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Modeling Volatility in Electrical Load Patterns in Sulaimani Governorate Using the Hyperbolic GARCH (HYGARCH) Model

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Abstract: Electricity load patterns are inherently volatile, driven by consumer demand patterns, seasonal patterns, and consumption behavior. Modeling and forecasting volatility in these patterns is critical to landscape planning and electricity system stability in areas such as the Sulaimani Governorate. The study uses the HYGARCH model on Sulaimani's electricity demand to better capture volatility patterns, improving forecast accuracy. It enhances energy planning and lays the groundwork for applying advanced GARCH models beyond finance and offers a new approach to support sustainability in regions with similar demand patterns. For this reason, the objective of the study is to model and assess the volatility of electricity load data using a hyperbolic GARCH (HYGARCH) model, a suitable model that provides reliable modeling of long memory and volatility clustering in time series data.

The hyperbolic GARCH (HYGARCH) model is an advanced variant within the GARCH family, tailored to more accurately depict volatility clustering and long-term dependencies in time series data. Its versatility makes it particularly apt for modeling asset return dynamics, as it incorporates both conditional heteroscedasticity and the empirically observed stylized facts. The HYGARCH model uses hourly load data to measure volatility patterns and provide information to inform decision-making for future resource allocation when managing energy. This study adds to the literature by applying the HYGARCH model to the energy sector, demonstrating the model is valuable and relevant when outside conventional financial applications. The results indicated that the model's coefficients indicate a significant long memory in volatility (d = 0.885422112), suggesting that shocks might have enduring impacts. The 24-hour-ahead estimates provide stability after an initial increase, assisting energy planners in managing demand unpredictability and enhancing resource allocation.

Keywords: Hyperbolic GARCH model, Volatility, Long memory, High frequency data.

نمذجة التقلبات في أنماط الحمل الكهربائي في محافظة السليمانية باستخدام نموذج HYGARCH الهايبربوليكي

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المستخلص: تتسم أنماط الحمل الكهربائي بطبيعتها المتقلبة، مدفوعة بأنماط طلب المستهلكين والأنماط الموسمية وسلوك الاستهلاك. إن نمذجة وتوقع التقلبات في هذه الأنماط أمر بالغ الأهمية لتخطيط المشهد واستقرار النظام الكهربائي في مناطق مثل محافظة السليمانية. تستخدم الدراسة نموذج HYGARCH على طلب الكهرباء في السليمانية لالتقاط أنماط التقلب بشكل أفضل، مما يحسن دقة التنبؤ. إنها تعزز تخطيط الطاقة وتضع الأسس لتطبيق نماذج GARCH المتقدمة خارج المالية وتقدم نهجاً جديداً لدعم الاستدامة في المناطق ذات أنماط الطلب المماثلة.
لهذا السبب، فإن هدف الدراسة هو نمذجة وتقييم تقلبات بيانات الحمل الكهربائي باستخدام نموذج GARCH الهايبربوليكي(HYGARCH) ، وهو نموذج مناسب يوفر نمذجة موثوقة للذاكرة الطويلة وتجميع التقلبات في بيانات السلاسل الزمنية.
نموذج GARCH الهايبربوليكي (HYGARCH) هو متغير متقدم ضمن عائلةGARCH ، مُصمم لتصوير تجميع التقلبات والتبعيات طويلة المدى في بيانات السلاسل الزمنية بدقة أكبر. تجعله مرونته مناسباً بشكل خاص لنمذجة ديناميكيات عوائد الأصول، حيث يدمج كلاً من عدم التجانس الشرطي والحقائق المُصممة المُلاحظة تجريبياً.
يستخدم نموذج HYGARCH بيانات الحمل بالساعة لقياس أنماط التقلب وتوفير معلومات لإرشاد صنع القرار لتخصيص الموارد المستقبلية عند إدارة الطاقة. تضيف هذه الدراسة إلى الأدبيات من خلال تطبيق نموذج HYGARCHعلى قطاع الطاقة، مما يُظهر أن النموذج قيم وذو صلة عند الخروج عن التطبيقات المالية التقليدية.
أشارت النتائج إلى أن معاملات النموذج تشير إلى ذاكرة طويلة مهمة في التقلب ($d = 0.885422112$)، مما يشير إلى أن الصدمات قد يكون لها تأثيرات دائمة. تقديرات الـ ٢٤ ساعة القادمة توفر استقراراً بعد زيادة أولية، مما يساعد مخططي الطاقة في إدارة عدم القابلية للتنبؤ بالطلب وتعزيز تخصيص الموارد.

الكلمات المفتاحية: نموذج GARCH الهايبربوليكي، التقلب، الذاكرة الطويلة، البيانات عالية التكرار.

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Introduction

Statistical modeling is beneficial for interpreting complex, temporal data, but especially for understanding and predicting how systems evolve over time. Time series analysis has emerged as robust tools forecast temporal patterns providing researchers and decision-makers an avenue for judgment on future outcomes using historical (past) data. In recent years, volatility modeling in time series has garnered considerable interest, as comprehending unpredictability is crucial across many applications.

The GARCH (Generalized Autoregressive Conditional Heteroskedasticity) family and its expansions, including HYGARCH (Hyperbolic-GARCH), are among the most prevalent models, adeptly capturing both heteroskedasticity and long memory behavior. This framework has demonstrated substantial effectiveness for high-frequency data applications (Davidson, 2004). The present study is based on hourly power load data. This data reflects energy use via includes energy usage data, providing multiple data points of information about demand, periods of peak usage and fluctuations in energy use, and associated demand. Statistical models help us with interpreting complex time series data and they are valuable in identifying and predicting changes in a temporal system. Time series have become the best available means of exploring time-related patterns to make forecasts, now allowing academics and decision-makers to understand future conditions based on prior analyses.

