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Comparing Different Fuzzy estimators of Reliability Function For two parameters Weibull distribution

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Abstract

This Paper deals With Comparing different estimators of Fuzzy Reliability Function of Two Parameters Weibull (f (t , p , θ)) , were first we estimate (p , θ) by Maximum likelihood Methods and Moments Method , and Method of percentile, and Compare decimators of Fuzzy Reliability taking different Sample Size and initial values of (p , θ)

Introduction

The Live Times Of machines are Considered random Variables , have probability distribution With Crisp Parameters . But in Many Situations These Parameters are difficult to determined due to uncertainties and imprecision of data . According to this the Reliability of this distribution of life time , may be also of fuzzy type which may be encoded as triangular or trapezoidal number

Fuzzy Reliability Function based on fuzzy number , and then , Many Researcher presented Fuzzy System Failure engineering (cai,k.y.1996) and Also (Cai , wen and Zhang (1991)[1] , Fuzzy introduce Variables as abasis for a theory of Fuzzy Reliability Function Also utkin (1999) discussed imprecise reliability Models for general life time distribution .

In 1996 (Cai,k.y)[2] present Fuzzy System Failure engineering Model and Also (Cai , Wen and Zhang 1991)[1] Introduce .

Fuzzy Variables as abasis for a theory of Fuzzy Reliability Function .

In our research We continue the Work of Comparing Fuzzy estimators of Reliability Function by Method of Maximum likelihood and Method of Moments and the third Method Analysis . is Method of Percentile Which depend on principle of Regression .

We Know that the life times of Units or System , are assumed to be random variables , have Probability distribution Called (life time density Function)
Have crisp parameters , and crisp hazard rate , and crisp reliability Function, but some times the parameters cannot be estimators perfectly due to (uncertainties) or due to (imprecision of data) , this lead To introduce the Concept of Fuzzy reliability , When the Context of Fuzzy measures is expanded , after the work of Fuzzy set theory due to Zadeh (1978) , and also Zimmerman (1986)[3] .
The Fuzzy Reliability Was proposed and developed by several authors , like Cai , Wen and Zhang (1991 , 1993) also Cai (1996) Mon (1993) and El-hawary (2000) also discusses

Theoretical Aspects

The two Parameters Weibull distribution (p , ϴ) , Were (p) is shape Parameter and (ϴ) is scale Parameter , is defined by

$$F(t,\theta,p)=\frac{p}{\theta}t^{p-1}e^{-\frac{t^p}{\theta}}\ldots(1) \qquad \theta>0\,,p>0\,,t>0$$

and the C D F of this distribution is

$$F_T^t=1-e^{-\frac{t^p}{\theta}}\ \ldots(2) \qquad (t,p,\theta)>0$$

While the Reliability Function is

$$R_T{}^{(t)}=1-F_T{}^{(t)}=e^{\frac{-t^p}{\theta}}\ldots(3)$$

$(t,p,\theta)>0$

Estimation Methods

Since the aim of research is to estimate the fuzzy Reliability Function for weibull Failure Model R $_T$ (t) , so here we explain some methods of estimation to estimate parameters (p , θ) :

1 – Maximum likelihood Estimator[9]

Let { T $_{(1)}$, T$_{(2)}$,…. T$_{(n)}$ } be ar.s from equation (1) , denote the Failure times from weibull failure Model with p.o.t

$$f(t,\theta,p)=\frac{p}{\theta}t^{p-1}e^{-\frac{t^p}{\theta}}$$

Then

L= $\prod_{i=1}^{n}f(ti)$ be likelihood function

Then

Then L $=\frac{p^n}{\theta^n}\ \prod_{i=1}^{n}ti^{p-1}\ e^{\frac{-\sum ti^p}{\theta}}\ldots(4)$

Log L = n Log p – n Log ϴ + (p – 1) $\sum_{i=1}^{n}Log\ ti-\frac{\sum_{i=1}^{n}ti^p}{\theta}\ldots(5)$

$$\frac{Log\ L}{p}=\frac{n}{p}+\sum_{i=1}^{n}Log\ ti-\sum_{i=1}^{n}\frac{ti^p(1)\log t}{\theta}$$

$$\frac{Log\ l}{\theta}=\frac{-n}{\theta}+\frac{\sum ti^p}{\theta^2}$$

Put $\frac{\partial Log\ L}{\partial p}=0$, $\frac{\partial Log\ L}{\partial\theta}=0$

This gives m.1.es and θ which are !

$$\hat{\theta}_{MLE}=\frac{\sum_{i=1}^{n}ti^p}{n}\ldots(6)$$

2- Moment estimators[10]

The estimators by this Method obtain from solving E (x^r)=Mr

$$mr=\frac{\sum_{i=1}^{n}xi^r}{n}$$

Since we have two parameter (p , θ) we need to find E(x) , V (x)

$$E(x) = \theta^{\frac{1}{p}} \Gamma\left(\frac{p+1}{p}\right) \dots (7)$$

$$V(x) = E(x^2) - (x)^2 = \frac{2}{\theta^p}\left[\Gamma(\frac{p+2}{p} - \Gamma^2(\frac{p+1}{p})\right] \dots (8)$$

Coefficient of variation (C.V)

