وقائع المؤتمر العلمي لكلية التربية الأساسية في مجال العلوم الصرفة
وتحت شعار
(العلوم الصرفة والتطبيقية بوابة لخدمة المجتمع)
يومي الاربعاء و الخميس 28-29/5/2025

Triple Integral Transform (SEE- Sadik- Shehu) and its Properties with Applications

Emad A. Kuffi[1] , Nour K. Salman[2] , ,Talat J. Aldhlki[3] , Shatha H. Theyab[4]
[1,2,3,4] Department of Mathematics, College of Basic Education, Mustansiriyah University, Iraq Baghdad.
[1]emad.kuffi@uomustansiriyah.edu.iq
[2]noorkareem94@uomustansiriyah.edu.iq
[3]dr.talatjassim.edbs@ uomustansiriyah.edu.iq
[4]shatha.haider1994@uomustansiriyah.edu.iq

Abstract.
In this paper, we introduce a new technique for solving some partial differential equations called triple SEE-Sadik-Shehu technique, some properties for this technique are introduced, and find SEE-Sadik-Shehu transform for important basic functions. In addition, we use this triple technique transform to evaluate the exact solution of Laplace, heat partial operator equations. The main aim of this combination of these three integral transforms is generality and obtaining an exact solution with simple and easy mathematical operations.

Keywords: Triple SEE-Sadik-Shehu transform, Sadik transform, Shehu transform, SEE transform , Partial differential equations.

1- Introduction

Due to the importance of integral transforms in finding accurate solutions to differential equations , authors were interested in proposing many integral transforms , including single ,double ,triple transforms. Every author began to address one of the problems that exist in life , whether engineering , physical ,medical, or astronomical ,and sometimes social and economic problems ,in order to obtain solutions using easier and technical methods that are far from the complexity[1,2,7,8,9,24,25,26,27,28,29].

In this paper, a triple transform was proposed that is more general than many of the triple transforms proposed previously. This is because both Sadik and SEE transforms are more general transforms due to the presence of the powers on the parameter in the kernel of these two transforms, and also that Shehu transform is a transform with two parameters, which gives this triple transform generality than other triple transforms[5,10-24].

Here, a novel triple integral transform technique is presented; it joins three interesting transforms, SEE, Sadik and Shehu. The fundamental properties regarding the existence conditions, linearity and the inverse of this new triple transform are displayed. Furthermore, we establish new results related to partial derivatives and the triple convolution theorem.

2. Definitions and Some Basic Useful Properties of Triple SEE Shehu Sadik Transform Technique

Definition (2-1) [24].

SEE technique of the function $f(z), z \geq 0$ is defined as : is known:

$$S_z\{f(z)\} = \frac{1}{\sigma^n}\int_0^\infty e^{-\sigma z} f(z)dz = F(\sigma).$$

Where σ is a complex parameter, n is any integer number Re(σ)>0.

If $S_z[f(z)] = F(\sigma)$ is the SEE integral transform, then $(z) = S^{-1}[F(\sigma)] = \frac{1}{2\pi j}\int_{\delta-j\infty}^{\delta+j\infty} \sigma^n\ F(\sigma)e^{\sigma z}\,d\sigma$, is called an inverse of the SEE integral transform.

Definition (2-2) [12].

Sadik transform technique of the function $f(t), t \geq 0$ is defined as :

$$S_t\,[f(t)] = \frac{1}{v^\beta}\int_0^\infty e^{-v^\alpha t} f(t)dt = F(v).$$

Where v is a complex parameter and $\alpha \neq 0$ and β are any real numbers. Here S_t is called the Sadik transform operator. The inverse Sadik transform technique is defined by:

$$S_t^{-1}[F(v)] = f(t) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} v^\beta\, e^{v^\alpha t}\, F(v)dv.$$

Where c is a real constant, and $i \in \mathbb{C}$.

Definition (2-3) [6]. Shehu transform technique of the real function $f(x)$ of exponential order is defined over the set of functions

$$M = \left\{ f(x): \exists N, \tau_1, \tau_2 > 0, |f(x)| < Ke^{\frac{|x|}{\tau_i}}, for\ x \in (-1)^i \times [0,\infty), i = 1,2 \right\}.$$

Via the following integral

$$S_x\,[f(x)] = \int_0^\infty e^{-\frac{\delta}{\mu}x} f(x)\,dx = F(\delta,\mu), \qquad \delta,\mu > 0.$$

Where $e^{-\frac{\delta}{\mu}x}$ is the kernel function, and S_x is the operator of Shehu transform. The inverse Shehu transform is defined as:

$$S_x^{-1}[F(\delta,\mu)] = f(x) = \frac{1}{2\pi i}\int_{d-i\infty}^{d+i\infty} e^{\frac{\delta}{\mu}}\, F(\delta,\mu)\, d\delta.$$

Where d is a real constant, and $i \in \mathbb{C}$.

Definition (2-4). Triple SEE Shehu Sadik transform of the function $f(z,t,x)$ be a continuous function of three variables $z,t,x > 0$ is denoted by $S_z\, S_t S_x[f(z,t,x)] = F(\sigma,v,\delta,\mu)$ and defined as:

$$S_z\, S_t S_x[f(z,t,x)] = F(\sigma,v,\delta,\mu) = \frac{1}{\sigma^n v^\beta}\int_0^\infty\int_0^\infty\int_0^\infty e^{-(\sigma z+v^\alpha t+\frac{\delta}{\mu}x)} f(z,t,x)dz\,dt\,dx \quad (1)$$

$$= \lim_{\substack{a\to\infty\\ b\to\infty\\ c\to\infty}} \frac{1}{\sigma^n v^\beta}\int_0^a\int_0^b\int_0^c e^{-(\sigma z+v^\alpha t+\frac{\delta}{\mu}x)} f(z,t,x)dz\,dt\,dx.$$

Where μ,δ,σ and $v > 0$, It converges if the limit of the integral exists, and diverges if not.

Definition (2-5): The inverse triple SEE Shehu Sadik transform technique of a function $F(\sigma,v,\delta,\mu)$ is given by

$$S_z^{-1}\, S_t^{-1} S_x^{-1}[F(\sigma,v,\delta,\mu)] = f(z,t,x).$$

Equivalently,

$$f(z,t,x) = S_z^{-1}\, S_t^{-1} S_x^{-1}[F(\sigma,v,\delta,\mu)]$$

$$= \frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty} e^{\sigma z}\, d\sigma\ \frac{1}{2\pi i}\int_{b-i\infty}^{b+i\infty} e^{v^\alpha t}\, dt\ \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} e^{\frac{\delta}{\mu}x}\ F(\sigma,v,\delta,\mu)\, d\delta. \quad (2)$$

Where a , b and c are real constants.

