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RESEARCH ARTICLE

A New Efficient Method to Solve Bi-Objective Transportation Problems Under Fuzzy Parameters

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ABSTRACT

The transportation problem (TP) is a classical optimization problem in operations research and logistics. Due to several factors, real-life situations may have inconsistent supply, demand, and unit transportation costs. The fuzzy numbers represent these inaccurate data. In the present scenario, the decision-maker handles several objectives simultaneously. This paper presents a simple method to solve linear Bi-Objective Transportation Problems (BOTP) using (η, ξ) interval-valued fuzzy numbers (IVFN), offering a more realistic way to model uncertainty than normal fuzzy numbers. Using the signed distance ranking, the Fuzzy BOTP was transformed into the equivalent crisp BOTP. In this paper, A method is developed based on assigning the allocation in the objective's minimum cost, corresponding to the row and column cells with the minimum objective value. A unique, efficient solution is obtained directly, leading to an optimal compromise solution that the decision-maker prefers using the proposed method. The proposed method aims to assign the allocation in a way that minimizes the total objective value. The compromise (η, ξ) fuzzy efficient solution as well as the crisp efficient solution of the fuzzy BOTP is provided by this method, which has a minimum distance from the ideal solution. This proposed method is less time-consuming and simple to use. A numerical example is used to illustrate our proposed method and to compare the results with some other existing methods. The proposed method provides a non-degenerate efficient compromise solution for the example, which has a minimum distance (33.95) from the ideal solution.

Keywords: Bi-objective transportation problem (BOTP), Compromise optimal solution, Efficient solution, Fuzzy transportation problem, Interval-valued fuzzy numbers

Introduction

A particular type of linear programming problem known as the "Transportation Problem" (TP) aims to deliver an item or provide services at the lowest cost from numerous supply sources to various demand destinations. TP was formulated mathematically for the first time.[1] Traditional methods, including the Vogel approximation method, the Matrix Minima approach, and the North West Corner method, are used to solve the transportation problem. Using a set of parameters whose values are decided by Decision Makers (DMs), TP simulates actual-life situations. With the traditional technique, DMs had to set exact values for the parameters. The parameters of the issue are typically described uncertainly since, in this case, DM is unsure of the parameters' exact values. The bi-objective transportation problem is an extension of the classical transportation problem which aims to optimize two conflicting objectives simultaneously, such as costs and time. Scenarios with uncertainty and imprecision interval-valued fuzzy numbers (IVFNs) are used to represent these uncertainties effectively. In many scientific disciplines, including operations research and systems analysis, a model must be built using only approximately known data. The fuzzy set theory was invented by L.A. Zadeh.[2] Numerical data with fuzzy characteristics can be represented by fuzzy subsets on the real line, also called fuzzy numbers. Fuzzy set theory enables the handling

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of uncertainty and imprecision by extending the principles of classical set theory. The use of IVFNs enhances the mathematical representation of BOTP.

In this paper, an efficient solution of (η, ξ) interval-valued fuzzy BOTP is obtained by the proposed method. The (η, ξ) BOTP is converted into crisp BOTP by using the Signed-distance ranking function. Our method is based on assigning the allocation in the v-th objective's minimum cost, corresponding to the row and column of cells with the minimum objective. By using this proposed method, a unique, fuzzy as well as crisp efficient solution is obtained directly that leads to a compromise solution of fuzzy BOTP. Our obtained efficient solution has a minimum distance from the Ideal solution. The proposed method is significant due to its efficient approach, empirical validation, and practical implications for decision-making in uncertain environments further fuzzy transportation problem-solving.

Literature review

Bi-Criteria Transportation Problem was solved by finding non-extreme points.[3] The two interactive algorithms that had been proposed to Solve the Multi-Objective Transportation problem (MOTP).[4] A fuzzy programming method is presented for solving multi-criteria decision-making TP.[5] The evaluation Programming was developed to solve Bi-Criteria TP.[6] A new method (dripping method) is proposed to find the set of efficient Solutions for BOTP.[7] The MMK method was proposed to solve BOTP using lexicographic programming.[8] A fuzzy binary multi-objective model was developed that, using structural assumptions, optimizes the inbound and outbound transportation expenses of a multi-echelon supply chain[9] (The Iranian steel supply chain). A novel method of MOCTP that has mixed constraints (linear and fractional), the trapezoidal fuzzy numbers are used to handle these uncertainties ranking functions, and fuzzy goal programming is employed to achieve the optimal solution.[10] An efficient approach was proposed for solving TP in which they first extended a basic feasible solution and then applied an existing optimality method to determine the cost of transportation.[11] A signed distance ranking function method was proposed to find a set of efficient solutions for fuzzy MOTP.[12] the location-arc routing problem (TLARP) addresses, what arises when suppliers must make transportation decisions to established depots and develop a bi-objective mathematical model using an augmented ϵ-constraint method to maximize the overall costs and makespan, and the real Pareto solutions are obtained.[13] A two-stage Flow shop scheduling model is presented, considering machine setup and processing time, to minimize waiting time for all jobs, compared to existing makespan approaches of Johnson and Palmer.[14] A study on an MOCTP is conducted in which some constraints are linear and some are fractional. They developed the concepts of linearizing fractional goals to solve MOCTP.[15] Two ranking functions were proposed to defuzzify and solve fuzzy multiple objectives (FMO) programming problems, and they used two types of membership functions (MF), namely ordinary fuzzy trapezoidal MF and weighted trapezoidal fuzzy MF, to compare the obtained results.[16] An iterative method was developed that Constructs a Comprehensive Set of efficient Solutions with bounded decision variables to Solve BOTP.[17] A two-stage flow shop fuzzy scheduling approach is proposed, aiming to minimize job waiting times in a structured model in which the processing time is represented by trapezoidal membership functions, here trapezoidal fuzzy numbers are defuzzified by Yager's ranking method, outperforming other heuristic approaches and achieving the better-desired objective.[18] a mathematical model was proposed for the multi-objective aspirational level fractional transportation problem (MOFTP) based on the highest value of every model objective of model.[19] Triangular fuzzy fractional programming problems were solved using a new ranking method for ordinary fuzzy numbers.[20] An Interval Robust Possibilistic Approach was developed to Solve Bi-Objective Productive Transportation problem.[21] A two-stage flow shop fuzzy scheduling approach for uncertain situations is developed, focusing on reducing waiting time in structured models in which the processing time is represented by triangular membership functions. The triangular fuzzy numbers are defuzzified by Yager's ranking method. The algorithm outperforms existing makes-pan approaches, achieving the desired objective.[22] The long-haul truck scheduling problem was studied.[23] An algorithm is provided to handle the problem on a road network, and the method produces a set of non-dominated pathways for the two objectives. This situation is framed as a bi-objective optimization problem, taking into account the minimization of fuel cost and the minimizing of path length. a Lagrangian Relaxation (LR) heuristic approach is presented that may offer both a lower limit and a nearly optimum solution for each of the single-objective issues is created due to the NP-hardness of the single-objective problems produced from BMTPP.[24] The logistics problem of the supply chain network to optimize the order allocation of products from multiple plants, warehouses, and distributors is studied. Their objective was to reduce total transportation and inventory costs by determining the most optimal locations, flows, shipment composition, and shipment cycle times.[25] A linear shipping model that focuses on decision-making is presented. It encompasses the setup of the shipping network, the selection of transportation means, and the transfer of individual customer shipments using a specific transport system presented in this paper. Their objective was to reduce total transportation and inventory costs by determining the optimal locations, flows, shipment composition, and shipment cycle times.[26] The problem studied and solved consists of the manufacturing of orders from customers being carried out in a job-shop environment, and order deliveries are made by various kinds of vehicles, each of which is allowed to make several journeys. This work investigates a new variation of the integrated production scheduling and vehicle routing problem.[27] A method was proposed to solve the bi-objective solid transportation problem (BOSTP), in which they defuzzify the fuzzy BOSTP by alpha cut method, converted BOSTP into STP, and solved STP by hierarchical goal programming method.[28] A local search method in the memetic algorithm had been developed to solve bi-objective CSP with interval type-2 fuzzy numbers, which represent the cost of edges in an uncertain environment.[29] This study aims to deal with the uncertainties in closed-loop supply chain (CLSC) networks by adopting a multi-objective approach. The main focus is on optimizing the integrated production and transportation operations.[30] An algorithm is presented to solve multi-objective transportation problems with generalized trapezoidal fuzzy numbers based on the proposed ranking function.[31]