C.V = $\frac{s}{\bar{x}}$This gives

$$\dots(9) C.V = \sqrt{\frac{\Gamma\left(\frac{p+2}{p}\right) - \Gamma^2\left(\frac{p+1}{p}\right)}{\Gamma^2\left(\frac{p+1}{p}\right)}}$$

Which is independent of θ, so equation (9)

Can be used to find Moment estimator of p , here p becomes known ($\hat{p}$) , then

$$\hat{\theta} = \frac{\sum_{i=1}^{n} ti^{\hat{p}}}{n}$$

3- Proposed Method (LSME)

The estimation by this method depending on Minimizing the total Sum – Square between $F_T(ti)$ and Some of it's non parametric estimator $\hat{f}(ti) = \frac{i}{n+1}$,

$$T = \sum_{i=1}^{n}\left[1 - e^{-\frac{ti^p}{\vartheta}} - \frac{i}{n+1}\right]^2.$$

$$T = \sum_{i=1}^{n}\left[\left(1 - \frac{i}{n+1}\right) - e^{-\frac{ti^p}{\theta}} - \frac{i}{n+1}\right]^2.$$

$$\frac{\partial T}{\partial \theta} = 2\sum_{i=1}^{n}\left[(1 - \frac{i}{n+1} - e^{-\frac{ti^p}{\theta}})\right]\left(-e^{-\frac{ti^p}{\theta}}.\left(\frac{ti^p}{\theta^2}\right)\right) = 0 \dots (10)$$

$$\frac{\partial T}{\partial \theta} = 0 \longrightarrow$$

$$\frac{1}{\theta^2}\left[\sum_{i=1}^{n} ti^p\, e^{\frac{-ti^p}{\theta}} - \sum_{i=1}^{n}\frac{i}{n+1}\, e^{-\frac{ti^p}{\theta}}.ti^p - \sum_{i=1}^{n} ti^p\, e^{\frac{-2ti^p}{\theta}}\right] = 0$$

Equation 10 solved numerically to find $\hat{\vartheta}$ MLE

And $\frac{\partial T}{\partial P} = 2\sum_{i=1}^{n}\left[\left(1 - \frac{i}{n+1}\right) - e^{\frac{-ti^p}{\theta}}\right] - e^{\frac{-ti^p}{\theta}}(-)\left(\frac{ti^p}{\theta}\right)(1)\log ti$

$$\frac{\partial T}{\partial P} = 0 \Longrightarrow$$

$$\sum_{i=1}^{n}\left(1-\frac{i}{n+1}\right)\frac{ti^{\hat{p}}}{\hat{\vartheta}}\log ti\ e^{\frac{-ti^{\hat{p}}}{\hat{\theta}}}$$

$$-\sum_{i=1}^{n} e^{\frac{-2ti^{p}}{\theta}}\left(\frac{ti^{p}}{\theta}\right)\log ti=0\ \ldots(11)$$

Solving $\frac{\partial T}{\partial P}=0(\,numerically)\ gives\ \hat{p}_{MLE}$

After we estimate (p , Ɵ) by different three Methods we work on estimating Function due different sample Size $\tilde{R}_{T}^{(ti)}=e^{\frac{-ti^{\hat{P}}}{\hat{\theta}}}$

Simulation procedures

Using C D f which is

$F_{T(ti)=1-}e^{\frac{-ti^{p}}{\theta}}$

Let F (ti) = ui 0≤ui≤1

$ui=1-e^{\frac{-ti^{p}}{\theta}}$

$e^{\frac{-ti^{p}}{\theta}}=1-ui$

$-\frac{ti^{p}}{\theta}=\log(1-ui)$

$ti^{p}=\theta\log(1-ui)$

$ti=[\theta\log(1-ui)]^{\frac{1}{p}}$

The values of random variable(t) is generated from formula

$ti=\ [-\theta\log(\,1-ui\,)]^{\frac{1}{p}}\qquad\qquad 0\ \le\ \text{ui}\ \le\ 1$

And then using initial values for the given parameters (p , θ) and Vague factor ($\grave{K}$) as follows :

n	25	50	100
p	2.5	2.5	----
θ	1.2	1.2	-----
$\hat{k}i$	0.6	0.6	-----

Table (1) fuzzy Reliability estimator of tow parameters weibull ($\widetilde{K}$ = 0.3 , θ = 0.8 , p = 1.5)

	t_i	Real	mle	Mom	LSM	Mse_mle	Mse_mom	Mse_LSM	BEST
'n'	0.45	0.992570	0.96571	0.918308	0.9483	0.001338	1.22E-02	0.003729	**MLE**
25	0.75	0.977457	0.934486	0.819181	0.914101	0.002521	5.15E-02	0.005845	**MLE**
P' '	1.05	0.944993	0.870206	0.657417	0.840896	0.00688	0.120688	0.013221	**MLE**