Property(2-1): (Linearity) If the Triple SEE Shehu Sadik transform Technique of functions $S_z\, S_t S_x[f(z,t,x)] = F(\sigma,v,\delta,\mu)$ and $S_z\, S_t S_x[\phi(z,t,x)] = \Phi(\sigma,v,\delta,\mu)$, then for any constants A and B, we have

$$S_z\, S_t S_x[A\, f(z,t,x) + B\, \phi(z,t,x)] = A S_z\, S_t S_x[\, f(z,t,x)] + B\, S_z\, S_t S_x[\, \phi(z,t,x)] \quad (3)$$

Proof:

$$S_z\, S_t S_x[A\, f(z,t,x) + B\, \phi(z,t,x)] = \frac{1}{\sigma^n v^\beta}\int_0^\infty\int_0^\infty\int_0^\infty e^{-(\sigma z+v^\alpha t+\frac{\delta}{\mu}x)}[A\, f(z,t,x) + B\, \phi(z,t,x)]dz\,dt\,dx,$$

$$= \frac{A}{\sigma^n v^\beta}\int_0^\infty\int_0^\infty\int_0^\infty e^{-(\sigma z+v^\alpha t+\frac{\delta}{\mu}x)} f(z,t,x)dz\,dt\,dx + \frac{B}{\sigma^n v^\beta}\int_0^\infty\int_0^\infty\int_0^\infty e^{-(\sigma z+v^\alpha t+\frac{\delta}{\mu}x)} \phi(z,t,x)dz\,dt\,dx,$$

$$= A S_z\, S_t S_x[\, f(z,t,x)] + B\, S_z\, S_t S_x[\, \phi(z,t,x)]. \quad (4)$$

property(2-2): (Shifting) If $S_z\, S_t S_x[f(z,t,x)] = F(\sigma,v,\delta,\mu)$ then for any real constant a , b and we have

$S_z\, S_t S_x[e^{az+bt+cx}\, f(z,t,x)] = F((\sigma-a),(v^\alpha-b),(\frac{\delta-\mu c}{\mu})).$ (5)

Proof: $S_z\, S_t S_x[e^{az+bt+cx}\, f(z,t,x)]=$

$\frac{1}{\sigma^n v^\beta}\int_0^\infty\int_0^\infty\int_0^\infty e^{-(\sigma z+v^\alpha t+\frac{\delta}{\mu}x)}(e^{az+bt+cx}\, f(z,t,x))dz\,dt\,dx,$

$$=\frac{1}{\sigma^n v^\beta}\int_0^\infty\int_0^\infty\int_0^\infty e^{-(\sigma-a)z-(v^\alpha-b)t-(\frac{\delta}{\mu}-c)x)}\, f(z,t,x)dz\,dt\,dx,$$

$$=F((\sigma-a),(v^\alpha-b),(\frac{\delta-\mu c}{\mu})).$$

Property(2-3): Let $f(z,t,x)=y(z)h(t)g(x),\quad z>0, t>0,\ x>0.$ then

$S_z\, S_t S_x[\,f(z,t,x)] = S_z\,[y(z)]\ \ S_t[h(t)]S_x[g(x)].$ (6)

Proof : By definition

$S_z\, S_t S_x[\,f(z,t,x)]$

$$=\frac{1}{\sigma^n v^\beta}\int_0^\infty\int_0^\infty\int_0^\infty e^{-(\sigma z+v^\alpha t+\frac{\delta}{\mu}x)}\,[y(z)h(t)g(x)]dz\,dt\,dx,$$

$=\frac{1}{\sigma^n}\int_0^\infty e^{-\sigma z}\, y(z)\, dz\, \frac{1}{v^\beta}\int_0^\infty e^{-v^\alpha t}\, h(t)\, dt\int_0^\infty e^{-\frac{\delta}{\mu}x}\, g(x)\, dx,$

$= S_z\,[y(z)]\ \ S_t[h(t)]S_x[g(x)]$

Now, we compute this triple technique for some essential functions,

1) Let $f(z,t,x)=1$ then

$$S_z\, S_t S_x[\,1] = \frac{1}{\sigma^n v^\beta}\int_0^\infty\int_0^\infty\int_0^\infty e^{-(\sigma z+v^\alpha t+\frac{\delta}{\mu}x)}\, dz\,dt\,dx.$$

Thus, we get using the properties of single transforms

$S_z\, S_t S_x f(z,t,x) = \frac{1}{\sigma^{n+1}}\,\frac{1}{v^{\alpha+\beta}}\,\frac{\mu}{\delta}.$

2) Let $f(z,t,x)=z^r t^\rho x^\omega,\ z>0, t>0\ and\ x>0$ and $r,\rho\ and\ \omega$ are positive constants. Then

$S_z\, S_t S_x[z^r t^\rho x^\omega] = \frac{1}{\sigma^n v^\beta}\int_0^\infty\int_0^\infty\int_0^\infty e^{-(\sigma z+v^\alpha t+\frac{\delta}{\mu}x)}[z^r t^\rho x^\omega]\, dz\,dt\,dx\,,$

$=\frac{1}{\sigma^n}\int_0^\infty e^{-\sigma z}\,[z^r]dz\,\frac{1}{v^\beta}\int_0^\infty e^{-v^\alpha t}\,[t^\rho]dt\,\int_0^\infty e^{-\frac{\delta}{\mu}x}\, x^\omega dx\ .$

Thus, we get using the properties of single transforms

$S_z\, S_t S_x[z^r t^\rho x^\omega] = \frac{\Gamma(r+1)}{\sigma^{n+r+1}}\,\frac{\Gamma(\rho+1)}{v^{\beta+\alpha\rho+\alpha}}\,\frac{\Gamma(\omega+1)}{(\frac{\mu}{\delta})^{\omega+1}}\ .$

3) Let $f(z,t,x)=e^{az+bt+cx}\,, z>0\,, t>0\,, and\ x>0$ and a , b and c are constants. Then,

$S_z\, S_tS_x[e^{az+bt+cx}]$

$$= \frac{1}{\sigma^n v^\beta}\int_0^\infty \int_0^\infty \int_0^\infty e^{-(\sigma z+v^\alpha t+\frac{\delta}{\mu}x)}\,[e^{az+bt+cx}]dz\,dt\,dx,$$

$= \frac{1}{\sigma^n}\int_0^\infty e^{-\sigma z}\,[e^{az}]\,dz\,\frac{1}{v^\beta}\int_0^\infty e^{-v^\alpha t}\,[e^{bt}]\,dt\int_0^\infty e^{-\frac{\delta}{\mu}x}\,[e^{cx}]\,dx,$

Thus, we get using the properties of single transforms

$S_z\, S_tS_x[e^{az+bt+cx}] = \frac{1}{\sigma^n(\sigma-a)}\,\frac{1}{v^\beta(v^\alpha-b)}\,\frac{\mu}{\delta-\mu c}.$

Similarly, $S_z\, S_tS_x\left[e^{i(az+bt+cx)}\right] = \frac{1}{\sigma^n(\sigma-ia)}\,\frac{1}{v^\beta(v^\alpha-ib)}\,\frac{\mu}{\delta-i\mu c}.$

Thus, one can obtain

$S_z\, S_tS_x\left[e^{i(az+bt+cx)}\right] = \frac{\mu^2\left(\sigma\, v^\alpha\frac{\delta}{\mu}-\sigma\, b\, c-av^\alpha c-a\, b\,\frac{\delta}{\mu}\right)+i\mu^2\left(\sigma\, v^\alpha c+\sigma\, b\,\frac{\delta}{\mu}+av^\alpha\frac{\delta}{\mu}-a\, b\, c\right)}{\sigma^n\, v^\beta\,(\sigma^2+a^2)\,(v^2v^\alpha+b^2)(\delta^2+\mu^2c^2)}$

.

