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Effect of Soil Friction on the Optimum Design of Cantilever Retaining Walls

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Abstract

In this study the Artificial Intelligent Method (AIM) was used to find safety factors against sliding, overturning and bearing capacity of cantilever type retaining wall. The method was also used to determine the optimum dimensions of retaining wall and consequently minimum retaining wall volume. Three parameters were investigated in this study: friction of the soil beneath and behind the retaining wall (ϕ), retaining wall height (H) and surcharge load (Q). approximately, zero constrains violation and constant volume of retaining wall were achieved through three iterations of solution. This results gave a reliability for using this method for optimum design of cantilever type retaining wall. The results demonstrated that, the significant increasing in sliding and bearing capacity factors of safety started at ($\phi = 35^o$), while the safety factor against overturning decreased with increasing soil friction up to (35^o) and then increased. On the other hand, there is no change in base width of retaining wall when (ϕ) excessed (35^o), and the optimum volume of retaining wall decreased significantly at this value of friction angle. Finally, the research concluded the possibility of using the results obtained from this study as a design charts to obtain the various retaining wall elements and different safety factors within the limits of studied parameters i.e, soil friction, height of retaining wall and surcharge load.

Keywords: Soil friction, Optimum design, Retaining walls , Artificial Intelligent, Safety factors

Nomenclature

L_h	Length of the heel slab (m)	B	Total width of the base slab (m)
M_1 to N_4	Moments acting due to W_1 to W_4 respectively (kN.m)	E	Eccentricity (m)
		$F.S._{Overturning}$	Safety factor against overturning.
M_5	Overturning moment (kN.m)	$F.S._{Sliding}$	Safety factor against sliding.
M_6	Resistance moment (kN.m)	H	Total height of the retaining wall
q_{max}	Maximum soil pressure (kN/m^2)	H_1	Stem height (m)
t	Base slab thickness (m)	H_2	Height of stem including backfill height(m)
W_1	Load subjected to the soil by rectangular part of steam (kN)	k_a	Coefficient of active earth pressure.
W_2 :	Load subjected to the soil by triangular part of stem (kN)	L_b	Triangle width at steam bottom
W_3 :	Load subjected to the soil by base slab (kN)	L_t	Length of the toe slab (m)
		L_s	Stem thickness at top (m)

μ	Friction coefficient between soil and concrete	W_4 :	Load subjected to the wall by trapezoidal backfill (kN)
ΣV	Sum of all loads (kN)	γ_c :	Concrete unit weight (kN/m^3)
$\sum Mr$	Total resisting moment (kN.m)	γ_s :	Backfill soil unit weight (kN/m^3)
$\sum Mo$	Total overturning moment (kN.m)	α	Backfill slope (°)

1. Introduction

Among the many types of retaining structures, a cantilever type retaining wall considers one of the most common earth retaining structures used in civil engineering practice because, of its low construction cost for the height up to 10 m. [1].

In addition to the variation in the backfill soil behind the retaining wall, the stability of retaining walls should be evaluated by taking into account many types of failure. In order to get the best geometrical design and, as a result, the lowest possible construction cost, construction portions must also be proportioned in some way.

Optimization is the process of structural design which give designs not only safety but also lowering in cost.

The safety factor of cantilever retaining wall is considered as a multi objective problem. For a high safety design, the retaining wall should be analyzed to a high value of safety factor especially in high seismic regions [2,3], at the same time to minimize the construction cost of retaining wall [4,5]. It also important to check the deflection of the steam of the retaining wall to ensure that the geometric members of the wall are within the appropriate proportion. Numerous studies have looked into this feasibility using various optimization techniques [6, 7, 8, 9].

The optimum design problems were solved by many researchers through the past decades. Some of them used numerical method to find the optimum solution, others derived some equations concerning particular optimization problems, while a few used artificial intelligent methods as a main approach to find the optimum design.

Saribas and Erbatur (1996) [10] used a special computer program (RETORT) to design cantilever type retaining wall, by considering the weight or the construction cost of the wall as the objective function. The results of design are presented with numerical examples.

Purohit. S [11] suggested a method for design retaining wall with low weight and construction cost by using optimization technique. This method of design depends on the technique of multiple objective scope, taking into account the various safety factors and construction cost of retaining wall as alternative solutions.

In 2017, V. Nandha Kumar and C.R. Suribabu [12] found the minimum weight of retaining walls using Differential Evolution Algorithm, they introduced the reinforcing area of steel as a design variable in addition to dimension of the wall. They achieved 15% reduction in the weight as compared to the manual solution. The selected design constraints took into account a variety of wall failure types.

Rasim Temür and Gebrail Bekdaş 2018 [13] suggested a few metaheuristic algorithms to determine the optimal cost of cantilever type retaining walls using stability and dimensional limitations as design constraints according to the ACI Code 2018.