Many researchers have worked with the HYGARCH framework. Conrad (2010) focused on the conditions required to maintain the conditional variance (that is, to satisfy the non-negativity conditions) for HYGARCH, and emphasized these conditions are critical for generating reliable multi-step-ahead forecasts (Conrad, 2010). Mohammadi and Rezakhah (2017) presented the Smooth Transition HYGARCH (ST-HYGARCH) model that uses the logistic function to model time-varying volatility regimes, and showed it improved forecasting accuracy dramatically for financial indexes like S&P 500 (Mohammadi and Rezakhah, 2017). Shi and Yang (2018) proposed the Adaptive Hyperbolic EGARCH (A-HYEGARCH) model that can appropriately model both long memory and structural shifts in high-frequency financial data (Shi and Yang, 2018). Li et al.

(2012) also evaluated the practical limits of HYGARCH using score tests and simulations and showed that the practical limitations made HYGARCH difficult to implement in practice, even while being very theoretically promising (Li et al., 2011).

Despite advancements in time series modelling, the application of the Hyperbolic GARCH (HYGARCH) model in the energy sector, particularly for modelling electricity load data, remains limited. In regions like the Sulaimani Governorate in Iraq, electricity demand is highly variable due to seasonality, consumer behaviour, and infrastructure constraints. Traditional models often overlook the long memory and volatility clustering that these data exhibit, leading to suboptimal forecasting and inefficient energy planning. In contrast, the HYGARCH model, which is better suited for capturing these complexities, offers a promising alternative for modelling long-memory systems.

However, its application to electrical load data in the energy domain remains underexplored. To address this gap, the current study poses the question: To what extent can the Hyperbolic GARCH (HYGARCH) model accurately capture and predict volatility, long memory, and clustering in electrical load patterns in the Sulaimani Governorate-Iraq, and how can it aid in energy planning and policy formulation? Therefore, this research aims to fill this gap by examining the efficacy of the HYGARCH model can accurately capture and predict volatility, long memory, and clustering in electricity load patterns in Sulaimani Governorate, Iraq, and it aims to demonstrate how the model can enhance energy planning and policy formulation by providing reliable forecasts.

Specifically, it focuses on analyzing hourly electrical load data to measure volatility, detect long memory characteristics, and generate accurate forecasts that can inform energy management strategies. In addition, the current study consists of four sections. The following section presents a theoretical overview of the HYGARCH model, while the third section discusses the data and results of forecasting volatility in electrical load data. The last part presents conclusions and further discussion on the findings.

1st: Theoretical Part and Model Specification

1- Hyperbolic GARCH (HYGARCH) Model

The Hyperbolic GARCH model is a suggested extension of the classic GARCH family suitable for describing volatility clustering and longtime characteristics (Conrad, 2010),(Baillie et al., 1996),(Shi and Yang, 2018). While the classic GARCH model assumes short memory of shocks to volatility (Bollerslev, 1986), the HYGARCH model incorporates long memory, much suited for financial data, where the volatility persists over a longer duration (Cont, 2007). The model thus represents a very good way of valuing returns of assets considering both conditional heteroscedasticity and the stylized aspects of volatility in financial settings and has also been extended into realized volatility frameworks (Sall et al., 2021), (P et al., 2006), this makes it particularly apt for modeling asset return dynamics, as it incorporates both conditional heteroscedasticity and the empirically observed stylized facts (Akgül and and Sayyan, 2008). HYGARCH Model is expressed as:

$$\boldsymbol{y\_t = \mu + \epsilon\_t}$$

$$\boldsymbol{\epsilon_t = \sigma_t z_t}$$

$$\sigma_t^2 = w + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i} + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 + \gamma \sigma_{t-1}^2 \left(\sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2 \right)^{\delta}$$

where y_t is the asset return at time t, μ is the mean return, ϵ_t is the innovation(residual), σ_t^2 is the conditional variance at time t, α_i, $\beta_j$are the GARCH parameters,δ represents the hyperbolic scaling factor and z_t is a white noise process ((Conrad, 2010), (Baillie et al., 1996)).

2- Stationary Process

Time series stationary is a core term in time series analysis referring to a process where the statistical nature-the mean, variance and autocovariance-are constant over time. Stationarity is required in a time series model for it to be valid (Manimaran et al., 2006). A constant conditional variance with respect to the time process refers to stationarity in volatility models such as GARCH and HYGARCH.

Types of Stationarity:

- Strict Stationarity: Moment in the distribution does not change when shifted with respect to time.
- Weak Stationarity: The first two moments (the mean and the variance) are constants, and the covariance between values depends only on their time difference, not the points themselves in time. Most volatility models assume weak stationarity (Tsay, 2005), which ensures that although the underlying time series itself varies, the conditional variance is constant through time.

A. Stationarity Conditions Specific to Hyperbolic GARCH

The Hyperbolic GARCH (HYGARCH) model extends the Fractionally Integrated GARCH (FIGARCH) model (Baillie et al., 1996)by permitting a convex combination of GARCH and FIGARCH conditional variances (Conrad, 2010), (Sall et al., 2021). The stationarity criteria of a HYGARCH model are established as follows:
Define the conditional variance equation of the HYGARCH model as follows:

$$\sigma_t^2 = w + (1-\beta L)^{-1}(1-\phi L)(1-L)^d \epsilon_t^2$$

When $0 \le d \le 1$ long-memory behavior is guaranteed, and stationarity conditions necessitate that $\sum \pi_j < \infty$where $\pi_j$are the weights in the infinite ARCH representation(Conrad, 2010), (Baillie et al., 1996).

B. Execution and Analysis of Augmented Dickey-Fuller (ADF) Tests

The ADF test is used to check the stationarity through null hypothesis of unit root (Manimaran et al., 2006).

$$\Delta y_{\ t} = \alpha + \beta_t + \gamma y_{t-1} + \sum_{1=1}^{p} \delta i \ \Delta y_{t-i} + \epsilon_t$$

Where: y_t isthe examined time series,$\Delta y_t = y_t - y_{t-1}$is the first difference of the time series, αisa constant term (also called the drift term),$\beta$is the coefficient of the deterministic trend t (if included),$t$isthe time index (used when testing for trend stationarity),γisthe coefficient ofy_{t-1} , which determines whether the series has a unit root,p is the number of lagged differences incorporated to adjust for serial correlation,$\delta_i$is the coefficients of the lagged first differences $\Delta y_{t-i}$and $\epsilon_t$is a white noise error term(Manimaran et al., 2006).