Managerial implications

In the real-world Scenario, a challenging task for the decision maker is to handle the uncertainties in different parameters like transportation cost, availabilities, requirements, cost, profit, fuel efficiency, etc. to address the transportation problem. In the proposed method interval-valued fuzzy numbers are used to handle these uncertainties. Some managerial implications of the proposed method are as follows:

1. **Enhanced Decision-Making in Uncertain Environments**

The method ensures that imprecise and variable data are more accurately represented, reducing the risk of suboptimal decisions due to inaccurate data assumptions. By using interval-valued fuzzy numbers, managers can better model and accommodate the inherent uncertainties in transportation logistics. This leads to more robust and reliable decision-making processes.

2. **Efficiency in Handling Multiple Objectives**

   - **Simultaneous Objective Management:** The decision maker often needs to balance multiple objectives, such as minimizing costs while meeting demand and supply constraints. The proposed method simplifies this process by transforming the fuzzy BOTP into an equivalent crisp problem, making it easier to handle and solve.
   - **Compromise Solutions:** The method provides a unique and efficient compromise solution that meets multiple objectives, which is crucial for decision-makers who need to find a balance between competing goals.

   This method not only enhances decision-making efficiency and cost-effectiveness but also supports strategic planning and risk management in dynamic and uncertain environments.

Theoretical Implication: The theoretical implications of the proposed method are as follows:

- **Improved Data Representation:** This approach allows for a more realistic representation of imprecise data, addressing the inherent variability in supply, demand, and transportation costs. This refinement can be theoretically extended to other areas of operations research where data uncertainty is a critical factor. The use of IVFN instead of normal fuzzy numbers provides a more nuanced and accurate modeling of uncertainty, enhancing the theoretical framework of fuzzy optimization problems.
- **Methodological Innovation:** The transformation of the Fuzzy Bi-Objective Transportation Problem (FBOTP) into an equivalent crisp Bi-Objective Transportation Problem using signed distance ranking introduces a novel methodology. This transformation simplifies the handling of fuzzy parameters, making it easier to apply traditional optimization techniques.
- **Optimal Compromise Framework:** The proposed method for obtaining an efficient compromise solution directly addresses the need for decision-makers to balance multiple objectives, this approach enhances the theoretical framework of preference-based optimization, which is crucial in practical decision-making scenarios.

  This method presents a framework designed to address various combinatorial optimization problems, including the Traveling Salesman Problem (TSP), by leveraging interval-valued fuzzy numbers and signed-distance ranking functions.

Some basic definitions

Fuzzy Number: A fuzzy set $\tilde{B}$ defined on the set of real numbers $\mathbb{R}$ is said to fuzzy number if its membership function $\tilde{B}: \mathbb{R} \rightarrow [0, 1]$ has the following properties:

(i) $\tilde{B}(\lambda x_1 + (1-\lambda)x_2) \geq \min\{\widetilde{B(x_1)}, \widetilde{B(x_2)}\}$
(ii) there exists a $x \in \mathbb{R}$ such that $\tilde{B}(x) = 1$.
(iii) $\tilde{B}$ is piece-wise continuous.

Level η of Fuzzy number: If the fuzzy set $\tilde{B}$ on $\mathbb{R}$ has a Membership Function (MF) that is given as

$$\mu_{\tilde{B}}(y) = \begin{cases} \frac{\eta(y-f)}{g-f}; & f < y \leq g, \\ \frac{\eta(h-y)}{h-g}; & g \leq y < h, \\ 0; & otherwise \end{cases}$$

Where $0 < \eta \leq 1$ is said to be level a fuzzy number, and it is denoted as $\tilde{B} = (f, g, h; \eta)$

Interval-valued fuzzy (IVF) number: An IVF set $\tilde{B}$ on $\mathbb{R}$ is given by $\tilde{B} = \{(y, [\mu_{B^-}(y), \mu_{B^+}(y)]) : y \in \mathbb{R}\}$, where $\mu_{B^-}(y), \mu_{B^+}(y) \in [0, 1]$ and $\mu_{B^-}(y) \leq \mu_{B^+}(y)$.

$$\text{Let } \mu_{B^-}(y) = \begin{cases} \frac{\eta(y-f)}{g-f}; & f < y \leq g, \\ \frac{\eta(h-y)}{h-g}; & g \leq y < h, \\ 0; & otherwise \end{cases}$$

Then, $\tilde{B}^- = (f, g, h; \eta)$

$$\text{Let } \mu_{B^+}(y) = \begin{cases} \frac{\xi(y-u)}{g-u}; & u < y \leq g, \\ \frac{\xi(w-y)}{w-g}: & g \leq y < w, \\ 0, & otherwise \end{cases}$$

Then, $\tilde{B}^+ = (u, g, w; \xi)$

It is clear that $0 < \eta \leq \xi \leq 1$ and $u < f < g < h < w$.