Sasidhar T et al [14] analyzed cantilever type retaining wall with different heights using MATLAB program. By comparison the results with other results obtained from conventional design method, the researchers found that the MATLAB is an effective program to get not only optimum geometric design but also minimum construction cost of cantilever retaining wall.

Esra et al. (2020) [15] used the colony of the bee algorithm technique to study the design of two different kinds of cantilever retaining walls. This study looked into the bearing capacity safety factor as well as the safety factor against sliding and overturning. The authors concluded that this optimization technique is appropriate for application in the design of cantilever type retaining walls.

The results of the optimization analysis show that when the angle of internal friction of the soil beneath the retaining wall foundation is less than 25°, the costs of cantilever retaining walls and cantilever retaining walls with key increase.

M. Shalchi Tousi et al (2021) [16] used the algorithm of optimization of gas motion in the optimum weight design and optimum cost of cantilever type retaining wall. This method is based on the motion of gaseous molecules. By employing this technique, the researchers were able significantly to lower the retaining wall's weight and construction costs, particularly when the lateral earth pressure of the retaining wall was determined using the Coulomb analysis method.

Nikhil D. G and Vishal S (2022) [17] used a non linear evolutionary optimization method to design two types of retaining wall: relieving platform retaining wall and conventional cantilever type retaining wall. The height of retaining wall changed from (3-10) m. they concluded that this optimization design method is more efficiency to design platform retaining wall rather than cantilever type retaining wall.

The current study aims to use Artificial Intelligent Method (AIM) to produce design charts for sliding, overturning and bearing capacity safety factors as well as optimum retaining wall volume within the limits of studied parameters i.e, soil friction, height of retaining wall and surcharge load.

2. Solution Methodology

The Artificial Intelligent Technique (pattern search method, MATLAB code) was used to find the minimum volume of the retaining wall. This technique had proved its efficiency through many challenging structural optimization problems. By limiting the issue using the sufficient constraints form that was retrieved from the code, the solution was achieved. [18]. However, the retaining wall's different structural components are among the design variables.

The design constraints of the problem were divided into two categories: the first one contains the main constraints represented by the factor of safety for sliding, overturning and bearing capacity of the retaining wall. While, the second category represent the design limitations for the elements dimensions of the retaining wall such as the dimensions of the design variables.

3. Failure Modes of Retaining Wall

Various optimization limitations to obtain the optimal design of cantilever type retaining wall have been proposed, including the three basic geotechnical constraints. Overturning, sliding and bearing failure modes have been taken into consideration. Considering a cantilever type retaining wall shown in Fig. 1 with a height of H, the various expressions of the wall are given in the following sections.

4. Constraints

Referring to the Bowles [19], the design constraints are classified into two categories: geotechnical and structural requirements. These requirements represent the failure modes as a function of the design variables. The failure modes include stability for overturning, sliding and bearing capacity.

5. Load Calculation

Fig. 1. shows the various loads acting on the retaining wall. The following formulas are traditionally used to calculate the loads acting on the retaining wall [20]:

$$W1 = \gamma c \times (H\text{-}t) \times Ls \quad (1)$$

$$W2 = \frac{1}{2} \times \gamma c \times (H\text{-}t) \times Lb \quad (2)$$

$$W3 = \gamma c \times t \times B \quad (3)$$

$$W4 = \frac{1}{2} \times (H1 + H2) \times \gamma 1 \times Lh \quad (4)$$

$$Pa = \frac{1}{2} \times \gamma 1 \times H^2 \times ka \quad (5)$$

$$\Sigma V = W1 + W2 + W3 + W4 + (Pa \times \sin \alpha) \quad (6)$$

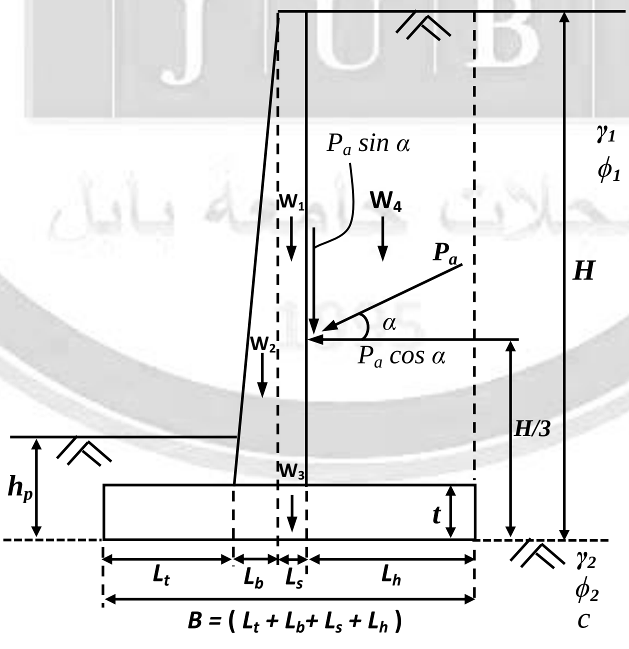

Fig. 1: Forces acting on a reinforced concrete cantilever retaining wall

6. Moment Calculation

The moment acting on each section due to various loads was calculated as follow:

$$M_1 = W_1 \times (L_t + L_b + \frac{Ls}{2}) \quad (7)$$

$$M_2 = W_2 \times (L_t + \frac{2}{3} L_b) \quad (8)$$

$$M_3 = W_3 \times \frac{B}{2} \quad (9)$$

$$M_4 = W_4 \times ((L_t + L_b + L_s) + \left(\frac{L_h}{2}\right)) \quad (10)$$

$$M_5 = P_a \times \left(\frac{H}{3}\right) \cos \alpha \quad (11)$$

$$M_6 = P_a \times \mathrm{B} \times \sin \alpha \quad (12)$$

$$\sum Mr = M_1 + M_2 + M_3 + M_4 + M_5 + M_6 \quad (13)$$

$$\sum Mo = M_5 \quad (14)$$

The constraint equations for sliding and overturning are given as:

$$F.S._{\ Overturning} = \frac{\sum Mr}{\sum Mo} \quad (15)$$

$$Fr = \sum V \times \mu \quad (16)$$

$$Fd = P_a \times \cos \alpha \quad (17)$$

$$F.S._{\ Sliding} = \frac{Fr}{Fd} \quad (18)$$

The equations for eccentricity and maximum soil pressure acting on soil are given as:

$$\mathrm{e} = \frac{B}{2} - \frac{\sum Mr - \sum Mo}{\sum V} \quad (19)$$

$$q_{max} = \left(\frac{\sum V}{B}\right) + \left(1 + \frac{6e}{B}\right) \quad (20)$$

7.Results and Discussion:

Three parameters of retaining wall, load and soil were studied in this paper, i.e. 1- height of retaining wall (H) taken as (4, 5, 6, 7 and 8)m. 2- surcharge load (Q) taken as (0, 5, 10, 15 and 20) kN/ m2, and finally soil friction (ϕ) which where (30, 35, 40°)

In addition to, the values of bearing capacity parameters (Nc, Nq, Nγ) were considered corresponding to the values of soil friction (ϕ). The height of passive side of retaining wall was (1.2) m. Since the soil is cohesionless, the cohesive of the soil is of (C=0) kN/ m2.

To check the effect of the design constrains, the solution was conducted, at first on one main design constrains i.e. factor of safety against sliding. In the next step, the factor of safety against overturning was added, and finally the bearing capacity factor of safety. The results of three times solution were achieved through three iteration with nearly zero constraints violation and with constant design volume of retaining wall. Figs. (2-5) show the variation of both of objective function (retaining wall height) and constraints violation with No. of iteration for all studied

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parameters listed above. These Figures (2-5) represent an increase in the degree of reliability of the solution. The objective function and constraints violations are reached to constant values and zero respectively at iteration number equal to 2.

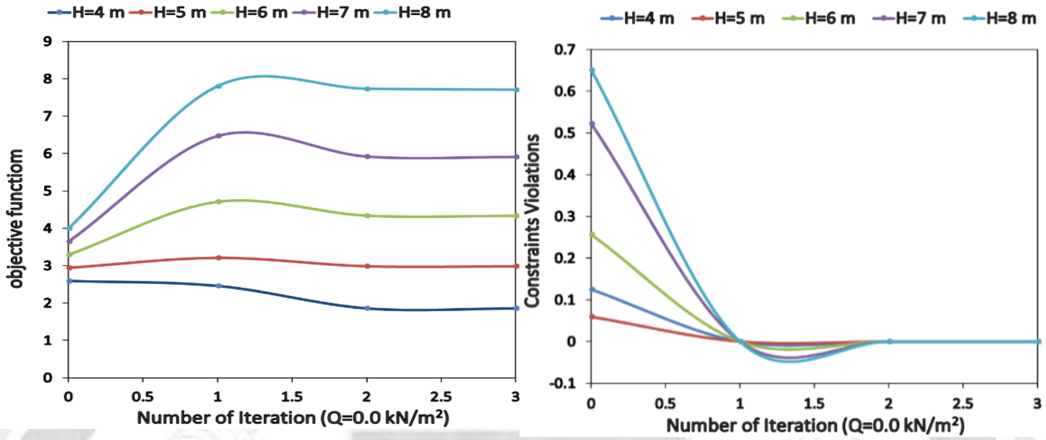

Fig. 2 : Number of iteration versus objective function and constraints violations for
surcharge pressure (Q = 0.0 kN/m^2)