Phillips-Perron Adjustment

The PP test adjusts the test statistics from the ADF regression to include serial correlation and heteroskedasticity in ϵ_t. Instead of augmenting the model through lagged differences, the PP test modifies the test statistic using nonparametric estimates the variance in the long term. The adjusted t-statistic is expressed as follows (Manimaran et al., 2006), (Tsay, 2005):

$$z_T = t_\gamma - \frac{(s^2 - \hat{\sigma}^2)}{2\hat{\sigma}^2}\left(T\sum_{t=1}^{T} y_{t-1}^2\right)^{-1/2}$$

Wheret_γ is the t-statistic from the ADF test,s^2 is a consistent estimator of the long-run variance of ϵ_t,$\hat{\sigma}^2$is the variance of ϵ_t and T is the sample size(Manimaran et al., 2006).
The long-run variance is estimated utilizing the Newey-West estimator:

$$s^2 = \hat{\sigma}^2 + 2\sum_{j=1}^{m}\left(1 - \frac{j}{m+1}\right)\hat{\gamma}_j$$

where$m$ is the truncation lag and $\hat{\gamma}_j$ is the sample autocovariance at lag j(Tsay, 2005).

3- Estimation Methods

A. Maximum Likelihood Estimation

The estimation of parameters of the ARCH-GARCH model is more complex than that of the CER model. The parameters of conditional variance do not have a simple plug-in principal estimator. On the other hand, parameters of the ARCH-GARCH models are often estimated using by another technique called maximum likelihood (ML) (Engle, 1982), (Bollerslev and and Wooldridge, 1992), calibration techniques such as MLE have been applied in many engineering contexts (Singh et al., 2024).This section summarizes ML estimation and shows how to use it to estimate the parameters of the ARCH-GARCH model.

The Likelihood Function

This constructs a joint density function that serves as a T-dimensional representation based on observations $X1, \dots, XT$, conditioned on parameters contained in basic vector θ.

$$f(x1, \dots, xT; \theta) = f(x1; \theta) \cdots f(xT; \theta) = \prod_{t=1}^{T} f(x_t; \theta).$$

The joint density must satisfy specific conditions: it should remain non-negative i.e., $f(x1, \dots, xT; \theta) \geq 0$, and should integrate to one over all its variables:

$$\int \cdots \int f(x1, \dots, xT; \theta) dx1 \cdots dxT = 1.$$

The probability function itself emerges from interpreting this joint density with respect to the parameters encapsulated in θ:

$$L(\theta | x1, \dots, xT) = f(x1, \dots, xT; \theta) = \prod_{t=1}^{T} f(x_t; \theta)$$

observe that the likelihood function is also a k dimensional function of θ, which is conditioned on data$x_1, \cdots, x_T$.It should be noted that the likelihood function, in terms of θ not the data, is not a true probability density function.

$$\int \cdots \int L(\theta | x1, \dots, xT) d\theta 1 \cdots d\theta k \neq 1.$$

Specifically, $\int \cdots \int L(\theta | x1, \dots, xT) d\theta 1 \cdots d\theta k$ is positive, but it is not 1.

For convenience, assume that the vector $x = (x1, \dots, xT)'$is the observed sample, and denote the joint probability density function as $f(x; \theta)$ and the likelihood function as $L(\theta | x)$, (Engle, 1982, Bollerslev and and Wooldridge, 1992, Kim et al., 1998)

Maximum Likelihood Estimation (MLE) for HYGARCH

Maximum Likelihood Estimation (MLE) is a popular method in estimating parameters for time-series models, including HYGARCH. Written into its definition is the probability distribution of the residuals $\boldsymbol{z_t}$, commonly chosen among a normal or t-distribution (Kim et al., 1998). Hence, the purpose is to optimize the likelihood function concerning the model parameters $\theta = (w, \alpha_i, \beta_j, \gamma, \delta)$.

The general form of the log-likelihood for HYGARCH is:

$$\ln L(\theta) = \sum_{t=1}^{T} \ln \left[f(\epsilon_t | \sigma_t^2, \theta) \right]$$

where $f(\epsilon_t | \sigma_t^2, \theta)$indicates the conditional density of $\boldsymbol{\epsilon_t}$, and σ_t^2 the conditional variance prescribed by the HYGARCH model(Manimaran et al., 2006), (Bollerslev and and Wooldridge, 1992).

B. Quasi-Maximum Likelihood Estimation (QMLE)

QMLE finds great use in cases where estimation of the likelihood function proves difficult, or the errors are difficult to model with a simple normal distribution. A quasi-likelihood function is used to approximate the real likelihood function. The application of the quasi-likelihood maximum-likelihood estimation method becomes, therefore, central in those kinds of models with non-normal errors (Bollerslev and and Wooldridge, 1992), (Kwan et al., 2012), (Nelson, 1991)where the likelihood function has no analytical tractability.

Log-Likelihood Function

The likelihood function $L(\theta)$for the entire sample (from $t = 1$ to $t = T$)is the joint probability of all residuals:

$$L(\theta) = \prod_{t=1}^{T} f(\epsilon_i | h_t)$$

Substituting the normal PDF into this product, we get:

$$L(\theta) = \prod_{t=1}^{T} \frac{1}{\sqrt{2\pi h_t}} \exp\left(-\frac{\epsilon_t^2}{2 \cdot h_t}\right)$$

Applying the natural logarithm of the likelihood function to get the log-likelihood function:

$$logL(\theta) = \sum_{t=1}^{T} \left[- \log\left(\sqrt{2\pi h_t}\right) - \frac{\epsilon_t^2}{2h_t}\right]$$

Simplifying the logarithm terms:

$$logL(\theta) = \sum_{t=1}^{T} \left[-\frac{1}{2}\log(2\pi h_t) - \frac{\epsilon_t^2}{2h_t}\right]$$

This represents the log-likelihood function for a normal distribution with conditional variance h_t, which is contingent upon the parameters θ of the HYGARCH model.
The ultimate log-likelihood function for the HYGARCH model is articulated as:

$$L(\theta) = -\frac{1}{2} \sum_{t=1}^{T} \left[\log(2\pi h_t) + \frac{\epsilon_t^2}{h_t}\right]$$

The initial equation utilized for estimating the model parameters $\theta = (\omega, \alpha, \beta, d)$involves maximizing the log-likelihood function(Bollerslev and and Wooldridge, 1992),(Kwan et al., 2012), (Nelson, 1991).