1.5	1.35	0.933250	0.846217	0.605881	0.814523	0.009345	0.142568	0.016775	**MLE**
'th'	1.65	0.924970	0.829443	0.569957	0.797103	0.011444	0.158718	0.019559	**MLE**
0.8	1.95	0.918949	0.818072	0.546521	0.784956	0.012734	0.16753	0.021509	**MLE**
'k̀'	2.25	0.896213	0.775472	0.467659	0.737013	0.118631	0.10026	0.13061	**MOM**
0.3	2.55	0.879255	0.750648	0.421002	0.710126	0.121055	0.120206	0.133893	**MOM**
	2.85	0.855425	0.718978	0.373198	0.675624	0.123662	0.240345	0.137574	**MLE**
	3.15	0.795140	0.645745	0.273352	0.605416	0.227661	0.25687	0.241336	**MLE**
'n'	0.45	0.998744	0.987837	0.98696	0.985315	0.000207	3.46E-04	0.000475	**MLE**
50	0.75	0.996072	0.974754	0.951916	0.975343	0.000513	2.69E-03	0.000639	**MLE**
'P'	1.05	0.992956	0.963347	0.912947	0.964283	0.000898	0.007467	0.00103	**MLE**
1.5	1.35	0.984747	0.942355	0.834323	0.943857	0.001832	0.029084	0.001949	**MLE**
'th'	1.65	0.973549	0.915332	0.748084	0.917273	0.003433	0.059513	0.003477	**MLE**
0.8	1.95	0.962945	0.890992	0.677754	0.892618	0.015365	0.08815	0.055600	**MLE**
'k̀'	2.25	0.952776	0.873502	0.630986	0.875180	0.016424	0.013248	0.006467	**LSM**
0.3	2.55	0.941119	0.848898	0.570833	0.850301	0.018885	0.004605	0.018958	**MOM**
	2.85	0.915521	0.80972	0.489228	0.810131	0.011902	0.191904	0.011929	**MLE**
	3.15	0.905383	0.791708	0.455998	0.791152	0.013808	0.211328	0.014004	**MLE**
'n'	0.45	0.99983	0.99536	0.999009	0.994938	2.63E-05	2.21E-06	3.78E-05	**MOM**
100	0.75	0.998448	0.980802	0.982228	0.978916	0.000443	8.34E-05	0.000722	**MOM**
'P'	1.05	0.99817	0.978231	0.978826	0.976534	0.000529	0.001074	0.000822	**MLE**
1.5	1.35	0.997349	0.965493	0.969524	0.965197	0.00105	0.001459	0.001244	**MLE**
'th'	1.65	0.997059	0.961914	0.96595	0.961805	0.001274	0.001618	0.001448	**MLE**
0.8	1.95	0.994007	0.940374	0.928399	0.940642	0.003052	0.005974	0.003197	**MLE**
'k̀'	2.25	0.993068	0.933311	0.916214	0.934034	0.003728	0.00712	0.003747	**MLE**
0.3	2.55	0.990676	0.92521	0.891669	0.925793	0.004334	0.012954	0.004335	**MLE**
	2.85	0.987089	0.909558	0.855677	0.910232	0.00616	0.021397	0.006211	**MLE**
	3.15	0.98254	0.894374	0.816586	0.894733	0.007861	0.033765	0.007757	**LSM**

Table (2) fuzzy reliability Estimator when (θ = 0.8 , $\widetilde{K}$ = 0.6 , p =1.5)

	t_i	**real**	**mle**	**Mom**	**LSM**	**Mse_mle**	**Mse_mom**	**Mse_ LSM**	**BEST**
'n'	0.45	0.994019	0.982525	0.928286	0.978971	3.42E-04	2.94E-03	5.52E-04	**MLE**
25	0.75	0.938618	0.904331	0.61939	0.900937	0.001788	1.33E-01	0.001952	**MLE**
'P'	1.05	0.928516	0.882723	0.552179	0.87654	0.00303	0.161344	0.003167	**MLE**

2.5	1.35	0.89859	0.831245	0.42459	0.818587	0.009907	0.029052	0.009405	**LSM**
'th'	1.65	0.876828	0.805099	0.373872	0.789614	0.00811	0.256175	0.01155	**MLE**
0.8	1.95	0.850517	0.77733	0.327906	0.758787	0.018536	0.012582	0.017607	**MOM**
'k̀'	2.25	0.833079	0.753815	0.290729	0.732672	0.010293	0.295676	0.015695	**MLE**
0.3	2.55	0.79836	0.714356	0.244241	0.691505	0.012099	0.30763	0.018098	**MLE**
	2.85	0.769441	0.682515	0.215214	0.657914	0.013155	0.307366	0.019733	**MLE**
	3.15	0.697616	0.607492	0.156192	0.579693	0.016142	0.294311	0.023761	**MLE**
'n'	0.45	0.987651	0.973351	0.868437	0.963062	3.66E-04	2.25E-04	1.08E-03	**MOM**
50	0.75	0.986011	0.968467	0.851037	0.957308	0.000582	2.58E-02	0.001548	**MLE**
'P'	1.05	0.980278	0.956952	0.798416	0.94489	0.000876	0.040347	0.001957	**MLE**
2.5	1.35	0.972442	0.944456	0.749033	0.931035	0.001195	0.061201	0.002474	**MLE**
'th'	1.65	0.964558	0.931526	0.6964	0.917018	0.001634	0.08352	0.003242	**MLE**
0.8	1.95	0.949948	0.911612	0.628362	0.896222	0.001991	0.11965	0.003849	**MLE**
'k̀'	2.25	0.925307	0.875995	0.524345	0.859608	0.003252	0.17521	0.005531	**MLE**
0.3	2.55	0.913494	0.861099	0.490929	0.844120	0.003581	0.194096	0.005994	**MLE**
	2.85	0.908103	0.853758	0.475575	0.836756	0.00382	0.202263	0.006312	**MLE**
	3.15	0.883438	0.816526	0.394829	0.799539	0.006171	0.245395	0.009196	**MLE**
'n'	0.45	0.998727	0.994065	0.985895	0.992845	4.88E-05	6.59E-04	8.14E-05	**MLE**
100	0.75	0.996813	0.985367	0.962236	0.983033	0.000156	1.70E-03	0.000251	**MLE**
'P'	1.05	0.992952	0.972056	0.914513	0.969104	0.000507	0.007556	0.000775	**MLE**
1.5	1.35	0.98992	0.963889	0.882277	0.960533	0.000806	0.014749	0.001195	**MLE**
'th'	1.65	0.986085	0.953924	0.84523	0.95053	0.001223	0.024773	0.00168	**MLE**
0.8	1.95	0.981487	0.942702	0.803106	0.939649	0.001694	0.036946	0.002201	**MLE**
'k̀'	2.25	0.979323	0.936886	0.783437	0.93462	0.002556	0.002444	0.062452	**MOM**
0.3	2.55	0.974556	0.925781	0.74106	0.924909	0.002543	0.055887	0.002749	**MLE**
	2.85	0.969144	0.915317	0.702352	0.914604	0.00309	0.072277	0.003241	**MLE**
	3.15	0.963643	0.905349	0.670241	0.904706	0.003525	0.088574	0.003694	**MLE**