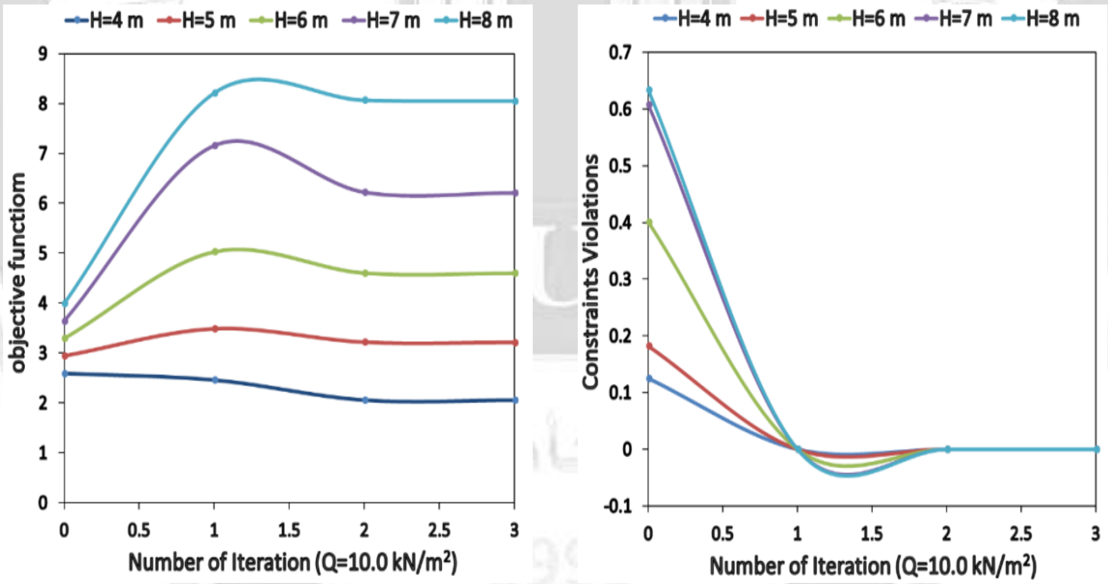

Fig. 3: Number of iteration versus objective function and constraints violations for
surcharge pressure (Q = 10.0 kN/m2)

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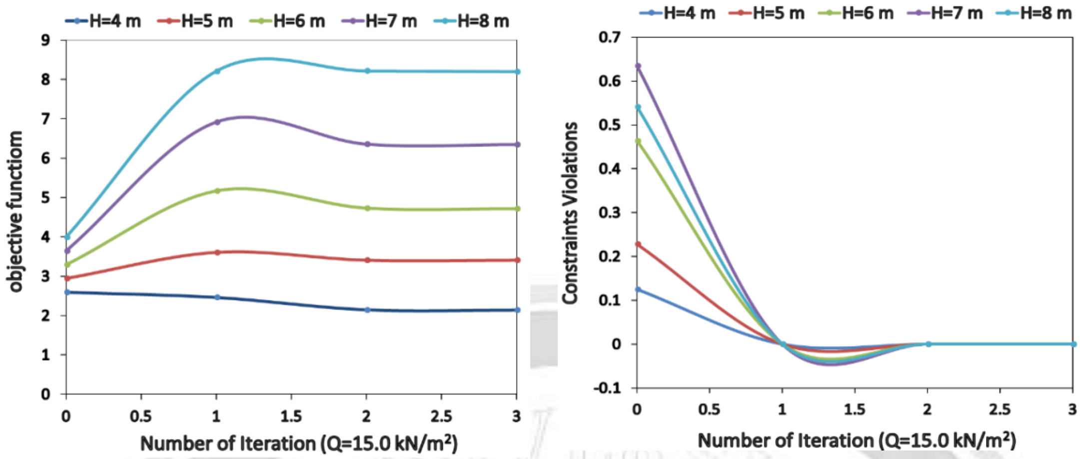

Fig. 4: Number of iteration versus objective function and constraints violations for surcharge pressure (Q = 15.0 kN/m2)

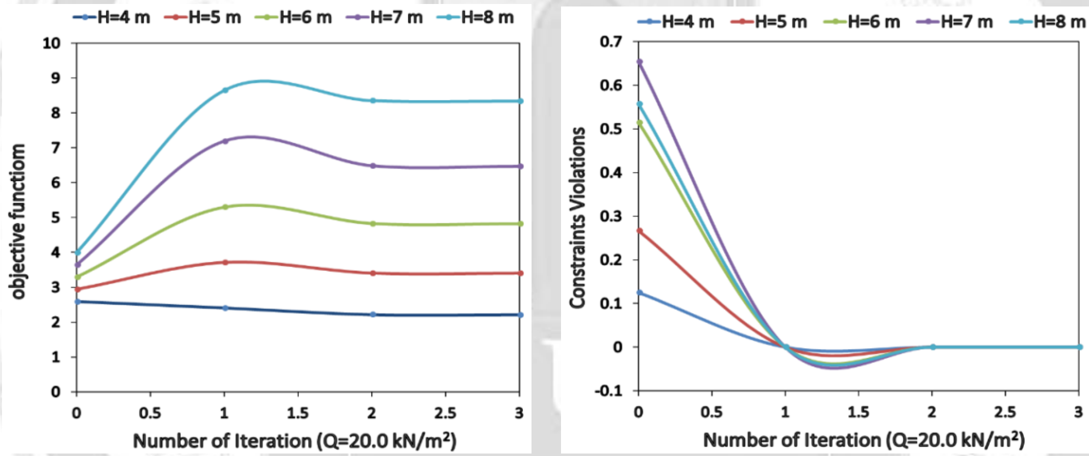

Fig. 5: Number of iteration versus objective function and constraints violations for surcharge pressure (Q = 20.0 kN/m^2)

