

A Proposed Method by Using the Adjacency Matrix to Solve the Edges Deletion Problem of Complete Graphs

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Abstract— The effectiveness and dependability of a network can be examined using graph theory techniques, and the dependability of the network can be evaluated using its connection. Removing some vertices or edges can enhance a network's diameter, which is a measure of its efficiency. In this paper, we calculate the maximum diameter $f_n(tt)$ of the modified graph G with n ($n \geq 4$) vertices, which resulted from removing t edges from the complete graph K_n , ($n \geq 4$) was determined using the adjacency matrix $adj(K_n)$.

Keywords— Edges deletion problem; Maximum diameter; Complete graph; Adjacency matrix .

I. INTRODUCTION

Due to its applications in a variety of mathematical and scientific fields, including biology, engineering, and social science, graph theory has seen a recent increase in popularity [6]. A graph is the mathematical representation of the network. In the majority of studies, a graph usually refers to a mathematical model, while a network usually refers to an actual system [2]. Graphs are well described by the concepts of adjacency lists and adjacency matrices, which makes them useful. Adjacency matrices are typically used to describe networks in order to study them using graph theory. A graph is saved on a computer as an adjacency matrix [3]. A key idea in communication network design is fault tolerance, which quantifies the maximum message delay that a message can encounter while moving between any two network vertices [4]. Communication is a key factor in network architecture. Because of this, a robust network is difficult to break and should continue to send and receive data even if nodes or links fail [5]. Graphs that can endure severe shocks when mistakes occur on the nodes or links are typically preferred by network designers [6]. A measure of network efficiency called diameter was used to examine the effects of connection failures in networks. Knowing how a graph is altered by deleting edges and vertices is helpful because a network represented by one of these graphs may become less effective. Adding or deleting edges can alter a graph's diameter [7].

Several authors have looked into how a graph's width changes when edges are removed. As an example,

The impact of decreasing and increasing edges on the diameter of specific hypercube graphs was investigated by Harary and Graham [8]. Bouabdallah et al. talked about altering a hypercube graph's diameter after deleting its maximum number of edges [9]. If D is an odd or even (greater than 2) diameter, a lower bound for the greatest diameter was determined by Schoone and associates and provided by $(D - 1)E + D$ or $E(D - 2) + D + 2$ respectively. They found an upper bound for $(ED + D)$. They also take the entire graph complete graph (K_n) as the particular case [10]. Najim and Jun-Ming investigated the maximum diameter $f(E, D)$ of the connected graph G , which is created by removing E edges. They demonstrated that $f(E, D) \geq D + (D - 1)E + 1$ were $7 + E \geq D \geq 4 + E$ and $E > 3, D = 2m + E(2m - 1)$ and $m \geq 1$, as well as $E = 4$ and $E = 1 + 10m$ [11]. Jasim and Najim were able to determine the precise values of the maximum diameter of a modified hypercube graph by eliminating specific edges from it while preserving the graph's connectedness [12].

The greatest diameter of the graph $GP(n, k, t)$ was determined by Jasim and Najim by removing t edges from $GP(n, k)$ with $k = 1, n \geq 3$ (where the generalized Petersen graph is denoted by $GP(n, k)$) [13]. Jasim and Najim looked into how the diameter of a full graph expanded after deleting a specific edge. By removing t edges from a complete graph, they were able to display the precise values of $f(t, d)$ for each connected graph G_i with n vertices and $i \geq 1$, accomplished by removing t edges from an entire graph [14].

This study proposes a new approach to use the adjacency matrix of K_n , ($n \geq 4$) to find the values of $f_n(tt)$ denotes the maximum diameter of an altered connected graphs G_i , $i \geq 1$ with order n . And G_i , $i \geq 1$ obtained from a graph G after deleting tt edges. A graph G obtained from a complete graph K_n , ($n \geq 4$) after deleting t edges.

