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Simulation of Lateral Distribution Functions of Gamma-Ray Showers: Classical Versus Modified Models for Proton-Induced Showers

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Abstract:
This work uses lateral distribution function (LDF) modeling to tackle the crucial problem of enhancing gamma/proton separation in cosmic ray showers. Even though the classical Nishimura-Kamata-Greisen (NKG) function has been applied extensively to electromagnetic showers, hadronic showers require more sophisticated methods due to their limitations. Three lateral distribution function (LDF) models, the classical NKG, a parametric model based on AIRES simulations, and a recently suggested modified NKG formulation are compared here. Simulation energy of range (10^{15}, 10^{18} and 10^{20}) and zenith angles (0º, 15º and 25º), employing both QGSHET-04-II and EPOS-LHC hadronic interaction models. Important results show that the modified NKG is better than the classical NKG, enhancing the gamma/proton separation power. The model's improved description of secondary particle dispersion, especially in shower periphery, is the source of these improvements. The findings have important ramifications for next-generation observatories like Cherenkov telescope array (CTA), where enhanced LDF parameterization may boost ultra-high-energy gamma detection effective sensitivity. This work lays the groundwork for future enhancements to machine learning-enhanced LDF reconstruction and nucleus-induced showers.
Keywords: Lateral Distribution, Aires simulations, Extensive air shower, Nishimura-Kamata-Greisen (NKG), gamma ray.

Introduction:

Nature and the beginning of the highest-energy cosmic rays above 10^{17} eV are among the most enigmas in particle astrophysics. The main way to learn about high-energy cosmic rays is to investigate large air showers created in our atmosphere by these particles. There are numerous methods for observing these air showers, such as detecting Cherenkov light and atmospheric fluorescence [1]. Following an extremely high-energy cosmic ray (CR) particles such as protons, pions, photon, and heavy nuclei like iron, initial collision with the atmosphere, a series of follow-up interactions result in particle multiplication, and decay processes produce a series of secondary particles known as extensive air showers (EAS) [2] .Electromagnetic cascades and hadronic multiparticle generation combine to form EAS in a complex way. To determine the characteristics of the main CRs that cause air showers, thorough numerical simulations of the phenomena are required [3]. As shown in figure 1, charged cosmic rays can also cause air showers in addition to rays. The primaries can be hadronic particles, primarily protons, or electrons or positrons that, like rays, introduce pure electromagnetic cascades. Hadronic air showers include both a muonic and a hadronic component in addition to an electromagnetic component. The generation of $\pi_{\circ}$ produces the electromagnetic component, whereas $\pi^{\mp}$ lead produces the muonic component.

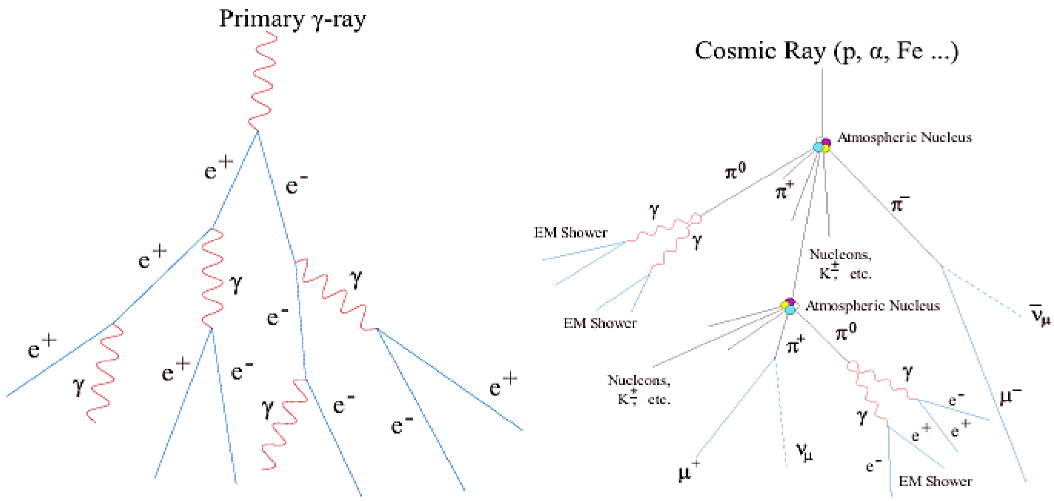

Figure 1: Diagram of a hadronic (right) and electromagnetic (left) air shower [3].

The Lateral Distribution function (LDF) is one of EAS's most crucial components. Determining the LDF of shower particles is therefore crucial. The particle density or signal regarding the distance to the shower core location must be ascertained [4,5,6]. LDF are influenced by air attenuation, multiple scattering, and the production height distribution of the arriving particles [7]. Furthermore, the transverse momentum distribution is used to form the EAS particles. Analysis of high-energy cosmic-ray events requires precise knowledge of the lateral distribution of cascade showers due to the multiple scattering hypothesis [8,9].
Fomin et al. (2008) examined the effects of various lateral distribution functions (LDFs) on extensive air shower (EAS) analysis by contrasting a more recent scaling function with the conventional Nishimura-Kamata-Greisen (NKG) model. It was demonstrated that the NKG offered respectable accuracy despite

its oversimplified assumptions, their work supports the need to evaluate alternative models, particularly for gamma-ray and proton-induced showers, by highlighting the impact of LDF choice on shower parameter estimation [10].