Euler's formulas implies that $\sin z = \frac{e^{iz}-e^{-iz}}{2i}$ and $\cos z = \frac{e^{iz}+e^{-iz}}{2}$.

And, $\sinh z = \frac{e^{z}-e^{z}}{2i}$ and $\cosh z = \frac{e^{z}+e^{-z}}{2}$

Consequently, the new triple technique of some essential functions can be obtained as :

$S_z\, S_tS_x[sin\,(az+bt+cx)] = \frac{\mu^2\left(\sigma\, v^\alpha c+\sigma\, b\,\frac{\delta}{\mu}+av^\alpha\frac{\delta}{\mu}-a\, b\, c\right)}{\sigma^n\, v^\beta\,(\sigma^2+a^2)\,(\,v^{2\alpha}+b^2)(\delta^2+\mu^2c^2)}$. (7)

$S_z\, S_tS_x[cos\,(az+bt+cx)] = \frac{\mu^2\left(\sigma\, v^\alpha\frac{\delta}{\mu}-\sigma\, b\, c-av^\alpha c-a\, b\,\frac{\delta}{\mu}\right)}{\sigma^n\, v^\beta\,(\sigma^2+a^2)\,(\,v^{2\alpha}+b^2)(\delta^2+\mu^2c^2)}$.
(8)

$S_z\, S_tS_x[sinh\,(az+bt+cx)] = \frac{\mu^2\left(\sigma\, v^\alpha\, c+\sigma\, b\,\frac{\delta}{\mu}+av^\alpha\frac{\delta}{\mu}-a\, b\, c\right)}{\sigma^n\, v^\beta\,(\sigma^2-a^2)\,(\,v^{2\alpha}-b^2)(\delta^2-\mu^2c^2)}$.
(9)

$S_z\, S_tS_x[cosh\,(az+bt+cx)] = \frac{\mu^2\left(\sigma\, v^\alpha\frac{\delta}{\mu}-\sigma\, b\, c-av^\alpha c-a\, b\,\frac{\delta}{\mu}\right)}{\sigma^n\, v^\beta\,(\sigma^2-a^2)\,(\,v^{2\alpha}-b^2)(\delta^2-\mu^2c^2)}.$
(10)

4)Let

$S_z\, S_tS_x[\sin(az\,)\,\sin(bt)\sin(cx)] =$

$\frac{1}{\sigma^n v^\beta}\int_0^\infty \int_0^\infty \int_0^\infty e^{-(\sigma z+v^\alpha t+\frac{\delta}{\mu}x)}\,[\sin(az)\,\sin(bt)\sin(cx)]dz\,dt\,dx,$

$=\frac{1}{\sigma^n}\int_0^\infty e^{-\sigma z}\sin(az\,)\,dz\,\frac{1}{v^\beta}\int_0^\infty e^{-v^\alpha t}\sin(bt)\,dt\int_0^\infty e^{-\frac{\delta}{\mu}x}\sin(cx)\,dx.$

Thus, we get using the properties of single transforms:

$$S_z\, S_t S_x[\sin(az)\sin(bt)\sin(cx)] = \frac{\mu^2 a\, b\, c}{\sigma^n\, v^\beta\, (\sigma^2+a^2)\,(v^{2\alpha}+b^2)(\delta^2+\mu^2 c^2)}.$$

Definition (2-6): If $f(z,t,x)$ is defined on $[0,Z]\times[0,T]\times[0,X]$, and satisfies the condition $|f(z,t,x)| \le Re^{hz+kt+\gamma x}$,$\exists R>0, \forall z>Z, \forall t>T and\ \forall x>X$.

Then $f(z,t,x)$ is said to be a function of exponential orders $h,k\ and\, \gamma$ as $z,t,x \to \infty$.

Theorem (1). The existence condition of TLSEESSHT of the continuous function $f(z,t,x)$ defined $[0,Z]\times[0,T]\times[0,X]$ is to be of exponential orders $h,k\ and\, \gamma$, for $[\sigma]>h, Re[\,v^\alpha]>k, and\ Re\left[\frac{\delta}{\mu}\right]>\gamma$.

Proof.. The definition of TSEESSHT implies that

$$\left|F\left(\sigma,\ v^\alpha,\frac{\delta}{\mu}\right)\right| = \left|\frac{1}{\sigma^n v^\beta}\int_0^\infty\int_0^\infty\int_0^\infty e^{-(\sigma z+v^\alpha t+\frac{\delta}{\mu}x)}\, f(z,t,x)dz\,dt\,dx\right|$$

$$\le \frac{1}{\sigma^n v^\beta}\int_0^\infty\int_0^\infty\int_0^\infty e^{-(\sigma z+v^\alpha t+\frac{\delta}{\mu}x)}\,|f(z,t,x)|dz\,dt\,dx\,,$$

$$\le \frac{R}{\sigma^n v^\beta}\int_0^\infty\int_0^\infty\int_0^\infty e^{-(\sigma z+v^\alpha t+\frac{\delta}{\mu}x)}\,e^{hz+kt+\gamma x}\,dz\,dt\,dx\,,$$

$$= \frac{R}{\sigma^n v^\beta}\int_0^\infty e^{-(\sigma-h)z}dz\int_0^\infty e^{-(v^\alpha-k)t}\int_0^\infty e^{-\left(\frac{\delta}{\mu}-\gamma\right)x}dx\,,$$

$$= \frac{R\ \mu}{\sigma^n v^\beta\,(\sigma-h)(\,v^\alpha-k)(\delta-\mu\gamma)}.$$

$Re[\sigma]>h, Re[\,v^\alpha]>k, and\ Re\left[\frac{\delta}{\mu}\right]>\gamma$.

