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RESEARCH ARTICLE

A New Strategy for Solving the Initial Basic Feasible Solution of the Transportation Problem Using Exponential Distribution and a New Penalty Model

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ABSTRACT

One of the key concepts in operations research (OR) is the transportation problem (TP). The transportation problem involves distributing products from several sources to different destinations while minimizing shipping expenses. To achieve minimum transportation costs, the transportation problem is examined in two phases. An initial basic feasible solution (IBFS) is the first phase for achieving the lowest price, with the help of IBFS, one can calculate the optimal solution derived as the second phase. The literature review indicates that the authors developed a novel algorithm for the IBFS, incorporating several mathematical tools and some new models yet possessing certain limitations. To overcome this, we proposed a new algorithm for the IBFS, a collaborating statistical tool and a new penalty model. An exponential distribution illustrates the probability of cost or time required to reach each source to the destination. Furthermore, a novel penalty model is formulated to distribute essential supply to requisite demand. A modified distribution method (MODI) is employed to calculate the optimal solution. Eleven numerical examples were illustrated to develop the proposed algorithm. The results are compared with classical Vogel's approximation method (VAM) and ten previously published papers. In addition, three randomly generated problems were performed. Finally, with the reduction in total transportation cost over Vogel's approximation method, the demand-based allocation method (DBAM) and the existing ten published studies. Our suggested method of getting 10 optimal solutions in IBFS recorded a 71.42% accuracy.

Keywords: Exponential distribution, Initial basic feasible solution, New penalty model, Optimal solution, Transportation problem

Introduction

Operations research is a field concerned with developing and using analytical methodologies to enhance decision-making. Operations research frequently focuses on determining the extreme values of certain real-world objectives, the maximum (gain, efficiency, output) or minimum (loss, risk, or price). Linear programming is a method for optimizing operations under certain limitations. Many practical concerns in OR may be stated as linear programming problems (LPPs). Linear programming issues are a type of optimization problem that aids in determining the feasible area and optimizing the solution to get the maximum or lowest value of the function. A transportation problem is a special case of a linear programming problem in which goods are transported from a set of sources to a set of destinations, depending on supply and demand constraints, to minimize transportation cost, time, and distance while avoiding prohibited roots. Transportation concept is a term used in mathematics and economics to describe the study of efficient transportation and

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resource allocation. Gaspard Monge, a French mathematician, formalized the problem in 1781. A basic feasible solution is defined as a solution with a minimum set of non-zero variables in a transportation problem. The role of IBFS is to identify a basic feasible solution capable of yielding optimum solutions. A TP can be either balanced or unbalanced. If all supply and demand constraints are equal, it is balanced otherwise, it is unbalanced. Three procedures are utilized to solve the IBFS, the northwest corner rule (NWCR), the lowest cost method (LCM), and Vogel's approximation method (VAM). To check the optimality condition a modified distribution method (MODI) or stepping-stone approach is required.

In recent years, many methodologies for determining the IBFS have been developed. Many authors are working to propose a new algorithm that will produce an optimum solution. Azad and Hossain [1] provided an Average Penalty for an initial basic feasible approach that used a mean row penalty and mean column penalty, from these, the author assigned an amount of supply-demand to the decision variable to obtain an optimal solution. Ahmed et al. [2] constructed an improved algorithm based on a previous strategy, an extremely different method formulated a maximum row cost difference (MRCD) and maximum column cost difference (MCCD) for allocating the amount to a decision variable, thus minimizing the price for Transportation. Harrath and Kaabi [3] proposed a global minimum method (GMM) for the initial solution to determine the best solution. Karagul and Sahin [4] implemented the karagul-Sahin approximation method (KSAM) for calculating the IBFS for a transportation problem. Amaliah et al. [5] suggested a method called total opportunity cost matrix-minimal total (TOCM-MT) to determine IBFS as the basic answer to transportation challenges.

Kamble and Kore [6] proposed an average opportunity cost method (AOCM) for designing a new method for the initial basic solution to obtain the optimum answer. Sathyavathy and Shalini [7] created a process for the first solution that uses four distinct means of statistical and rational calculations to find optimal results. Hossain [8] suggested an approach called an improved average penalty cost (IAPC), which is designed in such a way that minimizes overall production expenses. Jamali et al. [9] proposed a feasible initial solution for both balanced and unbalanced transportation issues so that the cost associated with transporting a certain amount of goods from source to destination is minimized also satisfying constraints. Hossain and Ahmed [10] developed a new approach, the least cost mean method (LCMM), to produce a better IBFS in which row and column penalties are calculated by taking the mean of the lowest and lowest values in each row and column of the cost matrix.

VAM is a classical and best algorithm for generating the initial solution. Babu et al. [11] demonstrate that VAM has restrictions and computational errors. To address this an improved Vogel's approximation method (IVAM) was introduced to generate an initial solution. Putcha [12] developed a code to identify an initial basic feasible answer to get optimal outcomes. Sam'an and Ifriza [13] created an algorithm that combines the total difference (TD) and the karagul-Sahin approximation method (KSAM) to get the IBFS. Shahi and Henry [14] suggested a matrix minimum approximation method (MMAM) for identifying an initial solution to the transportation issue. Ramakrishna and Ashok [15] generated a program that uses the northwest corner method to find the initial answer for TP. The program was created in C coding will handle balanced and unbalanced issues.