Table (3) fuzzy rrliability Estimator when (θ = 0.8 , $\widetilde{K}$ = 0.6 , p = 2.5)

	t_i	real	mle	Mom	pro	Mse_mle	Mse_mom	Mse_ pro	BEST
'n'	0.45	0.979011	0.971218	0.779522	0.962913	0.000615	0.000207	0.001393	**MOM**
25	0.75	0.944854	0.932602	0.593899	0.922168	0.000434	0.000675	0.001919	**MLE**
'P'	1.05	0.906278	0.893205	0.446862	0.880949	0.00055	0.003764	0.002018	**MLE**
1.5	1.35	0.883150	0.868322	0.385176	0.852462	0.000716	0.052485	0.002407	**MLE**
'th'	1.65	0.854104	0.838671	0.328184	0.819749	0.000847	0.080551	0.002772	**MLE**

0.8	1.95	0.822643	0.806426	0.280844	0.782568	0.000983	0.196585	0.003435	**MLE**
$'\grave{k}'$	2.25	0.779423	0.762089	0.227766	0.736475	0.003278	0.10512	0.002061	**SLM**
0.6	2.55	0.712435	0.694727	0.16975	0.662728	0.001456	0.295315	0.005012	**MLE**
	2.85	0.681008	0.662119	0.143489	0.626952	0.00161	0.289418	0.005679	**MLE**
	3.15	0.611387	0.5882	0.088194	0.558377	0.00232	0.274554	0.005242	**MLE**
'n'	0.45	0.985328	0.993752	0.859181	0.995075	0.000257	0.000503	0.000385	**MLE**
50	0.75	0.964558	0.97359	0.707125	0.979155	0.000601	0.000661	0.000994	**MLE**
'P'	1.05	0.941591	0.947628	0.56695	0.956045	0.001071	0.000762	0.001173	**MLE**
1.5	1.35	0.922324	0.925313	0.487506	0.935476	0.000986	0.001679	0.001378	**MLE**
'th'	1.65	0.912195	0.913593	0.451292	0.924383	0.001092	0.00368	0.001394	**MLE**
0.8	1.95	0.87144	0.870382	0.353476	0.884036	0.001559	0.001389	0.002233	**MOM**
$'\grave{k}'$	2.25	0.862138	0.860043	0.334902	0.873597	0.001652	0.279568	0.002304	**MLE**
0.6	2.55	0.849482	0.845612	0.311507	0.858978	0.001692	0.290597	0.002369	**MLE**
	2.85	0.831959	0.82505	0.280205	0.838099	0.001742	0.304864	0.002474	**MLE**
	3.15	0.815185	0.806488	0.254922	0.818795	0.001856	0.314198	0.002575	**MLE**
'n'	0.45	0.993234	0.994359	0.921306	0.994577	8.06E-06	0.008822	9.49E-06	**MLE**
100	0.75	0.988893	0.989236	0.875764	0.990489	2.45E-05	0.008021	2.61E-05	**MLE**
'P'	1.05	0.977451	0.979731	0.787138	0.983098	8.13E-05	0.001146	0.000188	**MLE**
1.5	1.35	0.972788	0.971272	0.739911	0.976135	0.00018	0.004535	0.000253	**MLE**
'th'	1.65	0.969322	0.967384	0.716562	0.972593	0.000223	0.001293	0.000306	**MLE**
0.8	1.95	0.957813	0.951569	0.641249	0.957828	0.000564	0.004367	0.000616	**MLE**
$'\grave{k}'$	2.25	0.947672	0.939627	0.587437	0.946673	0.00069	0.002557	0.000723	**MLE**
0.6	2.55	0.936331	0.928505	0.542047	0.936259	0.000755	0.000591	0.000817	**MOM**
	2.85	0.927213	0.918295	0.507173	0.926243	0.000996	0.005326	0.001037	**MLE**
	3.15	0.917194	0.907433	0.473131	0.915751	0.001105	0.006741	0.001128	**MLE**