4- Model Selection and Assessment

A. Goodness-of-Fit of Model

Goodness-of-fit (GOF) tests are statistical methods employed to evaluate the compatibility of a dataset with a designated probability distribution, with various tests exhibiting differing efficacy based on the type of divergence from the null hypothesis. The goodness-of-fit of a model quantifies its efficacy in representing the observed data. In the case of volatility models, such as HYGARCH, evaluating the goodness-of-fit guarantees (Meintanis et al., 2020)that the model represents important characteristics of financial time-series-like volatility clustering, persistence, and long memory. A variety of statistical tests and diagnostic methods exist for the evaluation of goodness-of-fit.

(1) Likelihood-Based Criteria (AIC, BIC)

The AIC and BIC are two perspectives often drawn on when measuring model fit quality. These criteria balance model fit with model complexity: the more complex the model, the more parameters it has, thus a higher penalty is given to avoid overfitting.

- AIC: AIC = -2 $ln(L)$+ $2k$
- BIC: BIC = -2 $ln(L)$+ $k\ ln(T)$

where ln(L) represents the log-likelihood, k denotes the total number of parameters examined, and T signifies the number of observations in the sample(Manimaran et al., 2006), (Tsay, 2005)

(2) Out-of-Sample Forecasting Performance

Out-of-sample forecasting refers to fundamental to model assessment. This is because, when assessing the model, data which has not been involved in its estimation or fitting is used. This tests to see if the model can generalize to never-before-seen data (Tashman, 2000).Standard metrics for evaluating out-of-sample forecasting ability include Mean Squared Error, Mean Absolute Error, and Theil U-statistic(Tashman, 2000). When creating out-of-sample tests for a single time series, the most important question to answer is how to separate the periods for fitting and testing. This separation determines how much data we can use to develop and fit a forecasting model, and how many forecasts will be available to complete the out-of-sample test of model performance. There are many issues to consider when determining the appropriate number of periods N to drop from the time series. The most important criterion is the length of the long-term forecast required. Let H be this maximum length forecast required. Then, N should be at least equal to H. It may also be worthwhile to extend the out-of-sample test period so that we may have M forecasts at lead time H. The out-of-sample duration will be defined as H + M - 1 predictions. Considering the minimum is M = 3, a rolling-origin assessment should be constructed with a test period of length H + 2 (Tashman, 2000). For example, if the desired long-term forecast is for five years ahead (H = 5), we would choose a test period of seven years so that the assessment of accuracy to forecast five years ahead can draw from at least three forecasts. To assess the distribution of forecast errors, a significantly higher number of forecasts is needed rather than simply the average error measures. Short time series limit the testing period, since truncating the data may not provide enough observations to fit the model properly. In this case, we can take full advantage of the rolling-origin approach for analyzing one-step ahead forecast errors while retaining enough information to fit the model, without letting the fit period become too short. We need to evaluate the model using out of sample predictions with accuracy metrics:

Root Mean Square Error (RMSE)

Calculates the average of the squared differences between predicted and actual values. A reduced RMSE signifies superior performance prediction accuracy (Tashman, 2000).

$$RMSE = \sqrt{\frac{1}{T}\Sigma(y_t - \hat{y}_t)^2}$$

Wherey_t is actual value, $\hat{y}_t$ is predicted value andT is number of observations

Mean Absolute Error (MAE)

Quantifies the mean absolute discrepancies between predicted and actual values (Tashman, 2000).

$$MAE = \frac{1}{T}\sum \mid y_t - \hat{y}_t \mid$$

- Less sensitive to extreme errors than RMSE.
- A lower MAE indicates better model performance(Tashman, 2000).

Diebold-Mariano (DM) Test

The Diebold-Mariano (DM) Test measures the forecasting accuracy of HYGARCH model forecasting compared with plausible similar models such as GARCH or FIGARCH (Tashman, 2000). DM Test Statistic:

$$DM = \frac{\bar{d}}{\sqrt{\frac{2\pi f_d(0)}{T}}}$$

Where

$\bar{d}$ is the mean difference between forecasting errors of two models, $f_d(0)$ is the spectral density at frequer
(Diebold and Mariano, 1995).
A notable DM test outcome indicates that one model substantially surpasses the other.

(3) Likelihood Ratio Test (LRT)

A Likelihood Ratio Test (LRT) is one of the methods used to evaluate competing models [12], one of which is a non-complex (more restricted) form of the other. Here, the likelihood ratio tests whether the additional complexity of a more flexible model is justified by a significantly better fit.

Hypothesis are H_0: The simple model is adequate and H_1: The more complex model has a significantly better fit. The likelihood ratio test (LRT) statistic is computed as:

$$LR = -2[\ln L(\theta_r) - \ln L(\theta u)]$$

Where $L(\theta_r)$ is the likelihood of the constrained model and $L(\theta u)$: is the likelihood of the unrestricted (full) model [12]. This statistic follows a chi-square (χ^2) distribution with its degrees of freedom equal to the different numbers of parameters between the models.
A significant test result indicates that the unrestricted model fits the data better. Other diagnostics, which express whether or not the residuals (errors) obtained from the model respect the expected behavior according to certain leads, are used. Some of the most crucial residual-based hypothesis tests regarding volatility specifications.