Then the IVF set is $\tilde{B} = \{(y, [\mu_{B^-}(y), \mu_{B^+}(y)]) : y \in \mathbb{R}\}$ that is also denoted by $\tilde{B} = [(f, g, h; \eta), (u, g, w; \xi)] = [\tilde{B}^-, \tilde{B}^+]$.

Signed distance ranking method

Let $F_{IVFN}(\eta, \xi) = \{[(f, g, h; \eta), (u, g, w; \xi)]$: for all $u < f < g < h < w$ and $0 < \eta \leq \xi \leq 1\}$ be the family of IVFN.

Let $\tilde{G} = [(f, g, h; \eta), (u, g, w; \xi)] \in F_{IVFN}(\eta, \xi)$, $0 < \eta \leq \xi \leq 1$. The signed distance ranking formula of $\tilde{G}$ from $\tilde{0}$ is defined as:

$$D(\tilde{G}, \tilde{0}) = \frac{1}{16}\left[6g + f + h + 4u + 4w + 3(2g - u - w)\frac{\eta}{\xi}\right].$$

Some properties of IVFN

Let $\tilde{G} = [(f, g, h; \eta), (u, g, w; \xi)] \in F_{IVFN}(\eta, \xi)$, and $\tilde{H} = [(f_0, g_0, h_0; \eta), (u_0, g_0, w_0; \xi)] \in F_{IVFN}(\eta, \xi)$

Then

(I) $\tilde{G}(+)\tilde{H} = [(f + f_0, g + g_0, h + h_0; \eta), (u + u_0, g + g_0, w + w_0; \xi)]$, And

$$k\tilde{G} = \begin{cases} [(kf, kg, kh; \eta), (ku, kg, kw; \xi)], & k > 0 \\ [(kh, kg, kf; \eta), (kw, kg, ku; \xi)], & k < 0 \\ [(0, 0, 0; \eta), (0, 0, 0; \xi)], & k = 0 \end{cases}$$

(II) The ranking of level (η, ξ) IVFN in $F_{IVFN}(\eta, \xi)$ using the signed distance function, D is given as:

$$\tilde{G} \prec \tilde{H} \Leftrightarrow D(\tilde{G}, \tilde{0}) < D(\tilde{H}, \tilde{0})$$

$$\tilde{G} \approx \tilde{H} \Leftrightarrow D(\tilde{G}, \tilde{0}) = D(\tilde{H}, \tilde{0})$$

And, $D(\tilde{H} + \tilde{G}, \tilde{0}) = D(\tilde{H}, \tilde{0}) + D(\tilde{H}, \tilde{0})$

$$D(k\tilde{G}, \tilde{0}) = kD(\tilde{G}, \tilde{0})$$

Mathematical model for fuzzy BOTP

$$(\tilde{S}) \quad \text{Min } \tilde{Z}_1(y) = \sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{a}_{ij}^{(1)} y_{ij},$$

$$\text{Min } \tilde{Z}_2(y) = \sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{a}_{ij}^{(2)} y_{ij}$$

Subject to

$$\sum_{j=1}^{n} y_{ij} = \tilde{s}_i, \; i = 1, 2, \ldots, m$$

$$\sum_{i=1}^{m} y_{ij} = \tilde{d}_j, \; j = 1, 2, \ldots, n,$$

$$y_{ij} \geq 0, \; i = 1, 2, \ldots, m; \; j = 1, 2, \ldots, n, \text{ and}$$

$$\tilde{s}_i > 0, \; \tilde{d}_j > 0.$$

Here,

$\tilde{s}_i$ = Available fuzzy amount of product at i-th source,

$\tilde{d}_j$ = Required fuzzy amount of product at j-th destination,

$\tilde{a}_{ij}^{(v)}$ = Fuzzy transportation cost of a unit transported from i-th source to j-th destination of v-th objectives, where v = {1,2}

y_{ij} = The transported amount of product from i-th source to j-th destination

Assume that, $\sum_{i=1}^{m} \tilde{s}_i = \sum_{j=1}^{n} \tilde{d}_j$ (i.e., Balanced BOTP)

And, $\tilde{s}_i, \tilde{d}_j, \tilde{a}_{ij}^{(v)} \in F_{IVFN}(\eta, \xi)$

Procedure for proposed method

Interval-valued fuzzy BOTP requires an efficient solution that is close to the ideal solution to solve. Here the proposed method provides a unique efficient solution, that leads to a compromise solution.

The steps of our proposed technique are following:

Step I: Fuzzy BOTP ($\tilde{S}$) is converted into Crisp BOTP(S) using the signed distance function.

Then

(S) The crisp BOTP is given as

$$\text{Min } Z_1(\text{y}) = \sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij}^{(1)} y_{ij},$$

$$\text{Min } Z_2(\text{y}) = \sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij}^{(2)} y_{ij}$$

Subject to

$$\sum_{j=1}^{n} y_{ij} = s_i, \text{i} = 1, 2, \ldots, \text{m}, \quad (1)$$

$$\sum_{i=1}^{m} y_{ij} = d_j, \text{j} = 1, 2, \ldots, \text{n}, \quad (2)$$

$$y_{ij} \geq 0, \text{i} = 1, 2, \ldots, \text{m}; \text{j} = 1, 2, \ldots, \text{n}, \quad (3)$$

and $s_i > 0$, $d_j > 0$.

Here,

s_i = Available amount of product at i-th source,

d_j = Required amount of product at j-th destination,

$a_{ij}^{(v)}$ = Transportation cost of a unit transported from i-th source to j-th destination of v-th objectives, where v = {1,2}

y_{ij} = The transported amount of product from i-th source to j-th destination

Definition 1: A Set $Y^0 = \{y_{ij}^0$, i = 1, 2, ..., m; j = 1, 2, ..., n} is called a feasible solution to problem (T) if Y^0 satisfies the Eqs. (1) to (3).

Definition 2: A feasible solution Y^0 is called an efficient solution to the problem (T) is there does not exist any feasible solution Y of BOTP such that $Z_1(Y) \leq Z_1(Y^0)$ and $Z_2(\text{Y}) < Z_2(Y^0)$ or $Z_2(\text{Y}) \leq Z_2(Y^0)$ and $Z_1(\text{Y}) \leq Z_1(Y^0)$.

Step II: first, the crisp BOTP (S) is represented in Tabular form given in Table 1.

Table 1. Representation of crisp BOTP in tabular form.