Definition (2-7): the convolution of $f(z,t,x)$ and $\xi(z,t,x)$ is denoted by $(f***\xi)(z,t,x)$ and defined by

$$(f***\xi)(z,t,x) = \int_0^z\int_0^t\int_0^x f(z-B,t-\varepsilon,x-A)\xi(B,\varepsilon,A)\,dB\,dt\,dA \quad . \quad (11)$$

Theorem (2): let $S_z\, S_t S_x[f(z,t,x)] = F\left(\sigma,\ v^\alpha,\frac{\delta}{\mu}\right)$. Then

$$S_z\, S_t S_x\,[f(z-B,t-\varepsilon,x-A)H(z-B,t-\varepsilon,x-A)] = e^{-\sigma B-\varepsilon v^\alpha t-\frac{\delta}{\mu}A} = F\left(\sigma,\ v^\alpha,\frac{\delta}{\mu}\right)$$

.

Where $H(z,t,x)$ denotes the Heaviside function defined by:

$$H(z-B,t-\varepsilon,x-A) = \begin{cases}1, & z>B, t>\varepsilon, x>A\\ 0, & \text{otherwise}\end{cases}.$$

Proof: From the definition, we have

$S_z\, S_t S_x\,[f(z-B,t-\varepsilon,x-A)H(z-B,t-\varepsilon,x-A)] =$

$\frac{1}{\sigma^n v^\beta}\int_0^\infty\int_0^\infty\int_0^\infty e^{-(\sigma z+v^\alpha t+\frac{\delta}{\mu}x)} f(z-B,t-\varepsilon,x-A)H(z-B,t-\varepsilon,x-A)\, dz\, dt\, dx$

,

$$=\frac{1}{\sigma^n v^\beta}\int_0^\infty\int_0^\infty\int_0^\infty e^{-(\sigma z+v^\alpha t+\frac{\delta}{\mu}x)} f(z-B,t-\varepsilon,x-A)\, dz\, dt\, dx \qquad (12)$$

Putting $-B=k$, $t-\varepsilon=w$ and $x-A=D$ in equation (11) , we obtain :

$$S_z\, S_t S_x\,[f(z-B,t-\varepsilon,x-A)H(z-B,t-\varepsilon,x-A)] = \frac{1}{\sigma^n v^\beta}\int_0^\infty\int_0^\infty\int_0^\infty e^{-\sigma(k+B)+v^\alpha(\varepsilon+w)+\frac{\delta}{\mu}(A-D)} f(k,w,D)\, dk\, dw\, dD. \qquad (13)$$

Thus, Equation (13) can be simplified into

$$S_z\, S_t S_x\,[f(z-B,t-\varepsilon,x-A)H(z-B,t-\varepsilon,x-A)] = e^{-\sigma B-v^\alpha\varepsilon-\frac{\delta}{\mu}A}\left(\frac{1}{\sigma^n v^\beta}\int_0^\infty\int_0^\infty\int_0^\infty e^{-\sigma k-v^\alpha w-\frac{\delta}{\mu}D} f(k,w,D)\, dk\, dw\, dD\right.$$

$$= e^{-\sigma B-v^\alpha\varepsilon-\frac{\delta}{\mu}A}\, F\left(\sigma,\ v^\alpha,\frac{\delta}{\mu}\right).$$

Theorem (3). (Convolution Theorem) If

$S_z\, S_t S_x[f(z,t,x)] = F\left(\sigma,\ v^\alpha,\frac{\delta}{\mu}\right)$ and $S_z\, S_t S_x[\rho(z,t,x)] = \mathrm{P}(\sigma,\ v^\alpha,\frac{\delta}{\mu})$, then

$$S_z\, S_t S_x[(f***\rho)(z,t,x)] = \frac{1}{\sigma^n v^\beta} F\left(\sigma,\ v^\alpha,\frac{\delta}{\mu}\right)\ \mathrm{P}(\sigma,\ v^\alpha,\frac{\delta}{\mu}) \qquad (14)$$

Proof: From the definition, we have

$$S_z\, S_t S_x[(f***\rho)(z,t,x)] = \frac{1}{\sigma^n v^\beta}\int_0^\infty\int_0^\infty\int_0^\infty e^{-(\sigma z+v^\alpha t+\frac{\delta}{\mu}x)} \left[\int_0^z\int_0^t\int_0^x f(z-B,t-\varepsilon,x-A)\xi(B,\varepsilon,A)\, dB\, dt\, dA\right] dz\, dt\, dx. \qquad (15)$$

The definition of Heaviside function, Equation (16) can be written as

$$S_z\, S_t S_x[(f***\rho)(z,t,x)] = \frac{1}{\sigma^n v^\beta}\int_0^\infty\int_0^\infty\int_0^\infty e^{-(\sigma z+v^\alpha t+\frac{\delta}{\mu}x)}\left[\int_0^\infty\int_0^\infty\int_0^\infty f(z-B,t-\varepsilon,x-A)H(z-B,t-\varepsilon,x-A)\xi(B,\varepsilon,A)\, dB\, dt\, dA\right] dz\, dt\, dx. \qquad (15)$$

Thus, Equation (16) can be written as
$S_z\, S_t S_x[(f *** \rho)(z,t,x)] =$
$\int_0^\infty \int_0^\infty \int_0^\infty \xi(B,\varepsilon,A)\, dB\, dt\, dA\, [\frac{1}{\sigma^n v^\beta} \int_0^\infty \int_0^\infty \int_0^\infty e^{-(\sigma z+v^\alpha t+\frac{\delta}{\mu}x)} f(z-B,t-\varepsilon,x-A) H(z-B,t-\varepsilon,x-A)\, dz\, dt\, dx]$

Using Theorem 3, we have :

$S_z\, S_t S_x[(f *** \rho)(z,t,x)] = \int_0^\infty \int_0^\infty \int_0^\infty \xi(B,\varepsilon,A)\, dB\, dt\, dA\, (e^{-\sigma B - v^\alpha \varepsilon - \frac{\delta}{\mu}A}\, F\left(\sigma,\, v^\alpha, \frac{\delta}{\mu}\right)),$

$= F\left(\sigma,\, v^\alpha, \frac{\delta}{\mu}\right) \int_0^\infty \int_0^\infty \int_0^\infty e^{-\sigma B - v^\alpha \varepsilon - \frac{\delta}{\mu}A}\, \xi(B,\varepsilon,A)\; dB\, dt\, dA\,,$

$= \sigma^n v^\alpha\, F\left(\sigma,\, v^\alpha, \frac{\delta}{\mu}\right) \mathrm{P}(\sigma,\, v^\alpha, \frac{\delta}{\mu}).$

The following theorem presents for the partial derivatives of orders 1 and 2.

