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Approximate solution to differential equations and how to find it using some numerical methods

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Abstract

Differential equations are an essential tool in the mathematical modeling of many physical, engineering, and economic phenomena, describing the relationships between variables and their rates of change. In many cases, finding an analytical (exact) solution to these equations is difficult or impossible, especially when they are nonlinear or complex in form. This is where numerical solutions become important, allowing for approximation using structured computational methods. These methods are characterized by their simplicity and feasibility using programming or tables, and they allow for increasing accuracy as the calculation step decreases. Thus, numerical methods are an essential tool in the modern era for understanding and simulating phenomena that are difficult to express with explicit mathematical solutions. Numerical methods rely on dividing the domain into small steps and then using mathematical rules to estimate the values of the function at these points, providing an effective means of dealing with equations that are difficult to solve using traditional methods. The most prominent of these methods are: Euler's method, the improved method (Heun), and the Runge-Kutta method. In this research, we used Euler's method, which represents the simplest and oldest numerical method for solving initial value problems. Then, we will use the modified Euler method, which is one of the prediction and correction methods. Finally, we will discuss the Runge-Kutta algorithms for finding the solution.

Keywords: differential equations, Euler's method, Taylor's method, improved Euler's method, Range-Kutta method.

الحل التقريبي للمعادلات التفاضلية وكيفية ايجاده باستخدام بعض الطرق العددية

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المستخلص:

تُعد المعادلات التفاضلية أداة أساسية في النمذجة الرياضية للعديد من الظواهر الفيزيائية والهندسية والاقتصادية، حيث تصف العلاقات بين المتغيرات ومعدلات تغيرها. وفي كثير من الحالات، يكون من الصعب أو المستحيل إيجاد حل تحليلي (دقيق) لهذه المعادلات، خاصةً عندما تكون غير خطية أو معقدة الشكل. وهنا تظهر أهمية الحلول العددية التي تتيح تقريب الحلول باستخدام أساليب حسابية منظمة. تميز هذه الطرق ببساطتها وقابليتها للتنفيذ باستخدام البرمجة أو الجداول، كما أنها تتيح دقة متزايدة كلما صغرت خطوة الحساب. وبذلك، تُعد الطرق العددية أداة ضرورية في العصر الحديث لفهم ومحاكاة الظواهر التي يصعب التعبير عنها بحلول رياضية صريحة. تعتمد الطرق العددية على تقسيم المجال إلى خطوات صغيرة، ومن ثم استخدام قواعد رياضية لتقدير قيم الدالة في هذه النقاط، مما يوفر وسيلة فعالة للتعامل مع المعادلات التي يصعب حلها بالطرق التقليدية. ومن أبرز هذه الطرق :طريقة أويلر، والطريقة المحسنة(Heun) ، وطريقة رانج-كوتا، في هذا البحث استخدمنا طريقة اويلر التي تمثل ابسط واقدم الطرق العددية لحل مسائل القيم الابتدائية ثم بعد ذلك سنستخدم طريقة اويلر المعدلة وهي من طرق التنبؤ والتصحيح واخيرا سنتطرق الى خوارزميات رانج- كوتا لايجاد الحل.

الكلمات المفتاحية: المعادلة التفاضلية، طريقة اويلر، طريقة اويلر المحسنة، طريقة رانج كوتا.

Introduction:

It can be said without exaggeration that differential equations occupy a prominent position in all branches of engineering and physics, as most of the relationships and laws governing the variables of a physical or engineering problem appear in the form of differential equations. To understand this problem, it is necessary to solve these differential equations or at least know many of the properties of this solution. This is not always an easy matter, but rather many differential equations are unsolvable, so numerical methods are an ideal solution for most differential equations. Numerical methods and their multiple types pose a difficulty in how to solve and how to differentiate between their different types, as well as the difficulty of applying them in various fields of physics, chemistry, and others. Due to the difficulty of these fields themselves, because it is not an easy matter, there are no general mathematical methods for solving differential equations, and there are some methods that can be generalized to a special group of differential equations. Even numerical methods and the method of finite elements are not general methods for solving all differential equations in all multi-branch and comprehensive sciences. Therefore, the application of fields and differential equations since the time of Newton has been used in understanding the physical, engineering, chemical, and biological sciences, in addition to its contribution to the study of mathematical analysis. Its uses have extended to the economic and social sciences, and differential equations have developed and their importance has increased in all fields of science and their applications.[6]