2nd: Practical Analysis, Discussion and Results

1- Data description

This study used a file from the General Directorate of power of Sulaimani, which compiled hourly power load data for Sulaimani from April 1, 2023 to December 31, 2023. This dataset contained hourly power load data for 275 consecutive days, a total of 6,600 observations (275 days times 24 hours). The load numbers are in amperes (A) and produce a measure of the instantaneous electric consumption in the Sulaimani Governorate. Thus, each observation signifies the electrical load at a certain hour of a day, with the data providing a complete observation for each hour of each day in the collection. This high-resolution, high-frequency time series reflects the daily, monthly, and seasonal nature of power usage - and is well suited for modelling and forecasting volatility. Since the data provided multiple seasons, it is ideal for modelling effects such as volatility clustering, long memory variables, and seasonal patterns in energy consumption. This complete picture forms a good foundation for the Hyperbolic GARCH (HYGARCH) modelling framework in advancing our understanding of load dynamics and effective energy management and planning.

A. Descriptive Statistics and Time Series Properties

In figureFigure *(1*, shows the raw time series of electric power load meanwhile the study period. There are very spasmodic fluctuations in electricity consumption as the load figures were reported to vary from roughly 20,000 up to 60,000 amperes. Electricity load shows variability in time of electricity demand as noted in the Sulaimani region's electricity load is based on patterns that include daily and seasonable trends, consumer practice, and supply chain demands.

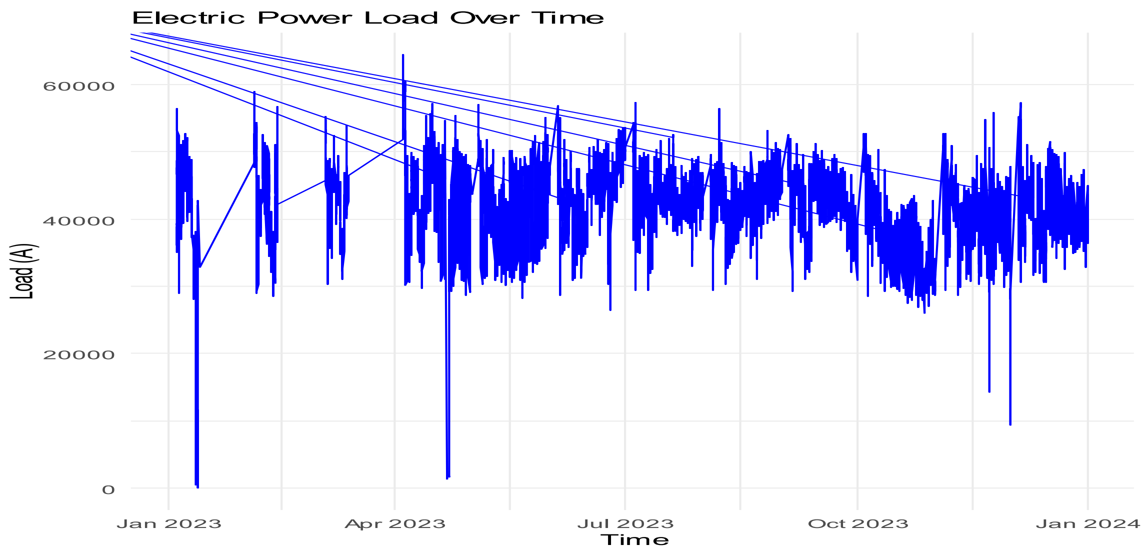

Figure (1): Electric Power Load Over Time, April 2023-January 2024

We initially assessed the stationarity characteristics of the series to prepare the data for volatility modelling. In accordance with typical procedures in time series analysis, we converted the raw load data into logarithmic returns:

$$r_t = ln(X_t) - ln(X_{t-1})$$

where X_t represents the load at time t, and r_t is the appropriate logarithmic return. This transformation seeks to stabilize variance and attain stationarity, a condition for implementing GARCH-type models.

In figure (2) illustrates the logarithmic returns of the power load data. Visual examination reveals that the returns seem stable, exhibiting volatility clustering across the series intervals of elevated volatility tend to succeed one another, as do intervals of diminished volatility. The clustering behavior is a fundamental attribute that GARCH-family models aim to represent.

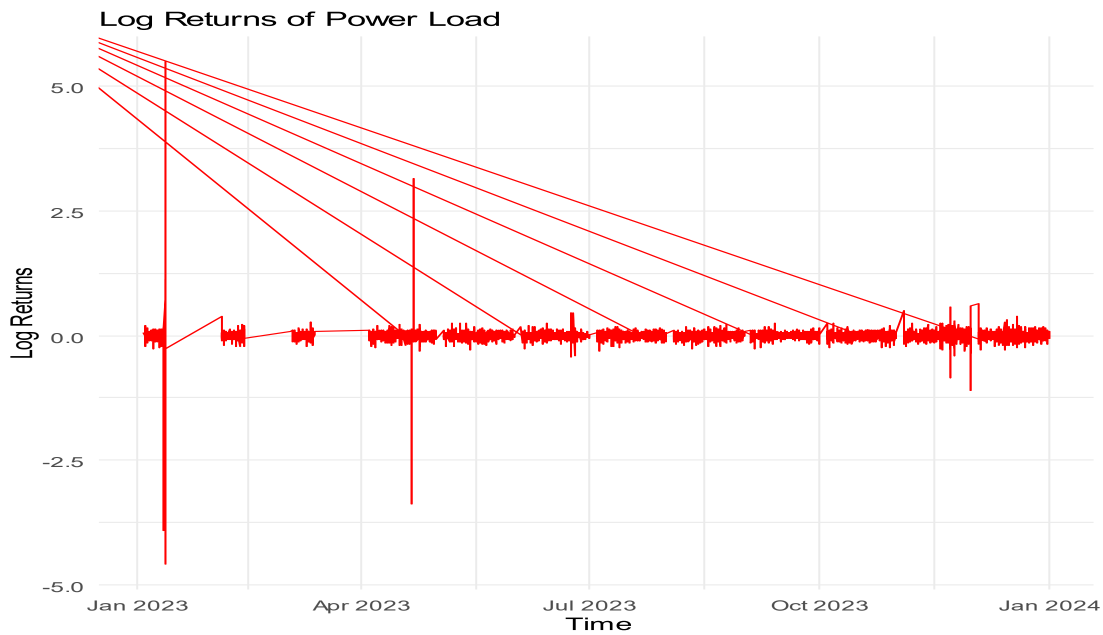

Figure (2): Log Returns of Power Load, April 2023-January 2024

B. Stationary Testing

We undertook the Augmented Dickey-Fuller (ADF) test to formally determine the stationary of our log returns series. The results shown in Table 1 clearly reject the unit-root null hypothesis (Dickey-

