Estimating an Exponentiated Expanded Power Function Distribution Using an Artificial Intelligence Algorithm

Maysam Sajit Khudhair **Dr. Emad Hazim Aboudi**

University of Baghdad - College of Administration and Economics – Department of Statistics

الملخص:-

تم تقديم توزيع دالة القوة الأسية الموسعة EEPF بأربعة معلمات ، بطريقة التوسعة الاسية باستعمال توزيع موسع لدالة القوة ، تتميز هذه الطريقة في الحصول على توزيع جديد ينتمي لعائلة الاسية ، كما حصلنا على دالة معدل البقاء ومعدل الفشل لهذا التوزيع ، وتم ايجاد بعض الخصائص الرياضية، ثم استعملنا طريقة المربعات الصغرى المطورة لتقدير المعلمات باستعمال الخوارزمية الجينية وأجريت دراسة محاكاة مونت كارلو لتقيم أداء تقديرات الإمكان بواسطة خوارزمية الجينية GA.

الكلمات المفتاحية: التوسع الأسّي ، توزيع دالة القوة الموسع ، البقاء ، معدل الفشل ، المربعات الصغرى المطورة، الخوارزمية الجينية ، محاكاة مونت كارلو.

Abstract:-

The distribution of the expanded exponentiated power function EEPF with four parameters, was presented by the exponentiated expanded method using the expanded distribution of the power function, This method is characterized by obtaining a new distribution belonging to the exponential family, as we obtained the survival rate and failure rate function for this distribution, Some mathematical properties were found, then we used the developed least squares method to estimate the parameters using the genetic algorithm, and a Monte Carlo simulation study was conducted to evaluate the performance of estimations of possibility using the Genetic algorithm GA.

Keywords: exponentiated expanded, expanded power function distribution, survival, failure rate, developed least squares, genetic algorithm, Monte Carlo simulation.

1- Introduction

The main objective of construction probabilistic statistical models is to identify the appropriate model that adequately describes the data set obtained from experiments, observational studies, field surveys, etc., most of these model building techniques depend on finding the most appropriate probability distribution that explains the basic structure of the data set. However, there is no single probability distribution suitable for the different data set, which led to the expansion of the current classical distributions or the development of new distributions.

Expanding the distributions was done by adding one or more parameters to the probability distribution, and this method leads to the creation of a new distribution that is more flexible in representing survival data. From previous studies that dealt with this method in the expansion in 2015 (Samir K. Ashour and Mahmoud A. Eltehiwy)[8] gave a more generalized Lindley distribution that generalizes the previous two called the Lindley distribution of exponentiated power, 2017 (Fernado A. Pena-Ramirez, et al) {16} proposed a new model called the Weibull distribution of generalized power (EPGW), 2018 (Demet Aydin) {9} presented a new distribution study called the five-parameter inverse exponentiated Weibull distribution DTEIW with known cut-off points, in 2020 (Dawlah Al-Sulami) {7} proposed the exponentiated Weibull distribution (EEWD), some statistical properties of the proposed distribution were studied, 2020 (Maha A. Aldahlan, et al) {3} present the family of power-series generalized Weibull power series (EPGWPS) distributions, obtained by aggregating the series and power generalized Weibull power distributions, in 2021 (Mahmoud Afshari,et al) {2} introduced a new distribution called the . distribution Expanded yen exponentiated (EE-C).

2- Exponentiated Expanded Power Function (EEPF)

In the past few decades, the power function (PF) distribution has been commonly used to explain finite-rare data sets, and the (PF) distribution is a special case of the beta distribution and is also the inverse of the Pareto distribution{10,11}.

This distribution is preferred for the best fit compared to (exponential distribution, Weibull distribution, log-normal distribution and other distributions) due to its simplicity and applicability. The extender (EPF) is given by the following formula {19}:

$$g(x,\xi\,,\omega,\varphi)=\frac{\omega\varphi x^{\omega\varphi-1}}{\xi^{\ \omega\varphi}}\qquad 0<x<\xi\ ;\ \ \xi\,,\omega,\varphi>0\quad \ldots..(1)$$

Where ω,φ is the shape parameter and $\xi$is the scale parameter, and the cumulative distribution function (CDF) is given by the following formula:

$$G(x)=\left(\frac{x}{\xi}\right)^{\omega\varphi}\qquad 0<x<\xi\qquad \ldots\ldots(2)$$