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Research problem:

Numerical methods and their multiple types pose a difficulty in how to solve and how to differentiate between their different types, as well as the difficulty of applying them in various fields of physics, chemistry, and others. Due to the difficulty of these fields themselves, because it is not an easy matter, there are no general mathematical methods for solving differential equations, and there are some methods that can be generalized to a special group of differential equations. Even numerical methods and the method of finite elements are not general methods for solving all differential equations in all multi-branch and comprehensive sciences. Therefore, the application of fields and differential equations since the time of Newton has been used in understanding the physical, engineering, chemical, and biological sciences, in addition to its contribution to the study of mathematical analysis. Its uses have extended to the economic and social sciences, and differential equations have developed and their importance has increased in all fields of science and their applications.

Objective of the research

Identify numerical methods for solving differential equations and expand their study to solve more difficult problems for numerical methods.

Search terms:

The equation:

It is a mathematical expression composed of mathematical symbols that states the equality of two mathematical expressions. This equality is expressed by the equal sign [1]. It is the similarity of any mathematical expression equal to another mathematical expression. When you write the equation, we will have an expression on the left side and another expression on the right side, such that there is an equal sign between them because the two expressions must be equal to each other. [2]

Differential equations:

It is a relationship that links one or more independent variables and the function being researched that is dependent on these variables which are assumed to be real variables as is the function. are a relationship that combines one or more functions and derivatives. They can also be defined as equations in which the variable is a function. In most applications, the function is a physical quantity. They are divided into two sections: ordinary and partial equations, and linear and nonlinear equations. Differential equations of the first order are considered to contain only first derivatives. [3][5]

Previous Studies:

1- Dr. Zainab Ali and Ms. Samia Rajab (2016)The two researchers presented a study of some numerical algorithms for solving a system of ordinary differential equations. They studied Taylor's method, which relies on Taylor's series. This method is not numerical in the absolute and explicit sense, but it is considered the cornerstone of other numerical methods. It is an accurate method but impractical due to its need for higher-order derivatives, which may be difficult to find for some functions with complex algebraic structures. The researchers also presented Euler's method, which is considered the oldest and simplest numerical method for solving initial value problems, and the modified Euler's method, which is a prediction and correction method because the calculated value is corrected using one of the previous methods. They also generalized the Runge-Kutta algorithms for solving an initial value problem [7].

2- (Jawad Kazem Al-Zarkani) (2015), In his book, he presented some basic definitions and concepts, such as the classification of differential equations in terms of order and degree, the concept of a general solution and anomalous solutions to differential equations. He also presented methods for solving first-order differential equations, such as the chapter The Complete Differential Equation, Solving the Homogeneous First-Order Differential Equation, Variables Bernoulli's Equation, Solving Incomplete Equations Using the Integrator

Before beginning to find a solution to any elementary problem, one must ensure that the problem is a variable, which means that $y(t); t > t_0$ is stable. An exact solution to a Cauchy problem means obtaining a function that satisfies the ordinary differential equation of the order show n in Figure [3].

$$y^n(t_0) = y_0\,; y(t_0) = \ y_0 \dots, y^{(n-1)}(t_0) = y_0(n-1) \quad \dots (1)$$

It also fulfills the initial conditions.

$$y(t_0) = \ Y_0\,, y(t_0) = \ y_0, \ \ y^{(n+1)}(t_0) = y^{(n-1)}(t_0) = y_0^{(n+1)} \dots . (2)$$

However, we focus our attention on finding numerical solutions to the first-order variation of the Cauchy elementary problem.

$$Y(t) = f(t, y); \ y(t_0) = \ y_0\,; t \in [a, b] \quad \dots (3)$$

Where the function f(t,y) is defined and continuous on the rectangle.

$$\{t - t_0 \leq a\,, y - y_0 \leq b\}$$

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Taylor method: [8]