Amulu Priya and Maheswari [16] proposed a new method that improved Vogel's approximation technique into a new Vogel's approximation method (NVAM) used to calculate the best solution. Ekanayake and Ekanayake [17] designed a model for assigning an amount to decision variables for both balanced and unbalanced issues, the model is based on the average unit cost value. Sharma and Goel [18] created a strategy that includes an arithmetic mean and assigned the shortest minimax method to address the initial solution and analyze the optimal cost of transportation issues. Many real-world applications rely on transportation to identify the best solution for product delivery from supply to demand. Amaliah et al. [19] introduced a novel algorithm known as the Bilqis Chastine Erma approach for finding the optimum answer to transportation issues. When tackling transportation challenges, it is necessary to reduce traveling time, while solving transportation problems. Raval [20] provided Raval's approximation method for the IBFS that lowers transportation costs and finds the best alternatives.

Zabiba et al. [21] developed a novel approach for maximizing the profit of transportation issues. The method is simple for learning and using as well as getting the best outcomes. Ackora-Prah et al. [22] focus on identifying a technique which is a Demand-Based Allocation Method (DBAM) to address the IBFs. DBAM is implemented in MATLAB and can handle large-scale transportation issues to meet industrial needs. Unbalanced issues represent the reality of supply chain and logistics circumstances in which the available supply of commodities does not match the demand in different areas. Rashid and Islam [23] established a novel strategy to turn an unbalanced into a balanced one, as a consequence to obtain the optimum answer. Hilal Abdelatli Abdelwali [24] proposes a unique approach for calculating an initial basic feasible solution while avoiding bigger costs

(ABC); this method is simple to get the best result. Jude and Udo[25] proposed improving the modified distribution approach and estimating the IBFS using a median approximation technique (MAT) to a transportation issue. Amreen and Venkateswarlu[26] developed a method based on exponential distribution and contraharmonic mean for finding the IBFs to achieve the best solution. Indira and Jayalakshmi[27] devised a triangular fuzzy number that is split into two interval integer transportation using the α-cut method. The fundamental objective is to minimize the overall fuzzy transportation cost.

This research is divided into parts. Each part gives the impact on the proposed study. Part 1: Describes the introduction and mathematical formulation of TP. Part 2: Presents a methodology for the new algorithm. Part 3: Includes several numerical examples. Part 4: Results and discussion compare the suggested approach to current ones. Part 5: The conclusion examines the study's strengths, limitations, and future scope.

Based on the discussion many writers proposed a novel algorithm for finding an initial, basic feasible solution, constantly using original cost and employing new penalty formulation tools or new Vogel approximation methods. According to the literature survey, a limited study has been done using statistical tools there is no unique method containing distribution and a new penalty model. This motivates the authors to suggest a new algorithm that alters the original cost and penalty analysis and, more frequently, provides the best answer for computing IBFS. The objective of this research is with the help of a new algorithm can reduce the cost, time, or distance for transporting goods.

The key contribution of this research:

- The original cost with new penalty models (improving the VAM) always depends on conditions. To restrict this, our study converted the original cost into an estimated cost with the help of distribution.
- Exponential distribution can estimate the time and cost of delivering logistics from one source to a destination.
- A new penalty model is designed from VAM to calculate the penalties that allocate the necessary supply required to a destination.
- The proposed approach is validated using both balanced and unbalanced issues.
- The suggested algorithm is compared with DBAM, NVAM, Raval's approximation, IVAM, IAPC, GMM, Average penalty, and traditional method VAM.
- To verify the performance, random data is performed with the current data.
- A sensitivity range is performed to validate the optimality of the numerical example.

Mathematical formulation

This section describes the transportation problem formulation shown in Fig. 1 and Table 1.

The mathematical formulation for the transportation problem is represented in the form of LPP is shown below.

$$\left.\begin{array}{l}\text{Minimum or Maximum (total cost)} = \sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij}c_{ij}\\ \text{Subject to Constraints}\\ \sum_{j=1}^{n} x_{ij} = a_i \;\; \text{i} = 1 \text{ to m} \;\; \text{(Supply Constraint)}\end{array}\right\} \quad (1)$$

$$\sum_{i=1}^{m} x_{ij} = b_j \;\; \text{j} = 1 \text{ to n} \;\; \text{(Demand Constraint)}$$

$x_{ij} \geq 0$ for all i and j (non-negativity requirement)

Here,

m; m sources (locations) or origins

n; n destinations or distribution centres

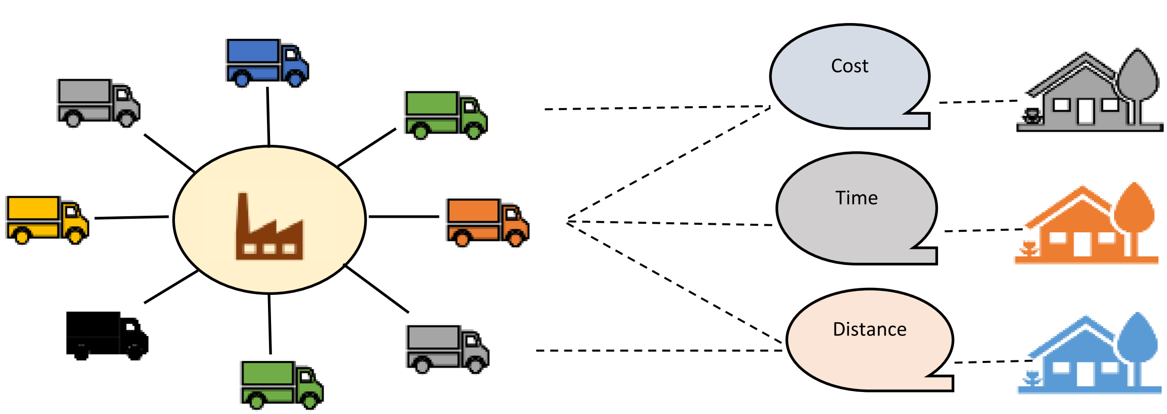

Fig. 1. Transporting products from factories to distribution centres.