Fuller =-24.675, p-value = 0.01), thus concluding that the log returns series is stationary. The stationary of the log returns is important to allow for confident implementation of our volatility modelling choice.

Table (1): Augmented Dickey-Fuller Test Results

Test Statistic	Lag Order	p-value	Conclusion
-24.675	18	0.01	Stationary

2- Volatility Characteristics Assessment

A. Volatility Clustering

In order to determine if volatility clustering exists in the log returns series, we calculate the autocorrelation function (ACF) of the squared returns. Figure (3) depicts the ACF of squared returns from 0 lag up to 30 lags with significant autocorrelations present over a wide range of lags. This evidence of volatility clustering suggests that there is strong persistence in volatility; past volatility serves as an effective predictor of future volatility, which is not accounted for in naive time series models.

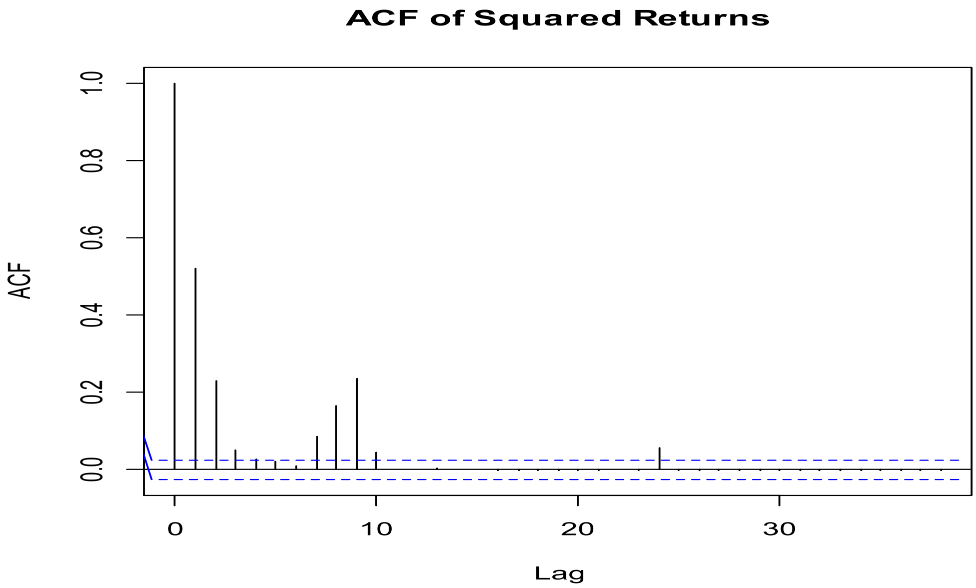

Figure (3): ACF of Squared Returns

B. ARCH Effects Testing

Prior to the estimate of the HYGARCH model, we performed Engle's ARCH Lagrange Multiplier (LM) test to explicitly ascertain the existence of ARCH effects in the dataset. In Table 2 The results (Chi-squared = 2240.2, df = 12, p-value < 2.2e-16) decisively reject the null hypothesis of the absence of ARCH effects, affirming that volatility in the electrical load data demonstrates time-varying conditional heteroskedasticity. This discovery corroborates our choice to utilize GARCH-family models for the analysis.

Table (1): ARCH LM Test Results

Chi-squared	Degrees of Freedom	p-value	Conclusion
2240.2	12	< 2.2e-16	ARCH effects present

C. Long Memory Assessment

There is a significant theoretical advantage for the HYGARCH model, which is that it can capture long memory features in volatility processes. In order to determine if our data has long range

dependence, we created a long-memory test. The value we calculated for the fractional differencing parameter d was 4.583013e-05 just over zero. This represents some long memory characteristics and makes us confident we can fit the HYGARCH model to our data.
Moreover, we also performed the Ljung-Box test on the squared residuals for residual serial autocorrelation. The stunning result (X-squared = 475.33, df = 20, p-value < 2.2e-16) indicates there is considerable year-autocorrelation in the squared residuals, thus supporting the need for a model that can simultaneously handle volatility clustering and long memory effects.

3- HYGARCH Model Estimation

A. Model Specification and Parameter Estimation

Following the prior work, we defined and estimated a HYGARCH model for the logarithmic returns of electrical load. The model was estimated via the maximum likelihood method, with optimization executed via a quasi-Newton algorithm. The parameter estimations are displayed in Table 3.

Table (2): HYGARCH Parameter Estimates

Parameter	Estimate	Interpretation
ω (omega)	0.001254639	Baseline volatility level
α (alpha)	0.227949752	Impact of short-term shocks
β (beta)	0.305158104	Persistence of volatility
d	0.885422112	Long memory parameter
λ (lambda)	0.859831125	Hyperbolic decay rate

Based on the HYGARCH coefficients we calculated, we can infer some meaningful characteristics of the volatility process of Sulaimani's electrical load data:

1. The low ω (0.001254639) value indicates that there is a relatively low level of unconditional volatility.
2. The value of α (0.227949752) reflects the impact of transitory shocks on volatility.
Its moderate magnitude suggests that recent shocks have a noticeable but not dominant effect on current volatility.
3. The value of β (0.305158104) represents the persistence of volatility. This mid-range
value implies that while volatility exhibits some degree of persistence, it is not excessively short-term.
4. The d parameter (0.885422112) is particularly significant, as it quantifies the degree of long memory in the volatility process. Its closeness to 1 signal strong long-memory behaviour, indicating that volatility shocks have a lasting influence on the electricity load data.
5. The λ parameter (0.859831125) controls the hyperbolic decay rate of the autocorrelation function. Since the value is below 1, it ensures the stationarity of the process while still allowing for substantial persistence.
The amalgamation of these factors suggests that the HYGARCH model proficiently encapsulates both the short-term dynamics and long-memory attributes of volatility in the electrical load data from Sulaimani.

