},\frac{103h}{3150}\right)
\end{aligned}\right\} \quad (6)$$

3. Properties of the block method

The properties of the block method shall be investigated to ensure the convergence of the block method when solving equation (1). The properties include the order, error constant, consistency, zero stability and convergence.

3.1. Order and error constant of the block method

Consider the linear operator $\ell\{y(\xi):h\}$ with second order block method of the form,

$$\ell\{y(\xi):h\}=\sum_{i=\frac{1}{8},\frac{3}{8}}(\phi_i(t)y_{n+i})-h^2\omega_0y''(x)-h^2\omega_{\frac{1}{8}}y''\left(x+\frac{1}{8}\right)-h^2\omega_{\frac{3}{8}}y''\left(x+\frac{3}{8}\right)-h^2\omega_{\frac{5}{8}}y''\left(x+\frac{5}{8}\right)-h^2\omega_{\frac{7}{8}}y''\left(x+\frac{7}{8}\right)-h^2\omega_1y''(x+1) \quad (7)$$

Using Taylor series expansion and comparing the coefficient of h according to Ref. [13] gives $C_0=C_1=\cdots=C_5=0$ and

$$C_6=\left[-6.1409\times10^{-8}\quad 9.1280\times10^{-8}\quad -3.1537\times10^{-8}\quad 1.2115\times10^{-7}\quad 5.9743\times10^{-8}\right]$$

3.2. Consistency of the Method

Definition: According to Ref. [17], a block method is said to be consistent if its order is greater than or equal to one. From the above analysis, it is obvious that our method is consistent.

3.3. Zero stability of the method

Definition: The numerical method is said to be zero-stable, if the roots $m_s, s=1,2,\cdots,k$ of the first characteristics polynomial $\rho(m)$ defined by $\rho(m)=\det\left(mA^{(0)}-E\right)$ satisfies $|m_s|\leq 1$ and every root satisfies $|z_s|=1$ have multiplicity not exceeding the order of the differential equation. The first characteristic polynomial is given by,

Using Taylor series expansion and comparing the coefficient of h according to Ref. [13] gives $C_0=C_1=\cdots=C_5=0$ and

$$C_6=\left[-6.1409\times10^{-8}\quad 9.1280\times10^{-8}\quad -3.1537\times10^{-8}\quad 1.2115\times10^{-7}\quad 5.9743\times10^{-8}\right]$$

$$\rho(m)=\left|m\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\0&0&0&1&0\\0&0&0&0&1\end{bmatrix}-\begin{bmatrix}0&0&0&0&1\\0&0&0&0&1\\0&0&0&0&1\\0&0&0&0&1\\0&0&0&0&1\end{bmatrix}\right|=\left|\begin{bmatrix}m&0&0&0&-1\\0&m&0&0&-1\\0&0&m&0&-1\\0&0&0&m&-1\\0&0&0&0&m-1\end{bmatrix}\right|=m^4(m-1)$$

Thus, solving for q in m^7-m^6 gives $m=0,0,0,0,1$. Hence the method is said to be zero stable [17].

3.4. Convergence of the block method

Theorem: the necessary and sufficient conditions for linear multistep method to be convergent are that it must be consistent and zero-stable. Hence our method is convergent according to [17].

3.5. Region of absolute stability of our method

Definition: The region of absolute stability for a method refers to the portion of the complex z plane where $z=\lambda h$ for which the method remains stable. In the case of the block method, the region of absolute stability is determined using an approach that avoids computations involving polynomial roots or solving simultaneous inequalities. This method, as described in Ref. [8], is known as the boundary locus method. Applying the method we obtain the region of absolute stability in as (see Fig. 1).

$$\rho(m)=\left|m\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\0&0&0&1&0\\0&0&0&0&1\end{bmatrix}-\begin{bmatrix}0&0&0&0&1\\0&0&0&0&1\\0&0&0&0&1\\0&0&0&0&1\\0&0&0&0&1\end{bmatrix}\right|=\left|\begin{bmatrix}m&0&0&0&-1\\0&m&0&0&-1\\0&0&m&0&-1\\0&0&0&m&-1\\0&0&0&0&m-1\end{bmatrix}\right|=m^4(m-1)$$

Thus, solving for q in m^7-m^6 gives $m=0,0,0,0,1$. Hence the method is said to be zero stable [17].