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Figs. (6-8) show the variation of factor of safety against sliding, overturning, and bearing capacity respectively with soil friction beneath and behind retaining

wall (ϕ). The Figs. also show the effect of retaining wall height and surcharge load on these safety factors. It can be seen that, generally, the significant increasing in factor of safety against sliding begins at ($\phi = 30^o$), especially with high value of surcharge load (Q). on the other hand, the factor of safety against overturning decreases with increasing soil friction up to (35^o), and then increases. Similarly to the factor of safety against sliding, the significant increasing of factor of safety of bearing capacity begins at ($\phi = 30^o$). Figs. 9 and 10 show the variation of base width and toe width of retaining wall respectively with soil friction (ϕ) for all height values of retaining wall (H) and surcharge load (Q). The curves indicate that almost there is no change in base width when (ϕ) excess than (35^o), except for (Q=20 kN/m2). The base width witnesses a slight decreasing at this value of (Q). Regarding the variation of toe width of retaining wall with (φ), it can be noted that, there is no change in toe width with (φ) except for high retaining walls (H= 7 and 8 m.) and sometimes (6m.). It is economically important to study the variation of optimum retaining wall volume (m3) per unit length with (φ) and with other studied parameters. Fig. (11) show that the change in optimum retaining wall volume decreases or sometimes vanishes when (ϕ) excess (35^o).

The current research represents the possibility of using the results obtained from this study as a design charts within the limits of studied parameters i.e, friction of the soil beneath and behind the retaining wall (ϕ) , retaining wall height (H) and surcharge load (Q).

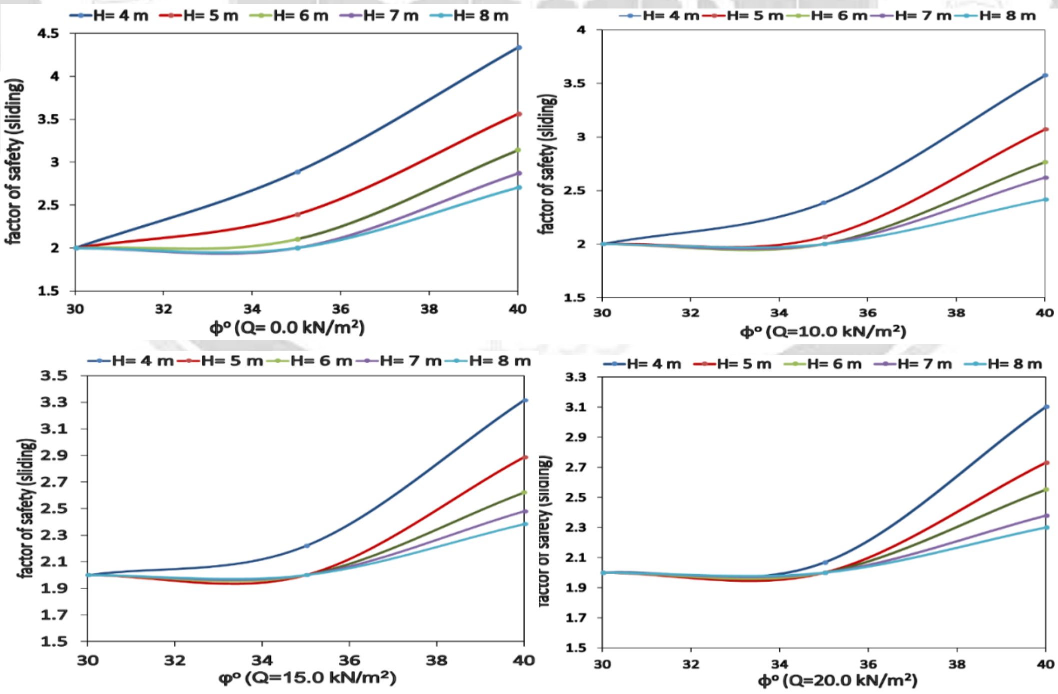

Fig. 6: Soil friction angle (φ) versus factor of safety against sliding for surcharge pressure (Q = 0.0 – 20.0 kN/m^2)