Morales-Soto et al. (2019) analyzed the lateral distribution function (LDF) of cosmic-ray-induced air showers using data from the HAWC observatory in Mexico. Their study focused on air showers with energies between 3 and 300 TeV, aiming to parameterize the LDF and explore its sensitivity to the primary cosmic ray mass composition and energy [11]. By creating an enhanced LDF parameterization that greatly improves air shower analysis, this work contributes to the field of cosmic ray research. Three major advantages for the field are offered by our modified NKG model: (i) increased gamma/hadron separation capability, especially important for gamma-ray astronomy; (ii) improved accuracy in reconstructing high-energy events above 10^{18} eV and inclined showers, addressing significant shortcomings of existing models; and (iii) direct applicability to next-generation observatories like CTA. These enhancements make it easier to identify astrophysical gamma-ray sources close to the PeV energy frontier and allow for more accurate investigations of ultra-high-energy cosmic rays.

Lateral Distribution Function:

The Monte Carlo system is used to simulate the lateral dispersion of an extended air shower with a specific main particle type and energy based on a specific interaction model [12]. The AIRES system can offer complete particles traveling across the actual medium of the atmosphere by observing the curvature of the Earth and the properties of the geomagnetic field. The lateral distribution of produced particles in EAS, which is represented as the photon density as a function of the core's distance, was simulated using the AIRES system [13]. The AIRES system is used with a specific main particle and energy based on a specific interaction model [14]. EPOS-LHC [15] and QGSJET-4-II [16] are models of high-energy hadronic interactions included in the AIRES Monte Carlo program version (10.04.04). Three main energies 10^{15}, 10^{18}, 10^{20}, and eV were considered. To assess the angular dependence of the lateral distribution, showers were simulated at three zenith angles: (0°, 15°, and 25°).

Python was used to collect and process the necessary parameters from the AIRES simulation data. These distributions were fitted and compared using three distinct models:

1. The Nishimura-Kamata-Greisen (NKG) functions in its classical form [17]

$$\rho_{classic}(r) = N.C(s).\left(\frac{r}{r_m}\right)^{s-4.5}.\left(1+\frac{r}{r_m}\right)^{(s-4.5)} \quad (1)$$

Where:
r: is the radial distance from the shower core.
N : is the total number of particles.
s: is the shower age parameter.
r_m: is the Moliere radius.
$C(s)$: is normalization factor.

Where the normalization factor is

$$C(s) = \frac{\Gamma(4.5 - s)}{\Gamma(s).\Gamma(4.5 - s)} \quad (2)$$

The classic NKG function after substituting $C(s)$is:

$$\rho_{classic}(r, N, s, r_m) = N.\left(\frac{\Gamma(s)}{\Gamma(s/2).\Gamma((4.5 - s)/2)}\right).r^{s-2}.\left(\frac{r}{r_m} + 1\right)^{(s-4.5)} \quad (3)$$

$\Gamma(z)$: is the Gamma function.

2. A modified NKG function whose parameters have been changed to better fit the simulated data.

$$\rho_{modified}(r, N, s, r_m, \beta) = N.\left(\frac{\Gamma(s)}{\Gamma(s/\beta).\Gamma((\beta - s)/\beta)}\right).r^{s-2}.\left(\frac{r}{r_m} + 1\right)^{(s-\beta)} \quad (4)$$

Where:
$r, N, s, r_m, and\, \Gamma(z)$: are the same as in the classical NKG function.
β : is an additional fitting parameter that was included to change the lateral distribution function's shape. In order to improve the agreement between the simulated data and the modified function, especially at long radial distances, its value was changed. The value of β was obtained through a fitting process using Python, based on AIRES simulation data

3. A model based on parameterization that uses an empirical fit customized for distributions caused by gamma (logistic function).

$$\log_{10}\rho = \frac{\lambda + (\alpha - \lambda)}{(1 + (\log_{10}\frac{x}{\eta})\ \hat{}\beta} \quad (5)$$

Where:

ρ: represents the photon density

λ: represents minimum photon density.

α: represents maximum photon density.

η: distance where the density trend changes.

β: rate of transition between asymptotes.

x is the distance from the shower core.

In this study, the lateral distribution function derived from AIRES simulations of gamma-ray primary were fitted to create a new empirical parameterization based on the logistic function. This model was created directly from the simulated data, not from a particular reference.