Table (4) fuzzy reliability Estimator when (θ = 1.2 , $\widetilde{K}$ = 0.3 , p = 1.5)

	t_i	real	mle	Mom	LSM	Mse_mle	Mse_mom	Mse_ LSM	BEST
'n'	0.45	1	0.969403	0.968719	0.953108	1.06E-03	0.004788	2.94E-03	**MLE**
25	0.75	0.999997	0.932829	0.961729	0.913465	4.87E-03	0.006973	7.81E-03	**MLE**
'P'	1.05	0.999994	0.906761	0.959004	0.883567	6.16E-03	0.007701	0.013947	**MLE**
2.5	1.35	0.999962	0.861313	0.942784	0.832446	0.020379	0.014825	0.029072	**MOM**
'th'	1.65	0.999883	0.814416	0.929501	0.782016	0.035069	0.016805	0.048054	**MOM**
0.8	1.95	0.999827	0.792896	0.923048	0.758121	0.043860	0.059182	0.058982	**MLE**

'k̀'	2.25	0.999673	0.77188	0.915261	0.736011	0.052992	0.021680	0.070319	**MOM**
0.6	2.55	0.999518	0.745688	0.907301	0.707300	0.032032	0.033842	0.086481	**MLE**
	2.85	0.998772	0.689209	0.883929	0.650505	0.016349	0.032608	0.121659	**MLE**
	3.15	0.996928	0.656424	0.863072	0.619484	0.01733	0.038275	0.143575	**MLE**
'n'	0.45	1	0.988792	0.968297	0.984937	1.70E-04	0.005025	3.80E-04	**MLE**
50	0.75	1	0.973067	0.96777	0.972085	7.55E-04	0.005188	9.28E-04	**MLE**
'P'	1.05	1	0.950846	0.959121	0.948320	2.83E-03	0.008347	0.003703	**MLE**
2.5	1.35	1	0.925302	0.952741	0.923660	0.005814	0.011042	0.006491	**MLE**
'th'	1.65	0.999999	0.897945	0.942957	0.895992	0.010489	0.015852	0.011352	**MLE**
0.8	1.95	0.999993	0.876349	0.940745	0.873701	0.016236	0.016926	0.016491	**MLE**
'k̀'	2.25	0.999988	0.85915	0.93965	0.855614	0.020519	0.017023	0.021081	**MOM**
0.6	2.55	0.999982	0.846127	0.938315	0.841947	0.024395	0.017122	0.025115	**MOM**
	2.85	0.999976	0.836744	0.936406	0.832069	0.017202	0.028151	0.028501	**MLE**
	3.15	0.999949	0.810756	0.929979	0.804975	0.026367	0.030357	0.038107	**MLE**
'n'	0.45	1	0.991372	0.999956	0.987686	1.51E-07	4.81E-09	3.73E-04	**MOM**
100	0.75	1	0.979255	0.999783	0.97398	4.93E-05	1.43E-07	9.21E-04	**MOM**
'P'	1.05	1	0.970399	0.999391	0.964944	9.03E-05	1.44E-06	0.001369	**MLE**
2.5	1.35	1	0.959822	0.998822	0.953422	0.001669	5.75E-02	0.00237	**MLE**
'th'	1.65	1	0.949042	0.998038	0.942112	0.002705	1.48E-02	0.00359	**MLE**
0.8	1.95	1	0.937445	0.997196	0.930093	0.003036	3.99E-03	0.005096	**MLE**
'k̀'	2.25	1	0.930888	0.997021	0.923093	0.005036	3.27E-03	0.006297	**MOM**
0.6	2.55	0.999999	0.921176	0.996024	0.913073	0.005623	6.96E-03	0.008033	**MLE**
	2.85	0.999999	0.91647	0.995713	0.908011	0.006383	7.43E-03	0.009084	**MLE**
	3.15	0.999999	0.905271	0.992188	0.896863	0.001166	0.001247	0.01112	**MLE**

Table (5) fuzzy reliability Estimator when (θ =0.8 ,$\tilde{K}$= 0.6 , p=1.5)

