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# Studying the nuclear structure for the $^{73}$Br isotope using NuShellX@MSU code

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**ABSTRACT**

Effective f5pvh interactions have been identified and the NuShellX@MSU code within the fp- shell was utilized to compute the energy levels and electromagnetic transition probability of the $^{73}$Br nucleus. Applying the nuclear shell model with f5pvh interaction within the (fp- shell) is successful, according to the agreement with the experimental and theoretical data.

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# دراسة التركيب النووي لنظير $^{73}$Br باستخدام كود NuShellX@MSU

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**الـــخُـــلاصـــة**

باستخدام التفاعل الفعال f5pvh و (NuShellX@MSU code) ضمن قشرة fp ، تم حساب مستويات الطاقة واحتمالية الانتقال الكهرومغناطيسي لنواة $^{73}$Br . وقد تم تطبيق نموذج القشرة بنجاح باستخدام التفاعل f5pvh ضمن قشرة fp عن طريق مقارنة البيانات العملية مع حساباتنا النظرية.

**الكلمات المفتاحية:**

التركيب النووي،
NuShellX@MSU ,
نموذج القشرة،
مستويات الطاقة،
احتماليات الانتقال

## 1. INTRODUCTION

The nuclear shells' model has proven to be a useful instrument for research on the nuclear structure. It can compute numerous parameters accurately and systematically by choosing the residual effective interaction appropriately[1]. The present work aims to lower electromagnetic transition probabilities, {B(E2), B(M1)}, and compute the energy levels for the $^{73}$Br isotope using harmonic oscillator potential (HO, b), b > 0. By utilizing the effective charges of both protons and neutrons, the calculations were considered the effects of core polarization.

Mayer and Jensen devised the shell model[2], which was effectively applied to evaluate the nuclear structural features of comparatively light and near-closed shell nuclei , including rotation, parity, magnetic moment, etc. It has been regarded as the source (macroscopic nuclear models) and is one of the essential microscopic nuclear models [3]. The Cohen-Kurath and USD effective for (p and sd- shells), are two examples of common effective interactions for light nuclei. In contemporary times, the examination of sd-nuclei rich in neutrons has innduced the scientists attention since it unveils novel facets of nuclear composition[4][5]. The nuclear shell model codes like Oxbash[6], Antoine[7], NuShell [8], and NuShellX [9] etc, were utilized extensively with shell model calculations in the fp- shell, sd- shell, and finally in the p- shell[10]. Five valence orbits exist in the f5p-model space: (1f7/2, 2p3/2, 1f5/2, 2p1/2 and 1g9/2). Within this mass-centric model space, finding the energy and the region(A=73) was computed throughout this electromagnetic transmission, research, and energy levels options that exist inside the fp-shell was extracted using f5pvh interaction. We have utilized the complete f5p as a single-particle space (SPS) in this work [9]. Additionally applying The nucleic structure code NuShellX@MSU is utilized for specific computations [9]. It has the following shells: 1f7/2, 2p3/2, 1f5/2and 2p1/2for protons, and 1f7/2, 2p3/2, 1f5/2and 2p1/2for neutrons. Complete π 2p3/2 and ν 2p3/2shells are used in the computations. The SPEs were obtained for certain shells from Grawe et al. Protons and neutrons move independently of one another in a common potential in the mean-field approximation, which provides many nuclear features of the nucleus[11]. Nucleon interaction results in the generation of electric and magnetic fields [12][13][14][15][16].