Table 1. Basic transportation model.

Sources/Destinations	*I*	*II*	…	*n*	*Supply* (a_i)
I	c_{11} x_{11}	c_{12} x_{12}	…	c_{1n} x_{1n}	$\boldsymbol{a_1}$
II	c_{21} x_{21}	c_{22} x_{22}		c_{2n} x_{2n}	$\boldsymbol{a_2}$
:	:	:	:	:	**:**
m	c_{m1} x_{m1}	c_{m2} x_{m2}	…	c_{mn} x_{mn}	$\boldsymbol{a_m}$
Demand (b_j)	$\boldsymbol{b_1}$	$\boldsymbol{b_2}$	**…**	$\boldsymbol{b_{mn}}$	$\sum \boldsymbol{a_i} = \sum \boldsymbol{b_j}$

$S_i(a_i)$; Available (supply) products in sources $S_i(a_i)$
$D_j(b_j)$; Required (demand) goods in destination $D_j(b_j)$
X_{ij} (decision variable); Several units of supply/demand are transported from S_i to D_j
C_{ij}; Cost of transporting one unit of item from S_i to D_j

Materials and methods

This section describes the specification of the proposed method. Initially, the new algorithm technique begins with an initial transportation problem. The original per-unit cost of transportation is converted into an estimated value for shipping from one point to another. The data was transformed through distribution.

Distribution is a key notion in statistics, allowing us to make sense of data and estimate future trends. The exponential distribution is a continuous distribution used in probability theory and statistics. An exponential distribution computes the elapsed time between occurrences (events). In our research exponential distribution acts as a source for converting the original transportation problem into an exponential distribution, also it calculates the estimated cost in the intervals from 0 to 1 for transporting the item.

A cumulative distribution function (CDF) in exponential distribution is used to cumulate the transportation cost and gives the probability cost for TP.

The general formula for the cumulative distribution function is given by

$$F\ (x;\lambda) = \begin{cases} 1 - e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases} \tag{2}$$

Three characters are needed to calculate in Excel Solver

X: Per unit cost of commodity

Lambda: E(x) (mean) $= \mu = 1/\lambda$

Convert, $\lambda = 1/\mu$

$$\text{Cumulative} \begin{cases} \text{True function denotes the cumulative distribution function (CDF)}. \\ \text{False function denotes probability density function (PDF)}. \end{cases}$$

The formula for computing exponential distribution in an Excel spreadsheet is presented below.

$$= \text{EXPON.DIST}\,(\text{x, lambda, True}) \tag{3}$$

A novel model for estimating the penalty for transportation issues has been refined from Vogel's approximation technique and is shown below.

$$\sqrt{|\text{Smallest Cost} - \text{Next Smallest Cost}|}\,/2 \tag{4}$$

Algorithm

This section explains the details of the proposed algorithm

Step 1: Select a transportation issue.

- ❖ Balanced transportation problem $\sum S_i = \sum D_j$
- ❖ Unbalanced transportation problem $\sum S_i \neq \sum D_j$. Convert the unbalanced into balanced TP by adding a dummy row or column cost of zero.

Step 2: Generate a new table by using Eq. (3).

Step 3: Calculate the penalties for each row using Eq. (4).

Step 4: Use Eq. (4) to determine the penalty for each column.

Step 5: Choose the highest penalty from row or column.

Step 6: Then, depending on the largest penalty, choose the cell with the lowest cost in that row or

Table 2. Transportation problem matrix.[16]

	S1	S2	S3	S4	S5	Supply
F1	4	1	2	6	9	**100**
F2	6	4	3	5	7	**120**
F3	5	2	6	4	8	**120**
Demand	**40**	**50**	**70**	**90**	**90**	

Table 3. Probability cost using exponential distribution.

	S1	S2	S3	S4	S5	Supply
F1	0.597	0.203	0.365	0.744	0.870	**100**
F2	0.698	0.550	0.451	0.632	0.753	**120**
F3	0.632	0.329	0.698	0.550	0.798	**120**
Demand	**40**	**50**	**70**	**90**	**90**	

*Penalty = $\sqrt{|0.203-0.365|/2} = 0.284$

Table 4. First iteration.

	S1	S2	S3	S4	S5	Supply	Penalty
F1	0.597	0.203	0.365	0.744	0.870	**100**	**0.284**
F2	0.698	0.550	0.451	0.632	0.753	**120**	**0.222**
		50					
F3	0.632	0.329	0.698	0.550	0.798	**120**	**[0.332]**
Demand	**40**	**50**	**70**	**90**	**90**		
Penalty	**0.132**	**0.250**	**0.207**	**0.202**	**0.150**		

Table 5. Second iteration.

	S1	S3	S4	S5	Supply	Penalty
		70				
F1	0.597	0.365	0.744	0.870	**100**	**[0.340]**
F2	0.698	0.451	0.632	0.753	**120**	**0.300**
F3	0.632	0.698	0.550	0.798	**70**	**0.202**
Demand	**40**	**70**	**90**	**90**		
Penalty	**0.132**	**0.207**	**0.202**	**0.150**		

column, and assign as many units of supply/demand as possible to the decision variable.

Step 7: Set the supply and demand constraints, then cross out the whole row or column.

Step 8: Continue with steps 3–7 until the supply and demand requirements are fulfilled.

Step 9: Once the supply-demand conditions have been met, return to the original transportation table and assign the supply and demand values from Table 9.

Step 10: Finally calculate the initial solution using the objection function Eq. (1) of the transportation problem.

Step 11: A probability cost using exponential distribution Table 9 is used to find the optimal solution.