II. PRELIMINARY

A graph G is a simple, undirected graph, containing $E(G)$ as the set of its edges and $V(G)$. Assume that $d(a_1, a_2)$ stands for the distance between the vertices a_1 and a_2 , then the greatest $d(a_1, a_2)$ in G is referred to as the graph's diameter G , denoted by D , such that $D = \max(d(a_1, a_2))$ and $a_1, a_2 \in V(G)$ [12]. Let $\{e\}$ be a set of edges in a graph G . Then the altered graph AG results by deleting $\{e\}$ from G and $G/\{e\} = AG$. The subgraph AG has the set of edges $E(G/\{e\}) = E(AG)$ and the set of vertices $V(G) = V(AG)$ [15]. A complete graph is an undirected and simple graph. Each pair of unique vertices in k_n are adjacent, it contains the greatest quantity of edges in all simple graphs. Note that k_n has n vertices and $\frac{1}{2}n(n-1)$ edges. A complete graph is $(n-1)$ regular graph with diameter $d = 1$ [16].

III. ADJACENCY MATRIX

In computer science and mathematics, an adjacency matrix is a method for displaying which vertices of a graph are next to which other vertices. In graph G , the adjacency matrix is dependent on the vertex ordering, that is a different adjacency matrix could be produced by a different vertex ordering [3]. In an undirected graph G of order n the adjacency matrix is a symmetric square matrix denoted by $adj(G)$, i.e. $adj(G) = [b_{ij}]$ for all $i, j \geq 1$ and $i \neq j$ [17]. Elements of $adj(G)$ are 0 or 1 and can be represented by the following:

$$b_{ij} = \begin{cases} 1 & \text{if } e \in E(G), e = (i, j) \\ 0 & \text{if } e \notin E(G), e = (i, j). \end{cases}$$

A. Adjacency matrix of complete graph

The following examples display adjacency matrices of complete graphs k_4 and k_5 .

$$adj(k_4) = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}, adj(k_5) = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

IV. EDGES DELETION PROBLEM OF COMPLETE GRAPH

Numerous researchers have tried using a range of techniques to calculate the graph's maximum diameter after removing a specific number of edges from the complete graph k_n while maintaining the resulting graph's connectivity, such as: Jasim and Najim determined the maximal diameter $f_n(tt)$ of altered connected graphs G_i , $i \geq 1$ obtained from a graph G after deleting tt edges. A graph G obtained from a k_n after deleting t edges and the maximal diameter of a graph G denoted by $f_n(t)$ and $f_n(t) \geq 2$. All graphs $(G_i, i \geq 1, G, k_n)$ with order n and $n \geq 4$. The Lemma (4.1), is given the general formula of $f_n(tt)$ [14].

Lemma (4.1):

If $1 \leq tt \leq \frac{1}{2}(n-1)(n-2) - 1$, and $n \geq 4$, then:

$$f_n(tt) = \left\lceil n + \frac{1}{2} - \sqrt{\left(n - \frac{3}{2}\right)^2 - 2tt} \right\rceil.$$

The examples that follow demonstrate the value of $f_n(tt)$ after edges deletion from k_n , $n \in \{4, 5\}$. Figure (1), demonstrates the edge deletion of complete graph k_4 [14].

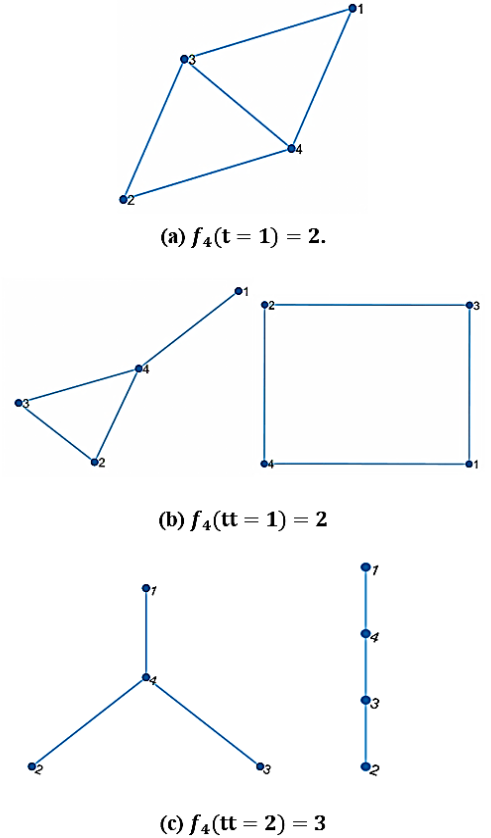


Fig. 1. (a) G obtained from K_4 . (b) and (c) G_i , $1 \leq i \leq 4$ obtained from G .