## 1. Theoretical framework

To achieve precise eigenvectors, energies, and spectroscopic overlaps with low-lying states in calculations utilizing the shell model Hamiltonian matrix with extraordinarily large scale, Bill Rae [8] had developed a set of computer programs known as NuShellX. It can account for J-scheme matrix dimensions until (100 mills.) and uses a J-coupled proton-neutron basis. The wrapper scripts known as NuShellX@MSU were developed by Alex Brown and produce Hamiltonian data files and model space as input for NuShellX [9]. Typically, in the classical shell model calculations, the energy stats of a single nucleon outer the magical core proportional to a locked shell are computed, rather than the system's total energy. Energy is taken to be a Hamiltonians' eigenvalue ($H_0$) when there are several nucleons outer the core, and can be written as the total Hamiltonian as follows[17]:

$$H = \sum_{K=1} (H_0)_K + \sum_{k<1} V_{kl} \quad (1)$$

If the residual two-body interaction, or $V_{kl}$, added to the usual shell model potential, then can be written as:

$$\sum_{K<1} V_{kl} = \sum_{JM} \sum j_a \geq j_b \; j_c \geq j_d \langle j_a \, j_b \, |V_{12}| j_c \, j_d \rangle_J \, a^{+}_{JM} ( j_a j_b ) \, a_{JM} ( j_c j_d ) \quad (2)$$

This < > refers to the matrix element of the residue 2- body interaction; $a_{JM} ( j_c j_d )$ this operator is the Hermitian adjoin operator to $a^{+}_{JM} ( j_a j_b )$; $a^{+}_{JM} ( j_a j_b )$ generates a pair of nucleons in the single-particle levels( $j_a$ & $j_b$) for total angular momentum JM; λ(σL) represented by transition probability to gamma-ray emission from multipolarity (L &σ) is as follows [18]:

$$\lambda( \sigma L, J_i \rightarrow J_f ) = \frac{8\pi ( L + 1)}{\hbar L[(2L + 1)!!]^2} \left(\frac{E_\gamma}{\hbar c}\right)^{2L+1} \times B( \sigma L, J_i \rightarrow J_f ) \quad (3)$$

where $E_\gamma$ is the γ-ray energy and B(σL) is the reduced transition probability .

B(σL) can be stated as follows using the reduced matrix element $<\psi_f||M(\sigma L)||\psi_i>$ [16]:

$$B( \sigma L, J_i \rightarrow J_f ) = \frac{1}{2J_i + 1} |< \psi f \, \| \, M(\sigma L) \, \| \, \psi i >|^2 \quad (4)$$

When a system with A nucleons is present, the density distribution is as follows[1]:

$$\rho_0(r) = \sum_{i=1}^{A} |\varphi_i (\dot{r} )|^2 \quad (5)$$

## 2. Results and discussions

Utilize the shell model in the light (NuShellX@MSU code) [9], the $^{73}$Br calculations were performed; the model space is f5p with f5pvh effective interaction, containing neutrons (N = 10) outer the $^{56}$Fe closed core to $^{73}$Br isotope.

### *3.1 Energy levels*

The $^{73}$Br nucleus's first state is a closed core $^{56}$Fe nuc. with (ten N) dispersed across the fp- shell outside the closed shell, while ($J=0^+$ & T=1). NushellX@MSUcode was utilized to compute the energy levels of the $^{73}$Br isotope by applying the interaction f5pvh.

By comparing the experimental data for this isotope in the table above with our theoretical findings using the f5pvh interaction, the following outcoms can be observed:

1. A comparison with the available empirical data revealed that the ground state parity and total angular momentum of the $\frac{1}{2}^-$ level were identical.
2. The theoretically computed energies were (0.719, 0.051) MeV, and comparing them with the available experimental data, we were able to obtain a good agreement for the angular momentums $7/2^-_1, 3/2^-_1$.