This method is based on the Taylor series, which is not a numerical method in the strict and explicit sense. The idea of this method is to use the function expansion$y(x)$ and about a We don't know but we knowIts derivatives from the initial value problem(1) The remaining derivatives are found at a and you write $y(x)$ As follows

$$Y(x) = y(a) + hy(a) + \frac{h}{2}y(a) + \cdots + \frac{h}{n}y^{(n)}(a)$$

Where $h = x - a$ It can be written in general terms.

$$Y(x^{n+1}) = y(x_n) + hy(x_n) + \frac{h}{2}y(x_n) + \cdots + \frac{h}{n}y^{(n)}(x_n)$$

Where $h = x_{n+1} - x_n$

Here the method is called the Taylor series of order(n)

Taylor's method for solving the system(1) Like Taylor's method for solving a single differential equation, This is done by changing the two functions $x = f(t), y = g(t)$ by knowing the first derivative For both functions, where:

$$\frac{dx}{dt} = f(t,x,y) \qquad \frac{dy}{dt} = g(t,x,y) \qquad about\ \ t = t_0\ \ as\ follows:$$

$$X(1) = x0 + (\,t - t_0)x + \frac{(t - t_0)2}{2}x + \cdots$$

$$Y(1) = y0 + (\,t - t_0)y + \frac{(t - t_0)2}{2}y + \cdots$$

and if then put $h = t - to$ then the two series above take the following form:

$$X(t) = x0 + hx + \frac{h}{2}x + \cdots$$

$$Y(t) = y0 + hy + \frac{h}{2}y + \cdots$$

Which are considered a solution to the system (1).

Example (1)

Using Taylor's method to solve the system

$\frac{dx}{dt} = 4x - 2y + 2t$, x(0)=0

$$\frac{dy}{dt} = 8x - 4y + 1 \qquad , \quad y(0) = 0$$

Step length $h = 0.1$ On the period, Compared to the actual solution

$$X(t) = \frac{4}{3}t^2 \qquad , \ Y(t) = t - 2t^2 + \frac{8}{3}t^3$$

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Euler's method:[4][9]

It is the simplest numerical method for solving the initial value (1) problem and is also called the tangent method. It is given by the formula: $(x, hf + y)$ and it is linear in $(x. y)$. and the solution of the system(1) using Euler's method: $X = x + hx$ is as follows: $Y = y + hy$

This method depends on giving the constant h as small as possible so that the terms of the Taylor series can be eliminated. Starting from the term containing x of the Taylor series for the point $x = h$, which is defined as follows:[3]

$$Y(x+h) = y(x) + hy(x)^{+} \frac{h2}{2}\ \mathrm{y}^{\mathrm{n}}(\mathrm{x}) + \cdots + \frac{h}{2}\ \mathrm{y}^{\mathrm{n}}(\mathrm{x})$$

By deleting the terms, considering it from the term h, n,y, we get (2)

$$Y(x+h) = y(x) + hy(x(= y(x) + hf(x,y)$$

We start from the point $(x0. y0)$ and by applying the above relationship we get that

$$Y1 = y(x0+h) = Y(x0) + hy(x0)$$

$$Y2 = y(x0+2h) = Y(x0) + 2hy(x0)$$

$$Yn+1 = yn + hf(xn, yn) \qquad n = 0,1,2,3, \dots \dots \dots \dots \dots, \mathrm{M}-1$$

That is, the general formula of Euler's law is:

$$Yn+1 = yn + hf(x_n . y_n)$$

When $x_n = x_0 + nh \qquad n = 0.1.2.3. \dots. \ \dots. M-1$

Example (2):

Find the approximate solution to the differential equation $y = x + y$ where $y = 0$, Take $0 \cdot 2 = h$ in the interval [1, 0]. Solution: Using the law (1) and the condition (y = 0) it results in $x = 0$, $y = 0$. Substituting into the law we find

$(fxn. Yn) = xn + Yn \qquad xn = xo + nh = nh$

$y_1 = yo + 0 \cdot 2\,(x + y_2) = 0$

$Y2 = y1 + 0 \cdot 2(x1 + y1) = 0 + 0 \cdot 2(0 \cdot 2 + 0) = 0 \cdot 4$

$y3 = ya + 0 \cdot 2(x2 + yz) = 0 \cdot 16 + 0 \cdot 2(0 \cdot 4 + 0 \cdot 04) = 0 \cdot 128$