Theorem (7). (Derivative properties) If
$S_z\, S_t S_x[f(z,t,x)] = F\left(\sigma,\, v^\alpha, \frac{\delta}{\mu}\right)$, Then

- $S_z\, S_t S_x\left[\frac{\partial f(z,t,x)}{\partial z}\right] = \sigma F\left(\sigma,\, v^\alpha, \frac{\delta}{\mu}\right) - \frac{1}{\sigma^n} S_t S_x[f(0,t,x)].$
- $S_z\, S_t S_x\left[\frac{\partial f(z,t,x)}{\partial t}\right] = v^\alpha F\left(\sigma,\, v^\alpha, \frac{\delta}{\mu}\right) - \frac{1}{v^\beta} S_z S_x[f(z,0,x)].$
- $S_z\, S_t S_x\left[\frac{\partial f(z,t,x)}{\partial x}\right] = \frac{\delta}{\mu} F\left(\sigma,\, v^\alpha, \frac{\delta}{\mu}\right) - S_z S_t[f(z,t,0)].$
- $S_z\, S_t S_x\left[\frac{\partial^2 f(z,t,x)}{\partial z^2}\right] = \sigma^2 F\left(\sigma,\, v^\alpha, \frac{\delta}{\mu}\right) - \frac{\sigma}{\sigma^n} S_t S_x[f(0,t,x)] - \frac{1}{\sigma^n} S_t S_x[f_z(0,t,x)].$
- $S_z\, S_t S_x\left[\frac{\partial^2 f(z,t,x)}{\partial t^2}\right] = v^{2\alpha} F\left(\sigma,\, v^\alpha, \frac{\delta}{\mu}\right) - \frac{v^\alpha}{v^\beta} S_z S_x[f(z,0,x)] - \frac{1}{v^\beta} S_z S_x[f_t(z,0,x)]$

.
- $S_z\, S_t S_x\left[\frac{\partial^2 f(z,t,x)}{\partial x^2}\right] = (\frac{\delta}{\mu})^2 F\left(\sigma,\, v^\alpha, \frac{\delta}{\mu}\right) - \frac{\delta}{\mu} S_z S_t[f(z,t,0)] - S_z S_t[f_x(z,t,0)]$

.

Proof:. Based on the definition, we get:

1. $S_z\, S_t S_x\left[\frac{\partial f(z,t,x)}{\partial z}\right] = \frac{1}{\sigma^n v^\beta} \int_0^\infty \int_0^\infty \int_0^\infty e^{-(\sigma z+v^\alpha t+\frac{\delta}{\mu}x)}\, f(z,t,x) dz\, dt\, dx.$

$$= \frac{1}{\sigma^n v^\beta} \int_0^\infty \int_0^\infty e^{-v^\alpha t - \frac{\delta}{\mu} x} dt\, dx \int_0^\infty e^{-\sigma z} \left[\frac{\partial f(z,t,x)}{\partial z}\right] dz .$$

Appling integrating by parts with:

$$u = e^{-\sigma z} \quad , \qquad du = -\sigma e^{-\sigma z} dz \ ,$$

$$dv = \frac{\partial f(z,t,x)}{\partial z} dz \ , \qquad v = f(z,t,x).$$

We obtain :

$$\int_0^\infty e^{-\sigma z} \left[\frac{\partial f(z,t,x)}{\partial z}\right] dz = -f(0,t,x) + \sigma \int_0^\infty e^{-\sigma z} f(z,t,x) dz .$$

$$Then\ S_z S_t S_x \left[\frac{\partial f(z,t,x)}{\partial z}\right] = \sigma S_z S_t S_x[f(z,t,x)] - \frac{1}{\sigma^n} S_t S_x[f(0,t,x)] \qquad (17)$$

$$2.\ S_z S_t S_x \left[\frac{\partial f(z,t,x)}{\partial t}\right] = \frac{1}{\sigma^n v^\beta} \int_0^\infty \int_0^\infty \int_0^\infty e^{-(\sigma z + v^\alpha t + \frac{\delta}{\mu} x)} f(z,t,x) dz\, dt\, dx$$

$$S_z S_t S_x \left[\frac{\partial f(z,t,x)}{\partial t}\right] = \frac{1}{\sigma^n v^\beta} \int_0^\infty \int_0^\infty e^{-\sigma z - \frac{\delta}{\mu} x} dz\, dx \int_0^\infty e^{-v^\alpha t} \left[\frac{\partial f(z,t,x)}{\partial t}\right] dt$$

Appling integrating by parts with:

$$u = e^{-v^\alpha t} \quad \rightarrow \quad du = -v^\alpha e^{-v^\alpha t} dt .$$

$$dv = \frac{\partial f(z,t,x)}{\partial t} dt \quad \rightarrow \quad v = f(z,t,x) .$$

Thus

$$\int_0^\infty e^{-v^\alpha t} \left[\frac{\partial f(z,t,x)}{\partial t}\right] dt = -f(z,0,x) + v^\alpha \int_0^\infty e^{-v^\alpha t} [f(z,t,x) dt ,$$

$$S_z S_t S_x \left[\frac{\partial f(z,t,x)}{\partial t}\right] = v^\alpha S_z S_t S_x[f(z,t,x)] - \frac{1}{v^\beta} S_z S_x[f(z,0,x)] \quad .$$

(18)

$$3.\ S_z S_t S_x \left[\frac{\partial f(z,t,x)}{\partial t}\right] = \frac{1}{\sigma^n v^\beta} \int_0^\infty \int_0^\infty \int_0^\infty e^{-(\sigma z + v^\alpha t + \frac{\delta}{\mu} x)} f(z,t,x) dz\, dt\, dx,$$

$$S_z S_t S_x \left[\frac{\partial f(z,t,x)}{\partial t}\right] =$$

$$\frac{1}{\sigma^n v^\beta} \int_0^\infty \int_0^\infty e^{-\sigma z - v^\alpha t} dz\, dt \int_0^\infty e^{-\frac{\delta}{\mu} x} \left[\frac{\partial f(z,t,x)}{\partial x}\right] dx .$$

Appling integrating by parts with:

$$u = e^{-\frac{\delta}{\mu} x} \quad \rightarrow \quad du = -\frac{\delta}{\mu} e^{-\frac{\delta}{\mu} x} dx .$$

$$dv = \frac{\partial f(z,t,x)}{\partial x} dx \quad \rightarrow \quad v = f(z,t,x) .$$

$$\int_0^\infty e^{-\frac{\delta}{\mu} x} \left[\frac{\partial f(z,t,x)}{\partial x}\right] dx = -f(z,t,0) + \frac{\delta}{\mu} \int_0^\infty e^{-\frac{\delta}{\mu} x} [f(z,t,x) dx.$$

Thus

$$S_z\, S_t S_x\left[\frac{\partial f(z,t,x)}{\partial x}\right]=\frac{\delta}{\mu}S_z\, S_t S_x[f(z,t,x)]-\ S_z S_t[f(z,t,0)] \qquad .$$
(19)