3. The practical energies' angular momentum and symmetry were established ( 3.967 ,4.280, 4.380, 4.460, 4.600, 4.640, 4.670, 4.830, 4.960, 5.170, 5.270, 5.340, 5.390, 5.450, 5.560, 5.610, 5.700, 5.750, 5.890, 6.240, 6.38, 6.403, 6.480) MeV corresponding to angular momentum ($13/2_5^-$, $13/2_7^-$, $13/2_8^-$, $13/2_9^-$, $13/2_9^-$, $17/2_1^-$, $15/2_2^-$, $15/2_3^-$, $15/2_4^-$, $15/2_5^-$, $15/2_6^-$, $15/2_7^-$, $17/2_2^-$, $15/2_8^-$, $15/2_9^-$, $15/2_{10}^-$, $17/2_3^-$, $19/2_1^-$, $17/2_4^-$, $17/2_5^-$, $17/2_6^-$, $17/2_7^-$, $19/2_2^-$, $17/2_8^-$). This result can reflect how well the practical value matches our theoretical value.
4. Just the total angular momentum for the experimentally uncertain energy (0.240) MeV, corresponding to angular momentum $5/2_1^-$, was confirmed because the experimental value is close to our theoretical value. The symmetry and total angular momentum of the experimentally uncertain energy MeV (0.943) corresponding to angular momentum $9/2_1^-$ were confirmed.
5. Using our calculations, we found that there are fifty-four levels with total angular momentum and symmetry for which there is currently no practical value.
6. We discovered that several energy values in the experimental data correspond to multiple values for the total angular momentum, which are consistent with our theoretical values.
7. According to our calculations, the maximum experimental energy value is 25.968MeV, where the highest predicted energy value is theoretically 9.575 MeV.

For the $^{73}$Br isotope, Table 1 presents a comparison utilizing the f5pvh interaction for the theoretical results and experimental results[19].

**Table1:** Offering a comparison for the experimental excitation energies and excitation energies for the $^{73}$Br isotope by using f5pvh interaction[19]

| Theoretical values | | Experimental values | |
|---|---|---|---|
| $J^-$ | E (MeV) | E (MeV) | $J^\pi$ |
| $1/2_1$ | 0.000 | 0.0 | $1/2^-$ |
| $3/2_1$ | 0.051 | 0.178 | $3/2^-$ |
| $5/2_1$ | 0.063 | 0.240 | $5/2^-$ |
| $3/2_2$ | 0.530 | 0.473 | $(3/2,5/2)^-$ |
| $5/2_2$ | 0.667 | 0.635 | $(1/2^-,3/2,5/2^-)$ |
| $7/2_1$ | 0.719 | 0.681 | $7/2^-$ |
| $1/2_2$ | 0.819 | 0.713 | $(1/2^-,3/2,5/2^-)$ |
| $3/2_3$ | 0.970 | ------ | ------ |
| $7/2_2$ | 1.126 | ------ | ------ |
| $9/2_1$ | 1.186 | 0.943 | $(9/2^-)$ |
| $5/2_3$ | 1.242 | 1.137 | $(1/2^-,3/2,5/2)$ |
| $3/2_4$ | 1.392 | 1.473 | $(1/2^-,3/2,5/2)$ |
| $5/2_4$ | 1.482 | ------ | ------ |
| $1/2_3$ | 1.588 | 1.542 | (1/2,3/2,5/2) |
| $3/2_5$ | 1.607 | ------ | ------ |
| $7/2_3$ | 1.713 | ------ | ------ |
| $9/2_2$ | 1.813 | ------ | ------ |
| $3/2_6$ | 1.826 | ------ | ------ |
| $7/2_4$ | 1.862 | ------ | ------ |