Figure (2), demonstrates the edge deletion of complete graph K_5 [14].

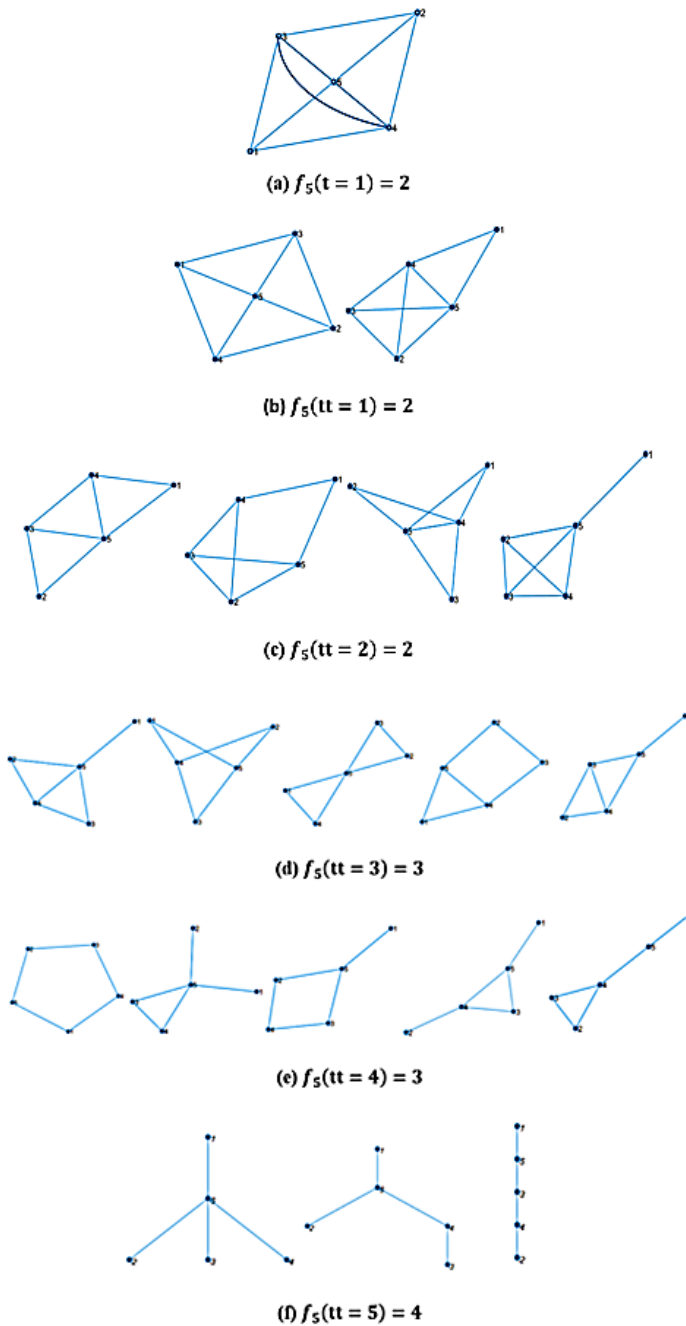


Fig. 2. (a) G obtained from K_5 . (b),(c),(d),(e) and (f) G_i , $1 \leq i \leq 19$ obtained from G .

V. PROPOSED METHOD OF EDGES DELETION PROBLEM

In this section, we studied how removing edges impacts the diameter of a complete graph by using a new proposed method to solve edges deletion problems. A proposed method finds $f_n(tt)$ of an altered graph derived from a

complete graph after a number of edges were removed by using the adjacency matrix of K_n . The following steps provide a summary of the suggested method's methodology:

- **Step (1).** determine the $adj(K_n)$ for K_n .
- **Step (2).** convert $adj(K_n)$ into upper triangular matrix, denoted by $up(adj(K_n))$.
- **Step (3).** calculate $\sum_{j=1}^n b_{ij}$ for each row i , denoted by $sumr_i = \sum_{j=1}^n b_{ij}$ with $1 \leq i \leq n$.
- **Step (4).** determine $sumr_i = 1$ with $1 \leq i \leq n$. Then in step (3) we note that the number of the formulas ($sumr_i = 1$) equal to diameter of K_n .
- **Step (5).** Process of deleting tt edges from K_n with:

$1 \leq tt \leq \frac{1}{2}(n-1)(n-2) - 1$ with $n \geq 4$, and figuring out the sum of ($sumr_i = 1$) with $1 \leq i \leq n$ after each deletion. The deletion process is done by: deleting one edge at a time.