A new distribution can be created using the extended exponentiated method by adding a new shape parameter to the cumulative function of the extended power distribution function (EPF). Thus, we can derive a new distribution with four parameters as follows:

$$F(x)=[G(x)]^{\vartheta}\qquad \ldots..(3)$$

Substituting formula (2) into (3), we get the following:

$$F(x)=\left[\left(\frac{x}{\xi}\right)^{\omega\varphi}\right]^{\vartheta}$$

$$F(x)=\left[\frac{x}{\xi}\right]^{\omega\varphi\vartheta}\qquad 0<x<\xi\qquad \ldots..(4)$$

By deriving formula (4), we get the probability density function of the new distribution, the distribution of the expanded exponential power function (EEPF), as follows:

$$f(x;\xi\,,\omega,\varphi,\vartheta)=\frac{\omega\varphi\vartheta x^{\omega\varphi\vartheta-1}}{\xi^{\ \omega\varphi\vartheta}}\qquad 0<x<\xi\ \ ;\ \ \xi\,,\omega,\varphi,\vartheta>0\qquad \ldots..(5)$$

The pdf function of the expanded exponential power function distribution has four parameters which are (ξ) the measurement parameter and the shape parameters are (ω,φ,ϑ) and the variable x depends on the measurement parameter.

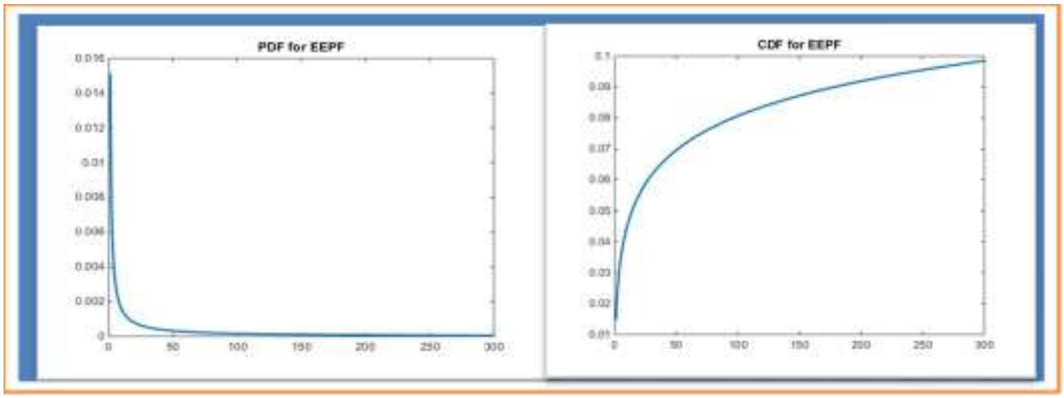

Figure (1-2) shows the CDF & PDF function for the new distribution (EEPF)

3- Properties of an Exponential Expanded Power Function Distribution

After obtaining the new probability distribution (EEPF), it is necessary to identify some of its properties as follows:-

1-mean

$$E(x) = \frac{\omega\varphi\vartheta\xi}{\omega\varphi\vartheta + 1} \qquad \ldots..(6)$$

2- variance

$$E(x - Ex)^2 = Var(x) = \frac{\omega\varphi\vartheta\xi^{\,2}}{(\omega\varphi\vartheta + 1)^2(\omega\varphi\vartheta + 2)} \qquad \ldots..(7)$$

3- standard deviation

$$\sigma = \frac{\xi\sqrt{\omega\varphi\vartheta}}{(\omega\varphi\vartheta + 1)\sqrt{\omega\varphi\vartheta + 2}} \qquad \ldots..(8)$$

4- Variation coefficient

$$C \cdot V = \frac{\sqrt{\omega\varphi\vartheta}}{\omega\varphi\vartheta\sqrt{\omega\varphi\vartheta + 2}} * 100 \qquad \ldots..(9)$$

4- Survival Function

It is the survival of the experimental unit for a period of not less than (x). In other words, if x is a random variable referring to the experimental unit, then $S(x)$ represents the probability of survival of the experimental unit for the next period. The survival function can be expressed by the following formula[11]:

$$S(x) = P_r(X \geq x) = \int_x^{\infty} f(u)du \qquad \ldots..(10)$$

$$= 1 - P_r(X \leq x)$$

$$S(x) = 1 - F(x) \qquad \ldots..(11)$$

Substituting formula (4) into formula (11), we get the general formula for the survival function for the new distribution (EEPF) as follows:

$$S(x) = 1 - \left(\frac{x}{\xi}\right)^{\omega\varphi\vartheta}$$

$$S(x) = \frac{\xi^{\ \omega\varphi\vartheta} - x^{\omega\varphi\vartheta}}{\xi^{\ \omega\varphi\vartheta}} \qquad \ldots..(12)$$

whereas

X: time of death

x: is the specified time between two time periods

5- Failure (Hazard Function)

It represents the percentage of failure or risk, which is the probability of failure in the subsequent period of time, knowing that the item was in good condition, and it is expressed mathematically as follows[14,19,17]:

$$h(x) = \frac{f(x)}{S(x)} \qquad \ldots..(13)$$

Substituting formula (5) and (12) into formula (13) we get the general formula for the failure rate function of the new distribution (EEPF) as follows:

$$h(x) = \frac{\omega\varphi\vartheta x^{\omega\varphi\vartheta-1}}{\xi^{\ \omega\varphi\vartheta} - x^{\omega\varphi\vartheta}} \qquad \ldots..(14)$$

The survival function is inversely proportional to the failure rate, and the failure rate function is directly proportional to time, as it increases with the increase in time.