Results and Dissection:

There were notable variations in the lateral distribution fitting between the parametrized, modified, and classic NKG models. The distributions of photon densities as a function of core distance are shown in Figures (2-7) for different primary energies (10^{15}, 10^{18}, and 10^{20}) eV and different zenith angles (0°, 15°, and 25°) with two models (QGSJET-04-II and EPOS-LHC). The outcomes of the Aires simulation are contrasted with the classic NKG and modified NKG functions. It is evident that, for all energy levels and zenith angles, the modified NKG offers a higher degree of agreement with the simulation results. This shows that the modified function is more accurate and performs better when representing the lateral distribution of photons in large air showers.

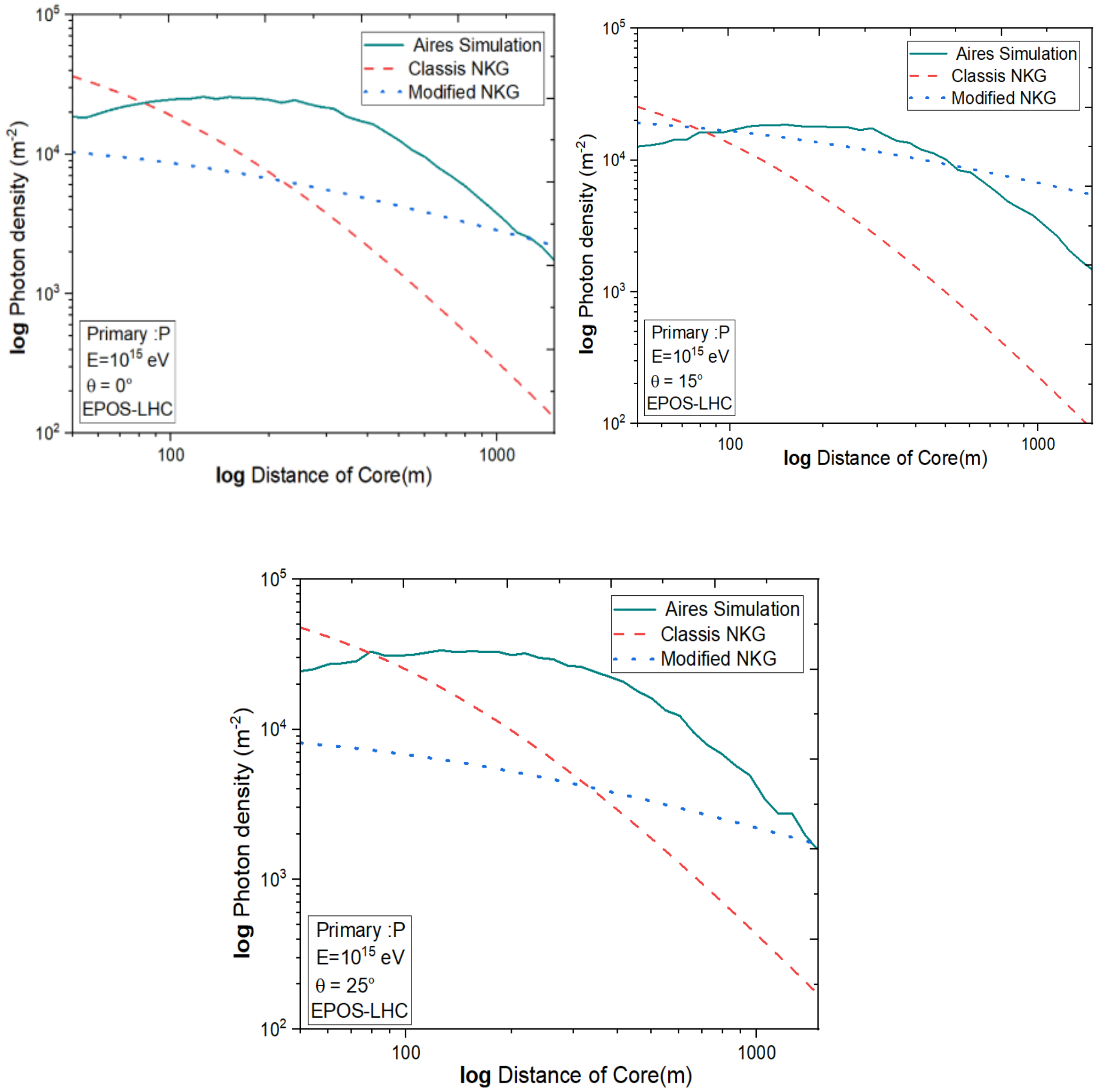

Figure 2: Lateral Distribution for a Primary proton with energy E=10^{15} eV at Different Zenith Angles using EPOS-LHC model: A Comparison between Aires Simulation (solid line), Classical NKG (dashed line), and Modified NKG (dotted line).