Compared with the results in [14], note that the summation of the formula ($sumr_i = 1$, $1 \leq i \leq n$) equal to $f_n(tt)$ after the deletion process.

VI. EXAMPLES ILLUSTRATING THE APPLICATION OF THE PROPOSED METHOD

By using the proposed method, we can determine the value $f_n(tt)$ values of an altered graph that is produced when a specific number of edges from K_4 and K_5 are removed while the resulting graph is still connected.

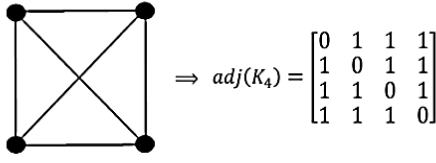
A. Illustrating the proposed method for K_4

Applying the edges deletion process in K_4 and finding $f_4(tt)$ using the proposed method.

- **Step (1).** determine the $adj(K_4)$ for K_4 , see Figure (3a).
- **Step (2).** convert $adj(K_4)$ into upper triangular matrix, denoted by $up(adj(K_4))$, see Figure (3b).
- **Step (3).** calculate $\sum_{j=1}^4 b_{ij}$ for each row i , denoted by $sumr_i = \sum_{j=1}^4 b_{ij}$ with $1 \leq i \leq 4$.
- **Step (4).** determine $sumr_i = 1$, $1 \leq i \leq 4$. Then in Step (3). we note that the number of the formula ($sumr_i = 1$) equal to diameter of K_n , see Figure (3c).
- **Step (5).** process of deleting $1 \leq tt \leq 2$ edges from K_4 , see Figure (3d).

Note that the formula ($sumr_i = 1$, $1 \leq i \leq 4$) in Figure (3c), equal to 1. Next, by applying the edges deletion process in Step (5) and finding $f_4(tt)$ for an altered connected graph It was acquired following the deletion of

($1 \leq tt \leq 2$) edges from k_4 , which can be shown in Figure (3d). Figure (3), displays the outcomes of the previous steps.



(a) Step (1).

$$adj(K_4) = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \Rightarrow up(adj(K_4)) = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(b) Step (2).

$$up(adj(K_4)) = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{matrix} sumr_1 = 3 \\ sumr_2 = 2 \\ sumr_3 = 1 \\ sumr_4 = 0 \end{matrix}$$

(c) Step (3,4).

	$tt = 0$	$tt = 1$	$tt = 2$
	$t = 1$		
$sumr_1 = 3$	3	2	1
$sumr_2 = 2$	2-1	1	1
$sumr_3 = 1$	1	1	1
$sumr_4 = 0$	0	0	0
$\sum_{i=1}^4 (sumr_i = 1)$	2	2	3
$f_4(tt)$	2	2	3

(d) Step (5).

Fig. 3. The application of the proposed method of K_4 .

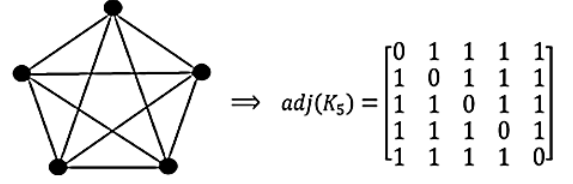
B. Illustrating the proposed method for K_5

Applying the edges deletion process in K_5 and finding $f_5(tt)$ using the proposed method.

- **Step (1).** determine the $adj(K_5)$ for K_5 , see Figure (4a).
- **Step (2).** convert $adj(K_5)$ into a upper triangular matrix, denoted by $up(adj(K_5))$, see Figure (4b).
- **Step (3).** calculate $\sum_{j=1}^5 b_{ij}$ for each row i , denoted by $sumr_i = \sum_{j=1}^5 a_{ij}$ with $1 \leq i \leq 5$.
- **Step (4).** determine $sumr_i = 1$ with $1 \leq i \leq 5$. Then in Step (3) we note that the number of the formulas ($sumr_i = 1$) equal to diameter of K_5 , see Figure (4c).
- **Step (5).** process of deleting $1 \leq tt \leq 5$ edges from K_5 , see Figure (4d).