	t_i	real	mle	Mom	LSM	Mse_mle	Mse_mom	Mse_ LSM	BEST
'n'	0.45	0.999999	0.97808	0.995551	0.973291	5.30E-04	9.83E-05	1.19E-03	MOM
25	0.75	0.999998	0.961579	0.993664	0.955079	1.83E-03	1.92E-04	3.61E-03	MOM
'P'	1.05	0.999976	0.929735	0.992131	0.927342	5.85E-03	2.13E-04	0.006569	MOM
1.5	1.35	0.999752	0.867805	0.977083	0.865644	0.019161	1.08E-03	0.020049	MOM
'th'	1.65	0.998815	0.786421	0.934638	0.783296	0.047655	6.64E-03	0.04793	MOM
1.2	1.95	0.997889	0.761346	0.915387	0.757111	0.059476	1.13E-02	0.059848	MOM
'$\grave{k}$'	2.25	0.993559	0.697069	0.869879	0.689252	0.091811	2.38E-02	0.094657	MOM
0.3	2.55	0.989492	0.666977	0.841359	0.657213	0.110423	3.37E-02	0.114718	MOM
	2.85	0.984857	0.620809	0.808533	0.614397	0.134510	4.02E-02	0.138903	MOM
	3.15	0.979513	0.602486	0.786392	0.595803	0.143998	0.046397	0.148729	MOM
'n'	0.45	1	0.890888	0.999178	0.988271	7.61E-02	1.77E-04	1.96E-04	MOM
50	0.75	0.999999	1.126261	0.993701	0.963274	5.95E-03	1.14E-04	1.83E-03	MOM
'P'	1.05	0.999997	1.151725	0.992314	0.945485	1.88E-03	1.64E-04	0.003371	MOM
1.5	1.35	0.999992	1.134661	0.990804	0.930396	0.000203	2.08E-04	0.005153	MLE
'th'	1.65	0.999974	1.107003	0.988668	0.912231	0.000134	2.69E-04	0.008102	MLE
1.2	1.95	0.999911	1.040075	0.985986	0.886795	0.006233	3.36E-04	0.013767	MOM
'$\grave{k}$'	2.25	0.999875	1.015654	0.981330	0.873031	0.00565	6.18E-04	0.016751	MOM
0.3	2.55	0.999814	0.97998	0.975370	0.854698	0.001008	1.15E-03	0.021891	MLE
	2.85	0.999607	0.941783	0.968028	0.831269	0.000516	1.65E-03	0.029054	MLE
	3.15	0.999405	0.915079	0.961516	0.816129	0.000460	0.002363	0.034056	MLE
'n'	0.45	1	0.990495	0.999981	0.990271	1.60E-04	1.73E-05	1.80E-04	MOM
100	0.75	0.999999	0.983843	0.999966	0.983593	3.07E-04	4.56E-05	3.36E-04	MOM
'P'	1.05	0.999999	0.97851	0.999948	0.978035	5.68E-04	8.50E-06	0.000629	MOM
1.5	1.35	0.999998	0.965871	0.99987	0.965537	0.001271	2.74E-05	0.001342	MOM
'th'	1.65	0.999997	0.958532	0.999737	0.958306	0.001809	1.25E-04	0.001855	MOM
1.2	1.95	0.999993	0.948229	0.99937	0.947918	0.002825	6.67E-04	0.002892	MOM
'$\grave{k}$'	2.25	0.999991	0.944737	0.999252	0.944398	0.003169	9.10E-03	0.003228	MLE
0.3	2.55	0.999952	0.924612	0.997719	0.924009	0.005787	9.90E-03	0.005816	MLE
	2.85	0.999856	0.913948	0.995712	0.91317	0.007605	4.08E-05	0.007677	MOM
	3.15	0.999795	0.901339	0.993909	0.900361	0.010078	7.13E-05	0.010192	MOM

Table (6) fuzzy reliability Estimator when (θ =1.2 , $\widetilde{K}$= 0.3 , p=2.5)

	t_i	real	mle	Mom	LSM	Mse_mle	Mse_mom	Mse_ LSM	BEST
'n'	0.45	0.999902	0.970594	0.99333	0.951438	1.04E-03	8.26E-05	2.96E-03	MOM
25	0.75	0.999563	0.931002	0.983116	0.902686	4.99E-03	4.08E-04	1.01E-02	MOM
'P'	1.05	0.999000	0.907122	0.972718	0.875834	9.15E-03	1.06E-03	0.016415	MOM
2.5	1.35	0.996827	0.867526	0.949108	0.832968	0.017249	3.76E-03	0.027401	MOM
'th'	1.65	0.996235	0.851723	0.942012	0.816269	0.021352	4.22E-03	0.032951	MOM
1.2	1.95	0.992491	0.802385	0.912583	0.765877	0.037005	7.70E-03	0.051846	MOM
'$\grave{k}$'	2.25	0.988814	0.775956	0.89017	0.739546	0.047064	1.14E-02	0.063346	MOM
0.3	2.55	0.981152	0.733104	0.855024	0.697626	0.062661	1.92E-02	0.081051	MOM
	2.85	0.970465	0.669125	0.803243	0.635540	0.091355	2.97E-02	0.112543	MOM
	3.15	0.956511	0.62282	0.75373	0.592071	0.012509	4.27E-02	0.133689	MLE
'n'	0.45	0.999988	1.029909	0.995254	0.983432	3.02E-04	1.05E-04	5.00E-04	MLE
50	0.75	0.999686	0.994904	0.984927	0.948382	1.31E-02	5.54E-04	2.68E-03	MOM
'P'	1.05	0.999360	0.978784	0.979911	0.930904	1.19E-02	7.93E-04	0.004952	MOM
2.5	1.35	0.999104	0.963795	0.971845	0.916718	0.019726	7.63E-03	0.006182	LSM
'th'	1.65	0.998746	0.94346	0.966079	0.894722	0.018287	1.95E-03	0.011109	MOM
1.2	1.95	0.998021	0.920319	0.953573	0.870764	0.022587	3.62E-03	0.016579	MOM
'$\grave{k}$'	2.25	0.997505	0.907226	0.947633	0.856999	0.01958	3.89E-03	0.020379	MOM
0.3	2.55	0.995866	0.883038	0.93072	0.831417	0.017762	5.92E-03	0.02786	MOM
	2.85	0.994499	0.866629	0.918319	0.814036	0.015711	7.64E-03	0.033915	MOM
	3.15	0.992572	0.850293	0.903309	0.796759	0.013129	9.92E-03	0.039183	MOM
'n'	0.45	0.999998	0.989108	0.999954	0.98548	1.69E-04	8.94E-05	2.91E-04	MOM
100	0.75	0.999993	0.980797	0.999801	0.977208	4.59E-04	1.73E-06	6.66E-04	MOM
'P'	1.05	0.999970	0.966284	0.999551	0.961304	1.19E-03	4.15E-05	0.001605	MOM
2.5	1.35	0.999960	0.961728	0.999435	0.956283	0.001521	5.78E-04	0.002061	MOM
'th'	1.65	0.999941	0.954103	0.999245	0.94776	0.002223	9.82E-05	0.00301	MOM
1.2	1.95	0.999925	0.943282	0.998839	0.937055	0.003268	1.58E-04	0.004192	MOM
'$\grave{k}$'	2.25	0.999899	0.936533	0.998295	0.930139	0.004074	3.38E-02	0.005089	MLE
0.3	2.55	0.999810	0.925428	0.997097	0.918464	0.005616	9.52E-03	0.006913	MLE
	2.85	0.999745	0.919233	0.996118	0.91206	0.001584	1.76E-03	0.007994	MLE
	3.15	0.999701	0.913848	0.995641	0.906361	0.007435	2.12E-05	0.00905	MOM

