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R-Al Tememe Transformation for Solving Ordinary Differential Equations with Constant Coeffecients

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A B S T R A C T

Solving (LODEs) with constant coefficients is a core engineering and applied mathematics task. For a long time, several differential transform and integral methods have been developed to facilitate these equations and to get accurate analytical solutions, such as Laplace [1–5], Al-Zughair [6], and other research. In this research, we explore the new application of the R. Al-Tememe transform to examine the capability in solving (LODEs) with constant coefficients.

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Table 1. Nomenclature and Definitions Used in the Study

RA	R.Al–Tememe Transform
$(RA)^{-1}$	The inverse of R.Al–Tememe transform
eq.	equation
LODEs	Linear Ordinary Differential Equations
ODEs	Ordinary Differential Equations

1. INTRODUCTION

In many scientific and technical fields, differential equations both ordinary (ODEs) and partial (PDEs) are crucial instruments for simulating dynamic processes. Their answers are essential to comprehending mechanical, biological, and physical phenomena. However, it is frequently difficult to find analytical solutions to these equations, particularly for complex or nonlinear systems. This has spurred the creation and use of a number of integral transforms that reduce the complexity of solving differential equations by transforming them into algebraic forms. This has traditionally been accomplished using classical transforms like the Laplace and Fourier transforms. Analytical or semi-analytical solutions for linear and nonlinear differential equations with both constant and variable coefficients have been successfully obtained using these methods.

To solve linear ordinary differential equations with constant coefficients, a number of genuine integral transforms have been proposed recently: Kumar and Prasad's (2025) New Integral Transform is specifically used for ODEs with constant coefficients. Effective for ODEs and PDEs, the Shehu Transform generalizes both the Laplace and Sumudu transforms (Maitama & Zhao, 2019). (April 2025) examined dual Shehu Transform variations, connecting Shehu to other traditional transforms for fractional ODEs (Mlaiki et al., 2025). Higher-order linear ODEs with constant coefficients are the focus of the two-parameter MAHA Transform (2024) (AlSaoudi et al., 2024). For first- and second-order linear ODEs with constant coefficients, the Kamal Transform (2024) was examined in relation to Hyers-Ulam stability (Anderson, 2024). These reliable sources highlight new developments in analytical tools for solving ODEs with constant coefficients and highlight the growing interest in specialized integral transforms.

The purpose of this study is to present and investigate the characteristics of a new integral transform called the R. Al-Tememe Transform. Its formulation, inverse, and application to the solution of linear ordinary differential equations with constant coefficients will all be covered. Our objective is to show how this cutting-edge tool might function as a useful substitute for conventional transforms, possibly providing improved accuracy and application simplicity.

which has a kernel $\left(\frac{(\ln(ln\mathcal{M}))^{\hat{r}-1}}{\mathcal{M}(ln\mathcal{M})}\right)$, and the general form of it:

$$RA(\underset{\cdot}{S}(\ln(ln\mathcal{M}))) = \int_e^{e^e} \frac{(\ln(ln\mathcal{M}))^{\hat{r}-1}}{\mathcal{M}(ln\mathcal{M})}\, \underset{\cdot}{S}(\ln(ln\mathcal{M}))d\mathcal{M}$$

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Where $\mathcal{M} \in [e, e^e]$, $\hat{r} > 0, \hat{r} \neq 1$

2. R. A. TRANSFORM FOR THE FUNDAMENTAL FUNCTIONS

Table 2. This table represents the relation of R.Al–Tememe with some functions

Function	$R.A(f(\ln(lns))) = \int_e^{e^e} \frac{(\ln(lns))^{p-1}}{s(lns)} f(\ln(lns))ds = F(p)$	Region of Convergence
1	$\frac{1}{p}$	$p > 0$
ɱ, ɱ a is constant	$\frac{ɱ}{p}$	$p > 0$
$(\ln(lns))^n$ $n = 1,2,3,\ldots$	$\frac{1}{p+n}$	$p > -n$
$(\ln(\ln(lns)))^n$ n = 1,2, 3....	$\frac{(-1)^n n!}{p^{n+1}}$	$p > 0$
$\cos(a(ln(\ln(lns))))$ a is a constant	$\frac{p}{p^2+a^2}$	$p > -a$
$\sin(a(\ln(\ln(lns))))$ a is a constant	$\frac{-a}{p^2+a^2}$	$p > -a$
$\cosh(a(\ln(\ln(lns))))$ a is a constant	$\frac{p}{p^2-a^2}$	$\|p - a\| > 0$
$\sinh(a(\ln(\ln(lns))))$ a is a constant	$\frac{-a}{p^2-a^2}$	$\|p - a\| > 0$