Note that the formula ($sumr_i = 1$, $1 \leq i \leq 5$) in Figure (4c), equal to 1. Next, by applying the edges deletion

process in Step (5) and finding $f_5(tt)$ for an altered connected graph It was acquired following the deletion of ($1 \leq tt \leq 5$) edges from k_5 , which can be shown in Figure (4d). Figure (4), displays the outcomes of the previous steps.



(a) Step (1).

$$adj(K_5) = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix} \Rightarrow up(adj(K_5)) = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(b) Step (2).

$$up(adj(K_5)) = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{matrix} sumr_1 = 4 \\ sumr_2 = 3 \\ sumr_3 = 2 \\ sumr_4 = 1 \\ sumr_5 = 0 \end{matrix}$$

(c) Step (3,4).

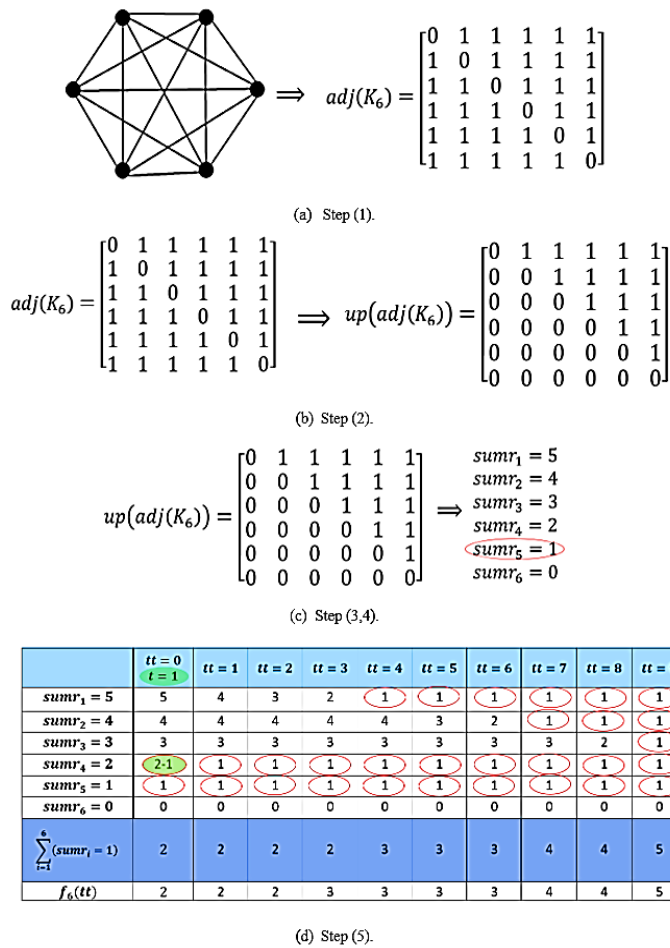
	$tt = 0$	$tt = 1$	$tt = 2$	$tt = 3$	$tt = 4$	$tt = 5$
	$t = 1$					
$sumr_1 = 4$	4	3	2	1	1	1
$sumr_2 = 3$	3	3	3	3	2	1
$sumr_3 = 2$	2-1	1	1	1	1	1
$sumr_4 = 1$	1	1	1	1	1	1
$sumr_5 = 0$	0	0	0	0	0	0
$\sum_{i=1}^5 (sumr_i = 1)$	2	2	2	3	3	4
$f_5(tt)$	2	2	3	3	3	4

(d) Step (5).

Fig. 4. The application of the proposed method of K_5 .

C. Illustrating the proposed method for K_6

Applying the edges deletion process in K_6 and finding $f_6(tt)$ using the proposed method as shown in Figure (5).

Fig. 5. The application of the proposed method of K_6 .

In the same way as in the above examples in Figures (3),(4) and (5), the value of $f_n(tt)$, $n \geq 4$ can be obtained using the proposed method.

VII. CONCLUSION

The paper's findings provide a method for calculating the maximum diameter of complete graph k_n , $n \geq 4$ in a shorter amount of time. In comparison to the method proposed by Anwar and Alaa in [13]. Actually, the utilization of the adjacency matrix connected to complete graph is what speeds up this process. We demonstrate our suggested methodology in section (VI) using the complete graph k_4 , k_5 and k_6 instances.

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