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$$Y4 = y3 + 0 \cdot 2(x3 + y3) = 0 \cdot 128 + 0 \cdot 4(0 \cdot 6 + 0 \cdot 128) = 0 \cdot 274$$

$$Y5 = y_4 + 0 \cdot 2(x_4 + y_4) = 0 \cdot 48 + 0 \cdot 4(0 \cdot 8 + 0 \cdot 489) = 1 \cdot 747$$

n	Y_n	$Y_{n-1} = y_n + 0 \cdot 2(x_n + y_n)$
0	0	0
1	0.2	0.04
2	0.4	0.128
3	0.6	0.275
4	0.8	0.489
5	1	1.747

Example (3):
Find the approximate solution to the initial condition problem
$x \leq 0$ and $y(0) = 0$,$y = -y + x + 1 \leq 1$, then compare it to the correct solution $y = x + ex$ (by comparison we mean finding the error committed and it represents the absolute value of the difference between the correct solution and the approximate solution)
$E = y_{n+1} - z_{n+1}$

the solution:

$x_n = X_0 + nh$, $h = \frac{b-a}{n}$

$(fxn.Yn) = -yn + xn + 1$

$Yn + 1 = Yn + 0.1\,(-yn + xn +)1$

$y_1 = y_1 + 0.1\,(-Yo + Xo + 1) = 10.1\,(-1 + 0 +)1 = 1$
$Y^2 = Y^1 + 0.1\,(-y^1 + x^1 + 1) = 10.1\,(-1 + 0.1 + 1) = 1.01$
Thus you get the approximate values shown in the table below.

n	Xn	$Y_{n+1} = Yn + 0.2\,(x + yn)$
0	0	1
1	0.1	1.101
2	0.2	1.029
3	0.3	1.016100
4	0.4	1.90490
5	0.5	1.1311441
6	0.6	1.78297
7	0.7	1.230467
8	0.8	1.287420
9	0.9	1.348578

To compare the approximate solution with the correct solution

$z_{n+1} = x_n + e^{(-xn)}$

$z_1 = x_0 + e^{x_0} = 0 + e^0 = 1$

$z_2 = x_1 + e^{x_1} = 0.1 + e^{-0.1} = 1.04837$

$z_3 = x_2 + e^{x_2} = 0.2 + e^{-0.2} = 1.018731$

$Z_1 = X_1 + e^{x_3} = 0.3 + e^{-0.3} = 1.04818$

And thus you get the rest of the correct values.

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n	y_{n+1}	z_{n+1}	$Error = y_{n+1} - z_{n+1}$
0	1	1	0.0000
1	1.01	1.04837	0.03837
2	1.029	1.0487	0.02176
3	1.01610	1.04081	0.02471
4	1.90490	1.07032	0.16542
5	1.13144	1.0653	0.06613
6	1.7829	1.1488	0.63415

Improved Euler's method (10)

Let the required solution be the differential equation.

$Y = f(x, y)$ ... (1)
In the closed period $[xo, xn\ ]$ whereas the initial condition

$Y(x_0) = y_0$ ... (2)

Where $y(x)$ it is the original function for (x) and from it we find
$y\ (x\ _1)\ =\ y\ (xo)\ +\int_{x0}^{x1} f\ \big(x\ y\ (x)\big)dx$ ... (3)

Now we can use numerical integration methods to approximate the definite integral on the right-hand side. from step (3) using the trapezoidal method of the relationship $h = \mathrm{x}_0 - \mathrm{x}_n$ we find

$$y(\mathrm{x}_1) = y(\mathrm{x}_0) + \frac{h}{2}[f(\mathrm{x}_0, y(\mathrm{x}_0) + f(\mathrm{x}_1, y(\mathrm{x}_1)]$$

From this relationship we find that $y(\mathrm{x}_1)$ It has been calculated in terms of itself, so there is an approximate value for it. according to some method. Let it be Euler's method and we will symbolize it as y_i the value is on the left side. $y+1=y+1$, which you replace again on the side and you get y4 thus we have

$$y_{i+1}=y_i+\frac{h}{2}[f(\mathrm{x}_i,y(\mathrm{x}_i)+f(\mathrm{x}_{i+1},y(\mathrm{x}_{i+1})] \qquad \dots(4)$$

$$i=0,1,2,\dots,n-1$$

$$whereas \quad y_{i+1}=y_i+hf(\mathrm{x}_i,\mathrm{y}_i) \quad , \quad \mathrm{x}_{i+1}=x+h \quad \dots(5)$$

Now for each step we use Euler's method to predict the initial value of the solution and then use the equation (4) and repeat the process.