The distribution is said to be IFR (increased failure rate) if h(x) is a non-decreasing function of x, and it is said to be DFR (decreased failure rate) if h(x) is non-increasing for x.

6- Parameters Estimation

6-1- Least Square Developed (LSD)

It is a method used to estimate the parameters of the probabilistic models by converting these models into the sum of squares of deviations formula (i.e., the formula adopted in the ordinary least squares method) [1,12].

Where we assume that $x_1, x_2, \ldots\ldots\ldots, x_n$ represents a random sample with a specific distribution $F(x_i)$and x_i represents the order statistics for the sample (See, Ashour et al. {8}).

$$E[F(x_i)] = E\big(P(X \le x_i)\big) = \frac{i}{n+1} \qquad \ldots..(15)$$

Using prediction on least squares developed (LSD) estimators, as follows:

$$\sum_{i=1}^{n} e_i^{\ 2} = \sum_{i=1}^{n}\left[F(x) - \frac{i}{n+1}\right]^2 \qquad \ldots..(16)$$

Substituting the formula (4) for the distribution (EEPF) into formula (16) we get

$$\sum_{i=1}^{n} e_i^{\ 2} = \sum_{i=1}^{n}\left[\frac{x^{\omega\varphi\vartheta}}{\xi^{\ \omega\varphi\vartheta}} - \frac{i}{n+1}\right]^2 \qquad \ldots..(17)$$

And by deriving formula (17) to find estimators of parameters for the new distribution (EEPF), and these are:

$$\frac{d\sum_{i=1}^{n} {e_i}^2}{d\xi} = 2\sum_{i=1}^{n}\left[\frac{x^{\omega\varphi\vartheta}}{\xi^{\ \omega\varphi\vartheta}} - \frac{i}{n+1}\right]\left[\frac{-\omega\varphi\vartheta x^{\omega\varphi\vartheta}}{\xi^{\ \omega\varphi\vartheta+1}}\right] \quad \ldots..(18)$$

$$\frac{d^2\sum_{i=1}^{n} {e_i}^2}{d\xi^{\ 2}} = 2\sum_{i=1}^{n}\left[\left(\frac{x^{\omega\varphi\vartheta}}{\xi^{\ \omega\varphi\vartheta}} - \frac{i}{n+1}\right)\left(\frac{\omega\varphi\vartheta x^{\omega\varphi\vartheta}(\omega\varphi\vartheta+1)}{\xi^{\ \omega\varphi\vartheta+2}}\right) + \left(\frac{\omega^2\varphi^2\vartheta^2 x^{2\omega\varphi\vartheta}}{\xi^{\ 2\omega\varphi\vartheta+2}}\right)\right] \quad \ldots..(19)$$

$$\frac{d\sum_{i=1}^{n} {e_i}^2}{d\omega} = 2\sum_{i=1}^{n}\left[\frac{x^{\omega\varphi\vartheta}}{\xi^{\ \omega\varphi\vartheta}} - \frac{i}{n+1}\right]\left[\frac{\varphi\vartheta x^{\omega\varphi\vartheta}(\ln x - \ln\xi\ )}{\xi^{\ \omega\varphi\vartheta}}\right] \quad \ldots..(20)$$

$$\frac{d^2\sum_{i=1}^{n} {e_i}^2}{d\omega^2} = 2\sum_{i=1}^{n}\left[\frac{\varphi^2\vartheta^2 x^{\omega\varphi\vartheta}(\ln x - \ln\xi\ )^2}{\xi^{\ \omega\varphi\vartheta}}\left(\frac{2x^{\omega\varphi\vartheta}}{\xi^{\ \omega\varphi\vartheta}} - \frac{i}{n+1}\right)\right] \quad \ldots..(21)$$

$$\frac{d\sum_{i=1}^{n} {e_i}^2}{d\varphi} = 2\sum_{i=1}^{n}\left[\frac{x^{\omega\varphi\vartheta}}{\xi^{\ \omega\varphi\vartheta}} - \frac{i}{n+1}\right]\left[\frac{\omega\vartheta x^{\omega\varphi\vartheta}(\ln x - \ln\xi\ )}{\xi^{\ \omega\varphi\vartheta}}\right] \quad \ldots..(22)$$