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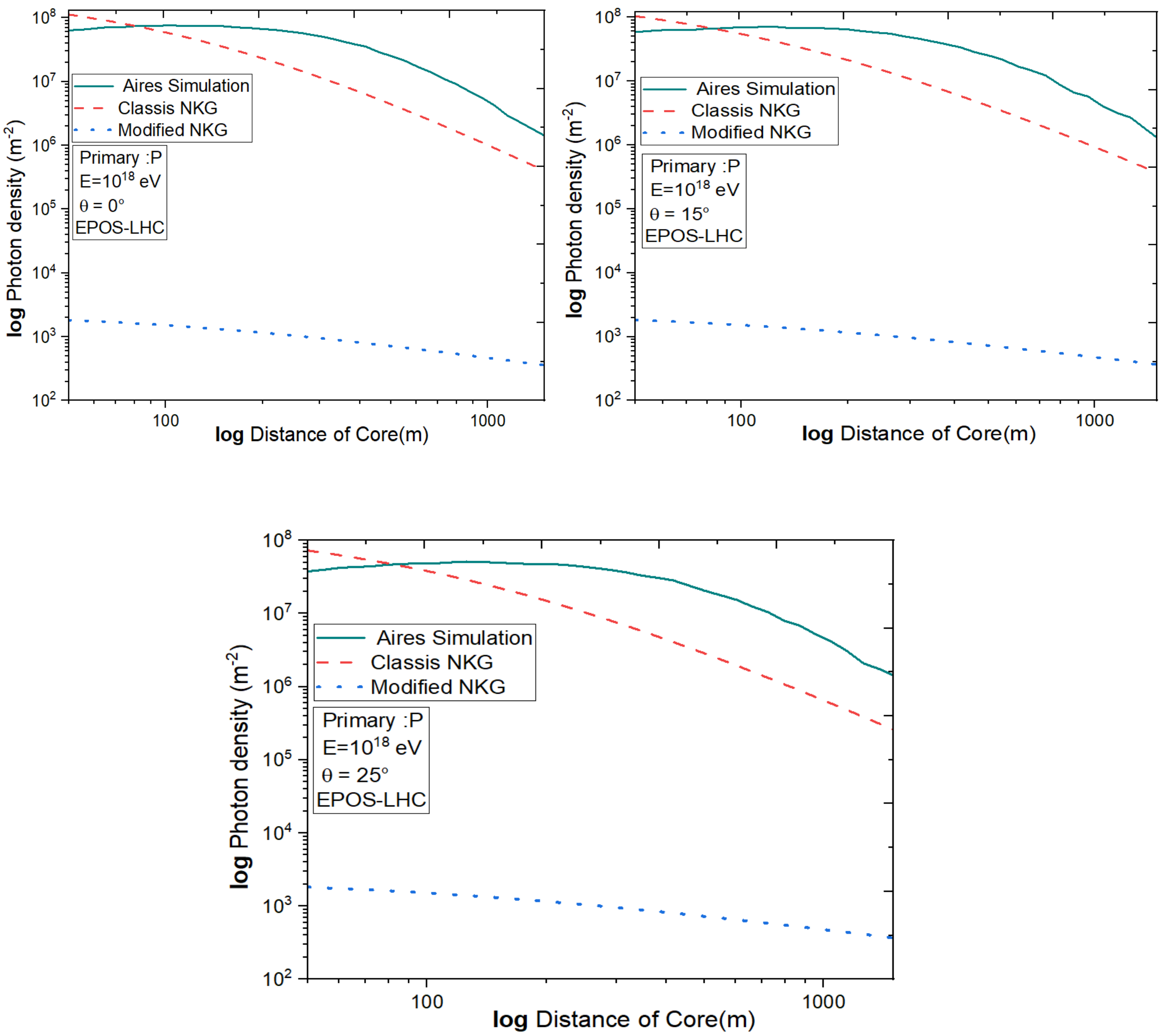

Figure3: Lateral Distribution for a Primary proton with energy $E=10^{18}$ eV at Different Zenith Angles using EPOS-LHC model: A Comparison between Aires Simulation (solid line), Classical NKG (dashed line), and Modified NKG (dotted line).