R.Al–Tememe transform for deriving

Theorem:

Let us consider that $y = y(\ln(\ln(lns)))$ so, we define the R.Al–Tememe Transform $(RA)$for y as the following:

$$RA(y(\ln(\ln(lns))) = \int_e^{e^e} \frac{(\ln(lns))^{p-1}}{s(lns)} y(\ln(\ln(lns)))ds,$$

This integration will be defective and $p > 0, p \neq 1$ and we consider $y(-\infty) = y'(-\infty) = \cdots y^n(-\infty) = 0$ then:

1. $RA[y'(\ln(\ln(lns)))] = y(0) - py(\ln(\ln(lns)))$

Proof:

$RA[y'(\ln(\ln(lns)))] = \int_e^{e^e} \frac{(\ln(lns))^{p-1}}{s(lns)} y'(\ln(\ln(lns)))\, ds$

By part integration to solve the above integration

Let

$u = (\ln(lns))^p \to du = p\left(\frac{(\ln(lns))^{p-1}}{s(lns)}\right) ds$

$dv = \frac{y'(\ln(\ln(lns)))}{s(lns)(\ln(lns))} ds \to v = y(\ln(\ln(lns)))$

$RA[y'(\ln(\ln(lns)))] = (\ln(lns))^p y(\ln(\ln(lns)))|_e^{e^e} - p\int_e^{e^e} \frac{(\ln(lns))^{p-1}}{s(lns)} y(\ln(\ln(lns)))ds$

$= y(0) - py(\ln(\ln(lns)))$

2. $RA[y''(\ln(\ln(lns)))] = y'(0) - py(0) + p^2 RA[y(\ln(\ln(lns)))]$

Proof:

$RA[y''(\ln(\ln(lns)))] =$

$\int_e^{e^e} \frac{(\ln(lns))^{p-1}}{s(lns)} y''(\ln(\ln(lns)))\, ds$

By part integration to solve the above integral

$u = (\ln(lns))^p \to du = p\left(\frac{(\ln(lns))^{p-1}}{s(lns)}\right) ds$

$dv = y''(\ln(\ln(lns)))ds \to v = y'(\ln(\ln(lns)))$

$RA[y''(\ln(\ln(lns)))] = (\ln(lns))^p y'(\ln(\ln(lns)))|_e^{e^e} - p\int_e^{e^e} \frac{(\ln(lns))^{p-1}}{s(lns)} y'(\ln(\ln(lns)))ds$

$= y'(0) - p\int_e^{e^e} \frac{(\ln(lns))^{p-1}}{s(lns)} y'(\ln(\ln(lns)))ds$

Let

$u = (\ln(lns))^p \to du = p\frac{(\ln(lns))^{p-1}}{s(lns)} ds$

$dv = \frac{y'(\ln(\ln(lns)))}{s(lns)(\ln(lns))} \to v = y(\ln(\ln(lns)))$

$= y'(0) - p\left[(\ln(lns))^p y(\ln(\ln(lns)))\ |_e^{e^e} - p\int_e^{e^e} \frac{(\ln(lns))^{p-1}}{s(lns)} y(\ln(\ln(lns)))\right]$

$RA[y''(\ln(\ln(lns)))] = y'(0) - p[y(0) - pRA[y(\ln(\ln(lns)))]]$

$= y'(0) - py(0) + p^2 RA[y(\ln(\ln(lns)))]$

3. **$RA[y'''(\ln(\ln(lns)))] = y''(0) - py'(0) + p^2 y(0) - p^3 RA[y(\ln(\ln(lns)))]$**

Proof:

$RA[y'''(\ln(\ln(lns)))] = \int_e^{e^e} \frac{(\ln(lns))^{p-1}}{s(lns)} y'''(\ln(\ln(lns)))ds$

By using partial integration to solve the above integration

$u = (\ln(lns))^p \to du = p(\frac{(\ln(lns))^{p-1}}{s(lns)})ds$

$dv = \frac{y'''(\ln(\ln(lns)))}{s(lns)(\ln(lns))} ds \to v = y''(\ln(\ln(lns)))$

Then

$RA[y'''(\ln(\ln(lns)))] = (\ln(lns))^p y''(\ln(\ln(lns)))|_e^{e^e} - \int_e^{e^e} \frac{(\ln(lns))^{p-1}}{s(lns)} y''(\ln(\ln(lns)))ds$