$$\frac{d^2\sum_{i=1}^{n} {e_i}^2}{d\varphi^2} = 2\sum_{i=1}^{n}\left[\frac{\omega^2\vartheta^2 x^{\omega\varphi\vartheta}(\ln x - \ln\xi\ )^2}{\xi^{\ \omega\varphi\vartheta}}\left(\frac{2x^{\omega\varphi\vartheta}}{\xi^{\ \omega\varphi\vartheta}} - \frac{i}{n+1}\right)\right] \quad \ldots..(23)$$

$$\frac{d\sum_{i=1}^{n} {e_i}^2}{d\vartheta} = 2\sum_{i=1}^{n}\left[\frac{x^{\omega\varphi\vartheta}}{\xi^{\ \omega\varphi\vartheta}} - \frac{i}{n+1}\right]\left[\frac{\omega\varphi x^{\omega\varphi\vartheta}(\ln x - \ln\xi\ )}{\xi^{\ \omega\varphi\vartheta}}\right] \quad \ldots..(24)$$

$$\frac{d^2\sum_{i=1}^{n} {e_i}^2}{d\vartheta^2} = 2\sum_{i=1}^{n}\left[\frac{\omega^2\varphi^2 x^{\omega\varphi\vartheta}(\ln x - \ln\xi\ )^2}{\xi^{\ \omega\varphi\vartheta}}\left(\frac{2x^{\omega\varphi\vartheta}}{\xi^{\ \omega\varphi\vartheta}} - \frac{i}{n+1}\right)\right] \quad \ldots..(25)$$

$$\frac{d^2\sum_{i=1}^{n} {e_i}^2}{d\xi\ \ d\omega} = 2\sum_{i=1}^{n}\left[\left(\frac{x^{\omega\varphi\vartheta}}{\xi^{\ \omega\varphi\vartheta}} - \frac{i}{n+1}\right)\left(\frac{-\varphi\vartheta x^{\omega\varphi\vartheta} - \omega\varphi^2\vartheta^2(\ln x - \ln\xi\ )}{\xi^{\ \omega\varphi\vartheta+1}}\right) + \frac{\omega\varphi^2\vartheta^2 x^{2\omega\varphi\vartheta}(\ln x - \ln\xi\ )}{\xi^{\ 2\omega\varphi\vartheta+1}}\right] \quad \ldots..(26)$$

$$\frac{d^2\sum_{i=1}^{n} {e_i}^2}{d\xi\ \ d\varphi} = 2\sum_{i=1}^{n}\left[\left(\frac{x^{\omega\varphi\vartheta}}{\xi^{\ \omega\varphi\vartheta}} - \frac{i}{n+1}\right)\left(\frac{-\omega\vartheta x^{\omega\varphi\vartheta} - \varphi\omega^2\vartheta^2(\ln x - \ln\xi\ )}{\xi^{\ \omega\varphi\vartheta+1}}\right) + \frac{\varphi\omega^2\vartheta^2 x^{2\omega\varphi\vartheta}(\ln x - \ln\xi\ )}{\xi^{\ 2\omega\varphi\vartheta+1}}\right] \quad \ldots..(27)$$

$$\frac{d^2\sum_{i=1}^{n} {e_i}^2}{d\xi\ \ d\vartheta} = 2\sum_{i=1}^{n}\left[\left(\frac{x^{\omega\varphi\vartheta}}{\xi^{\ \omega\varphi\vartheta}} - \frac{i}{n+1}\right)\left(\frac{-\omega\vartheta x^{\omega\varphi\vartheta} - \vartheta\omega^2\vartheta^2(\ln x - \ln\xi\ )}{\xi^{\ \omega\varphi\vartheta+1}}\right) + \frac{\vartheta\omega^2\varphi^2 x^{2\omega\varphi\vartheta}(\ln x - \ln\xi\ )}{\xi^{\ 2\omega\varphi\vartheta+1}}\right] \quad \ldots..(28)$$

$$\frac{d^2\sum_{i=1}^{n} {e_i}^2}{d\omega\, d\varphi} = 2\sum_{i=1}^{n}\left[\left(\frac{x^{\omega\varphi\vartheta}}{\xi^{\ \omega\varphi\vartheta}} - \frac{i}{n+1}\right)\left(\frac{\vartheta x^{\omega\varphi\vartheta}(\ln x - \ln\xi\ ) + \omega\varphi\vartheta^2 x^{\omega\varphi\vartheta}(\ln x - \ln\xi\ )^2}{\xi^{\ \omega\varphi\vartheta}} + \frac{\omega\varphi\vartheta^2 x^{2\omega\varphi\vartheta}(\ln x - \ln\xi\ )^2}{\xi^{\ 2\omega\varphi\vartheta}}\right)\right] \quad \ldots..(29)$$