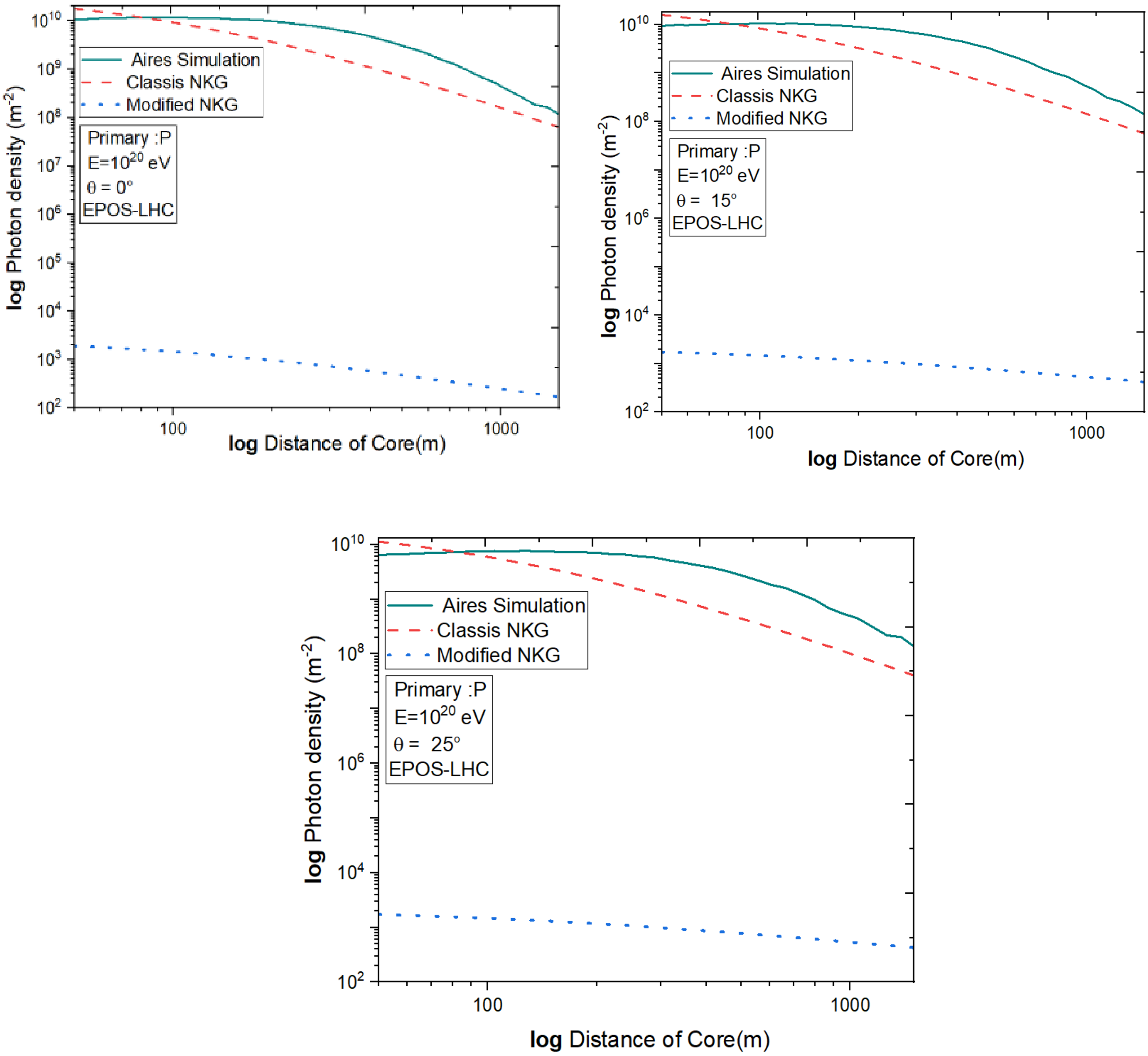

Figure4: Lateral Distribution for a Primary proton with energy $E=10^{20}$ eV at Different Zenith Angles using EPOS-LHC model: A Comparison between Aires Simulation (solid line), Classical NKG (dashed line), and Modified NKG (dotted line).

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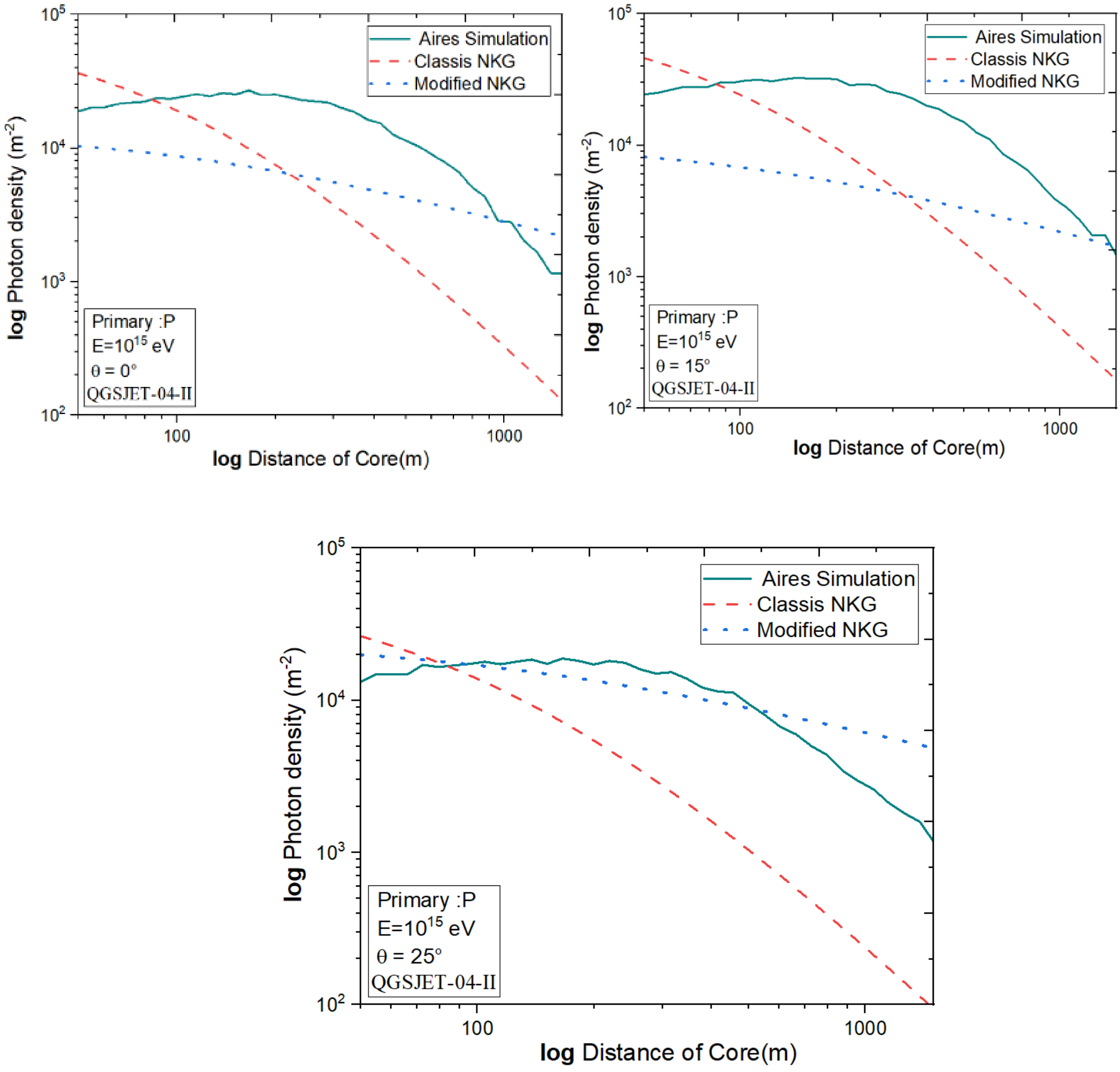