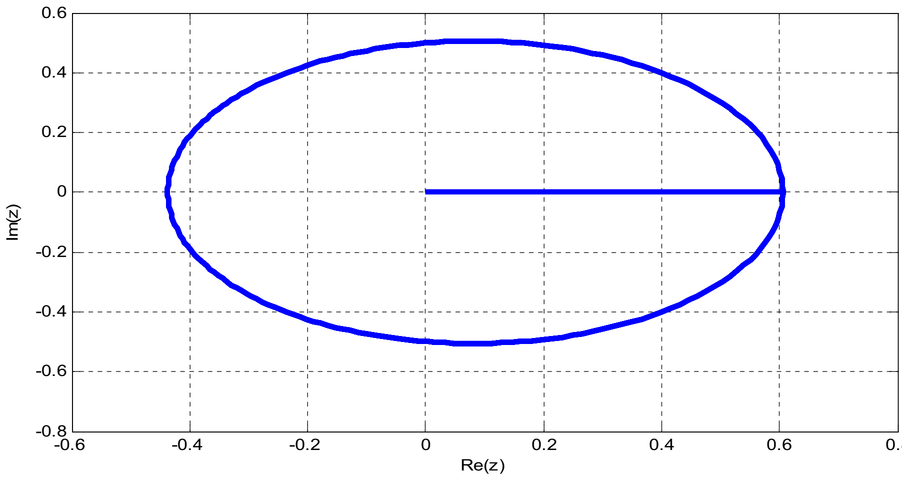

Fig. 1. Region of Absolute Stability of our method.

4. Numerical application of the method

The accuracy and convergence of the block method will be studied using some highly stiff second order initial value problems. The results are shown in tabular form and graphically. The following notation will be used in the tables and graphs.

Problem 1. Consider the second order cooling of a body.
The temperature y degrees of a body t minutes after being placed in a certain room, satisfies the differential equation $3\frac{d^2y}{dt^2}+\frac{dy}{dt}=0$. By using the substitution $z=\frac{dy}{dt}$ or otherwise, find y in terms of t given that $y=60$ when $t=0$ and $y=35$ when $t=\ 6\ln 4$. Find after how many minutes the rate of cooling of the body will have fallen below one degree per minute, giving your answer correct to the nearest minute. How cool does the body get?
Formulation of the Problem.

$$y''=\frac{-y'}{3}, y(0)=60, y'(0)=-\frac{80}{9}, h=0.1$$

With analytic solution (see Tables 1–3).

$$y(x)=\frac{80}{3}e^{-\left(\frac{1}{3}\right)x}+\frac{100}{3}.$$

Problem 2. The second order initial value problem. $y''(u)=3y'+8\exp(2u), y(0)=y'(0)=1, h=\frac{5}{1000}$ is consider with exact solution

$$y=4\exp(2u)+3\exp(3x)+2$$

Problem 3:
The second order initial value problem

$$y''-y'=0, y(0)=0, y'(0)=-1, h=0.1$$

Is consider with analytic solution is given by

$$y(g)=1-\exp(g)$$

Table 1. Result for problem 1.