Exsample (4):

Find the solution of the differential equation

$$Y=\frac{x-y}{2} \quad , \quad y(o)=1 \quad where\ h=0.25$$

according to the improved Euler method

sol:

$$F(\mathrm{x}_0,\mathrm{y}_0)=\frac{\mathrm{x}_0-\mathrm{y}_0}{2}=-0.5$$

First, we apply Euler's method and find

$$\mathrm{y}_1=\mathrm{y}_0+Hf((\mathrm{x}_0,\mathrm{x}_0)=1+0.25(0.5)=0.3125$$

$$F\left(\mathrm{x}_0,\mathrm{y}_0{}^{(1)}\right)=\frac{\mathrm{x}_1-\mathrm{y}_1}{2}=\frac{0.25-0.875}{2}=-0.3125$$

and so we get

$$\mathrm{y}_1{}^{(1)}=\mathrm{y}_0+\frac{\mathrm{H}}{2}\left[f(\mathrm{x}_0,\mathrm{x}_0)+f\left(\mathrm{x}_1,\mathrm{y}_1{}^{(0)}\right)\right]$$

$$=1+\frac{0.25}{2}[-0.5-0.3125]=0.898438$$

By repeating the process again, we get the following:

$$F\left(x_1, {y_1}^{(1)}\right) = \frac{x_1 - y_1}{2} = \frac{0.25 - 0.898438}{2} = -0.324219$$

$${y_1}^{(2)} = y_0 + \frac{H}{2}\left[f(x_0, x_0) + f\left(x_1, {y_1}^{(1)}\right)\right]$$

$$=1+\frac{0.25}{2}\,[-0.5 - 0.324219] = 0.896974$$

**** The second method: Fourth-order Range-Kutta method**

Euler's method is not used practically because it requires a small valuable step h, As for The Ring-Konna method is considered one of the most important methods used to find an approximate solution to differential equations. It enables us to obtain high accuracy while avoiding the need to derivate the function(x), It depends on Function substitution f(x,y) At selected points equations of the fourth order Ringe-Konna method It is known as follows:

$$Y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_n + k_n) \quad \ldots (1)$$

Whereas

$$k_1 = hf(x_n, y_n)$$

$$k_2 = hf\left(x_n + \frac{h}{2}, y_n + \frac{1}{2}k_1\right)$$

$$k_3 = hf\left(x_n + \frac{h}{2}, y_n + \frac{1}{2}k_2\right)$$

$$k_4 = hf(x_n + h, y_n + hk_3)$$

Example (5):

Using the method R.K.4, find the approximate solution to the initial condition problem,

$$Y = -y + x + 1 \qquad 0 \le x \le 1 \qquad h = 0.1\,, y(0) = 1$$

Sol:

The Range-Kutta equations can be written as follows:

$$Y_{n+1} = y_n + \frac{h}{6}(k_1 + 2k_2 + 3k_3 + k_4) \quad \dots (*)$$

By applying Range KUNA equations, we get:

$F(x, y) = -y + x + 1$

$k_1 = f(x_n , Y_n)$

$k_2 = f(x_n + Y_n + k_1\ h\ h)$

$k_3 = f(x_n + z_1 Y_n + k_2\ h\ h)$

$k_4 = f(x_n + h, Y_n + hk_3\ )$

$k_1 = -y_n + x_n + 1$

$k_2 = f\left(x_n \frac{h}{2}, y_n + 2\frac{h}{2}\right) = \frac{h}{2}\left(\frac{h}{2} - 1\right) y_n + \left(-1\frac{h}{2}\right) x + 1$