B. Model Diagnostics

We also ran diagnostic tests of the standardized residuals to assess whether the fitted HYGARCH model was appropriate. The ACF for the standardized residuals from the fitted HYGARCH model can be found in Figure (4).

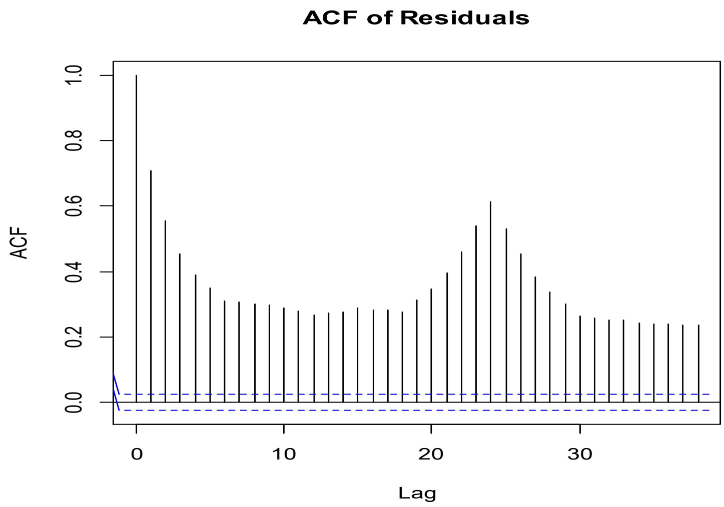

Figure (4): ACF of HYGARCH Standardized Residuals

The ACF shows that for most lags, the autocorrelations of the standardized residuals fall within the confidence bands. This means that the fitted HYGARCH model has captured serial dependency in the volatility process. Thus, we feel that it fits the data sufficiently well.

We ran Ljung-Box tests for both the residuals and the squared residuals. The test for the residuals produced Chi-squared statistic of 109.56 (df = 20, p = 2.365e-14), indicating that there is residual serial correlation. Additionally, the test of the squared residuals resulted in a chi-squared statistic of 0.37071 (df = 20, p = 1), meaning the HYGARCH specification includes the volatility structure. The different results for the residuals and squared residuals tests imply that while the model is a reasonably good characterization of the volatility structure it is possible there is some unobserved pattern that is impacting the mean process.

4- Volatility Forecasting

A. Forecast Generation

The central goal for the research reported here was to forecast variation in electrical load journeys to aid in energy planning and policymaking. Using the estimated HYGARCH model, we provided 24-hour ahead estimates for the conditional standard deviation or the forecasted volatility of electrical load next day.

Table 4 presents the hour-ahead forecasts of standard deviations and depict expected changes in volatility during the 24-hour forecast horizon.

Table (3): Hour-Ahead Forecasted Standard Deviations

Hour	Forecasted SD
1	0.08249913
2	0.08379172
3	0.08412759
4	0.08421549
5	0.08423854
6	0.08424459
…	…
24	0.08424674

In Figure (5) depicts conditional standard deviation forecasts during the 24-hour horizon. You can see that volatility is predicted to slightly increase after the first hour and stabilize, meaning that volatility is anticipated to be somewhat stable over the forecast period.

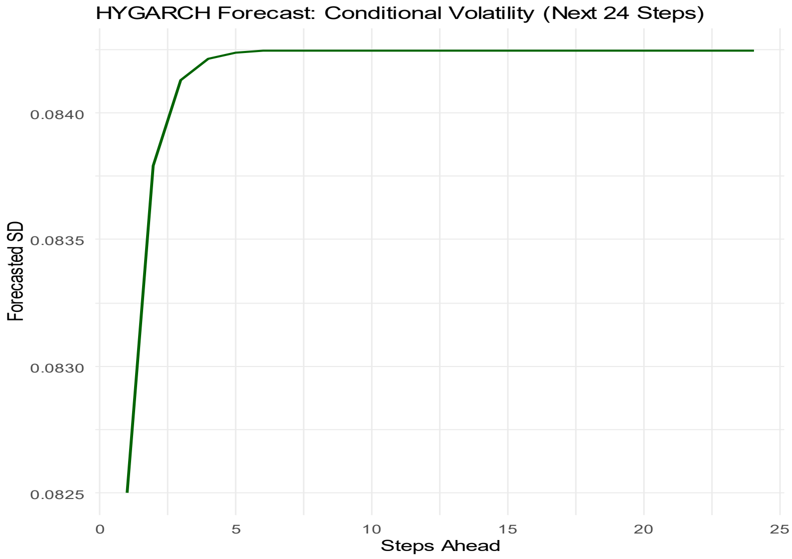

Figure (5): HYGARCH Forecast: Conditional Volatility (Next 24 Steps)

B. Forecast Evaluation

To assess the precision of the HYGARCH volatility projections, we conducted post-sample validation by juxtaposing the predicted volatilities with the actual volatilities during a holdout period. The residual analysis demonstrated that the model yields rather precise predictions. The summary statistics of the residuals (Minimum: -5.17882, Maximum: 4.68697, Mean: 0.07352, Standard Deviation: 0.9973465) indicate that the forecast errors are roughly centered around zero, with a standard deviation near 1, implying that the model is well-calibrated.

5-Implications for Energy Planning

The findings from the HYGARCH model provide several implications for energy planning in the Sulaimani Governorate.