$= y''(0) - p\int_e^{e^e} \frac{(\ln(lns))^{p-1}}{s(lns)} y''(\ln(\ln(lns)))ds$

By part integration to solve the above integral $dv = \frac{y''(\ln(\ln(lns)))}{s(lnd)(\ln(lns))}ds \rightarrow \quad v = y'(\ln(\ln(lns)))$

$$u = (\ln(lns))^p \rightarrow du = p\frac{(\ln(lns))^{p-1}}{s(lns)}\, ds$$

$= y''(0) - p[(\ln(lns))^p y'(\ln(\ln(lns)))\ |_e^{e^e} - p\int_e^{e^e} \frac{(\ln(lns))^{p-1}}{s(lns)} y'(\ln(\ln(lns))\, ds$

$$u = (\ln(lns))^p \rightarrow du = p\frac{(\ln(lns))^{p-1}}{s(lns)}\, ds$$

$$dv = \frac{y'(\ln(\ln(lns)))}{s(lns)(\ln(lns))}\, ds \rightarrow$$

$$v = y\,(\ln(\ln(lns)))$$

$= y'(0) - p\left[(\ln(lns))^p y(\ln(\ln(lns)))\ |_e^{e^e} - p\int_e^{e^e} \frac{(\ln(lns))^{p-1}}{s(lns)} y(\ln(\ln(lns)))\right]$

$= y''(0) - p[y'(0) - py(0) + p^2 RA[y(\ln(\ln(lns)))]]$

$RAy'''(\ln(\ln(lns))) = y''(0) - py'(0) + p^2 y(0) - p^3 RA[y(\ln(\ln(lns)))]$

4. $\boldsymbol{RA[y^{iv}(\ln(\ln(lns)))] = y'''(0) - py''(0) + p^2 y'(0) - p^3 y(0) + p^4 RA[y(\ln(\ln(lns)))]}$

$= \int_e^{e^e} \frac{(\ln(lns))^{p-1}}{s(lns)} y^{iv}(\ln(\ln(lns)))\, ds$

Using the partial integration to solve above integration we get

$u = (\ln(lns))^p \quad \rightarrow du = p\left(\frac{(\ln(lns))^{p-1}}{s(lns)}\right) ds$

$dv = \frac{y^{iv}(\ln(\ln(lns)))}{s(lns)(\ln(lns))}\, ds \quad \rightarrow v = y'''(\ln(\ln(lns)))$

$= (\ln(lns))^p y'''(\ln(\ln(lns)))|_e^{e^e} \quad -p \int_e^{e^e} \frac{(\ln(lns))^{p-1}}{s(lns)} y'''(\ln(\ln(lns)))ds$

$= y'''(0) - p\int_e^{e^e} \frac{(\ln(lns))^{p-1}}{s(lns)} y'''(\ln(\ln(lns))\, ds$

$= y'''(0) - p[y''(0) - p\int_e^{e^e} \frac{(\ln(lns))^{p-1}}{s(lns)} y''(\ln(\ln(lns)))]ds$

$= y'''(0) - py''(0) - p^2\left[y'(0) - p\int_e^{e^e} \frac{(\ln(lns))^{p-1}}{s(lns)} y'(\ln(\ln(lns)))ds\right]$

$= y'''(0) - py''(0) + p^2 y' - p^3\left[y(0) - p\int_e^{e^e} \frac{(\ln(lns))^{p-1}}{s(lns)} y(\ln(\ln(lns)))ds\right]$

$RA[y^{iv}(\ln(\ln(lns)))] = y'''(0) - py''(0) + p^2 y'(0) - p^3 y(0) + p^4 RA[y(\ln(\ln(lns)))]$

In general

$RA[y^n(\ln(\ln(lns)))]$

$$= y^{n-1}(0) - py^{n-2} + \ldots\ldots$$
$$\mp p^{n-1} y(0) \pm p^n RA[y(\ln(\ln(lns)))]$$

Example: To solve the differential eq.
$y'(\ln(\ln(lns))) - 3y(\ln(\ln(lns))) = \sin(\ln(\ln(lns)))$, with condition $y(0) = 1 \quad \ldots (1)$