Destination→ Source↓	A_1	A_2	......	A_n	Availability (s_i)
B_1	$a_{11}^{(1)}$ $a_{11}^{(2)}$	$a_{12}^{(1)}$ $a_{12}^{(2)}$	......	$a_{1n}^{(1)}$ $a_{1n}^{(2)}$	s_1
B_2	$a_{21}^{(1)}$ $a_{21}^{(2)}$	$a_{22}^{(1)}$ $a_{22}^{(2)}$	......	$a_{2n}^{(1)}$ $a_{2n}^{(2)}$	s_2
: :	: :	: :	......	: :	: :
B_m	$a_{m1}^{(1)}$ $a_{m2}^{(2)}$	$a_{m2}^{(1)}$ $a_{m2}^{(2)}$	......	$a_{mn}^{(1)}$ $a_{mn}^{(2)}$	s_m
Requirement (d_j)	d_1	d_2	......	d_n	

Step III: Now the minimum cost of rows/columns is given as:

Minimum cost of rows (Υ) as $\Upsilon_i^{(v)} = \min(a_{ij}^{(v)})$, for fixed i, $1 \leq j \leq n$ and $1 \leq v \leq 2$

Minimum cost of columns (δ) as $\delta_i^{(v)} = \min(a_{ij}^{(v)})$, for fixed j, $1 \leq i \leq m$ and $1 \leq v \leq 2$

Where, $\Upsilon = \{\Upsilon_1^{(1)}, \Upsilon_1^{(2)}, \Upsilon_2^{(1)}, \Upsilon_2^{(2)}, \ldots, \Upsilon_m^{(1)}, \Upsilon_m^{(2)}\}$ and $\delta = \{\delta_1^{(1)}, \delta_1^{(2)}, \delta_2^{(1)}, \delta_2^{(2)}, \ldots, \delta_n^{(1)}, \delta_n^{(2)}\}$.

The representation of Υ and δ in crisp BOTP (S) in Table 2.

Table 2. The representation of Υ and δ in crisp BOTP (S).

Destination→ Source↓	A_1	A_2	......	A_n	Availability (s_i)	Υ
B_1	$a_{11}^{(1)}$ $a_{11}^{(2)}$	$a_{12}^{(1)}$ $a_{12}^{(2)}$	......	$a_{1n}^{(1)}$ $a_{1n}^{(2)}$	s_1	$\Upsilon_1^{(1)}$ $\Upsilon_1^{(2)}$
B_2	$a_{21}^{(1)}$ $a_{21}^{(2)}$	$a_{22}^{(1)}$ $a_{22}^{(2)}$	......	$a_{2n}^{(1)}$ $a_{2n}^{(2)}$	s_2	$\Upsilon_2^{(1)}$ $\Upsilon_2^{(2)}$
: :	: :	: :	......	: :	: :	
B_m	$a_{m1}^{(1)}$ $a_{m2}^{(2)}$	$a_{m2}^{(1)}$ $a_{m2}^{(2)}$	......	$a_{mn}^{(1)}$ $a_{mn}^{(2)}$	s_m	$\Upsilon_m^{(1)}$ $\Upsilon_m^{(2)}$
Requirement (d_j)	d_1	d_2	......	d_n		
δ	$\delta_1^{(1)}$ $\delta_1^{(2)}$	$\delta_2^{(1)}$ $\delta_2^{(2)}$	......	$\delta_n^{(1)}$ $\delta_n^{(2)}$		

Step IV: Select

$$\text{P} = \min_{1\leq i\leq m, 1\leq j\leq n}\left(\Upsilon_i^{(v)}, \ \delta_i^{(v)}\right), \text{ for v} = \{1, 2\}$$

Step V: Select the cell (A) with the "P" as one of its objectives. Choose the Cell(A) with the maximum cost for another objective if there is more than one cell (A).

Step VI: Choose the cell that is selected in Step V that contains $\min(\sum_{j=1}^{n} a_{ij}^{(v)}$, for fixed i) in the corresponding row or column of the cell.

Step VII: Give the Maximum possible allocation to the selected cell in Step VI and ignore the row or column whose Availability/Requirement is satisfied.

Step VIII: Repeat the process for remaining sources and destinations until the whole availability or requirement has not been met. After applying all the steps, an efficient solution is obtained for the IVF BOTP and a crisp BOTP.

Pseudo Code:

1. Start
2. Initialize the fuzzy BOTP ($\tilde{S}$) in mathematical form.
3. Convert $\tilde{S}$ into Crisp BOTP (S) using Signed-distance function D ($\tilde{G}$, $\tilde{0}$).
4. Represent the Crisp BOTP (S) in tabular form
5. Calculate the minimum Cost of (Υ) as $\Upsilon_i^{(v)} = \min(a_{ij}^{(v)})$, for fixed, $1 \le j \le n$ and $1 \le v \le 2$ and columns $\delta_i^{(v)} = \min(a_{ij}^{(v)})$, for fixed j, $1 \le i \le m$ and $1 \le v \le 2$.
6. Calculate

$$P = \min_{1 \le i \le m, 1 \le j \le n} \left(\Upsilon_i^{(v)}, \ \delta_i^{(v)}\right), \text{ for v} = \{1, 2\}$$

7. Select the cell in tabular form of Crisp BOTP (S) that has P as one of its objective values. If more than one cell has P as one of its objective values, select the cell that has the maximum value for another objective.
8. Allocate the maximum possible allocation (requirements/availabilities) to the selected cell.
9. Ignore that row/column whose availability/requirement is fulfilled.
10. Repeat the process until all requirements/ availabilities are not satisfied i.e. all possible allocation (transported amount) y_{ij} is not obtained.
11. Stop.

The proposed method is illustrated with an example

Example: A (0.4,0.6) IVF BOTP With the following characteristics is considered:

Availability: $\tilde{s}_1 = [(6,8,10;0.4), (4,8,12;0.6)]$, $\tilde{s}_2 = [(10,19,24;0.4), (7,19,33;0.6)]$,

$$\tilde{s}_3 = [(10, 17, 20; 0.4), \ (6, 17, 30; 0.6)]$$

Requirements: $\tilde{d}_1 = [(9,10,17;0.4), (6,10,19;0.6)]$, $\tilde{d}_2 = [(1.5,3,4.5,0.4), (0.5,3,5.5;0.6)]$

$$\tilde{d}_3 = [(8, 14, 16; 0.4), \ (6, 14, 24; 0.6)],$$

$$\tilde{d}_4 = [(14, 16, 18; 0.4), \ (12, 16, 20; 0.6)]$$

Penalties:

Penalties for cost ($\tilde{a}_{ij}^{(1)}$: $1 \le i \le 3$, $1 \le j \le 4$)

$$\tilde{a}_{11}^{(1)} = [(0.5, 1, 1.5; 0.4), \ (0.3, 1, 1.7; 0.6],$$

$$\tilde{a}_{12}^{(1)} = [(1, 2, 3; 0.4), \ (0.5, 2, 3.5; 0.6)]$$

$$\tilde{a}_{13}^{(1)} = [(5, 7, 9; 0.4), \ (3, 7, 11; 0.6)],$$

$$\tilde{a}_{14}^{(1)} = [(4, 7, 10; 0.4), \ (3.5, 7, 10.5; 0.6)]$$

$$\tilde{a}_{21}^{(1)} = [(0.75, 1, 1.25; 0.4), \ (0.2, 1, 1.8; 0.6)],$$

$$\tilde{a}_{22}^{(1)} = [(8, 9, 10; 0.4), \ (6, 9, 12; 0.6)]$$

$$\tilde{a}_{23}^{(1)} = [(2, 3, 4; 0.4), \ (1.2, 3, 4.8; 0.6)],$$

$$\tilde{a}_{24}^{(1)} = [(2, 4, 6; 0.4), \ (1.75, 6, 6.25; 0.6)]$$

$$\tilde{a}_{31}^{(1)} = [(6.5, 8, 9.5; 0.4), \ (4.5, 8, 11.5; 0.6)],$$

$$\tilde{a}_{32}^{(1)} = [(7, 9, 11; 0.4), \ (6.5, 9, 11.5; 0.6)]$$

$$\tilde{a}_{33}^{(1)} = [(3, 4, 5; 0.4), \ (2, 4, 6; 0.6)],$$

$$\tilde{a}_{34}^{(1)} = [(2.5, 6, 9.5; 0.4), \ (1.5, 6, 10.5; 0.6)]$$

Penalties for time ($\tilde{a}_{ij}^{(2)}$: $1 \le i \le 3$, $1 \le j \le 4$)

$$\tilde{a}_{11}^{(2)} = [(2.25, 4, 5.75; 0.4), \ (1.5, 4, 6.5; 0.6)],$$

$$\tilde{a}_{12}^{(2)} = [(2.5, 4, 5.5; 0.4), \ (1, 4, 7; 0.6)]$$

$$\tilde{a}_{13}^{(2)} = [(1.75, 3, 4.25; 0.4), \ (1.5, 3, 4.5; 0.6)],$$

$$\tilde{a}_{14}^{(2)} = [(1.5, 4, 6.5; 0.4), \ (0.75, 4, 7.25; 0.6)$$

$$\tilde{a}_{21}^{(2)} = [(3, 5, 7; 0.4), \ (2, 5, 8; 0.6)],$$

$$\tilde{a}_{22}^{(2)} = [(5, 8, 11; 0.4), \ (3, 8, 13; 0.6)]$$

$$\tilde{a}_{23}^{(2)} = [(7.5, 9, 10.5; 0.4), \ (5.5, 9, 12.5; 0.6)],$$

$$\tilde{a}_{24}^{(2)} = [(9, 10, 11; 0.4), \ (8, 10, 12; 0.6)]$$

$$\tilde{a}_{31}^{(2)} = [4, 5, 10; 0.4), \ (3, 5, 13; 0.6)],$$

$$\tilde{a}_{32}^{(2)} = [(1, 2, 3; 0.4), \ (0.5, 2, 3.5; 0.6)]$$

$$\tilde{a}_{33}^{(2)} = [(4, 5, 6; 0.4), \ (3.6, 5, 6.4; 0.6)],$$

$$\tilde{a}_{34}^{(2)} = [(0.6, 1, 1.4; 0.4), \ (0.3, 1, 1.7; 0.6)]$$

Step I: first, the following fuzzy BOTP is converted into Crisp BOTP By using the signed distance function. Convert $\tilde{s}_1 = [(6,8,10;0.4), (4,8,12;0.6)]$ into crisp quantity as

$$s_1 = \frac{1}{16}\left[6 \times 8 + 6 + 10 + 4 \times 4 + 4 \times 12 + 3\,(2 \times 8 - 4 - 12)\,\frac{0.4}{0.6}\right]$$

$$= \frac{1}{16}\,[48 + 16 + 16 + 48 + 3 \times 0]$$

$$= 8$$

Similarly, all $\tilde{s}_i$, $\tilde{d}_j$, $\tilde{a}_{ij}^{(v)}$ for $1 \le i \le 3$, $1 \le j \le 4$ and v = {1, 2} is converted, then

Availabilities: $s_1 = 8$, $s_2 = 19$, $s_3 = 17$

Requirements: $d_1 = 11$, $d_2 = 3$, $d_3 = 14$, $d_4 = 16$

Penalties:

Penalties for cost

$a_{11}^{(1)} = 1, a_{12}^{(1)} = 2, a_{13}^{(1)} = 7, a_{14}^{(1)} = 7, a_{21}^{(1)} = 1, a_{22}^{(1)} = 9,$

$a_{23}^{(1)} = 3, a_{24}^{(1)} = 4, a_{31}^{(1)} = 8, a_{32}^{(1)} = 9, a_{33}^{(1)} = 4, a_{34}^{(1)} = 6.$

Penalties for time

$a_{11}^{(2)} = 4, a_{12}^{(2)} = 4, a_{13}^{(2)} = 3, a_{14}^{(2)} = 4, a_{21}^{(2)} = 5, a_{22}^{(2)} = 8,$

$a_{23}^{(2)} = 9, a_{24}^{(2)} = 10, a_{31}^{(2)} = 6, a_{32}^{(2)} = 2, a_{33}^{(2)} = 5, a_{34}^{(2)} = 1$

Step II: The crisp BOTP is represented in tabular form given in Table 3.

Table 3. Representation of crisp BOTP (Example) in tabular form.

Destination→ Source↓		A_1	A_2	A_3	A_4	Availability (s_i)
B_1	Cost	1	2	7	7	**8**
	Time	4	4	3	4	
B_2	Cost	1	9	3	4	**19**
	Time	5	8	9	10	
B_3	Cost	8	9	4	6	**17**
	Time	6	2	5	1	
Requirement (d_j)		**11**	**3**	**14**	**16**	

Step III: Now, calculate the minimum cost of row (Υ) as

$\Upsilon_1^{(1)} = \min\{1, 2, 7, 7\} = 1, \Upsilon_1^{(2)} = \min\{4, 4, 3, 3\} = 3,$

$\Upsilon_2^{(1)} = \min\{1, 9, 3, 4\} = 1, \Upsilon_2^{(2)} = \min\{5, 8, 9, 10\} = 5,$

$\Upsilon_3^{(1)} = \min\{8, 9, 4, 6\} = 4, \Upsilon_3^{(2)} = \min\{6, 2, 5, 1\} = 1.$

Calculate the minimum cost of column (δ) as

$\delta_1^{(1)} = \min\{1, 1, 8\} = 1, \delta_1^{(2)} = \min\{4, 5, 6\} = 4,$

$\delta_2^{(1)} = \min\{2, 9, 9\} = 2, \delta_2^{(2)} = \min\{4, 8, 2\} = 2,$

$\delta_3^{(1)} = \min\{7, 3, 4\} = 3, \delta_3^{(2)} = \min\{3, 9, 5\} = 3,$

$\delta_4^{(1)} = \min\{7, 4, 1\} = 1, \delta_4^{(2)} = \min\{4, 10, 1\} = 1.$

The representation of Υ and δ in crisp BOTP (S) is in Table 4.