Table (7) fuzzy reliability Estimator when (θ =1.2 ,$\widetilde{K}$= 0.6 , p=2.5)

	t_i	real	mle	LSM	Pro	Mse_mle	Mse_mom	Mse_LSM	BEST
'n'	0.45	0.999721	0.965745	0.898302	0.946079	1.18E-03	2.14E-04	2.94E-03	**MOM**
25	0.75	0.999062	0.941016	0.875143	0.91486	3.41E-03	2.66E-04	7.35E-03	**MOM**
'P'	1.05	0.997797	0.918983	0.856675	0.889082	6.79E-03	3.07E-03	0.013305	**MOM**
1.5	1.35	0.994474	0.886257	0.82764	0.852838	0.012522	3.95E-02	0.021841	**MLE**
'th'	1.65	0.992248	0.840729	0.797782	0.804314	0.024358	5.43E-02	0.037218	**MLE**
1.2	1.95	0.987069	0.771319	0.755571	0.733086	0.048312	6.58E-02	0.065757	**MLE**
'k̀'	2.25	0.979511	0.739809	0.731900	0.701990	0.059076	7.35E-02	0.078136	**MLE**
0.6	2.55	0.974600	0.703063	0.708442	0.665412	0.075281	8.22E-02	0.096753	**MLE**
	2.85	0.967984	0.675316	0.690637	0.638505	0.087179	8.98E-02	0.109726	**MLE**
	3.15	0.955328	0.627693	0.658983	0.592531	0.110378	1.01E-01	0.100987	**LSM**
'n'	0.45	0.999997	0.989687	0.974515	0.985443	1.87E-04	1.14E-03	4.63E-04	**MLE**
50	0.75	0.999978	0.974511	0.952805	0.971775	6.65E-04	2.51E-03	1.06E-03	**MLE**
'P'	1.05	0.999839	0.954295	0.932493	0.950551	2.37E-03	4.75E-03	0.00326	**MLE**
1.5	1.35	0.999388	0.928583	0.901869	0.927058	0.005419	1.06E-02	0.006424	**MLE**
'th'	1.65	0.998992	0.911595	0.885385	0.910162	0.008032	1.40E-02	0.009205	**MLE**
1.2	1.95	0.998621	0.900406	0.875515	0.899297	0.010084	1.61E-02	0.011136	**MLE**
'k̀'	2.25	0.997311	0.880256	0.855635	0.879918	0.014703	2.21E-02	0.015811	**MLE**
0.6	2.55	0.994082	0.843659	0.821147	0.844294	0.02407	3.45E-02	0.023759	**LSM**
	2.85	0.99087	0.807088	0.790738	0.807454	0.03446	4.32E-02	0.033841	**LSM**
	3.15	0.990333	0.803393	0.787587	0.803663	0.035675	4.44E-02	0.035007	**LSM**
'n'	0.45	1	0.996401	0.99538	0.995128	2.18E-05	4.44E-05	5.31E-05	**MLE**
100	0.75	0.999995	0.987106	0.98983	0.983854	2.35E-04	1.54E-04	4.00E-04	**MOM**
'P'	1.05	0.999985	0.978002	0.984343	0.975258	5.56E-04	3.54E-04	0.000817	**MLE**
1.5	1.35	0.999920	0.963185	0.973127	0.959441	0.001627	1.03E-03	0.002248	**MOM**
'th'	1.65	0.999849	0.944361	0.965630	0.942155	0.003403	1.48E-03	0.003681	**MOM**
1.2	1.95	0.999797	0.936309	0.959610	0.933584	0.004196	1.93E-03	0.004633	**MLE**
'k̀'	2.25	0.999642	0.923219	0.950444	0.920648	0.006004	2.95E-03	0.006347	**MOM**
0.6	2.55	0.999431	0.913075	0.943067	0.9104	0.007594	3.91E-03	0.007987	**MOM**
	2.85	0.999265	0.906675	0.938916	0.903939	0.008753	4.46E-03	0.009129	**MOM**
	3.15	0.998916	0.892445	0.929051	0.889492	0.01139	5.82E-03	0.012053	**MOM**