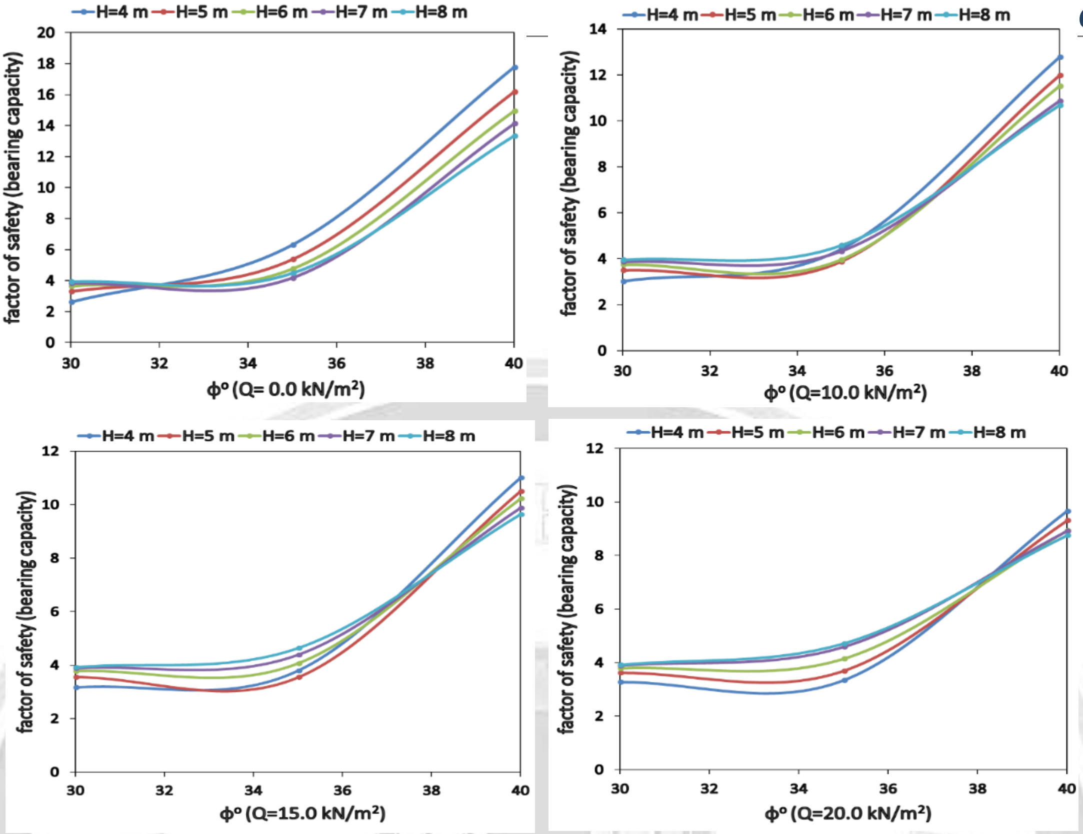

Fig. 8: Soil friction angle (φ) versus bearing capacity safety factor for surcharge pressure (Q = 0.0 – 20.0 kN/m^2)