Table 4. The representation of Υ and δ in crisp BOTP (S).

Destination→ Source↓		A_1	A_2	A_3	A_4	Availability (s_i)	Υ
B_1	Cost	1	2	7	7	**8**	1
	Time	4	4	3	4		3
B_2	Cost	1	9	3	4	**19**	1
	Time	5	8	9	10		5
B_3	Cost	8	9	4	6	**17**	4
	Time	6	2	5	1		1
Requirement (d_j)		**11**	**3**	**14**	**16**		
δ		1	2	3	4		
		4	2	3	1		

Step V: calculate

$$P = \min_{1 \le i \le 3, 1 \le j \le 4}\left(\Upsilon_i^{(v)}, \delta_j^{(v)}\right) \text{ for v } = \{1, 2\}$$

$$= \min\{1, 2, 3, 4, 5\}$$

$$P = 1$$

Step VI: Here $a_{11}^{(v)}$, $a_{21}^{(v)}$, $a_{34}^{(v)}$ have one of its objective value P (=1), but only one cell has to be chosen. So according to our method, the cell with the maximum cost for another objective is selected. From the selected cells, $a_{34}^{(v)}$ has the maximum cost for another objective, so the cell $a_{34}^{(v)}$ is selected.

Step VII: here, cell $a_{34}^{(v)}$ has the minimum cost $(1 + 6 = 7)$, so the cell $a_{34}^{(v)}$ is selected to give possible allocation.

Step VIII: In this step, the allocation min (16,17) = 16 is allocated to the cell $a_{34}^{(v)}$ and delete the fourth (A_4) column of Table 5, which requirement (d_4) is satisfied.

Table 5. Table after applying the Step V to Step VIII.

Destination→ Source↓		A_1	A_2	A_3	A_4	Availability (s_i)	Υ
B_1	Cost	1	2	7	7	**8**	1
	Time	4	4	3	4		3
B_2	Cost	1	9	3	4	**19**	1
	Time	5	8	9	10		5
B_3	Cost	8	9	4	(16) 6	**17**	4
	Time	6	2	5	1		1
Requirement (d_j)		**11**	**3**	**14**	**16**		
δ		1	2	3	4		
		4	2	3	1		

Step IX: Apply the same procedure from Step II to Step VIII, for making the possible allocation in the remaining rows and columns, 2^{nd}, 3^{rd}, 4^{th}, 5^{th}, and 6^{th} allocations as 11,3,8,5,1 at cells $a_{21}^{(v)}$, $a_{12}^{(v)}$, $a_{23}^{(v)}$, $a_{13}^{(v)}$, $a_{33}^{(v)}$ is obtained. Table 6 is generated, after applying all the steps.

Results and discussion

Result analysis: The obtained allocations and solution of the given problem are given in Table 7.

Comparison of Result: The same crisp BOTP are solved by some existing methods in the literature, so the results obtained by the proposed method are compared with the results obtained by the existing methods.

Comparison of our Proposed Method result with Some existing method Results is given in the Table 8.

The results are compared for the above example obtained using the proposed method and other existing methods (methods given by Bit et al.,[5] Yang & Gen,[6] P. Pandian and Anuradha,[7] and Abdul Quddoos et al.[8] Table 8 and Fig. 1 show the comparison of crisp results obtained for example. Due to the conflicting objectives, the cost is a little high. Still, the time is relatively optimized and the nature of the optimal compromised solution obtained by the proposed method is non-degenerate. In contrast, the nature of the optimal compromised solution obtained by other existing methods is degenerate. In sum, our proposed method provides a fuzzy and crisp compromise solution.

Table 6. The final solution table with optimal allocations.

Destination→ Source↓		A_1	A_2	A_3	A_4	Availability (s_i)	ϒ ϒ
B_1	Cost	1	**(3)** 2	**(5)** 7	7	8	1
	Time	4	4	3	4		3
B_2	Cost	**(11)** 1	9	**(8)** 3	4	**19**	1
	Time	5	8	9	10		5
B_3	Cost	8	9	**(1)** 4	**(16)** 6	**17**	4
	Time	6	2	5	1		1
Requirement (d_j)		**11**	**3**	**14**	**16**		
δ		1	2	3	4		
		4	2	3	1		

Table 7. Result analysis table.

Obtained allocations	$y_{11} = 0, y_{12} = 3, y_{13} = 5, y_{14} = 0, y_{21} = 11,$ $y_{22} = 0, y_{23} = 8, y_{24} = 0, y_{31} = 0, y_{32} = 0,$ $y_{33} = 1, y_{34} = 16,$
Obtained Cost for IVF BOTP ($\tilde{Z}_1, \tilde{Z}_2$) [Cost, Time]	[{(95.25,176,256.75;0.4), (54.3,176,297.7;0.6)}, {(122.85,175,227.15;0.4), (84.9,175,265.1;0.6)}]
Obtained cost for converted crisp BOTP (Z_1, Z_2) (Cost, Time)	(176,175)
Ideal cost for BOTP (Z_1, Z_2)	(143,167)

Table 8. Result comparison table.

Method used	Compromise Optimal Solution (cost, time)	Distance from Ideal Solution (cost, time) i.e. (143,167)	Nature of the Solution
Bit et al.[5]	(160,195)	32.75	Degenerate
Yang and Gen[6]	(168,185)	30.80	Degenerate
P. Pandian and Anuradha[7]	(168,185)	30.80	Degenerate
Abdul Quddoos et al.[8]	(176,175)	33.95	Non-Degenerate
Our Proposed Method	(176,175)	33.95	Non-Degenerate