Figure 5: Lateral Distribution for a Primary proton with energy E=10^{15} eV at Different Zenith Angles using QGSJET-04-II model: A Comparison between Aires Simulation (solid line), Classical NKG (dashed line), and Modified NKG (dotted line).

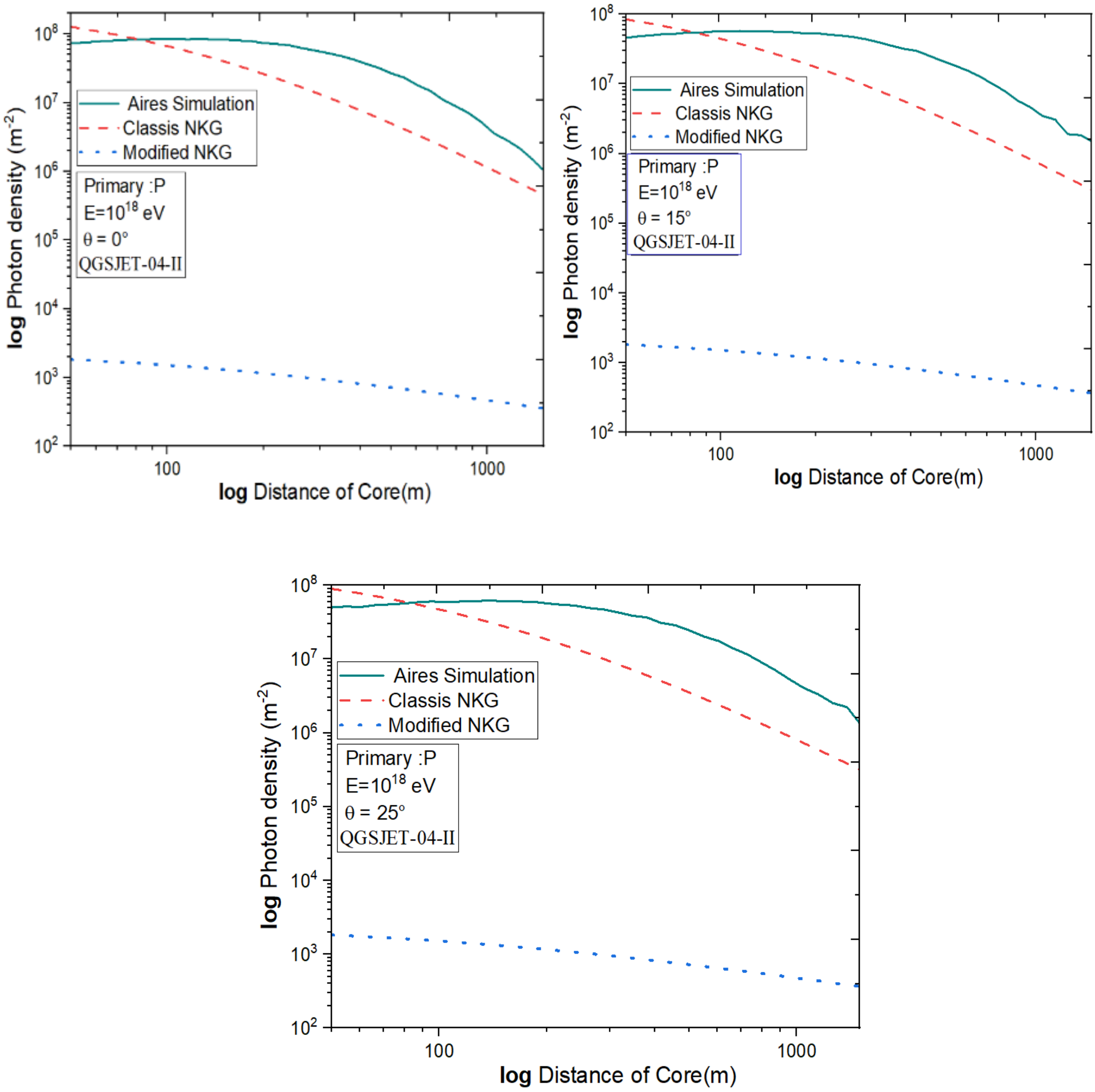

Figure6: Lateral Distribution for a Primary proton with energy $E=10^{18}$ eV at Different Zenith Angles using QGSJET-04-II model: A Comparison between