Similarly, we can prove that:

- $S_z\, S_t S_x\left[\frac{\partial^2 f(z,t,x)}{\partial z^2}\right]=\sigma^2 S_z\, S_t S_x[f(z,t,x)]-\frac{\sigma}{\sigma^n}\, S_t S_x[f(0,t,x)]-\frac{1}{\sigma^n}\, S_t S_x[f_z(0,t,x)]$
.
- $S_z\, S_t S_x\left[\frac{\partial^2 f(z,t,x)}{\partial t^2}\right]=\ v^{2\alpha} S_z\, S_t S_x[f(z,t,x)]-\frac{v^\alpha}{v^\beta}\, S_z S_x[f(z,0,x)]-\frac{1}{v^\beta}\, S_z S_x[f_t(z,0,x)].$
- $S_z\, S_t S_x\left[\frac{\partial^2 f(z,t,x)}{\partial x}\right]=\frac{\delta^2}{\mu}\, S_z\, S_t S_x[f(z,t,x)]-\frac{\delta}{\mu}\, S_z S_t[f(z,t,0)]-S_z S_t[f_x(z,t,0)]$
.

Corollary 1. If $S_z\, S_t S_x[f(z,t,x)]=F\left(\sigma,\ v^\alpha,\frac{\delta}{\mu}\right)$, then

i. $S_z\, S_t S_x\left[\frac{\partial^2 f(z,t,x)}{\partial z\, \partial t}\right]=\sigma\, v^\alpha S_z\, S_t S_x[f(z,t,x)]-\frac{\sigma}{v^\beta}\, S_z S_x[f(z,0,x)]+(\frac{1}{\sigma^n\ v^\beta})^2 S_x[f(0,0,x)]-\frac{v^\alpha}{\sigma^{2n}\ v^\beta}\, S_t S_x[f_z(0,t,x)]$
.

ii. $S_z\, S_t S_x\left[\frac{\partial^2 f(z,t,x)}{\partial z\, \partial x}\right]=\sigma\frac{\delta}{\mu}S_z\, S_t S_x[f(z,t,x)]-\sigma\, S_z S_t[f(z,t,0)]+\frac{1}{\sigma^n}S_t[f(0,t,0)]-\frac{\delta}{\sigma^{2n}\mu}\, S_t S_x[f(0,t,x)]$
.

iii. $S_z\, S_t S_x\left[\frac{\partial^2 f(z,t,x)}{\partial t\, \partial x}\right]=\ v^\alpha\frac{\delta}{\mu}S_z\, S_t S_x[f(z,t,x)]-\ v^\alpha\, S_z S_t[f(z,t,0)]+\frac{1}{\sigma^n\ v^{2\beta}}S_z[f(z,0,0)]-\frac{\delta}{v^{2\beta}\mu}\, S_z S_x[f(z,0,x)]$
.

The proof can be obtained by direct applications of partial derivatives in Theorem 7

3- Applications

In this section, we use the proposed triple transform for solving some types of linear PDEs.

Example (3-1). Consider the following nonhomogeneous heat equation ,[4]:

$f_x(z,t,x) = f_{zz}(z,t,x) + f_{tt}(z,t,x) + 2\cos(z+t), \quad (z,t) \in R_+^2 \quad ,x > 0$
(20)

Subject to the boundary and initial conditions:

$$\begin{cases} f(0,t,x) = e^{-2x}\, sint + cos\,t\,, & f_z(0,t,x) = e^{-2x}\, cos\,t - sint \\ f(z,0,x) = e^{-2x}\, sinz + cos\,z\,, & f_t(z,0,x) = e^{-2x}\, cos\,z - sinz \\ f(z,t,0) = \sin(z+t) + \cos(z+t)\,. \end{cases} \quad (21)$$

Applying the triple transform on both sides of Equation (19), we have

$S_z\, S_t S_x[f_x(z,t,x)] = S_z\, S_t S_x[f_{zz}(z,t,x) + f_{tt}(z,t,x) + 2\cos(z+t)].$
(22)

Using the linearity and partial derivative properties of this triple transform, we get

$\frac{\delta}{\mu} F\left(\sigma,\, v^{\alpha}, \frac{\delta}{\mu}\right) - S_z S_t[f(z,t,0)] =$
$\sigma^2 F\left(\sigma,\, v^{\alpha}, \frac{\delta}{\mu}\right) - \frac{\sigma}{\sigma^n} S_t S_x[f(0,t,x)] - \frac{1}{\sigma^n} S_t S_x[f_z(0,t,x)] +$
$v^{2\alpha} F\left(\sigma,\, v^{\alpha}, \frac{\delta}{\mu}\right) - \frac{v^{\alpha}}{v^{\beta}} S_z S_x[f(z,0,x)] - \frac{1}{v^{\beta}} S_z S_x[f_t(z,0,x)] +$
$S_z\, S_t S_x[2\cos(z+t)]$
(23)

Rearranging the terms, we have

$F\left(\sigma,\, v^{\alpha}, \frac{\delta}{\mu}\right) = \frac{\mu}{\delta - \sigma^2 - v^{2\alpha}} \Big(S_z S_t[f(z,t,0)] - \frac{\sigma}{\sigma^n} S_t S_x[f(0,t,x)] -$
$\frac{1}{\sigma^n} S_t S_x[f_z(0,t,x)] - \frac{v^{\alpha}}{v^{\beta}} S_z S_x[f(z,0,x)] - \frac{1}{v^{\beta}} S_z S_x[f_t(z,0,x)] +$
$\frac{2\mu(\sigma\, v^{\alpha} - 1)}{\sigma^n\;\; v^{\beta}\delta(\sigma^2+1)(v^{2\alpha}+1)}$
. (24)

Substituting the transformed values:

$$S_z S_t[f(z,t,0)] = \frac{(\sigma\, v^{\alpha} + \sigma + v^{\alpha} - 1)}{\sigma^n\, v^{\beta}\, (\sigma^2+1)\, (v^{2\alpha}+1)} + \frac{(\sigma\, v^{\alpha} - \sigma - v^{\alpha} - 1)}{\sigma^n\, v^{\beta}\, (\sigma^2+1)\, (v^{2\alpha}+1)}.$$

$$S_t S_x[f(0,t,x)] = \frac{\mu^2 (1 + v^{\alpha})}{v^{\alpha} v^{2\beta} \delta(v^{2\alpha}+1)(\delta+2\mu)} + \frac{\mu(v^{\alpha}-1)}{v^{\beta}\delta(v^{2\alpha}+1)}.$$

$$S_t S_x[f_z(0,t,x)] = \frac{\mu^2 (v^{\alpha}-1)}{v^{\alpha} v^{2\beta} \delta(v^{2\alpha}+1)(\delta+2\mu)} - \frac{\mu(v^{\alpha}+1)}{v^{\beta}\,\delta(v^{2\alpha}+1)}.$$

$$S_z S_x[f(z,0,x)] = \frac{2\mu^2}{\sigma^{2n}\sigma\delta(\delta+2\mu)(\sigma^2+1)} + \frac{\mu(\sigma-1)}{\sigma^n \delta(\sigma^2+1)}.$$

$$S_z S_x[f_t(z,0,x)] = \frac{\mu^2(\sigma-1)}{\sigma^{2n}\sigma\delta(\delta+2\mu)(\sigma^2+1)} - \frac{2\mu}{\sigma^n\delta(\sigma^2+1)}.$$