| | | | |
|---|---|---|---|
| $5/2_5$ | 1.864 | ------ | ------ |
| $9/2_3$ | 1.906 | ------ | ------ |
| $11/2_1$ | 1.909 | ------ | ------ |
| $1/2_4$ | 1.944 | 2.154 | (1/2$^-$,3/2,5/2) |
| $5/2_6$ | 1.963 | ------ | ------ |
| $7/2_5$ | 1.971 | ------ | ------ |
| $5/2_7$ | 2.062 | 2.154 | (1/2$^-$,3/2,5/2) |
| $3/2_7$ | 2.126 | 2.154 | (1/2$^-$,3/2,5/2) |
| $7/2_6$ | 2.178 | ------ | ------ |
| $1/2_5$ | 2.230 | 2.261 | (1/2,3/2,5/2$^-$) |
| $9/2_4$ | 2.279 | ------ | ------ |
| $5/2_8$ | 2.322 | 2.261 | (1/2,3/2,5/2$^-$) |
| $11/2_2$ | 2.327 | ------ | ------ |
| $3/2_8$ | 2.359 | 2.261 | (1/2,3/2,5/2$^-$) |
| $3/2_9$ | 2.418 | ------ | ------ |
| $5/2_9$ | 2.419 | ------ | ------ |
| $3/2_{10}$ | 2.448 | 2.555 | (1/2$^-$,3/2,5/2) |
| $7/2_7$ | 2.475 | ------ | ------ |
| $1/2_6$ | 2.492 | 2.555 | (1/2$^-$,3/2,5/2) |
| $9/2_5$ | 2.521 | ------ | ------ |
| $7/2_8$ | 2.522 | ------ | ------ |
| $5/2_{10}$ | 2.522 | 2.555 | (1/2$^-$,3/2,5/2) |
| $1/2_7$ | 2.605 | ------ | ------ |
| $9/2_6$ | 2.606 | ------ | ------ |
| $11/2_3$ | 2.637 | ------ | ------ |

| | | | |
|---|---|---|---|
| $7/2_9$ | 2.672 | ------ | ------ |
| $7/2_{10}$ | 2.683 | ------ | ------ |
| $13/2_1$ | 2.692 | ------ | ------ |
| $9/2_7$ | 2.707 | ------ | ------ |
| $9/2_8$ | 2.807 | ------ | ------ |
| $11/2_4$ | 2.864 | ------ | ------ |
| $9/2_9$ | 2.887 | ------ | ------ |
| $1/2_8$ | 2.910 | ------ | ------ |
| $9/2_{10}$ | 2.950 | ------ | ------ |
| $1/2_9$ | 2.972 | 3.017 | (1/2,3/2,5/2) |
| $1/2_{10}$ | 3.041 | 3.252 | (1/2$^-$,3/2,5/2) |
| $11/2_5$ | 3.205 | ------ | ------ |
| $13/2_2$ | 3.376 | ------ | ------ |
| $11/2_6$ | 3.394 | ------ | ------ |
| $15/2_1$ | 3.421 | ------ | ------ |
| $11/2_7$ | 3.502 | ------ | ------ |
| $11/2_8$ | 3.531 | ------ | ------ |
| $11/2_9$ | 3.589 | ------ | ------ |
| $11/2_{10}$ | 3.746 | ------ | ------ |
| $13/2_3$ | 3.809 | ------ | ------ |
| $13/2_4$ | 3.842 | ------ | ------ |
| $13/2_5$ | 3.974 | 3.967 | ------ |
| $13/2_6$ | 4.229 | ------ | ------ |
| $13/2_7$ | 4.268 | 4.280 | ------ |
| $13/2_8$ | 4.393 | 4.380 | ------ |
| $13/2_9$ | 4.481 | 4.460 | ------ |
| $13/2_{10}$ | 4.630 | 4.600 | ------ |
| $17/2_1$ | 4.633 | 4.640 | ------ |
| $15/2_2$ | 4.697 | 4.670 | ------ |
| $15/2_3$ | 4.834 | 4.830 | ------ |