Aires Simulation (solid line), Classical NKG (dashed line), and Modified NKG (dotted line)**.**

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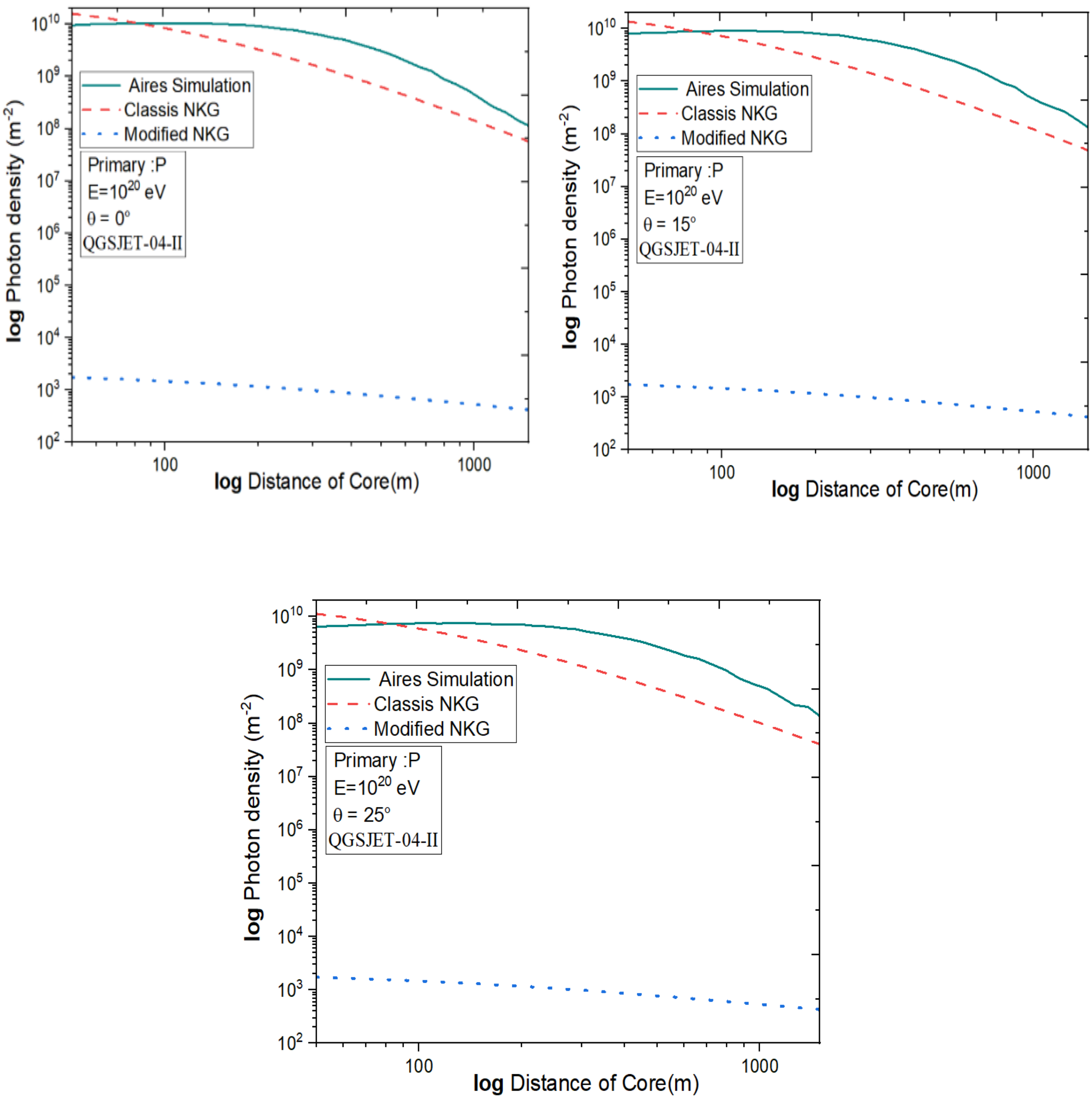

Figure 7: Lateral Distribution for a Primary proton with energy $E=10^{20}$ eV at Different Zenith Angles using QGSJET-04-II model: A Comparison between Aires Simulation (solid line), Classical NKG (dashed line), and Modified NKG (dotted line).