u	Exact Result	Computed Result	Error in our Method	Error in [18]	Error in [19]	Error in [20]
0.1	59.12576267952015738700	59.12576267952015738700	0.0000(00)	3.5500(-11)	7.4764(-06)	2.3000(-17)
0.2	58.28018626750980633900	58.28018626750980633500	4.0000(-18)	4.5800(-11)	2.9394(-05)	1.7100(-16)
0.3	57.46233114762558861700	57.46233114762558860800	9.0000(-18)	7.0000(-11)	6.4802(-05)	4.3700(-16)
0.4	56.67128850781193210600	56.67128850781193208900	1.7000(-17)	6.5000(-11)	1.1279(-05)	8.1300(-16)
0.5	55.90617933041637530700	55.90617933041637528100	2.6000(-17)	3.3300(-11)	1.7250(-04)	1.2910(-15)
0.6	55.16615341541284956400	55.16615341541284952600	3.8000(-17)	4.2000(-11)	2.4310(-04)	1.8640(-15)
0.7	54.45038843564751105000	54.45038843564751099900	5.1000(-17)	4.3800(-11)	3.2383(-04)	2.5250(-15)
0.8	53.75808902305729847200	53.75808902305729840700	6.5000(-17)	1.0700(-10)	4.1393(-04)	3.2690(-15)
0.9	53.08848588484580976200	53.08848588484580968100	8.1000(-17)	6.5800(-11)	5.1271(-04)	4.0890(-15)
1.0	52.44083494863438001100	52.44083494863437991400	9.7000(-17)	1.6900(-10)	6.1951(-04)	4.9800(-15)

See [18–20].

Table 2. Result for problem 2.

u	Exact Result	Computed Result	Error in our Method	Error in [21]	Error in [22]
0.005	1.00513852551048470790	1.00513852551048470760	3.0000(-19)	3.1590(-07)	4.4409(-16)
0.01	1.01055824175352732600	1.01055824175352732620	2.0000(-19)	1.2709(-06)	8.8878(-16)
0.015	1.01626544391208340570	1.01626544391208340560	1.0000(-19)	8.6554(-06)	2.2205(-16)
0.02	1.02226654286652595940	1.02226654286652595960	2.0000(-19)	2.5915(-05)	6.6613(-16)
0.025	1.02856806714979844830	1.02856806714979844810	2.0000(-19)	3.3951(-05)	4.4409(-16)
0.03	1.03517666493419258490	1.03517666493419258450	4.0000(-19)	5.9904(-05)	1.3323(-15)
0.035	1.04209910605024978050	1.04209910605024978030	2.0000(-19)	–	–
0.04	1.04934228403829279690	1.04934228403829279630	6.0000(-19)	8.8858(-05)	1.7764(-15)
0.045	1.05691321823310203150	1.05691321823310203100	5.0000(-19)	–	–
0.05	1.06481905588225886860	1.06481905588225886790	7.0000(-19)	–	–

Source: [21,22].

Table 3. Result for problem 3.

u	Exact Result	Computed Result	Error in our Method	Error in [22]	Error in [23]
0.1	−0.1051709180756476248	−0.10517091807564730491	3.1989(-16)	3.2482e-12	–
0.2	−0.2214027581601698339	−0.22140275816016846620	1.3677(-15)	8.5643e-11	3.4602e-09
0.3	−0.3498588075760031040	−0.34985880757599981095	3.2931(15)	3.4401e-10	5.6760e-09
0.4	−0.4918246976412703178	−0.49182469764126404899	6.2688(-15)	7.4251e-10	7.6413e-09
0.5	−0.6487212707001281468	−0.64872127070011765208	1.0495(-14)	1.3785e-09	1.0497e-08
0.6	−0.8221188003905089749	−0.82211880039049277398	1.6201(-14)	2.2193e-09	1.4495e-08
0.7	−1.0137527074704765216	−1.01375270747045286980	2.3652(-14)	3.3875e-09	1.8782e-08
0.8	−1.2255409284924676046	−1.22554092849243445310	3.3152(-14)	4.8470e-09	2.2799e-08
0.9	−1.4596031111569496638	−1.45960311115690461560	4.5048(-14)	6.7518e-09	2.8258e-08
1.0	−1.7182818284590452354	−1.71828182845898549400	5.9741(-14)	9.0628e-09	3.5547e-08

Source [22,23].

5. Conclusion

This paper discussed and analysed a single-step block hybrid method utilizing the linear block approach for solving second-order initial value problems. The numerical properties of the block method are established, demonstrating its consistency, convergence, and zero-stability. To validate the superiority of the block method, it is tested on three mathematical problems involving second-order initial value problems. The obtained results surpass those found in existing literature, showcasing the enhanced accuracy and performance of the proposed block method.

Funding

Self-funding.