In Equation (24) and simplifying, we obtain

$$F\left(\sigma,\, v^{\alpha}, \frac{\delta}{\mu}\right) = \frac{\mu}{\delta - \sigma^2 - v^{2\alpha}} \left(\frac{[\mu(\sigma\, v^{\alpha} - 1) + (\sigma\, v^{\alpha} - 1)(\delta+2\mu)](\delta - \sigma^2 - v^{2\alpha})}{\sigma^n v^{\beta}\delta(\delta+2\mu)(\sigma^2+1)(v^{2\alpha}+1)} \right. \quad (25)$$

Taking the inverse transform $S_z^{-1}\, S_t^{-1} S_x^{-1}$ for Equation (25), we get

$$f(z,t,x) = S_z^{-1}\, S_t^{-1} S_x^{-1}\left[\frac{\mu^2(\sigma\, v^{\alpha}-1)}{\sigma^n v^{\beta}\delta(\sigma^2+1)(\, v^{2\alpha}+1)(\delta+2\mu)} + \frac{\mu(\sigma\, v^{\alpha}-1)}{\sigma^n v^{\beta}\delta(\sigma^2+1)(\, v^{2\alpha}+1)}\right]$$

$$= e^{-2x}\sin(z+t) + \cos(z+t)$$

Example (2). Consider the following Laplace operator equation ,[3]:

$$f_{zz}(z,t,x) + f_{tt}(z,t,x) + f_{xx}(z,t,x) = 0, \qquad (z,t,x) \in R_+^3$$

(26)

Subject to the boundary and initial conditions:

$$\begin{cases} f(0,t,x) = 0\,, & f_z(0,t,x) = \sin t\, \sinh\sqrt{2}t \\ f(z,0,x) = 0, & f_t(z,0,x) = \sin z\, \sinh\sqrt{2}t \\ f(z,t,0) = 0\,, & f_t(zt,0) = \sqrt{2}\sin z\, \sin t\,. \end{cases}$$

(27)

Applying the triple transform on both sides of Equation (26), we have

$$S_z\, S_t S_x[f_{zz}(z,t,x) + f_{tt}(z,t,x) + f_{xx}(z,t,x)] = 0.$$

(28)

Using the linearity and partial derivative properties of this triple transform, we get

$$\sigma^2 F\left(\sigma,\, v^{\alpha},\frac{\delta}{\mu}\right) - \frac{\sigma}{\sigma^n}\, S_t S_x[f(0,t,x)] - \frac{1}{\sigma^n}\, S_t S_x[f_z(0,t,x)] + v^{2\alpha} F\left(\sigma,\, v^{\alpha},\frac{\delta}{\mu}\right) - \frac{v^{\alpha}}{v^{\beta}}\, S_z S_x[f(z,0,x)] - \frac{1}{v^{\beta}}\, S_z S_x[f_t(z,0,x)] + \left(\frac{\delta}{\mu}\right)^2 F\left(\sigma,\, v^{\alpha},\frac{\delta}{\mu}\right) - \frac{\delta}{\mu}\, S_z S_t[f(z,t,0)] - S_z S_t[f_x(z,t,0)] = 0$$

. (29)

Rearranging the terms, we have

$$F\left(\sigma,\, v^{\alpha},\frac{\delta}{\mu}\right) = \frac{1}{\sigma^2+v^{2\alpha}+(\frac{\delta}{\mu})^2}\Big(\frac{\sigma}{\sigma^n}\, S_t S_x[f(0,t,x)] + \frac{1}{\sigma^n}\, S_t S_x[f_z(0,t,x)] + \frac{v^{\alpha}}{v^{\beta}}\, S_z S_x[f(z,0,x)] + \frac{1}{v^{\beta}}\, S_z S_x[f_t(z,0,x)] + \frac{\delta}{\mu}\, S_z S_t[f(z,t,0)] + S_z S_t[f_x(z,t,0)]\Big).$$

(30)

Substituting $S_t S_x[f(0,t,x)] = 0$,

$$S_t S_x[f_z(0,t,x)] = \frac{\mu(\, v^{\alpha}+1)}{v^{\beta}\delta(\, v^{2\alpha}+1)}\; \frac{\sqrt{2}+\frac{\delta}{\mu}}{v^{\beta}\; v^{\alpha}((\frac{\delta}{\mu})^2-2)}.$$

$$S_z S_x[f(z,0,x)] = 0\,, \quad S_z S_x[f_t(z,0,x)] = \frac{\mu(\sigma+1)}{\sigma^n\delta(\sigma^2+1)}\; \frac{(\sqrt{2}+\sigma\, \frac{\delta}{\mu})}{\sigma^{n+1}((\frac{\delta}{\mu})^2-2)}.$$

$S_zS_t[f(z,t,0)] = 0, \quad S_zS_t[f_x(z,t,0)] =$
$\sqrt{2}\frac{(\sigma+1)}{\sigma^n\, v^\beta\, v^\alpha\, (\sigma^2+1)} \quad \frac{(v^\alpha+1)}{\sigma^n\, v^\beta \sigma(\, v^{2\alpha}+1)}$.

In Equation (30) and simplifying, we obtain

$$F\left(\sigma,\, v^\alpha, \frac{\delta}{\mu}\right) = \frac{1}{\sigma^2+v^{2\alpha}+(\frac{\delta}{\mu})^2}\left(\frac{\sqrt{2}\,\mu^2(\sigma^2+v^{2\alpha}+(\frac{\delta}{\mu})^2)}{\sigma^{3n}\, v^{3\beta}\, v^{2\alpha}\sigma^2\, \delta^2(\sigma^2+1)(\, v^{2\alpha}+1)((\frac{\delta}{\mu})^2-2)}\right)$$
(31)

Taking the inverse transform $S_z^{-1}\, S_t^{-1}S_x^{-1}$ for Equation (31), we get

$$f(z,t,x) = S_z^{-1}\, S_t^{-1}S_x^{-1}\left[\frac{\sqrt{2}\,\mu^2}{\sigma^{3n}\, v^{3\beta}\, v^{2\alpha}\sigma^2\, \delta^2(\sigma^2+1)(\, v^{2\alpha}+1)((\frac{\delta}{\mu})^2-2)}\right.$$
$$= \sin z \sin t \sinh \sqrt{2}\, x\,.$$

4- Conclusion

This article introduced a new triple integral transform called SEE Sadik Shehu transform technique. The definition of this transform and its useful properties, including linearity, existence, inverse and specific values for importance functions were discussed and proved. Moreover, new results concerning the triple convolution theorem and partial derivatives were introduced. The effectiveness of the novel technique was proven via using it to find the solution of various kinds of PDEs. The results obtained in this study highlighted the simplicity and practicality of this triple approach, achieving the aim of providing new techniques to evaluate the solution of PDEs with significant scientific applications. Moving forward, our aim is to extend the application of this technique to handle non-linear PDEs and systems of differential equations.