$$\frac{d^2\sum_{i=1}^{n} {e_i}^2}{d\omega\, d\vartheta} = 2\sum_{i=1}^{n}\left[\left(\frac{x^{\omega\varphi\vartheta}}{\xi^{\ \omega\varphi\vartheta}} - \frac{i}{n+1}\right)\left(\frac{\varphi x^{\omega\varphi\vartheta}(\ln x - \ln\xi\ ) + \omega\vartheta\varphi^2 x^{\omega\varphi\vartheta}(\ln x - \ln\xi\ )^2}{\xi^{\ \omega\beta\theta}} + \frac{\omega\vartheta\varphi^2 x^{2\omega\varphi\vartheta}(\ln x - \ln\xi\ )^2}{\xi^{\ 2\omega\varphi\vartheta}}\right)\right] \quad \ldots..(30)$$

$$\frac{d^2\sum_{i=1}^{n} {e_i}^2}{d\varphi\, d\vartheta} = 2\sum_{i=1}^{n}\left[\left(\frac{x^{\omega\varphi\vartheta}}{\xi^{\ \omega\varphi\vartheta}} - \frac{i}{n+1}\right)\left(\frac{\omega x^{\omega\varphi\vartheta}(\ln x - \ln\xi\ ) + \varphi\vartheta\omega^2 x^{\omega\varphi\vartheta}(\ln x - \ln\xi\ )^2}{\xi^{\ \omega\varphi\vartheta}} + \frac{\varphi\vartheta\omega^2 x^{2\omega\varphi\vartheta}(\ln x - \ln\xi\ )^2}{\xi^{\ 2\omega\varphi\vartheta}}\right)\right] \quad \ldots..(31)$$

We will use the method of artificial intelligence genetic algorithm to estimate nonlinear parameters.

6-2- Genetic Algorithm (GA)

This algorithm is considered one of the important modern techniques in the field of searching for the optimal solution from among the available solutions by passing the good characteristics of the sequential generation processes, the production of optimal offspring and the repetition of cycles to improve the offspring with modern phases and patterns.

6-2-1- Steps to Apply The Genetic Algorithm

The steps of the genetic algorithm are as follows:

1- The beginning {6}: is a random population enumeration of chromosomes (spatial search), that is, it is the set of solutions to the problem.

2- Initialization {6}: It is the creation of the primary generation, which includes the generation of random chromosomes with the size of the community and according to the nature of the problem being studied.

3- The evaluation function (Fitness Function) {4,18}: It is the function whose results give the probability of an individual entering the test and inheriting his characteristics, and that the optimal solutions give a greater chance of entering into the process of reproduction and inheritance of characteristics or change, and the value of this function is calculated for each chromosome It is based on the nature of the objective function. When the objective function is a reduction, the evaluation function is as follows:

Fitness Function (String) = (objective function (String i))*(-1)
But if the objective function is maximization, then the evaluation function is as follows:
Fitness Function (String) = objective function (String i)

4- Selection [18]: All successive generations have a percentage of the current chromosomes, which are selected for the production of a new generation. These chromosomes are selected depending on the optimal function, where the percentage of selection is optimal. There is another way by choosing a random set of chromosomes, but this process may It takes a very long time, and here are a group of ways to choose:

- Roulette wheel method [5]: This method depends on the best chromosomes in the population, that is, depending on the efficiency of each chromosome, by calculating the probability of sharing (Probability Sheering) for each chromosome from the community, which in turn depends on the fitness function, where The probability of participation for a chromosome is calculated from the product of dividing the chromosome evaluation function (f_i) (by the sum of the values of the evaluation functions (population members) as in the following mathematical formula:

$$P_i = \frac{f_i}{\sum_{i=1}^{n} f_i}$$

Where as
P_i: Chromosome (i) probability
f_i: the value of the value function of the chromosome (i)
n: the number of chromosomes in the current generation.

- Stochastic Universal Sampling[14]: It is very similar to the roulette wheel, but instead of choosing one fixed point, multiple points are chosen, thus giving an opportunity for all chromosomes to be one of the parents.

5- Reproduction [5]: It is a new generation of paternal chromosomes that have been selected from the process of selection. In its turn, these chromosomes go through two processes, crossover and mutation to produce a new generation of chromosomes (sons).