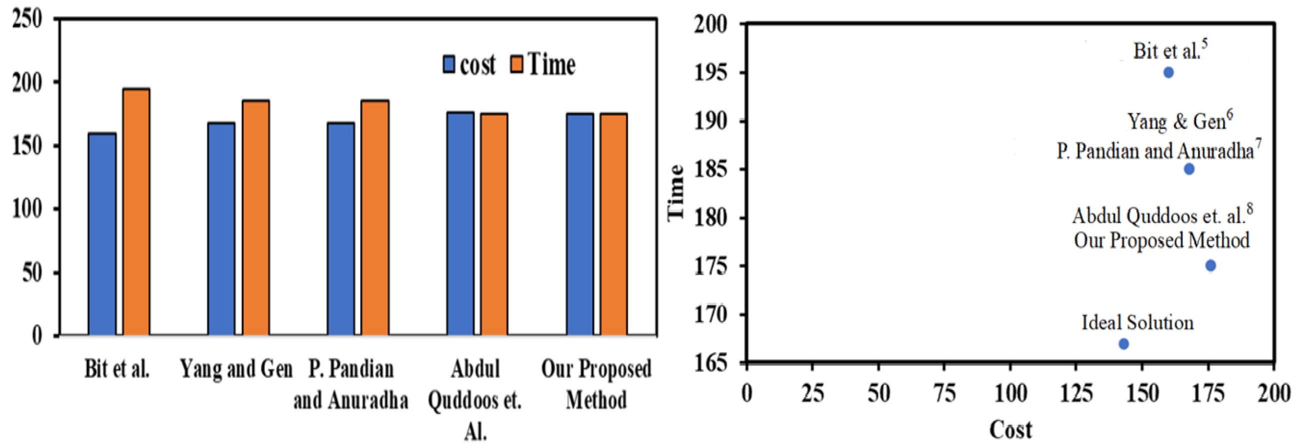

Fig. 1. Comparison of crisp results for example.

Conclusion

In this paper, the Bi-Objective Transportation Problem (BOTP) with (η, ξ) interval-valued fuzzy (IVF) number is studied and the IVF BOTP is converted into crisp BOTP by using signed distance ranking and then applied the proposed method that gives a unique efficient solution that leads to obtaining an efficient compromise solution of IVF BOTP as well as crisp BOTP. The obtained efficient compromise solution has the minimum distance from the Ideal solution. By the Proposed Method, an efficient compromise solution of a given IVF BOTP is obtained directly, and it takes less time and is easy to apply. The proposed method provides an efficient compromise solution of a given IVF BOTP directly that takes less time and is easy to apply. The proposed work provides a framework that can be implemented to address various types of other combinatorial optimization problems such as traveling salesman problems by leveraging interval-valued fuzzy numbers and signed-distance ranking functions.

Authors' declaration

- Conflicts of Interest: None.
- We hereby confirm that all the Figures and Tables in the manuscript are ours. Furthermore, any Figures and images, that are not ours, have been included with the necessary permission for re-publication, which is attached to the manuscript.
- No animal studies are present in the manuscript.
- No human studies are present in the manuscript.
- Ethical Clearance: The project was approved by the local ethical committee at the SRM Institute of Science of Technology Delhi-NCR Campus Modinagar, Ghaziabad.

Authors' contribution statement

R. Sh. and S. L. T. contributed to the design and implementation of the research, the analysis of the results, and the writing of the manuscript.

References

1. Hitchcock FL. The distribution of a product from several sources to numerous localities J Math Phys. 1941 Apr;20(1–4):224–30. https://doi.org/10.1002/sapm1941201224.
2. Zadeh LA. Fuzzy sets Inf. Control. 1965 Jun 1;8(3):338–53. https://doi.org/10.1016/S0019-9958(65)90241-X.
3. Aneja YP, Nair KP. Bicriteria transportation problem. Manage Sci. 1979 Jan;25(1):73–8. https://doi.org/10.1287/mnsc.25.1.73.
4. Ringuest JL, Rinks DB. Interactive solutions for the linear multiobjective transportation problem. Eur J Oper Res. 1987 Oct 1;32(1):96–106. https://doi.org/10.1016/0377-2217(87)90274-8.
5. Bit AK, Biswal MP, Alam S. Fuzzy programming approach to multicriteria decision making transportation problem. Fuzzy Sets Syst. 1992 Sep 10;50(2):135–41. https://doi.org/10.1016/0165-0114(92)90212-M.
6. Yang X, Gen M. Evolution program for bicriteria transportation problem. Comput Ind Eng. 1994 Sep 1;27(1–4):481–4. https://doi.org/10.1016/0360-8352(94)90339-5.
7. Pandian P, Anuradha D. A new method for solving bi-objective transportation problems. Aust J Basic Appl. Sci. 2011;5(10):67–74.
8. Quddoos A, Javaid S, Khalid MM. A new method to solve bi-objective transportation problem Int J Appl Math. 2013 Dec 21;26(5):555–63. http://dx.doi.org/10.12732/ijam.v26i5.4.
9. Mahdiraji HA, Beheshti M, Hajiagha SH, Zavadskas EK. A fuzzy binary bi objective transportation model: Iranian steel supply network. Transport. 2018 Oct 2;33(3):810–20. https://doi.org/10.3846/transport.2018.5800.