5- References

[1] Aggarwal, S., Singh, A., Kumar, A., Kumar, N., Application of Laplace transform for solving improper integrals whose integrand consisting error function, Journal of Advanced Research in Applied Mathematics and Statistics, 4(2019), 1–7.

[2] Ahmed, S.A., Elzaki, T.M., Elbadri, M., Mohamed, M., Solution of partial differential equations by new double integral transform (Laplace - Sumudu transform), Ain Shams Engineering Journal, 12(2021), 4045–4049.

[3] Albukhuttar, A., Jubear, B., Neamah, M., Solve the Laplace, Poisson and telegraph equations using the Shehu transform, Turkish Journal of Computer and Mathematics Education, Turkey, 10(2021), 1759–1768.

[4] Albukhuttar, A., Ridha, Z., Kadhim, H., Applications of a Shehu transform to the heat and transport equations, International Journal of Psychosocial Rehabilitation, 24(2020), 4254–4263.

[5] Alfaqeih, S., Misirli, E., On double Shehu transform and its properties with applications, International Journal of Analysis and Applications, 3(2020), 381–395.
[6] Alkaleeli, S., Mtawal, A., Hmad, M., Triple Shehu transform and its properties with applications, African Journal of Mathematics and Computer, 14(2021), 4–12.
[7] Debnath, L., The double Laplace transforms and their properties with applications to functional, integral and partial differential equations, Int. J. Appl. Comput. Math, 2(2016), 223–241.
[8] Dhunde, R., Waghmare, G., Double Laplace transform method in mathematical physics, International Journal of Theoretical and Mathematical Physics, 7(2017), 14–20.
[9] Eltayeb, H., Kilicman, A., A note on solutions of wave, Laplace's and heat equations with convolution terms by using a double Laplace transform, Applied Mathematics Letters 21(2008), 1324–1329.
[10] Lakmon, K., Mensah, Y., The Plancherel formula for the Shehu transform, Annals of Pure and Applied Mathematical Sciences, 1(2021) 14–19.
[11] Maitama, S., Zhao, W., New integral transform: Shehu transform a generalization of Sumudu and Laplace transform for solving differential equations, International Journal of Analysis and Applications, 17(2019), 167–190
[12] Aggarwal S., Bhatnagar K., Sadik Transform for Handling Population Growth and Decay Problems, Journal of Applied Science and Computations, 24 July 2019, 1076-5131.
[13] Hunaiber M., Al-aati A., On Double Laplace-Shehu Transform and its Properties with Applications, Turk. J. Math. Comput. Sci. 15(2)(2023) 218–226.
[14] Turq S.M. , Kuffi E.A., Anew complex Double Integral transform, and Applications in partial differential , AL-Muthanna Journal of pure science, vol.(10,No.(1), (2023).
[15]. Turq S.M. , Kuffi E.A., On the Double of the Emad - Falih Transformation and Its Properties with Applications, Ibn Al-Haitham Journal for Pure and Applied Sciences, October 2022, 10.30526/35.4.2938.
[16] Kuffi E.A. ,Abbas E.S., Maktoof S.F., Applying "Emad-Sara" Transform on Partial Differential Equations, Springer Nature Singapore,2022/5/11.
[17] Mansour E.A., Kuffi E.A., Mehdi S.A., Solving Partial differential equations using double complex SEE integral transform, AIP Conference Proceedings, 2591/1, 2023/3/29.
[18] Issa A., Kuffi E.A., On the Double Integral Transform(Complex SEE Transform)and Their Properties and Applications , Ibn AL-Haitham Journal For Pure and Applied Sciences, 37/1,2024/1/20.
[19] Mehdi S.A., Kuffi E.A., Jasim J.A., Solving the Partial differential equations by using SEJI integral transform, Journal of Interdisciplinary Mathematics, 26/6, 2023.

[20] Jasim J.A., Mehdi S.A., Kuffi E.A., Double SEJI Integral Transform and its Applications differential equations, General Letters in Mathematics (GLM), 13/4, 2023/12/1.
[21] Mehdi S.A., Kuffi E.A., Jasim J.A., Solving the Delay differential equations by using SEJI integral transform, IEEE, 2022/8/31.
[22]Mohammed N.S., Kuffi E.A., Al-Nuaimi D.H., Utilize the Complex Sadik Transform to Solution Volterra Integro-Differential, Mathematics Analysis and Numerical Methods IACMC 2023,Zaqa, Jordan, May 10-12.
[23]Alsaoudi M., Gharib G., Kuffi E.A., Guitatni A., New Two Parameter Integral Transform "MAHA Transform" and its Applications in Real Life, WSEAS Transactions on Mathematics, 23, 2024.
[24] Eman A. Mansour, Emad A. Kuffi, Sadiq A. Mehdi, The new integral transform " SEE transform " and its applications, Periodicals of Engineering and Natural Science, Vol. 9, No. 2, June 2021, PP. 1016-1029.
[25] Theyab Sh.Haider. ,Solution of the delay differential equations via applying Rohit transform technique, Journal of Interdisciplinary Mathematics, Vol. 28 (2025), No. 1, pp. 195–201
[26] Salman N.K, Kuffi E.A, Mansour E.A, Issa A.J., SEE Transform Technique in Control Theory, Journal of Kufa for Mathematics and Computer Vol.11, No.2, Aug., 2024, pp 147-149.
[27] Salman N.K., Jassim A.T., Kuffi E.A ., Issa A.J., Solution of Partial Differential Equations and Their Application by Three Parameters of Integral Transform, dvances in Nonlinear Variational Inequalities Vol 28 No. 1s (2025).
[28]Theyab Sh.Haider, Kuffi E.A., Salman N.K, Mansour E.A, Issa A.J Solution of First Order Complex Differential Equations by complex Sadik Transform Technique, NTU Journal for Administrative and Human Sciences / Special Issue, Published online : 01, 11, 2024.
[29] Salman N.K., Integral Transform of three Parameters with its Applications ,Kurdish Studies, Jan 2024 Volume: 12, No: 1, pp. 3743-3752.