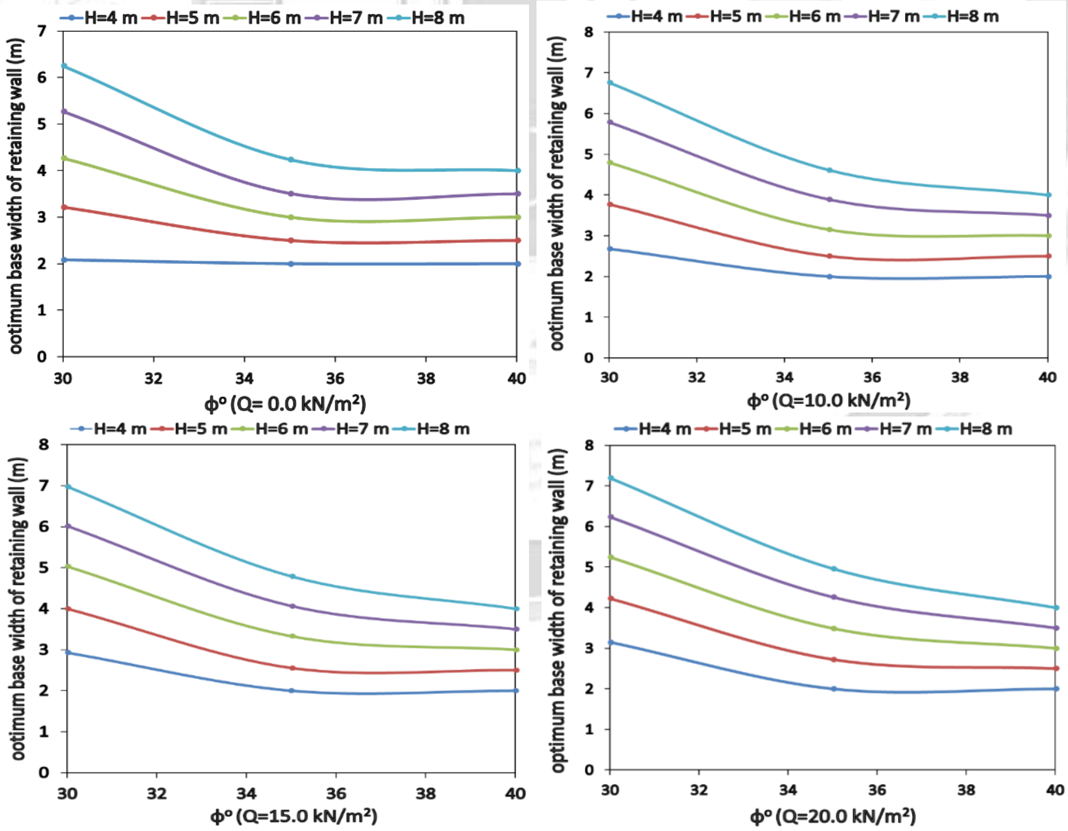

**Fig. 9: Soil friction angle (φ) versus optimum base width of retaining wall for
surcharge pressure (Q = 0.0 – 20.0 kN/m^2)**

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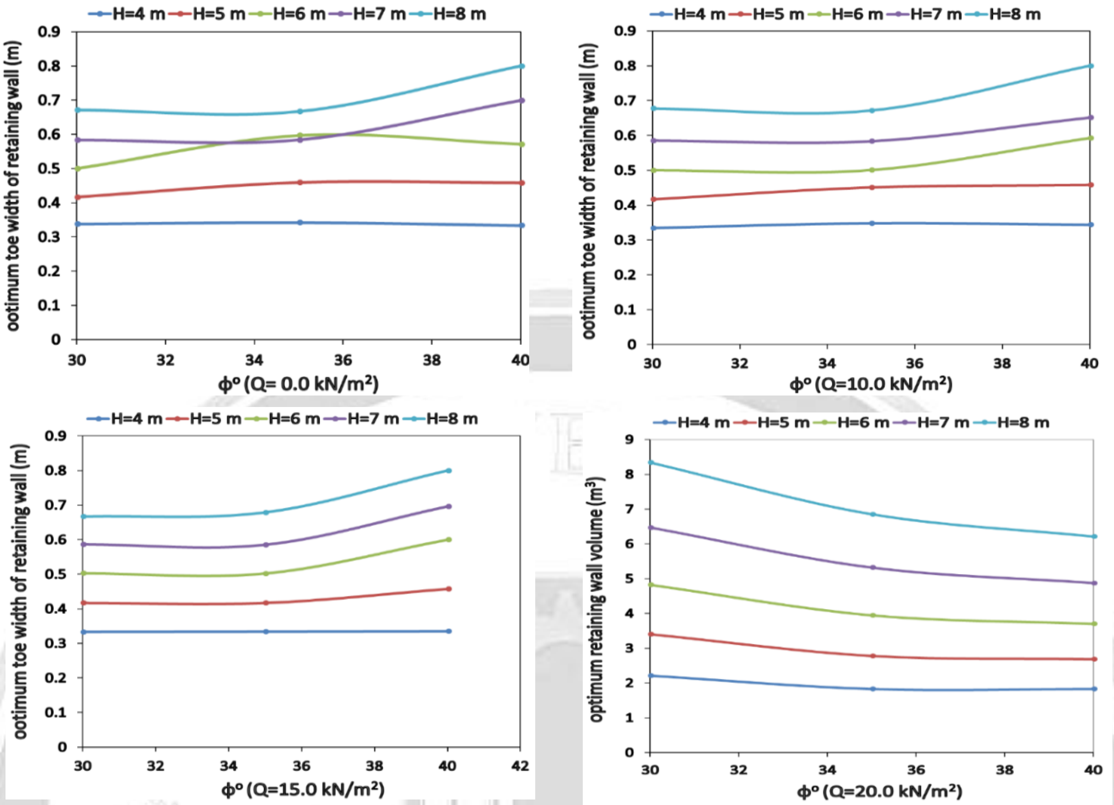

Fig. 10: Soil friction angle (φ) versus optimum toe width of retaining wall for surcharge pressure (Q = 0.0 – 20.0 kN/m^2