References

[1] Lambert JD. Computational methods in ordinary differential equations. In: Introductory mathematics for scientists and engineers, Wiley; 1973.
[2] Ross SL. Introduction to ordinary differential equations. fourth ed. Singapore: John Wiley & Sons, Inc; 1989.
[3] Awoyemi DO. An algorithm collocation method for direct solution of special and general fourth order initial value problems of ordinary differential equation. J NAMP 2005;6. 271-238.
[4] Awoyemi DO. A class of continuous methods for general second order initial value problems in ordinary differential equations. Int J Comput Math 1999;72(1):29−37.
[5] Kayode JS. An efficient zero stable method for fourth order differential equations. Int J Math Sci 2009:1−10.
[6] Onumanyi P, Sirisena UW, Dauda Y. Toward uniformly accurate continuous finite difference approximation of ordinary differential equation. Bagalel J Pure Appl Sci 2001;1:5−8.
[7] Omar Z, Abdullahi YA, Kuboye JO. Predictor-corrector block method of order seven for solving third order ordinary differential equations. Intern J Math Anal 2016;10(5):223−35.
[8] Fatunla SO. Block methods for second order ODEs. Int J Comput Math 1991;41(1−2):55−63.
[9] Olabode BT. Some linear multistep methods for special and general third order initial value problems of ordinary differential equation. PhD Thesis. Akure: Federal University of Technology; 2007 (Unpublished).
[10] Milne WE. Numerical solution of differential equations19. New York: Wiley; 1953. 3.
[11] Adesanya AO, Anake TA, Oghonyon GJ. Continuous implicit method for the solution of general second order ordinary differential equations. J Nigerian Assoc Math Phys 2009; 15:71−8.
[12] Raymond D, Pantuvu TP, Lydia A, Sabo J, Ajia R. An optimized half-step scheme third derivative methods for testing higher order initial value problems. African Sci Rep 2023; 2(76):1−8. https://doi.org/10.46481/asr.2023.2.1.76.
[13] Sabo J, Kyagya TY, Ayinde AM, Otaide IJ. I. J. Mathematical simulation of the linear block algorithm for modeling third-order initial value problems. BRICS J Educ Res 2022;12(3): 88−96.
[14] Donald JZ, Skwame Y, Sabo J, Ayinde AM. The use of linear multistep method on implicit one-step second derivative block method for direct solution of higher order initial value problems. Abacus (Math Sci Series) 2021;48(2): 224−37.

[15] Sabo J, Ayinde AM, Ishaq AA, Ajileye G. The simulation of one-step algorithms for treating higher order initial value problems. Asian Res J Math 2021;17(9):34−47.
[16] Lanczos C. Applied analysis, prentice hall. 1965. New Jersey.
[17] Dahlquist G. Convergence and stability in the numerical integration of ordinary differential equations. Math Scandinavia 1959;4:33−53.
[18] Omole EA, Ogunware BG. 3-point single hybrid block method (3PSHBM) for direct solution of general second order initial value problem of ordinary differential equations. J Sci Res Rep 2018;20(3):1−11.
[19] Olanegan OO, Ogunware BG, Alakofa CO. C. Implicit hybrid points approach for solving general second order ordinary differential equations with initial values. J Adv Math Comp Sci 2018;27(3):1−14.
[20] Skwame Y, Donald JZ, Kyagya TY, Sabo J, Bambur AA. The numerical applications of implicit second derivative on second order initial value problems of ordinary differential equations. Dutse J Pure Appl Sci 2020;6(4):1−14.
[21] Badmus AM, Yahaya YA. An accurate uniform order 6 block method for direct solution of general second order ordinary differential equations. Pacific J Sci Technol 2009;10(2): 248−54.
[22] Adeyefa EO, Adeniyi RB, Udoye AM, Odafi NO. Orthogonal based zero-table numerical integrator for second order IVPs in ODEs. Int J Pure Appl Math 2018;3:329−37.
[23] Kayode SJ, Adegboro JO. Predictor-corrector linear multistep method for direct solution of initial value problems of second order ordinary differential equations. Asian J Phys Chem Sci 2018;6(1):1−9.