Step 12: The non-degeneracy criteria are determined using a modified distribution approach.

Numerical examples

Example 1: *A commodity is manufactured at factories. These factories supply the items to stores. Unit transportation cost in rupees from each factory and each store is given in Table 2.*

Generate a new table by using Eq. (3)(excel solver formula for exponential distribution)

Here,

Mean (μ) = (4, 1, 2, 6, 9) = 4.4

$$\lambda = 1/4.4 = 0.227273$$

$$* = \text{EXPON.DIST}(4, 0.227273, \text{True}) = 0.597$$

The probability cost of transportation problem using exponential distribution is shown in Table 3.

Eq. (4) computes the penalty for each row/column is displayed in the Tables 4 to 8.

Table 9 shows the final allocation using the proposed method.

Step 9: Once the supply-demand conditions have been met, return to the original transportation table and assign the supply and demand values from Table 9.

In the given transportation problem, the proposed allocation is displayed in Table 10.

After calculating the initial basic feasible solution using the suggested method, the transportation cost comes out to 1,430.

Step 11: A probability cost using exponential distribution Table 9 is used to find the optimal solution.

Step 12: The non-degeneracy criteria are determined using a modified distribution approach.

Table 6. Third iteration.

	S1	S4	S5	Supply	Penalty
	30				
F1	0.597	0.744	0.870	**30**	**[0.271]**
F2	0.698	0.632	0.753	**120**	**0.181**
F3	0.632	0.550	0.798	**70**	**0.202**
Demand	**40**	**90**	**90**		
Penalty	**0.132**	**0.202**	**0.150**		

Table 7. Fourth iteration.

	S1	S4	S5	Supply	Penalty
F2	0.698	0.632	0.753	**120**	**0.181**
		70			
F3	0.632	0.550	0.798	**70**	**[0.202]**
Demand	**10**	**90**	**90**		
Penalty	**0.181**	**0.202**	**0.150**		

Table 8. Fifth iteration.

	S1	S4	S5	Supply	Penalty
	10	**20**	**90**		
F2	0.698	0.632	0.753	**120**	**0.181**
Demand	**10**	**20**	**90**		
Penalty	**0.181**	**0.202**	**0.150**		

Table 9. Final iteration.

	S1	S2	S3	S4	S5	Supply
	30		**70**			
F1	0.597	0.203	0.365	0.744	0.870	**100**
	10			**20**	**90**	
F2	0.698	0.550	0.451	0.632	0.753	**120**
		50		**70**		
F3	0.632	0.329	0.698	0.550	0.798	**120**
Demand	**40**	**50**	**70**	**90**	**90**	

Table 10. Final transportation problem with allocation.[16]

	S1	S2	S3	S4	S5	Supply
	30		**70**			
F1	4	1	2	6	9	**100**
	10			**20**	**90**	
F2	6	4	3	5	7	**120**
		50		**70**		
F3	5	2	6	4	8	**120**
Demand	**40**	**50**	**70**	**90**	**90**	

Table 11. Optimal solution.

	S1	S2	S3	S4	S5	Supply
	40	**20**	**40**			
F1	0.597	0.203	0.365	0.744	0.870	**100**
			30		**90**	
F2	0.698	0.550	0.451	0.632	0.753	**120**
		30		**90**		
F3	0.632	0.329	0.698	0.550	0.798	**120**
Demand	**40**	**50**	**70**	**90**	**90**	

Table 9 is used to investigate the non-degeneracy criteria and the optimal solution using the MODI method in MATLAB software.

Non-degeneracy criteria are determined as

(m + n-1) = number of allocated cells

Where m = number of rows, n = number of columns

(m + n-1) = 7, number of allocated cells = 7

Tables 11 and 12 show the optimal results of our proposed study.

Using the MODI approach the best (optimal) answer, which comes out to 1,400.

Sensitivity analysis for numerical example

Probability cost using an exponential distribution table is performed to validate the sensitivity range for allocated cells. Table 13 describes the sensitivity range for the assigned cells, and

Table 12. Optimal solution in given transportation problem.[16]

	S1		S2		S3		S4		S5		Supply
		40		**20**		**40**					
F1	4		1		2		6		9		**100**
						30				**90**	
F2	6		4		3		5		7		**120**
				30				**90**			
F3	5		2		6		4		8		**120**
Demand		**40**		**50**		**70**		**90**		**90**	

Table 13. Sensitivity analysis for the objective function.

Sensitive analysis
$0.506 \leq \mu \leq 0.612$
$-\infty \leq \mu \leq 0.294$
$0.350 \leq \mu \leq 0.461$
$0.355 \leq \mu \leq 0.466$
$-\infty \leq \mu \leq 0.849$
$0.601 \leq \mu \leq 0.723$
$-\infty \leq \mu \leq 0.581$

it always provides the best answer within that range.

Time minimization transportation problem

Table 14 shows the most time-efficient method for delivering the items; following the indicated methodology, the optimal result of 292.

Distance minimization transportation problem

Table 15 indicates the most efficient distance for delivering goods: utilizing the suggested approach, the optimal solution is 1,430.

One of the DBAM problem

Table 16 displayed the most efficient cost for transporting by using the suggested technique; the optimal solution is given as 412.

Table 17 contains eight balanced problems collected from various publications to solve our proposed approach.

Table 18 shows three unbalanced issues from diverse publications that can be solved using our suggested technique.

Random issues

Issue 13: Goods must be carried from source to destination. Table 19 shows the transportation cost per unit, source capacities, and destination requirements.

Using the proposed method and calculating the initial solution, the transportation cost is 14,830, and applying the MODI technique yields the best solution of 14,740.