A. Long memory: The long memory result shows significant long-run effects of electric demand using a very high-order long memory value (d = 0.885). Therefore, planning must be long-term.
B. Volatility is Predictable: Load or demand volatility has some predictability to it so the results can help with resource storage and (dis-)patching to manage the grid.
C. Enhanced Forecasting Accuracy: The HYGARCH model identified and accounted for long memory and hyperbolic decay in volatility, which can help improve the accuracy of energy demand forecasts.
D. Consistent Long-Term Perspective: Volatility will continue to decline over time, as the rate of development for infrastructure can improve.
E. Enhanced Risk Management: The ability to produce valid results representing volatility will allow planners to anticipate and plan for contingencies and spare capacity.

3rd: Conclusion

This research used the Hyperbolic GARCH (HYGARCH) model to examine and predict the volatility of electrical load data in the Sulaimani Governorate. The findings demonstrated significant volatility clustering and robust long memory in the data, with the parameter d = 0.8854 suggesting enduring impacts of shocks. With errors clustered around zero, the model generated exact 24-hour-ahead estimates suggesting its suitability for modeling and predicting volatility in energy demand.

These findings have important implications for energy planning and policy development. The HYGARCH model provides a pathway for being able to make decisions that improve grid reliability, resource allocation, and allow for the forecast variability of demand. It also aids long-term planning, captures a long-run component of diversity due to the impact of volatility shocks, and allows better risk management processes in the energy sector.

This study was conducted in one area and time period. Future research could expand the study to new areas and/or further time periods, with larger datasets. Use of external factors (e.g. weather, economic activity) and/or model extensions (e.g. regime-switching, asymmetries) may improve forecasting performance. This framework should also be applicable to related areas, including volatility in electricity prices or renewable energy generation.

Supplementary information Researchers who are interested can get the information that was used in this work if they make a good case for it. To get access to the raw or edited data, please get in touch with the author who wrote the paper.

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References

1- AKGÜL, I. & AND SAYYAN, H. 2008. Modelling and forecasting long memory in exchange rate volatility vs. stable and integrated GARCH models. Applied Financial Economics, 18, 463-483.
2- BAILLIE, R. T., BOLLERSLEV, T. & MIKKELSEN, H. O. 1996. Fractionally integrated generalized autoregressive conditional heteroskedasticity. Journal of Econometrics, 74, 3-30.
3- BOLLERSLEV, T. & AND WOOLDRIDGE, J. M. 1992. Quasi-maximum likelihood estimation and inference in dynamic models with time-varying covariances. Econometric Reviews, 11, 143-172.
4- BOLLERSLEV, T. 1986. Generalized autoregressive conditional heteroskedasticity. Journal of Econometrics, 31, 307-327.
5- CONRAD, C. 2010. Non-negativity conditions for the hyperbolic GARCH model. Journal of Econometrics, 157, 441-457.
6- CONT, R. 2007. Volatility Clustering in Financial Markets: Empirical Facts and Agent-Based Models. In: TEYSSIÈRE, G. & KIRMAN, A. P. (eds.) Long Memory in Economics. Berlin, Heidelberg: Springer Berlin Heidelberg.
7- DAVIDSON, J. 2004. Moment and Memory Properties of Linear Conditional Heteroscedasticity Models, and a New Model. Journal of Business & Economic Statistics, 22, 16-29.
8- DIEBOLD, F. X. & MARIANO, R. S. 1995. Comparing Predictive Accuracy. Journal of Business & Economic Statistics, 13, 253-263.
9- ENGLE, R. F. 1982. Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation. Econometrica, 50, 987-1007.
10- KIM, S., SHEPHARD, N. & CHIB, S. 1998. Stochastic Volatility: Likelihood Inference and Comparison with ARCH Models. The Review of Economic Studies, 65, 361-393.
11- KWAN, W., LI, W. K. & LI, G. 2012. On the estimation and diagnostic checking of the ARFIMA–HYGARCH model. Computational Statistics & Data Analysis, 56, 3632-3644.
12- LI, M., LI, G. & LI, W. K. 2011. Score Tests for Hyperbolic GARCH Models. Journal of Business & Economic Statistics, 29, 579-586.
13- MANIMARAN, P., PARIKH, J. C., PANIGRAHI, P. K., BASU, S., KISHTAWAL, C. M. & PORECHA, M. B. Modelling Financial Time Series. In: CHATTERJEE, A. & CHAKRABARTI, B. K., eds. Econophysics of Stock and other Markets, 2006// 2006 Milano. Springer Milan, 183-191.
14- MEINTANIS, S., MILOSEVIC, B. & OBRADOVIC, M. 2020. Goodness-of-fit tests in conditional duration models. Statistical Papers, 61.
15- MOHAMMADI, F. & REZAKHAH, S. 2017. Smooth transition HYGARCH model: Stability and forecasting. arXiv preprint arXiv:1701.05358.
16- NELSON, D. B. 1991. Conditional Heteroskedasticity in Asset Returns: A New Approach. Econometrica, 59, 347-370.
17- P, M., PARIKH, J., PANIGRAHI, P., BASU, S., KISHTAWAL, C. & PORECHA, M. 2006. Modelling Financial Time Series.
18- SALL, E. H. M., DEME, E. H. & DIONGUE, A. K. 2021. Modeling Risk via Realized HYGARCH Model. arXiv preprint arXiv:2104.12409.
19- SHI, Y. & YANG, Y. 2018. Modeling High Frequency Data with Long Memory and Structural Change: A-HYEGARCH Model. Risks, 6, 26.
20- SINGH, R. R., HAIDER, S. W. & BRYCE, J. 2024. Local calibration of JPCP transverse cracking and IRI models using maximum likelihood estimation. Journal of Road Engineering, 4, 433-445.
21- TASHMAN, L. 2000. Out-of-sample tests of forecasting accuracy: An analysis and review. International Journal of Forecasting, 16, 437-450.
22- TSAY, R. S. 2005. Analysis of financial time series, John wiley & sons.