10. Gupta S, Ali I, Ahmed A. Multi-objective capacitated transportation problem with mixed constraint: a case study of certain and uncertain environment. Opsearch. 2018 Jun;55:447–77. https://doi.org/10.1007/s12597-018-0330-4.
11. Kumar R, Edalatpanah SA, Jha S, Singh R. A Pythagorean fuzzy approach to the transportation problem. Complex Intell Syst 2019 Jun;5(2):255–63. https://doi.org/10.1007/s40747-019-0108-1.
12. Khalifa AE. A signed distance method for solving multi-objective transportation problems in fuzzy environment. Int J Res Ind Eng. 2019 Sep 1;8(3):274–82. http://dx.doi.org/10.22105/riej.2019.193041.1091.
13. Amini A, Tavakkoli-Moghaddam R, Ebrahimnejad S. A bi-objective transportation-location arc routing problem. Transp Lett. 2020 Oct 20;12(9):623–37. https://doi.org/10.1080/19427867.2019.1679405.
14. Goyal BH, Gupta DE, Rani DE, Rani RO. Special structures in flowshop scheduling with separated set-up times and concept of job block: Minimization of waiting time of jobs. Adv Math Sci J. 2020;9:4607–19. https://doi.org/10.37418/amsj.9.7.29.
15. Gupta S, Ali I, Ahmed A. An extended multi-objective capacitated transportation problem with mixed constraints in fuzzy environment. Int J Oper Res. 2020;37(3):345–76. https://doi.org/10.1504/IJOR.2020.105443.
16. Mitlif RJ, Hussein IH. Ranking Function to Solve a Fuzzy Multiple Objective Function. Baghdad Sci J. 2021 Mar 10;18(1):0144-. http://dx.doi.org/10.21123/bsj.2021.18.1.0144.
17. Sharma S, Arora S. Bi-Objective Capacitated Transportation Problem with Bounds over Distributions and Requirement Capacities Int J Appl Comput Math. 2021 Jun;7(3):115. https://doi.org/10.1007/s40819-021-00989-5.
18. Goyal B, Kaur S. Specially structured flow shop scheduling models with processing times as trapezoidal fuzzy numbers to optimize waiting time of jobs. In Soft Computing for Problem Solving: Proceedings of SocProS 2020, 2 (2021):27–42. Springer Singapore. https://doi.org/10.1007/978-981-16-2712-5_3.
19. Sharma MK, Kamini, Dhiman N, Mishra VN, Rosales HG, Dhaka A, *et al.* A fuzzy optimization technique for multi-objective aspirational level fractional transportation problem. Symmetry. 2021 Aug 10;13(8):1465. https://doi.org/10.3390/sym13081465.
20. Mitlif RJ. An Efficient Algorithm for Fuzzy Linear Fractional Programming Problems via Ranking Function. Baghdad Sci J. 2022 Feb 1;19(1):0071-. http://dx.doi.org/10.21123/bsj.2022.19.1.0071.
21. Shahrabi F, Tavakkoli-Moghaddam R, Triki C, Pahlevani M, Rahimi Y. Modelling and solving the bi-objective production–transportation problem with time windows and social sustainability. IMA J Manag Math. 2022 Oct;33(4):637–62. https://doi.org/10.1093/imaman/dpab008.
22. Goyal B, Kaur S. Flow shop scheduling-especially structured models under fuzzy environment with optimal waiting time of jobs. Int J Des Eng. 2022;11(1):47–60. https://doi.org/10.1504/IJDE.2022.127075.
23. Mor A, Archetti C, Jabali O, Simonetto A, Speranza MG. The bi-objective long-haul transportation problem on a road network. Omega. 2022 Jan 1;106:102522. https://doi.org/10.1016/j.omega.2021.102522.
24. Li Z, Chen H, Liu Y, Jin K. A Lagrangian Relaxation Heuristic for a Bi-Objective Multimodal Transportation Planning Problem. IEEE Trans Intell Transp. Syst. 2022 Nov 24;24(1):382–99. https://doi.org/10.1109/TITS.2022.3216273.
25. Gupta S, Vijaygargy L, Sarkar B. A bi-objective integrated transportation and inventory management under a supply chain network considering multiple distribution networks. RAIRO-Oper Res. 2022 Nov 1;56(6):3991–4022. https://doi.org/10.1051/ro/2022164.
26. Gupta S, Chaudhary S, Chatterjee P, Yazdani M. An efficient stochastic programming approach for solving integrated multi-objective transportation and inventory management problem using goodness of fit. Kybernetes. 2022 Feb 7;51(2):768–803. https://doi.org/10.1108/K-08-2020-0495.
27. Yağmur E, Kesen SE. Bi-objective coordinated production and transportation scheduling problem with sustainability: formulation and solution approaches. Int J Prod Res. 2023 Feb 1;61(3):774–95. https://doi.org/10.1080/00207543.2021.2017054.
28. Prathyusha G, Udaya Kumara KN, Vatsala GA. Optimizing bi-objective solid transportation problem using hierarchical order goal programming technique: a case study problem. Soft Comput. 2023 Oct 10:1–9. https://doi.org/10.1007/s00500-023-09253-y.
29. Tripathy SP. Bi-objective covering salesman problem with uncertainty. J Decis Anal Intell Comput. 2023 Aug 15;3(1):122–38. https://doi.org/10.31181/jdaic10015082023t.
30. Varshney N, Gupta S, Ahmed A. Addressing uncertainty in closed-loop supply chain networks: a multi-objective approach to integrated production and transportation problems. J Model Manag. 2024;19(6):1849–1882. https://doi.org/10.1108/JM2-01-2024-0011.
31. Sharma R, Tyagi SL. A New Algorithm to Solve Multi-Objective Transportation Problem with Generalized Trapezoidal Fuzzy Numbers. Reliability Theory Appl. 2024;19(1 (77)):531–43. https://doi.org/10.24412/1932-2321-2024-177-531-543.

طريقة جديدة فعالة لحل مشكلة النقل ثنائي الهدف بموجب معلمات غامضة

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المستخلص

مشكلة النقل (TP) هي مشكلة تحسين كلاسيكية في أبحاث العمليات واللوجستيات. نظرًا لعدة عوامل، قد يكون لحالات الحياة الواقعية عدم اتساق في تكاليف العرض والطلب والنقل بالوحدة. تمثل الأرقام الغامضة هذه البيانات غير الدقيقة. في السيناريو الحالي، يتعامل صانع القرار مع العديد من الأهداف في وقت واحد. تقدم هذه الورقة نهجًا بسيطًا للحصول على حل وسط فعال لمشكلة النقل الخطية ثنائية الهدف (BOTP) مع معايير غامضة. (ξ، η) يتم استخدام الأرقام الغامضة ذات القيمة الفاصلة (IVFN) بدلاً من الأرقام الغامضة العادية كمعايير غامضة. يمكن أن يؤدي استخدام الأرقام الغامضة ذات القيمة الفاصلة إلى تعزيز بيانات النمذجة غير الدقيقة، مما يؤدي إلى تمثيل أكثر واقعية لعدم اليقين. باستخدام تصنيف المسافة الموقع، تم تحويل مشكلة النقل Fuzzy Bi-Objective إلى مشكلة نقل ثنائية الهدف المكافئة. في هذه الورقة، يتم تطوير طريقة على أساس تخصيص الحد الأدنى لتكلفة الهدف، المقابل لخلايا الصف والعمود مع الحد الأدنى من القيمة الموضوعية. يتم الحصول على حل فريد وفعال بشكل مباشر، مما يؤدي إلى حل وسط أمثل بالطريقة المقترحة التي يفضلها صانع القرار. تهدف الطريقة المقترحة إلى تخصيص المخصصات بطريقة تقلل من القيمة الموضوعية الإجمالية. يتم توفير الحل المثالي (ξ، η) الغامض والفعال بالإضافة إلى الحل الفعال الهش لمشكلة النقل ثنائي الهدف الغامضة من خلال هذه الطريقة التي لها مسافة دنيا من الحل المثالي. هذه الطريقة المقترحة أقل استهلاكًا للوقت وسهلة الاستخدام. يُستخدم مثال رقمي لتوضيح طريقتنا المقترحة ومقارنة النتائج ببعض الأساليب الموجودة الأخرى. وتوفر الطريقة المقترحة الحل التوفيقي الفعال غير المنحط للمثال الذي يبعد مسافة دنيا (33.95) عن الحل المثالي.

الكلمات المفتاحية: مشكلة النقل ثنائي الهدف (BOTP)، الحل الوسط الأمثل، الحل الفعال، مشكلة النقل الغامضة، الأرقام الغامضة ذات القيمة الفاصلة.