Issue 14: A company has factories that supply products to warehouses. Weekly factory capacities, warehouse requirements, and unit shipping costs are given in Table 20.

Calculating the IBFS using a suggested technique the transportation cost is 4,208. After checking the optimality using the modified distribution method the optimal solution is 4,208.

Issue 15: A dairy firm has three plants located in a state. The dairy milk production at each plant each day, the firm must fulfill the needs of its distribution centers. Unit shipping cost from each plant to each distribution center is given in Table 21

For computing the initial solution using a proposed technique the transporting cost is 1,126 and applying the Modi technique produces the optimum answer of 1,126.

Results and discussion

This work proposed a novel technique for identifying the IBFS for a transportation problem that minimizes transportation costs and provides an optimal answer. A novel algorithm, based on probability theory and the statistical formula of exponential distribution, improved from VAM is a new penalty model. CDF is utilized in exponential distribution. Eq. (2) is the general formula for CDF. Eq. (3) is an Excel solver exponential distribution that makes it easy to compute shipping cost in terms of probability. The CDF will aggregate the transportation cost and provide a probability value within the interval [0,1] for each source to the destination. Table 3 displays the probability of the expense of transportation. Furthermore, Eq. (4) is modified from VAM. VAM has some limits and computational errors during calculations. A new penalty model is utilized to compute the penalties for each row and column. Combining the two, a new algorithm for IBFS is proposed. The main findings of the suggested algorithm are

Table 14. Time minimization problem.

	I	II	III	IV	Available
		8		**7**	
A	10	0	20	11	**15**
	12		**13**		
B	1	7	9	20	**25**
			2	**3**	
C	12	14	16	18	**5**
Required	**12**	**8**	**15**	**10**	

Table 15. Distance minimization problem.

	D1	D2	D3	Capacity
			14	**14**
S1	60	80	40	
	6	**5**	**1**	**12**
S2	40	90	80	
		5		**5**
S3	10	20	60	
Required	**6**	**10**	**15**	

Table 16. Balanced DBAM problem.

	1	2	3	4	Supply
	4			**4**	
A	13	18	30	8	**8**
		4	**6**		
B	55	20	25	40	**10**
		3		**8**	
C	30	6	50	10	**11**
Demand	**4**	**7**	**6**	**12**	

Table 17. Balanced transportation matrix.

Balanced transportation issue	
Example:2[1] $[C_{ij}]_{3x4}$ = [9 8 5 7; 4 6 8 7; 5 8 9 5] $[S_{ij}]_{3x1}$ = [12, 14, 16] $[D_{ij}]_{1x4}$ = [8, 18, 13, 3]	Example:3[25] $[C_{ij}]_{3x4}$ = [4 6 8 8; 6 8 6 7; 5 7 6 8] $[S_{ij}]_{3x1}$ = [40, 60, 50] $[D_{ij}]_{1x4}$ = [20, 30, 50, 50]
Example:4[25] $[C_{ij}]_{3x3}$ = [4 3 5; 6 5 4; 8 10 7] $[S_{ij}]_{3x1}$ = [90, 80, 100] $[D_{ij}]_{1x3}$ = [70, 120, 80]	Example:5[3] $[C_{ij}]_{3x4}$ = [15 4 6 15; 5 2 15 4; 6 5 3 14] $[S_{ij}]_{3x1}$ = [70, 47, 33] $[D_{ij}]_{1x4}$ = [52, 78, 15, 5]
Example:6[20] $[C_{ij}]_{3x4}$ = [6 3 5 4; 5 9 2 7; 5 7 8 6] $[S_{ij}]_{3x1}$ = [22, 15, 8] $[D_{ij}]_{1x4}$ = [7, 12,17, 9]	Example:7[8] $[C_{ij}]_{3x3}$ = [4 3 5; 6 5 4; 8 10 7] $[S_{ij}]_{3x1}$ = [9, 8, 10] $[D_{ij}]_{1x3}$ = [7, 12, 8]
Example:8[11] $[C_{ij}]_{4x5}$ = [4 4 9 10 13; 7 9 8 10 4; 9 3 7 10 6; 11 4 10 6 9] $[S_{ij}]_{4x1}$ = [100, 90, 80, 70] $[D_{ij}]_{1x5}$ = [60, 40, 90, 70, 80]	

Table 18. Unbalanced transportation matrix.

Unbalanced transportation issue	
Example:9[2] $[C_{ij}]_{3x3}$ = [4 8 8; 16 24 16; 8 16 24] $[S_{ij}]_{3x1}$ = [76, 82, 77] $[D_{ij}]_{1x3}$ = [72, 102, 41, 20]	Example-10[2] $[C_{ij}]_{3x5}$ = [5 4 8 6 5; 4 5 4 3 2; 3 6 5 8 4] $[S_{ij}]_{3x1}$ = [600, 400, 1000] $[D_{ij}]_{1x5}$ = [450, 400, 200, 250, 300]
Example-11[17] $[C_{ij}]_{4x5}$ = [5 1 8 7 5; 3 9 6 7 8; 4 2 7 6 5; 7 11 10 4 9] $[S_{ij}]_{4x1}$ = [15, 25, 42, 35] $[D_{ij}]_{1x5}$ = [30, 20, 15, 10, 20]	

Table 19. Random issue.

	D1	D2	D3	D4	D5	capacities
S1	40	10	25	60	90	**100**
S2	63	41	33	50	72	**120**
S3	50	26	64	43	88	**120**
requirements	**40**	**50**	**70**	**90**	**90**	

Table 20. Random matrix.

	I	II	III	IV	Capacities
A	130	182	300	87	**8**
B	550	200	257	440	**10**
C	300	66	525	100	**11**
requirements	**4**	**7**	**6**	**12**	

Table 21. Random problem.