Table (8) fuzzy reliability Estimator when (θ =1.2 $\widetilde{K}$= 0.6 , p=2.5)

	t_i	real	mle	Mom	LSM	Mse_mle	Mse_mom	Mse_ LSM	BEST
'n'	0.45	0.999197	0.982129	0.921738	0.975512	3.39E-04	1.42E-02	7.09E-04	**MLE**
25	0.75	0.996939	0.951728	0.881619	0.939596	2.22E-03	2.56E-02	3.87E-03	**MLE**
'P'	1.05	0.989117	0.899423	0.82921	0.884974	8.92E-03	4.01E-02	0.012024	**MLE**
2.5	1.35	0.978852	0.847593	0.783297	0.830031	0.017984	5.29E-02	0.023059	**MLE**
'th'	1.65	0.96024	0.795691	0.73477	0.776158	0.027346	6.30E-02	0.034577	**MLE**
1.2	1.95	0.952933	0.775935	0.718627	0.755848	0.031715	6.67E-02	0.039392	**MLE**
'$\grave{k}$'	2.25	0.939582	0.746040	0.692053	0.725310	0.038102	7.14E-02	0.046200	**MLE**
0.6	2.55	0.911378	0.696723	0.646764	0.674810	0.046646	1.89E-02	0.056179	**MOM**
	2.85	0.895318	0.672840	0.623744	0.650950	0.050036	8.22E-02	0.059911	**MLE**
	3.15	0.871357	0.627550	0.589522	0.605464	0.060483	4.89E-02	0.071429	**MOM**
'n'	0.45	0.999505	0.98649	0.935267	0.979853	2.32E-04	4.34E-03	7.83E-04	**MLE**
50	0.75	0.997903	0.972504	0.908446	0.96433	8.32E-04	8.66E-03	1.88E-03	**MLE**
'P'	1.05	0.995431	0.95653	0.883091	0.948757	1.68E-03	1.28E-02	0.002767	**MLE**
2.5	1.35	0.992922	0.933676	0.850914	0.927861	0.003565	2.08E-02	0.00266	**LSM**
'th'	1.65	0.99102	0.923249	0.838922	0.91688	0.004661	2.37E-02	0.003206	**LSM**
1.2	1.95	0.987362	0.901898	0.816296	0.89506	0.007659	2.97E-02	0.010161	**MLE**
'$\grave{k}$'	2.25	0.981564	0.871494	0.786705	0.865317	0.012646	3.89E-02	0.015085	**MLE**
0.6	2.55	0.979965	0.863588	0.779613	0.857488	0.014287	4.15E-02	0.01643	**MLE**
	2.85	0.974443	0.842255	0.76025	0.83599	0.018634	4.78E-02	0.020296	**MLE**
	3.15	0.963944	0.815601	0.736696	0.809242	0.022615	5.30E-02	0.02447	**MLE**
'n'	0.45	0.999583	0.987475	0.980853	0.986968	1.90E-04	5.20E-04	2.26E-04	**MLE**
100	0.75	0.999458	0.984644	0.978151	0.984043	2.71E-04	6.54E-04	3.18E-04	**MLE**
'P'	1.05	0.998986	0.974665	0.968181	0.973866	6.37E-04	1.20E-03	0.000717	**MLE**
2.5	1.35	0.998321	0.964613	0.959093	0.963576	0.001200	1.80E-03	0.001314	**MLE**
'th'	1.65	0.997556	0.956138	0.950801	0.955108	0.001818	2.44E-03	0.001922	**MLE**
1.2	1.95	0.996267	0.944768	0.939082	0.942804	0.002733	3.73E-03	0.003068	**MLE**
'$\grave{k}$'	2.25	0.995295	0.936816	0.931387	0.934658	0.003517	4.58E-03	0.003909	**MLE**
0.6	2.55	0.99379	0.926853	0.921726	0.924281	0.004619	5.86E-03	0.005178	**MLE**
	2.85	0.989376	0.907899	0.902211	0.904755	0.006841	8.76E-03	0.007426	**MLE**
	3.15	0.988033	0.901039	0.895774	0.89764	0.007782	9.78E-03	0.008513	**MLE**

Conclusions

1- From table the best fuzzy Reliability estimator is MLE (is first) with percentage ($\frac{23}{30}$ * 100) =76%

and second one is Moments Estimator with percentage ($\frac{5}{30}$) * 100=17%, while the third one is Least square Method with ($\frac{2}{30}$) * 100=7%

2 – from table two and three we find the best estimator is MLE (first) with percentage ($\frac{26}{30}$)* 100=87% , and Moments is best with percentage ($\frac{3}{30}$) * 100 =10%, Finally LSM is best ($\frac{1}{30}$) * 100=3%, i.e

all the percentage of preferring estimator is the same .

3 – from table (7) , the preferred fuzzy estimator of ($\hat{R}$) is MLE ($\frac{16}{30}$)100%=53.3% , first and Second is Moment ($\frac{10}{30}$) * 100=33.3% and third one ($\frac{4}{30}$) * 100=13.3%

4 - Finally from table (8) , the preferred fuzzy estimator ($\hat{R}$) is MLE ($\frac{26}{30}$) * 100=86.8%, and moment is best with ($\frac{2}{30}$) * 100=6.6% and LSM is best with ($\frac{2}{30}$) * 100= 6.6

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