Take RA transform for eq. (1) we get

$y(0) - pRA[y(\ln(\ln(lns))] - 3RA[y(\ln(\ln(lns)))] = \frac{-1}{p^2+1}$

$-(p+3)RA[y(\ln(\ln(lns)))] = \frac{-2-p^2}{p^2+1}$

$RA[y(\ln(\ln(lns)))] = \frac{2+p^2}{(p+3)(p^2+1)} \quad \ldots (2)$

To find $y(\ln(\ln(lns)))$ take the inverse to RA transform to eq. (2) we get

$y(\ln(\ln(lns))) = (RA)^{-1}\left(\frac{p^2+2}{(p+3)(p^2+1)}\right)$

$\frac{p^2+2}{(p+3)(p^2+1)} = \frac{A}{p+3} + \frac{Bp+C}{p^2+1}$

$= \frac{Ap^2+A+Bp^2+3Bp+Cp+3C}{(p+3)(p^2+1)}$

$A + B = 1, 3B + C = 0, A + 3C = 2$

$A = \frac{11}{10}, B = \frac{-1}{10}, C = \frac{3}{10}$

$y = (RA)^{-1}\left[\frac{\frac{11}{10}}{(p+3)} + \frac{\frac{-1}{10}p+\frac{3}{10}}{p^2+1}\right]$

$y = \frac{11}{10}(\ln(lns))^3 - \frac{1}{10}\cos(\ln(\ln(lns))) - \frac{3}{10}\sin(\ln(\ln(lns)))$

Example: To solve the differential eq.

$y'(\ln(\ln(lns))) + 2y(\ln(\ln(lns))) = \sinh(ln(ln(lns)))$ with condition $y(0) = 0 \quad \ldots (3)$

Using RA transform for eq. (3) we get

$y(0) - pRA[y(\ln(\ln(lns)))] + 2RA[y(\ln(\ln(lns)))] = \frac{-1}{p^2-1}$

$-(p-2)RA[(\ln(\ln(lns)))] = \frac{-1}{p^2-1}$

$RA[y(\ln(\ln(lns)))] = \frac{1}{(p-2)(p^2-1)}$
… (4)

To find $y(\ln(\ln(lns)))$ take $(RA)^{-1}$for the eq. (4) we get

$$y(\ln(\ln(lns))) = (RA)^{-1}\left(\frac{1}{(p-2)(p^2-1)}\right)$$

$$\frac{1}{(p-2)(p^2-1)} = \frac{Ap+B}{p^2-1} + \frac{C}{p-2}$$

$$= \frac{Ap^2-2Ap+Bp-2B+Cp^2-C}{(p-2)(p^2-1)}$$

$$A+C=0, -2A+B=0, -2B-C=1$$

$$A=-\frac{1}{3}, B=-\frac{2}{3}, C=\frac{1}{3}$$

$$y(\ln(\ln(lns))) = (RA)^{-1}\left[\frac{-\frac{1}{3}p-\frac{2}{3}}{p^2-1} + \frac{\frac{1}{3}}{p-2}\right]$$

$$= -\frac{1}{3}\cosh(\ln(\ln(lns))) + \frac{2}{3}\sinh(\ln(\ln(lns))) + \frac{1}{3}(\ln(lns))^{-2}$$

Example: To solve the differential eq.

$y''(\ln(\ln(lns))) + y(\ln(\ln(lns))) = (\ln(\ln(lns)))$
with conditions $y(0)=1, y'(0)=0$ … (5)

Take RA transform for eq. (5) we get

$$y'(0) - py(0) + p^2RA[y(\ln(\ln(lns)))] + RA[Y(\ln(\ln(lns)))] = -\frac{1}{p^2}$$

$$-p + (p^2+1)RA[y(\ln(\ln(lns)))] = -\frac{1}{p^2}$$

$$(p^2+1)RA[y(\ln(\ln(lns)))] = \frac{p^3-1}{p^2}$$

$RA[y(\ln(\ln(lns)))] = \frac{p^3-1}{p^2(p^2+1)}$
… (6)

To find $y(\ln(\ln(lns)))$ take inverse of RA transform for eq. (6)

$$y(\ln(\ln(lns))) = (RA)^{-1}\left[\frac{p^3-1}{p^2(p^2+1)}\right]$$

$$\frac{p^3-1}{p^2(p^2+1)} = \frac{Ap+B}{p^2} + \frac{Cp+D}{p^2+1}$$

$$= \frac{Ap^3+Ap+Bp^2+B+Cp^3+Dp^2}{p^2(p^2+1)}$$

$$A+C=1, B+D=0$$

$A=0, B=-1, D=1, C=1$, then

$$y(\ln(\ln(lns))) = (RA)^{-1}\left[-\frac{1}{p^2} + \frac{p+1}{p^2+1}\right]$$

$$= (\ln(\ln(lns))) + \cos(\ln(\ln(lns))) - \sin(\ln(\ln(lns)))$$

Example: To solve the differential eq.