	D1	D2	D3	supply
P1	45	51	63	**12**
P2	30	10	52	**11**
P3	32	44	44	**7**
demand	**9**	**8**	**13**	

Table 22. Comparative study.

		Initial basic feasible solution				
No.	Methods	VAM	Existing	DBAM	Proposed	Optimal
1	NVAM (2022)	1,580	1,490	1,550	1,430	1,400
2	Average Penalty (2017)	248	248	241	240	240
3	MAT (2024)	960	920	920	920	920
4	MAT (2024)	1,500	1,440	1,660	1,390	1,390
5	GMM (2018)	821	821	1,091	821	639
6	Raval's approximation (2023)	149	149	149	149	149
7	IAPC (2020)	150	144	166	139	139
8	IVAM (2020)	1,820	1,780	2,340	1,780	1,780

Table 23. Unbalanced problems.

			Initial basic feasible solution			
No.	Status	Methods	VAM	Existing	Proposed	Optimal
9	Unbalanced	MRCD/MCCD (2017)	2,752	2,424	2,424	2,424
10	Unbalanced	MRCD/MCCD (2017)	6,000	6,000	6,000	5,600
11	Unbalanced	New approach (2022)	374	374	374	374

Table 24. Random cases.

No.	Random problems	VAM	DBAM	Proposed	Optimal
12	Balanced	16,660	16,540	14,830	14,740
13	Balanced	5,032	4,208	4,208	4,208
14	Balanced	1,126	1,162	1,126	1,126

Table 25. Accuracy results.

No.	Method	Optimal solution	Accuracy (%)
1	VAM	3	20
2	DBAM	3	27.27
3	Existing	6	54.54
4	Proposed	11	71.42

to examine the best cost-effective, time-efficient, or distance-efficient methods for delivering goods and get optimal answers. There were no difficulties found in calculating the suggested technique comparing the other existing approaches. Table 22 comprises balanced numerical examples, whereas Table 23 includes unbalanced instances from other sources to evaluate our proposed method. The results are compared with a conventional technique referred to as Vogel's approximation method, many contemporary techniques, and the DBAM method. Fig. 2 illustrates the progression of the proposed results in graphical form. Along with the existing data, random issues are displayed in Table 24 and Fig. 3. Table 25 explains the accuracy results. In addition, some limitations occur while investigating the suggestion technique. This study cannot fit the probability density function to calculate the probability of transportation cost. It

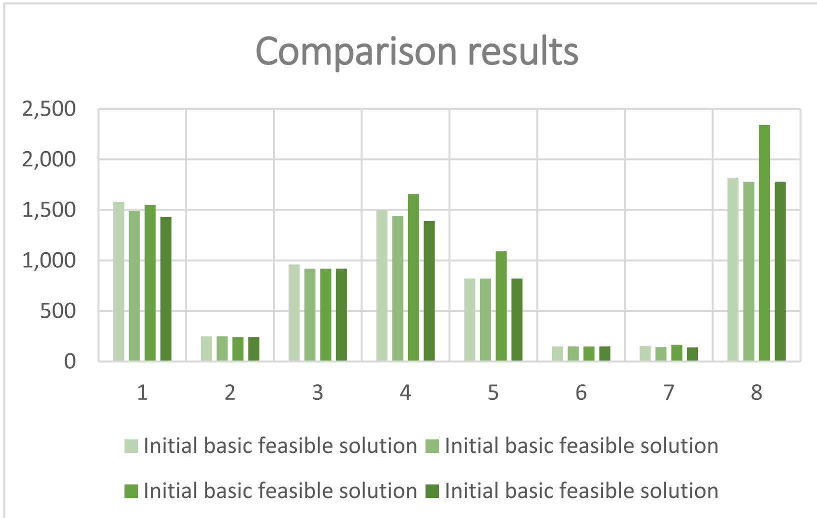

Fig. 2. Comparison graph of numerical examples.

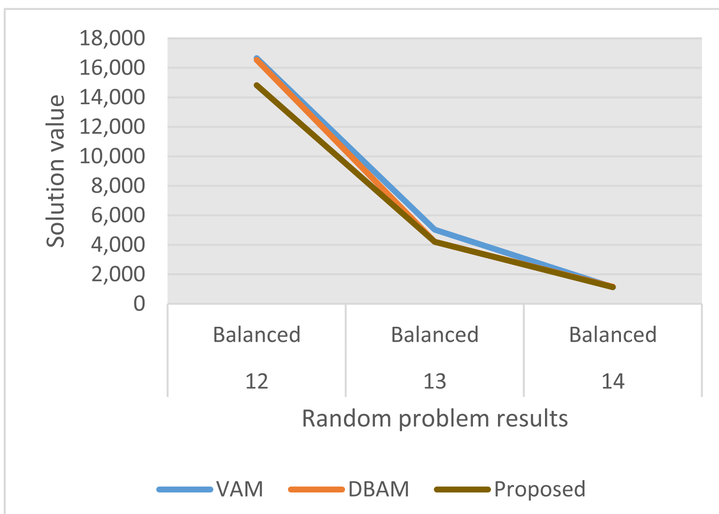

Fig. 3. A graph for the random issues.

is not suitable for cases in which supply is less than demand.

$$\text{Accuracy} = \frac{\text{Number of times IBFS gives optimal solution}}{\text{Total number of numerical examples}} \times 100 \quad (5)$$