Three different energies (10^{15}, 10^{18}, and 10^{20}) eV and zenith angles (0°, 15°, and 25°) have been used to study the lateral distribution of photon density as a function of core distance for primary protons. The results computed using the

Classical NKG function (dashed line) and the suggested Modified NKG function (dotted line) were contrasted with the Aires simulation results (solid line). In comparison to both analytical functions, the Aires simulation consistently forecasts a larger photon density near the core across all energies and zenith angles [18,19]. In contrast to the Classical NKG, the Modified NKG function shows a notably better agreement with the simulation results, particularly at broader radial distances, the Modified NKG function demonstrated improved agreement with the simulated data at 10^{15} eV. However, the Classic NKG function had a better overall match at higher energies (10^{18} and 10^{20})eV. This discrepancy in performance might indicate that each model's applicability varies according to the energy scale, which might prompt more research or better parameterization. Additionally, because of the longer air path and increased attenuation, photon density decreases at all core distances when the zenith angle is increased from 0° to 25°. At lower energy, the effect is more pronounced, while at higher energy, it diminishes. From every aspect, the Modified NKG function consistently exceeds the Classic NKG in terms of agreement with the simulation findings. Furthermore, the performance of the EPOS-LHC and QGSJET-04-II models in characterizing the lateral distribution of gamma is similar, with only slight variations between them.

Conclusion

This study shows that the modified NKG model offers a substantial advancement in air shower analysis. Compared to the classical NKG formulation, the modified function performs better, especially in three important areas. First, it offers improved gamma/hadron separation capabilities, making investigations of the composition of cosmic rays more trustworthy. Second, even at higher core distances where classical NKG models usually deviate from simulation data, the improved LDF exhibits consistent behavior in reconstructing inclined showers with longer atmospheric route lengths and high-energy showers (beyond 10^{17} eV) with greater precision. Third, the updated function more accurately represents the lateral photon density distribution, particularly in the critical area outside the shower center, according to a comparison with Monte Carlo simulations. The enhanced performance is a result of the model's increased capacity to depict the evolution of electromagnetic cascades in large air showers. For next-generation observatories like the Cherenkov Telescope Array (CTA), where accurate reconstruction of ultra-high-energy cosmic rays (UHECRs) is crucial, this development is especially pertinent. Because of its compatibility with contemporary detector systems, the modified NKG function is a useful tool for upcoming studies in cosmic-ray astrophysics. Its entire potential for expanding our knowledge of high-energy particle events in the environment will be established with additional confirmation through experimental data and inclusion into reconstruction frameworks.

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محاكاة دوال التوزيع الجانبي للزخات الهوائية الناتجة من اشعة كاما: النماذج الكلاسيكية مقابل النماذج المعدلة للزخات البروتونية

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مستخلص البحث:

تتناول هذه الدراسة تحسين تمييز زخات أشعة كاما عن الزخات البروتونية في الأشعة الكونية باستخدام نمذجة دالة التوزيع الجانبي (LDF). على الرغم من التطبيق الواسع لدالة NKG الكلاسيكية على الزخات الكهرومغناطيسية، إلا أن الزخات الهادرونية تتطلب منهجيات أكثر تطورًا بسبب محدودية النماذج التقليدية. تم في هذا العمل مقارنة ثلاثة نماذج لدوال التوزيع الجانبي: دالة NKG الكلاسيكية، ونموذج قائم على معاملات مستخلصة من محاكاة AIRES، وصيغة معدلة حديثة لدالة NKG. شملت الدراسة محاكاة زخات هوائية بطاقة تتراوح بين (10^{15} و 10^{18}و 10^{20}) إلكترون فولت وزوايا سمتية مقدارها (0درجة و15درجة و25درجة) باستخدام نماذج التفاعل الهادروني QGSJET-II-04 وEPOS-LHC . أظهرت النتائج تفوق النموذج المعدل على النموذج الكلاسيكي في تحسين قدرة الفصل بين أشعة كاما والبروتونات، ويعزى ذلك إلى وصفها الأكثر دقة لتوزيع الجسيمات الثانوية، خاصة في المناطق الطرفية الخارجية للزخات. تمتلك هذه النتائج تطبيقات مهمة في مراصد الجيل القادم مثل مصفوفة تلسكوبات شيرينكوف (CTA)، حيث يمكن لتحسين معاملات دالة التوزيع الجانبي أن يعزز حساسية الكشف عن أشعة كاما فائقة الطاقة. تُشكّل هذه الدراسة أساسًا لتحسينات مستقبلية تعتمد على تعلم الآلة في إعادة بناء دوال التوزيع الجانبي وتحليل زخات النوى الذرية.

الكلمات المفتاحية:التوزيع الجانبي، محاكاة AIRES، الزخات الهوائية الواسعة ، دالة NKG ،اشعة كاما