$= (1 - \frac{h}{2}) \left(1 - \frac{h}{2}\right) (\ x_n, y_n) + 1$

All n=0,1,2,….,9

Such as n=0

$k_1 =$ -$y_0 + x + 1 = 0$

$k_2 = (-0 \cdot 95y + 0 \cdot 95x + 1 = 0 \cdot 05$

$k_3 = -0.9525(x_n - y_n) + 1 = 0 \cdot 09525$

$k_4 = 1 - 0 - 19525 = 0 \cdot 09525$

To compensate the values by the equation (*) we get

$Y_{1=}Y_1 + (k_1 + 2k_2 + 2k_2 + k_1)$

$$(0.09525 + (1.09525) \times 2 + 0.05 \times 2 + 0) - 0.1 + 2021 + 1 = 1.0048375009$$

This is how you can get the values and the table below. It represents some calculated values. Try to find the rest of the values

X_n	Y_{n+1}
0	1.00000
0.1	1.0048375000
0.2	1.187309014
0.3	………
0.4	………

Example (6):

The correct solution to the differential equation

$y = 3x + \frac{y}{2}$ he $y = 13^{ex/2} - 6x - 12$

Find up Approximate For the equation at the primary value

$\mathrm{y}(0) = 1$ and take $\mathrm{h} = 0.1$ For the period $0 \leq \mathrm{x} \leq 1$

and prove that The accuracy increases the period decreases

Sol:

Applying the Range-Kutta equations where

$f(x, y) = 3x + \frac{y}{2}$

We get

$k_1 = hf(x_0, y_0) = h(0 \cdot 1) = 0 \cdot 05$

$k_2 = hf(x_0 + \frac{h}{2}.y_0 + \frac{1}{2}k_2 = hf(0 \cdot 05.1 \cdot 25) = 0 \cdot 06625$

$k_3 = hf(x_0 + \frac{h}{2}.y_0 + \frac{1}{2}k_2 = hf(0 \cdot 05.1 \cdot 033125) = 0.6665625$

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$k_4 = hf(x_0 + h.y_0 + \ h\ k_3 = hf(0 \cdot 1.1106665625) = 0.0833328$

Substituting these values into the Runge-Coat equation

$y_{n+1} = y_n + (k_1 + 2k_2 + 2k_3 + k_4)$

$y_1 = y_0(0.05 + 2 \times 0 \cdot 06625 + 0.083328)$

And the correct solution $y = 13^{ex/2} - 6x - 12$ We find the correct value of the point by substituting the value of x_1

She $y(0.1) = 1.06652424866$ That is, the accuracy reaches eight significant figures by taking h=0.1 By comparing the value of no resulting from the correct solution with the value of no resulting from the numerical solution with an increase in the value of h We will find that the error value increases, as shown in the table below.

hالفترة المختارة	قيمة لا من الحل الصحيح	قيمة ل بطريقة رنج كونا
H=0.1	Z(0.1)=1.06665242486	Y(0.1)= 1.06652421856
H=0.2	Z(0.2)=1.16722193718	Y(0.2(=1.1672208333
H=0.3	Z(0.3)=1.47823585939	Y(0.4)=1.4782

Example (7):

Use the fourth order Ringe -Konna method to solve the differential equation.

$$Y = \frac{(x - y)}{2}$$

At the period [0,3] Initial condition (0) =1 and he resides h=1 , Then compare the result with the correct solution.

$y = 13^{-x/2} - 2 - x$

Sol.

Applying the Range-Kutta equations

$Y_{n+1} = y_n + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_1)$

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$K_1 = f(x_n, Y_n)$

$k_2 = f(x_2 + Y_n + k_2 hh)$

$k_3 = f(x_2 + y_2 + k_2)$

$k_4 = f(x + h.y + k_3 h)$

We get

$$K_1 = \frac{0.0 - 1.0}{2} = -0.5$$

$$k_2 = \frac{0.125 - \big(1 + 0.25(0.5)(-0.5)\big)}{2} = -0.40625$$

$$k_3 = \frac{0.125 - \big(1 + 0.25(0.5)(-0.40625)\big)}{2} = -0.4121094$$

$$Y_1 = Y_n + \frac{0.25}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$Y_1 = 1.0 + 0.04167(-0.5 + 2(-0.40625) + 2(-0.4121094) - 0.3234863) = 0 \cdot 8974915$$