6- Process the Crossover [18]: It takes place through the mating of every two fathers to produce two new sons, and this process continues until a new set of chromosomes is found added to the group of fathers, and there are many methods of cross-breeding, namely:

- Point only crossover [15]: The cross is carried out by replacing either a point from a specific place of one parental chromosome against a point from the same place from the second paternal chromosome to obtain new offspring.
- Crossover with more than one point (Multi Point crossover) [15]: Replacement of several points from a specific place of the chromosome of one of the parents in exchange for the same number of points of the second paternal chromosome in order to obtain new offspring.
- Crossover of cut and aplice [18]: In this method, the process of cross-breeding is done by cutting the genes in the first chromosome from a site different from the site of the cut in the second chromosome, and this in turn will lead to a difference in chromosome lengths.

7- Mutation [5]: They are random changes in the genes of chromosomes through a change in one or more genes in the (son) chromosome, in order to obtain a new solution and reach the optimal solution, which leads to the chromosomes retaining the good traits between the genes.

Changes in the mutation process do not occur on the chromosomes resulting from offspring, but rather new chromosomes are generated and produced in the cloning process, and the evaluation function is applied to them and a new generation is produced.

8- Termination of the genetic algorithm {4}: The algorithm is terminated when the Reaching the optimal solution.

7- Simulation

The Monte Carlo method is one of the most important of these methods, which is considered the most common and widely used in the search and analysis of parameter capabilities in several ways for the model under study.

7-1- Stages of Simulation

1- Setting initial values, and this stage is very important for other stages to depend on .
2- Random observations (data) are generated, which follow the new distribution of the EEPF represented by four parameters: (α) measurement parameter and (λ, φ, ϑ) parameters.
3- The model parameters for the new distribution are estimated by the developed least squares method (LSD), using the genetic algorithm (GA) to estimate the four parameters.
4- Estimation of the survival and failure rate of the new distribution of the expanded exponential force function (EEPF) using LSD estimators.
5- To reach the best estimate of the survival and failure (risk) criterion, the mean squared integral error (MISE) criterion is used, which is the integration of the total area of xi and its reduction to a single value that represents a year of total time and is calculated according to the following formula{13}:

$$MISE\left(\hat{S}(x)\right) = \frac{1}{r}\sum_{i=1}^{r}\left[\frac{1}{n_x}\sum_{j=1}^{n_x}\left(\hat{S}(x) - s(x_j)\right)^2\right] \quad \ldots\ldots(32)$$

$$MISE\left(\hat{h}(x)\right) = \frac{1}{r}\sum_{i=1}^{r}\left[\frac{1}{n_x}\sum_{j=1}^{n_x}\left(\hat{h}(x) - h(x_j)\right)^2\right] \quad \ldots\ldots(33)$$

Where as :

r: the number of repetitions of the experiment (1000) times.

n_x: the number of data generated for each sample.

$\hat{S}(x), \hat{h}(x)$: the estimated survival and failure functions, respectively.

$s(x_j), h(x_j)$: the function of survival and failure according to the initial values and respectively.

Table (7-1) represents the simulation results using the LSD genetic algorithm

LSD								
N	$\xi = 10$	$\varphi = 2.5$	$\omega = 3$	$\vartheta = 5.5$	Survival rate	MISE(SF)	failure rate	MISE(HF)
	$\hat{\xi}$	$\hat{\varphi}$	$\hat{\omega}$	$\hat{\vartheta}$				
١٥	9.2027	2.5000	2.9999	5.5000	0.4926	2.0438E-07	1.2071	2.4317E-06
٥٠	9.2002	2.4969	2.9985	5.4996	0.4146	5.3182E-07	1.0579	1.2422E-08
١٠٠	9.2017	2.4999	3.0000	5.4999	0.4865	5.5711E-10	1.5070	9.2118E-09