| | | | |
|---|---|---|---|
| $15/2_4$ | 4.957 | 4.960 | ------ |
| $15/2_5$ | 4.972 | ------ | ------ |
| $15/2_6$ | 5.176 | 5.170 | ------ |
| $15/2_7$ | 5.282 | 5.270 | ------ |
| $17/2_2$ | 5.342 | 5.340 | ------ |
| $15/2_8$ | 5.403 | 5.390 | ------ |
| $15/2_9$ | 5.488 | 5.450 | ------ |
| $15/2_{10}$ | 5.535 | 5.560 | ------ |
| $17/2_3$ | 5.618 | 5.610 | ------ |
| $19/2_1$ | 5.701 | 5.700 | ------ |
| $17/2_4$ | 5.729 | 5.750 | ------ |
| $17/2_5$ | 5.896 | 5.890 | ------ |
| $17/2_6$ | 6.264 | 6.240 | ------ |
| $17/2_7$ | 6.396 | 6.380 | ------ |
| $19/2_2$ | 6.414 | 6.403 | ------ |
| $17/2_8$ | 6.481 | 6.480 | ------ |
| $17/2_9$ | 6.541 | ------ | ------ |
| $17/2_{10}$ | 6.632 | ------ | ------ |
| $21/2_1$ | 6.895 | ------ | ------ |
| $19/2_3$ | 6.931 | ------ | ------ |
| $19/2_4$ | 7.217 | 7.249 | ------ |
| $19/2_5$ | 7.226 | ------ | ------ |
| $21/2_2$ | 7.370 | ------ | ------ |
| $19/2_6$ | 7.401 | ------ | ------ |
| $19/2_7$ | 7.494 | ------ | ------ |
| $19/2_8$ | 7.698 | ------ | ------ |
| $19/2_9$ | 8.031 | ------ | ------ |
| $19/2_{10}$ | 8.217 | ------ | ------ |
| $21/2_3$ | 8.728 | ------ | ------ |
| $21/2_4$ | 8.960 | ------ | ------ |
| $21/2_5$ | 9.277 | ------ | ------ |
| $21/2_6$ | 9.575 | 9.511 | ------ |

### *3.2 Electromagnetic transition probability B(E2) and B(M1)*

It is possible to think of gamma rays as a type of electromagnetic radiation in which the electric field changes, causing the magnetic field to change as well. Radiation can be created by a fluctuating external magnetic field caused by an oscillating charge, or it can be produced by a changing magnetic field caused by a change in current or magnetic moment[19].

We observed a reasonable agreement between the available experimental data and the theoretical data for the electrical transitions B(E2) $3/2_{-1}\rightarrow 1/2_{-1}$ and B(E2) $3/2_{-2}\rightarrow 5/2_{-1}$ with using the (f5pvh) interaction. Our calculations revealed new transitions that had no experimental values before.

**Table 2.** Comparison of the B(E2) and B(M1) results by using ( f5pvh) interaction in units $e^2fm^4$ for the $^{73}$Br isotope with the experimental data [19]

| $J_i$ | → | $J_f$ | B(M1) $(\mu_N^2)$ | | B(E2) $e^2fm^4$ | |
|---|---|---|---|---|---|---|
| | | | Theory | Exp. | Theory | Exp. |

| | | | | | | |
|---|---|---|---|---|---|---|
| $3/2_{-1}$ | → | $1/2_{-1}$ | 0.6798 | 0.0143 | 137.9 | 906.122 |
| $5/2_{-1}$ | → | $1/2_{-1}$ | ----- | ----- | 33.14 | ----- |
| $5/2_{-1}$ | → | $3/2_{-1}$ | 0.001078 | 0.0019 | 123.6 | ----- |
| $3/2_{-2}$ | → | $1/2_{-1}$ | 0.146 | ----- | 39.63 | ----- |
| $3/2_{-2}$ | → | $3/2_{-1}$ | 0.483 | ----- | 89.97 | ----- |
| $3/2_{-2}$ | → | $5/2_{-1}$ | 0.0000144 | ----- | 247.6 | 13.954 |
| $5/2_{-2}$ | → | $1/2_{-1}$ | ----- | ----- | 145.9 | ----- |
| $5/2_{-2}$ | → | $3/2_{-1}$ | 0.01352 | ----- | 106.2 | ----- |
| $5/2_{-2}$ | → | $5/2_{-1}$ | 0.01637 | ----- | 133.4 | ----- |
| $5/2_{-2}$ | → | $3/2_{-2}$ | 0.001235 | ----- | 0.1601 | ----- |

## 4. Conclusions

The current work demonstrated that the energy levels and electromagnetic transition probability were computed using the interaction f5pvh and the results were shown to be in reasonable agreement with the existing experimental data. Numerous energy stats have been confirmed using the interaction and additional energy levels have been acquired in our computations. The B(E2) and B(M1) values also showed some degree of compatibility with the experimental results.

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