Conclusion

Moving objects from one location to another with minimal expenses is recognized as a challenge in today's environment. In response to these challenges, many researchers have devised innovative shipping methodologies that incorporate mathematical tools and new models, or enhance classical VAM techniques. However, a restriction invariably arises. This study introduces a novel algorithm developed through the application of statistical tools and enhancements to the VAM methodology. This approach presents a distinctive methodology and an effective foundation for tackling the transportation issue. The elements employed in our proposed methodology include exponential distribution and an innovative penalty model. Moreover, these materials are utilized in our innovative algorithm for the application of IBFS. Furthermore, the existing MODI method is implemented within MATLAB software to employ the most effective solution which gives quick results and fewer iterations to reach an optimum answer. The proposed methodology presented eight balanced examples, three unbalanced instances, and three randomly generated numerical cases to elucidate the findings. The findings of this research indicate that our proposed method minimizes iterations and achieves the optimal solution, or a solution closely approaching optimality, in the initial phase of the transportation problem. An experimental study is presented in Eq. 5, which reveals that the 71.42% accuracy of our proposed method provides an optimal solution in IBFS, in contrast to the previously published study, which achieved 54.54%, while the DBAM method recorded 27.27%, along with the classical approach recorded as 20%. The advantages of this study are that, in addition to other approaches (16.88%), more provides the ideal solution in the first step and analyzes the sensitivity range. Moreover, the proposed algorithm achieves an optimal solution in eleven iterations, while DBAM requires eleven iterations and VAM completes the process in ten iterations. The future scope may emphasize practical applications and various specific contexts of linear programming challenges.

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Authors' declaration

- Conflicts of Interest: None.
- We hereby confirm that all the Figures and Tables in the manuscript are ours. Furthermore, any Figures and images, that are not ours, have been included with the necessary permission for re-publication, which is attached to the manuscript.
- No animal studies are present in the manuscript.
- No human studies are present in the manuscript.
- Ethical Clearance: The project was approved by the local ethical committee at Vellore Institute of Technology, Vellore, India.

Authors' contribution

M. A. contributed to the design, performance, analysis, interpretation, drafting, and writing of the manuscript. V. B contributed to supervising and validating the manuscript.

References

1. Azad SMAK, Hossain MB. A New Method for Solving Transportation Problems Considering Average Penalty. IOSR J Math. 2017;13(1):40–43. https://doi.org/10.9790/5728-1301044043
2. Ahmed MM, Sultana N, Khan AR, Uddin MS. An Innovative Approach to Obtain an Initial Basic Feasible Solution for the Transportation Problems. J Phys Sci. 2017;22:23–42.
3. Harrath Y, Kaabi J. New Heuristic to Generate an Initial Basic Feasible Solution for the Balanced Transportation Problem. Int J Syst Eng. 2018;30(2):193–204. https://doi.org/10.1504/ijise.2018.094842.
4. Karagul K, Sahin Y. A Novel Approximation Method to Obtain Initial Basic Feasible Solution of a Transportation Problem. J King Saud Univ Eng Sci. 2020;32(3):211–218. https://doi.org/10.1016/j.jksues.2019.03.003.
5. Amaliah B, Fatichah C, Suryani E. Total Opportunity Cost Matrix – Minimal Total: A New Approach to Determine Initial Basic Feasible Solution of a Transportation Problem. Egypt Inform J. 2019;20(2):131–141. https://doi.org/10.1016/j.eij.2019.01.002.
6. Kamble SV, Kore BG. A New Method to Obtain an Initial Basic Feasible Solution of Transportation Problem with the Average Opportunity Cost Method. Int J Eng Adv Technol. 2019;9(2):206–209. https://doi.org/10.35940/ijeat.F9283.129219
7. Sathyavathy M, Shalini M. Solving Transportation Problem with Four Different Proposed Mean Method and Comparison with Existing Methods for Optimum Solution. J Phys Conf Ser. 2019;1362:012088. https://doi.org/10.1088/1742-6596/1362/1/012088.
8. Hossain MM. Improved Average Penalty Cost (IAPC) Method to Obtain Initial Basic Feasible Solution of Transportation Problem. Glob J Sci Front Res. 2020;20(F8):23–36.
9. Jamali S, Soomro AS, Shaikh MM. The Minimum Demand Method-A New and Efficient Initial Basic Feasible Solution Method for Transportation Problems. J Mech Continua Math Sci. 2020;15(10):94–109. https://doi.org/10.26782/jmcms.2020.10.00007.
10. Hossain MM, Ahmed MM. A Comparative Study of Initial Basic Feasible Solution by a Least Cost Mean Method (LCMM) of Transportation Problem. Am J Oper Res. 2020;10(4):122–131. https://doi.org/10.4236/ajor.2020.104008.
11. Babu MA, Hoque MA, Uddin MS. A Heuristic for Obtaining the Better Initial Feasible Solution to the Transportation Problem. OPSEARCH. 2020;57(1):221–245. https://doi.org/10.1007/s12597-019-00429-5.
12. Putcha C, Shekaramiz A. A Comprehensive Method for Arriving at Initial Feasible Solution for Optimization Problems in Engineering with Illustrative Examples. Turk J Comput Math Educ. 2021;12(5):1189–1205.
13. Sam'an M, Ifriza YN. A Combination of TDM and KSAM to Determine Initial Feasible Solution of Transportation Problems. J Soft Comput Explor. 2021;2(1):17–24. https://doi.org/10.52465/joscex.v2i1.16.
14. Shahi S, Henry VV. A Matrix Minimum Approximation Method to Obtain Initial Basic Feasible Solution of Transportation Problem. Int J Mech Eng. 2022;7(2):3842–3852.
15. Ramakrishna M, Ashok G. An Initial Basic Feasible Transportation Solution Based on the North-West Corner Rule Programming in c. J Multidiscip Res. 2022;2(2):19–23. https://doi.org/10.37022/tjmdr.v2i2.395.
16. Amulu Priya S, Maheswari V, Balaji V. New Vogel's Approximation Method (NVAM) to Determine Better Feasible Solution of Transportation Problem. Math Stat Eng Appl. 2022;71(3):1385–1397. https://doi.org/10.17762/msea.v71i3.493.
17. Ekanayake E, Ekanayake EMUSB. A Novel Approach Algorithm for Determining the Initial Basic Feasible Solution