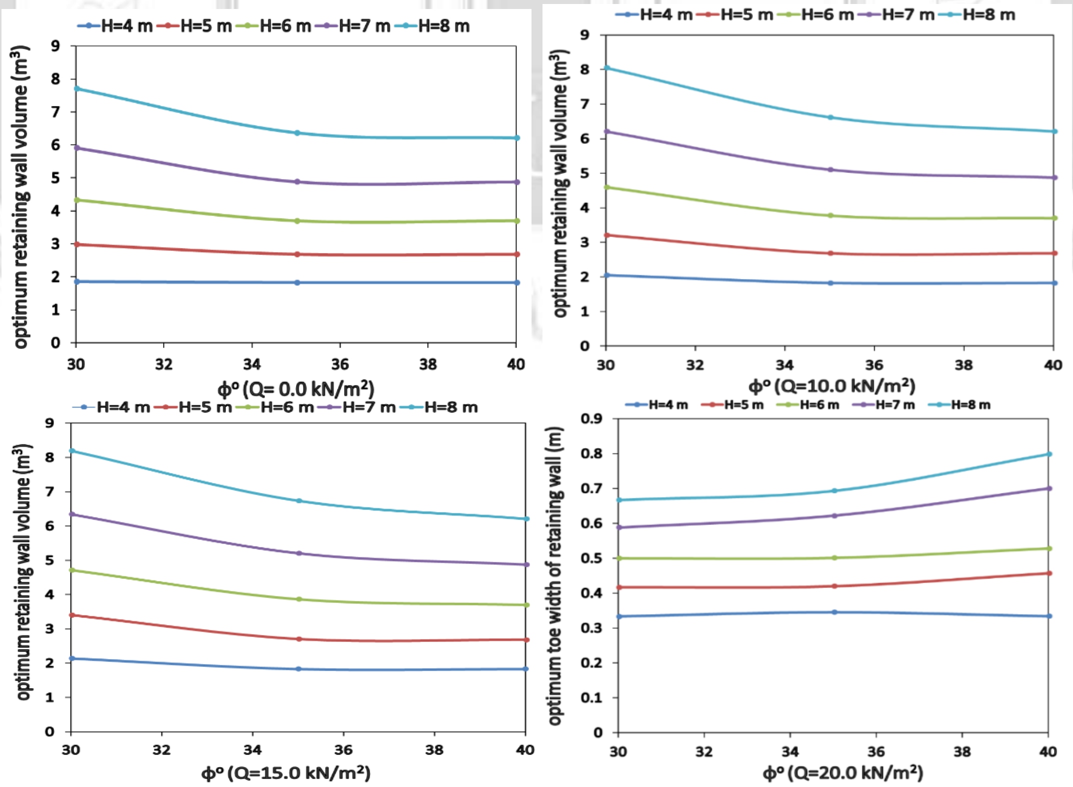

Fig. 11: Soil friction angle (φ) versus optimum volume of retaining wall for surcharge pressure (Q = 0.0 – 20.0 kN/m^2)

8. Conclusion

Based on the results of the study of optimum design of cantilever retaining walls using artificial intelligent method, the following conclusion can be pointed;

1. The nearly zero constrains violation and constant design volume of retaining wall were achieved through three iteration solution.
2. The significant increasing of factor of safety against sliding and bearing capacity safety factor started at ($\phi = 30^o$).
3. The factor of safety against overturning decreased with increasing soil friction up to (35^O) and then increased.
4. There is no change in base width of retaining wall (ϕ) excess (35^o) except for (Q = 20 kN/m^2).
5. The optimum volume of retaining wall decreased significantly (ϕ) excess (35^o).
6. The current research represents the possibility of using the results obtained from this study as a design charts within the limits of studied parameters i.e, friction of the soil beneath and behind the retaining wall (ϕ) , retaining wall height (H) and surcharge load (Q).

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تأثير احتكاك التربة على التصميم الأمثل للجدران الساندة الناتئة

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الخلاصة

في هذه الدراسة، تم استخدام طريقة الذكاء الاصطناعي (AIM) لإيجاد عوامل الأمان ضد الانزلاق والانقلاب وقابلية التحمل للجدران الساندة الناتئة. كما تم استخدام هذه الطريقة لتحديد الأبعاد المثلى للجدران الساندة وبالتالي الحد الأدنى لحجم هذه الجداران. تم التحقق من ثلاثة معاملات في هذه الدراسة : احتكاك التربة أسفل وخلف الجدار الساند (ϕ) ، وارتفاع الجدار (H) والحمل الإضافي (Q). حيث تم تحقيق هذه المتطلبات بدون أي حيود وحجم ثابت للجدار الساند من خلال ثلاث محاولات تكرارية للحمل. أعطت هذه النتائج موثوقية في استخدام هذه الطريقة للتصميم الأمثل للجدران الساندة الناتئة، حيث أظهرت النتائج أن الزيادة الكبيرة في عوامل الامان للانزلاق و قابلية التحمل بدأت عند ($\phi = 30^o$)، بينما انخفض معامل الأمان ضد الانقلاب مع زيادة احتكاك التربة حتى ($\phi = 35^o$) ثم بدا بالزيادة. من ناحية أخرى، لا يوجد تغيير في عرض قاعدة الجدار عند تجاوز ($\phi = 35^o$) وانخفض الحجم الأمثل للجدار الساند بشكل كبير عند هذه القيمة لزاوية الاحتكاك. وأخيراً خلص البحث إلى إمكانية استخدام النتائج المستحصل عليها من هذه الدراسة كمخططات تصميمية للحصول على الابعاد المثلى لعناصر الجدران الساندة المختلفة وعوامل الأمان المختلفة ضمن حدود المتغيرات المدروسة وهي احتكاك التربة وارتفاع الجدران الساندة و الحمل الإضافي.

الكلمات الدالة : احتكاك التربة، التصميم الأمثل، الجدران الساندة ، الذكاء الصناعي ، عوامل الأمان.