N	$\xi = 10$	$\varphi = 2.5$	$\omega = 3$	$\vartheta = 6$	Survival rate	MISE(SF)	failure rate	MISE(HF)
	$\hat{\xi}$	$\hat{\varphi}$	$\hat{\omega}$	$\hat{\vartheta}$				
١٥	9.0012	2.4999	2.9998	5.9999	0.3551	5.1074E-07	1.7090	3.0548E-06
٥٠	9.0011	2.5000	2.9989	5.9995	0.4697	1.8450E-11	1.3875	1.3923E-08
١٠٠	9.0010	2.4982	3.0000	6.0000	0.4687	2.7126E-10	1.4671	5.5032E-08
N	$\xi = 10$	$\varphi = 2.5$	$\omega = 5$	$\vartheta = 7.5$	Survival rate	MISE(SF)	failure rate	MISE(HF)
	$\hat{\xi}$	$\hat{\varphi}$	$\hat{\omega}$	$\hat{\vartheta}$				
١٥	9.8001	2.4989	4.9999	7.4998	0.3761	5.5733E-08	3.0189	5.9790E-06
٥٠	9.8002	2.4998	4.9999	7.4941	0.4720	5.5276E-11	1.2591	8.0391E-08
١٠٠	9.8001	2.5000	4.9954	7.5000	0.4852	2.5440E-08	1.3013	4.8443E-08
N	$\xi = 10$	$\varphi = 3$	$\omega = 4$	$\vartheta = 6$	Survival rate	MISE(SF)	failure rate	MISE(HF)
	$\hat{\xi}$	$\hat{\varphi}$	$\hat{\omega}$	$\hat{\vartheta}$				
١٥	9.0418	2.9999	3.9979	5.9982	0.4957	0.4957	1.2339	8.5644E-06
٥٠	8.8910	2.9984	3.9999	5.9981	0.4577	1.3079E-09	1.1274	2.1734E-08
١٠٠	8.8180	2.9946	3.9993	5.9795	0.5116	1.2812E-09	1.1418	3.1437E-06
N	$\xi = 11$	$\varphi = 2.5$	$\omega = 3$	$\vartheta = 5.5$	Survival rate	MISE(SF)	failure rate	MISE(HF)
	$\hat{\xi}$	$\hat{\varphi}$	$\hat{\omega}$	$\hat{\vartheta}$				
١٥	9.9058	2.9999	4.0000	5.9998	0.5087	1.7520E-08	0.9150	8.2399E-06
٥٠	10.0001	2.4975	2.9985	5.4998	0.4794	2.8483E-10	1.0967	4.2503E-09
١٠٠	10.0015	2.5000	3.0000	5.4998	0.4889	6.6090E-14	0.8580	5.1023E-07
N	$\xi = 11$	$\varphi = 3$	$\omega = 4$	$\vartheta = 6$	Survival rate	MISE(SF)	failure rate	MISE(HF)
	$\hat{\xi}$	$\hat{\varphi}$	$\hat{\omega}$	$\hat{\vartheta}$				
١٥	11.2	2.5	3.5	5.5	0.5747	1.0161E-08	0.7602	9.1310E-09
٥٠	11.2	3.0000	3.9999	5.9999	0.5090	2.3357E-10	0.6728	3.4806E-06
١٠٠	11.2	2.9999	4.0000	6.0000	0.4648	4.8631E-09	1.0198	1.3019E-08

Table (7-2) represents the simulation results using the LSD genetic algorithm

LSD								
N	$\xi = 11$	$\varphi = 3$	$\omega = 4$	$\vartheta = 7.5$	Survival rate	MISE(SF)	failure rate	MISE(HF)
	$\hat{\xi}$	$\hat{\varphi}$	$\hat{\omega}$	$\hat{\vartheta}$				
١٥	10.9928	3.0000	4.0000	7.49999	0.4747	1.3291E-08	1.0072	1.1787E-07
٥٠	10.9842	3.0000	3.9984	7.49995	0.4982	5.5264E-11	1.2266	5.4168E-09
١٠٠	10.9891	2.9989	4.0000	7.50000	0.4775	5.9502E-11	1.2895	3.6961E-09
N	$\xi = 11$	$\varphi = 3$	$\omega = 5$	$\vartheta = 7.5$	Survival rate	MISE(SF)	failure rate	MISE(HF)
	$\hat{\xi}$	$\hat{\varphi}$	$\hat{\omega}$	$\hat{\vartheta}$				
١٥	10.9010	3.0000	5.0000	7.4998	0.3598	3.8786E-09	1.6731	1.3758E-06