Thus, you get the rest of the values, and the table below shows the calculated values:

xn	Yn+1				Zn+1
	h=1	$h=\frac{1}{2}$	$h=\frac{1}{4}$	$h=\frac{1}{8}$	
0	1.0	1.0	1.0	1.0	1.0
1.125				0.9432392	0.9432392
0.25			0.8974915	0.8974908	0.8974917
0.375				0.8620874	0.8320874
0.50			0.83644037	0.8364024	0.8364023

0.75		0.8364258	0.8118696	0.8118679	0.8118678
1.00	0.8203125		0.8195940	0.8195921	0.8195920
1.50		1.9171423	0.9171021	0.9170998	0.9170997
2.00	1.1045125	1.1036826	1.1036408	1.1036385	0.1036383
2.50		1.3595575	1.3955168	1.3595145	1.3595144
3.00	1.6701860	1,6694308	1.6693928	1.6693906	1.6693905

By integrating the equation $f(x,y)$ Using the Smithson method of

$\mathrm{x}-1$ to $\mathrm{x}_{\mathrm{i}+1}$ We get

$$\int_{x_{i-1}}^{x_{i+1}} f(x)dx = \int_{x_{i-1}}^{x_{i+1}} f(x,y)dx \quad \dots(1)$$

$$F(F(x_i+1)F(x_i-1) = \frac{w}{3}[f(x_{i-1}.y_{i-1}) + 4f$$

$(x_i.y_i) + f(x_{i+1}.y_{i+1})$ ... (2)

Assuming

Then $x_{i-1} \approx y_{i-1}$, $y_{i+1} \approx F(x_{i+1})$

the equation (2) becomes

$$y_{i+1} = y_{i+1} + \frac{w}{3}[2f(x_i,y_i) - f(x_{i-1},y_{i-1}) + 2f(x_{i+1}.y_{i-1})4(\dots)1$$

To find value y_{i+1}

$$Y_{i+1} = y_{i-3} + \frac{4w}{3}[2F(x,y) - F(x_{-1},y_{i-1}) + 2F(x_{1-2}.y_{1-})4(\dots)2$$

The two equations (3), (4) They are way of Milne and it is clear that the method of knowing four values $Y(i-3)$ $Y(i-2)$ $Y(i-1)$ Yi Therefore, And so the way of Milne that you do not need to use before to get it values (y) (Oppler for example).

That is why we find an uncommon method, although it gives good results. [7]

Results

1- Numerical methods are a science concerned with deriving and analyzing methods for finding numerical solutions to mathematical problems that are difficult to solve using analytical algebraic methods. In some problems, we cannot obtain an exact solution using conventional analytical methods and theories. In such problems, we resort to searching for algorithms, i.e., specific-step methods, to obtain numerical solutions that approximate exact solutions based on the given data.
2- The fourth-order Range-Kutta method is an uncommon method but gives good results.
3- The important flaws in Euler's method are that, while it's easy to see, the error is very large. The error tends to accumulate the further we move away from the point, the greater the discrepancy between the approach and the truth. This can be explained by the principle Euler used as the basis for his method: line segments that are parallel to the graph of a function are tangent to the points. This fact, incidentally, can be clearly seen from the figure.

Conclusions:

From the previous study of the subject of numerical methods for solving ordinary differential equations, a set of conclusions was reached.

1- Ordinary differential equations are of great importance in many fields in various sciences (physics, chemistry, engineering).
2- Numerical solutions of differential equations are resorted to when a closed-form expression cannot be found.
3- There are several numerical methods for solving ordinary differential equations, such as the Euler method, the Runge-Kutta method, and others.
4- A numerical solution represents an approximate solution to the differential equation, as it contains a margin of error.
5- A numerical solution represents an approximate solution to the differential equation, as it contains a margin of error.

Suggestions:

1- The student should have a clear idea of ordinary differential equations and partial differential equations.
2- The student must have prior knowledge of the rules of integration and the laws of derivation
3- Care and focus when choosing numerical methods when solving ordinary differential equations.
4- Continuous practice in solving ordinary differential equations using numerical methods raises the student's level in this field.

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