for Transportation Problems. Indones J Innov App Sci. 2022;2(3):234–246. https://doi.org/10.47540/ijias.v2i3.529
18. Sharma SK, Goel K. Analysis of IBFS for Transportation Problem by Using Various Methods. Math Stat. 2022; 10(4):741–746. https://doi.org/10.13189/ms.2022.100404.
19. Amaliah B, Fatichah C, Suryani E. A New Heuristic Method of Finding the Initial Basic Feasible Solution to Solve the Transportation Problem. J King Saud Univ.-Comput Inf Sci. 2022;34(5):2298–2307. https://doi.org/10.1016/j.jksuci.2020.07.007.
20. Raval S. New Approach to Find Initial Basic Feasible Solution (IBFS) for Optimal Solution in Transportation Problem. Open J Appl Sci. 2023;13(2):207–211. https://doi.org/10.4236/ojapps.2023.132017.
21. Zabiba MS, Al-Dallal HA, Hashim KH, Mahdi MM, Shiker MA. A New Technique to Solve the Maximization of the Transportation Problems. AIP Conf Proc. 2023;2414(1):040042. https://doi.org/10.1063/5.0114806.
22. Ackora-Prah J, Acheson V, Owusu-Ansah E, Nkrumah SK. A Proposed Method for Finding Initial Solutions to Transportation Problems. Pak J Stat Oper Res. 2023; 19(1):63–75. https://doi.org/10.18187/pjsor.v19i1.4196.
23. Rashid A, Islam MA. An Efficient Approach for Transforming Unbalanced Transportation Problems into Balanced Problems in Order to Find Optimal Solutions. Am J Oper Res. 2024;14(1):74–86. https://doi.org/10.4236/ajor.2024.141004.
24. Abdelwali MHA. A New Approach for Finding an Initial Basic Feasible Solution to a Transportation Problem. J Adv Eng Trends. 2024;43(1):77–85. https://doi.org/10.21608/jaet.2022.133374.1151.
25. Jude O, Udo OC. Transportation Problem via Median Approximation Technique for IBFS and Revised Modi's Technique. Innov Sci Technol. 2024;3(2):32–43. https://doi.org/10.56397/ist.2024.03.05.
26. Amreen M, Venkateswarlu B. A New Way for Solving Transportation Issues Based on the Exponential Distribution and the Contraharmonic Mean. J Appl Math Inform. 2024;42(3):647–661. https://doi.org/10.14317/jami.2024.647
27. Indira P, Jayalakshmi M. Evaluation of the Minimum Transportation Cost of Asymmetric/Symmetric Triangular Fuzzy Numbers with-Cut by the Row-Column Minima Method. Baghdad Sci J. 2024;21(11):3528–3542. https://doi.org/10.21123/bsj.2024.10029.

استراتيجية جديدة لايجاد الحل الأساسي الأبتدائي الممكن لمشكلة النقل باستخدام التوزيع الأسي ونموذج جزاء جديد

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المستخلص

أحد المفاهيم الرئيسية في أبحاث العمليات (OR) هو مشكلة النقل (TP). تتضمن مشكلة النقل توزيع المنتجات من عدة مصادر إلى وجهات مختلفة مع تقليل نفقات الشحن إلى الحد الأدنى. ولتحقيق الحد الأدنى من تكاليف النقل، يجري بحث مشكلة النقل على مرحلتين. الحل الأولي الممكن (IBFS) هو المرحلة الأولى لتحقيق أقل سعر، بمساعدة IBFS، يمكن حساب الحل الأمثل المشتق كمرحلة ثانية. تشير مراجعة الأدبيات إلى أن المؤلفين طوروا خوارزمية جديدة لـ IBFS، تتضمن العديد من الأدوات الرياضية وبعض النماذج الجديدة التي تمتلك قيودًا معينة. للتغلب على هذا، اقترحنا خوارزمية جديدة لـ IBFS، وأداة إحصائية متعاونة ونموذج عقوبة جديد. يوضح التوزيع الأسي احتمال التكلفة أو الوقت اللازم للوصول إلى كل مصدر إلى المقصد. وعلاوة على ذلك، صيغ نموذج جديد للجزاء لتوزيع العرض الأساسي حسب الطلب المطلوب. يتم استخدام طريقة توزيع معدلة (MODI) لحساب الحل الأمثل. تم توضيح أحد عشر مثالًا رقميًا لتطوير الخوارزمية المقترحة. تمت مقارنة النتائج مع طريقة التقريب الكلاسيكية لـ Vogel (VAM) وعشر أوراق بحثية منشورة سابقًا. بالإضافة إلى ذلك، تم إجراء ثلاث مشاكل عشوائية. أخيرًا، مع انخفاض تكلفة النقل الإجمالية عن طريقة Vogel التقريبية، وطريقة التخصيص القائمة على الطلب (DBAM) والدراسات العشر المنشورة الحالية. سجلت طريقتنا المقترحة للحصول على 10 حلول مثالية في IBFS دقة 71.42٪.

الكلمات المفتاحية: التوزيع الأسي، الحل الأولي الأساسي الممكن، نموذج جزاء جديد، الحل الأمثل، مشكلة النقل.