N								
٥٠	10.9000	2.9999	4.9979	7.5000	0.4402	1.4434E-09	1.9842	1.2510E-09
١٠٠	10.9001	2.9989	5.0000	7.4948	0.4177	5.2813E-10	1.2378	1.0756E-08
N	$\xi = 12$	$\varphi = 3$	$\omega = 4$	$\vartheta = 6$	**Survival rate**	**MISE (SF)**	**failure rate**	**MISE (HF)**
	$\hat{\xi}$	$\hat{\varphi}$	$\hat{\omega}$	$\hat{\vartheta}$				
١٥	11.9516	3.19860	4.0999	5.9999	0.4368	5.8018E-08	1.1703	6.1633E-08
٥٠	11.9723	3.19999	4.1000	5.9999	0.4635	2.5351E-10	2.0315	7.9269E-09
١٠٠	11.9869	3.19998	4.0999	5.9988	0.5178	2.0142E-12	1.3371	1.5261E-08
N	$\xi = 12$	$\varphi = 3.5$	$\omega = 3$	$\vartheta = 5.5$	**Survival rate**	**MISE (SF)**	**failure rate**	**MISE (HF)**
	$\hat{\xi}$	$\hat{\varphi}$	$\hat{\omega}$	$\hat{\vartheta}$				
١٥	11.9737	3.4999	2.9999	5.4993	0.5010	4.7519E-11	1.0304	1.6943E-07
٥٠	11.9689	3.4978	2.9999	5.4994	0.4388	1.5893E-09	1.6686	1.2093E-06
١٠٠	11.9763	3.5000	2.9999	5.5000	0.4845	2.6324E-12	1.1725	6.5651E-12
N	$\xi = 12$	$\varphi = 3.5$	$\omega = 4$	$\vartheta = 6$	**Survival rate**	**MISE (SF)**	**failure rate**	**MISE (HF)**
	$\hat{\xi}$	$\hat{\varphi}$	$\hat{\omega}$	$\hat{\vartheta}$				
١٥	11.9984	3.5000	3.9975	6.0000	0.4996	1.9192E-07	1.2030	1.6284E-06
٥٠	11.9811	3.5000	3.9986	5.9997	0.4485	3.9098E-09	1.7063	3.3430E-07
١٠٠	11.9865	3.4999	3.9999	5.9988	0.4642	2.5440E-10	1.1450	3.5846E-10
N	$\xi = 12$	$\beta = 3.5$	$\lambda = 5$	$\theta = 7.5$	**Survival rate**	**MISE (SF)**	**failure rate**	**MISE (HF)**
	$\hat{\xi}$	$\hat{\varphi}$	$\hat{\omega}$	$\hat{\vartheta}$				
١٥	11.9962	3.4989	4.9999	7.4994	0.3215	4.5401E-07	2.1021	1.859E-07
٥٠	11.9824	3.4989	4.9960	7.4947	0.4621	1.2168E-09	1.2662	3.014E-08
١٠٠	11.9883	3.5000	5.0000	7.5000	0.4721	5.9333E-10	1.0420	4.209E-08

Table (7-1), (7-2) represents the simulation results for the developed least squares method LSD using the GA algorithm to distribute the exponential expanded power function (EEPF).

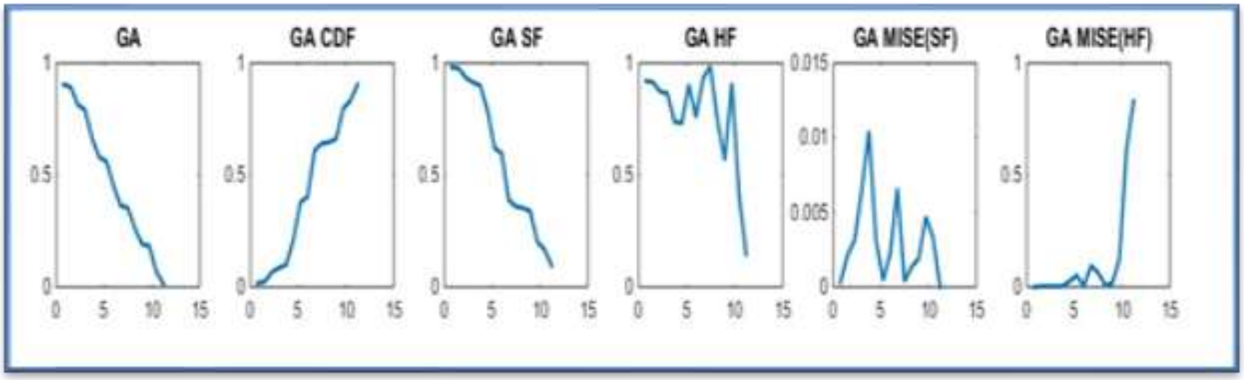

Figure (7-1) shows the PDF&CDF, survival and failure (LSD) function of the EEPF distribution.

8- Conclusions

Parameter estimates were approximate to the initial values assumed for the EEPF distribution of the GA algorithm at the volumes assigned to each model .We noticed that the hypothetical model (ξ=11,φ=3,ω=4,ϑ=6) in Table (7-1) when n=15 is the best model as it had a high survival rate and a low risk rate as well as the MISE criterion for survival rate and rate Low failure .We find that the survival rate was close to half as well as the failure rate is relatively close to its stability at (n = 50,100) and the MISE criterion for both survival and failure rate is low, and this indicates the efficiency of LSD method estimates , As for the results of the application of the developed least squares method (LSD) of the (GA) algorithm, We conclude that the proposed new distribution was efficient in representing survival data.

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