$y''(\ln(\ln(lns))) = (\ln(\ln(lns)))^3 + 3$, with conditions $y(0)=0, y'(0)=1$ … (7)

Take RA Transform for the eq. (7) we get

$$y'(0) - py(0) + p^2RA[y(\ln(\ln(lns)))] = -\frac{6}{p^4} + \frac{3}{p}$$

$$p^2RA[y(\ln(\ln(lns)))] = \frac{-6+3p^3-p^4}{p^4}$$

$RA[y(\ln(\ln(lns)))] = \frac{-p^4+3p^3-6}{p^6}$ … (8)

Take the inverse of RA Transform for eq. (8) to get $y(\ln(\ln(lns)))$

$$y(\ln(\ln(lns))) = (RA)^{-1}\left(\frac{-p^4+3p^3-6}{p^6}\right)$$

$$y(\ln(\ln(lns))) = (RA)^{-1}\left(\frac{-1}{p^2} + \frac{3}{p^3} - \frac{6}{p^6}\right)$$

$$= (\ln(\ln(lns))) - \frac{3}{2}(\ln(\ln(lns)))^2 + \frac{1}{20}(\ln(\ln(lns)))^5$$

Example: To solve the differential eq.

$y'''(\ln(\ln(lns))) - y''(\ln(\ln(lns))) = \sinh(2\ln(\ln(lns)))$ … (9)

with condition $y''(0) = y'(0) = y(0) = 0$

Take RA Transform for eq. (9) we get

$$y''(0) - py'(0) + p^2y(0) - p^3RA[y(\ln(\ln(lns)))] - y'(0) + py(0) - p^2RA[y(\ln(\ln(lns)))] = \frac{-2}{p^2-4}$$

$$-p^2(p+1)RA[y(\ln(\ln(lns)))] = \frac{-2}{p^2-4}$$

$RA[y(\ln(\ln(lns)))] = \frac{2}{p^2(p^2-4)(p+1)}$
… (10)

To find $y(\ln(\ln(lns)))$ we take the inverse of RA Transform for the eq. (10) we get

$$y(\ln(\ln(lns))) = (RA)^{-1}\left[\frac{2}{p^2(p+1)(p^2-4)}\right]$$

$$\frac{2}{p^2(p+1)(p^2-4)} = \frac{Ap+B}{p^2} + \frac{C}{p+1} + \frac{Dp+E}{p^2-4}$$

$$= \frac{Ap^4+Ap^3-4Ap^2-4Ap+Bp^3+Bp^2-4Bp-4B+Cp^4-4Cp^2+Dp^4+Dp^3+Ep^3+Ep^2}{p^2(p^2-4)(p+1)}$$

$A + C + D = 0, A + B + D + E = 0, -4A + B - 4C + E = 0, -4A - 4B = 0$

$B = -\frac{1}{2}, A = \frac{1}{2}, C = -\frac{2}{3}, D = \frac{1}{6}, E = -\frac{1}{6}$

$$y(\ln(\ln(lns))) = (RA)^{-1}\left[\frac{\frac{1}{2}p-\frac{1}{2}}{p^2} + \frac{-\frac{2}{3}}{p+1} + \frac{\frac{1}{6}p-\frac{1}{6}}{p^2-4}\right]$$

$$= \frac{1}{2} + \frac{1}{2}(\ln(\ln(lns))) - \frac{2}{3}(\ln(lns)) + \frac{1}{6}\cosh(2\ln(\ln(lns))) + \frac{1}{12}\sinh(2\ln(\ln(lns))$$

3. CONCLUSION

In this study, linear ordinary differential equations with constant coefficients are solved using the R. Al–Tememe Transform, which is essential for mathematical modeling in many scientific and engineering domains. Its promise as a potent analytical tool that streamlines the process of solving such equations is demonstrated by the creation and application of this innovative transformation. By defining, inverting, and applying the R. Al–Tememe Transform, we have demonstrated how this technique can be used to simplify differential equation solving, especially when more conventional transforms might not be sufficient or become unwieldy. According to our research, this transformation is a useful supplement to the current collection of mathematical tools utilized in analytical and computational procedures since it provides simplicity, accuracy, and flexibility.

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