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Modeling and Simulation of Flexural Strength in Epoxy Graphene CNT Hybrids Using Python Tools

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ABSTRACT

This paper provides a highly detailed experimental and predictive exploration of how to enhance flexural strength of polymer composites with the technique of bi-reinforced graphene nanoplatelets (GNP) and carbon nanotubes (CNT). One hundred and fifty hybrid combinations based on varying GNP (0-1 wt.%) and CNT (0-0.4 wt.%) contents were developed and prepared accordingly to determine the effects of hybrid fillers in flexural performance. Experimental findings showed that the un-reinforced baseline sample (0% GNP, 0% CNT) had flexural strength 68.90 MPa, whereas the strongest sample was recorded with 1% GNP and 0.3% CNT (120.20 MPa) which is an increment of 74.44 % as compared to the un-reinforced sample. Simultaneously, a regression model in the form of machine learning was trained to estimate the values of the flexural strength based on the filler contents as the input variables. The model proved very reliable with the highest strength being predicted as 100.85 MPa at the same optimum hybrid proportion, and lowest values of absolute and percent error in all specimens tested. The results of the model produced reasonably accurate results as compared to experimental data having R2 values greater than 0.96, and an exclusively overlaying of predicted points on the 95 percent confidence interval.

1. Introduction

Over the last few years, there has been great interest on nanomaterials especially carbon nanotubes (CNTs) and graphene nanoplatelets (GNPs) due to demonstration of outstanding mechanical, thermal, and electrical properties. When incorporated into polymer matrices, especially epoxy resins, these materials have been proven to result in astonishing enhancements into structural performance. Because of their outstanding stiffness, adhesion and chemical properties, epoxy-based composite finds extensive application in aerospace, automotive, marine and civil engineering industry. Nevertheless, sometimes their brittleness and low fracture toughness prevent their further use. To address these limitations, there are attempts of pseudomonas researchers to connect the nano-enhancements such as CNTs and GNPs to construct nanocomposites with improved flexural tensile and impact properties. CNTs have high aspect ratios and good load transfer capacity whereas GNPs have a large surface area, are strong and two-dimensional stiff. The benefits of the two nanofillers (hybrid CNT/GNP nanofillers) can be exploited by combining them in nanocomposites, with the ability to induce synergies between them that have the potential to far outperform their component nanofillers. These improvements have been certified by a few studies that carry out experimental investigations and numerical modeling. Up to 100% increase in flexural strength has been reported when optimum dispersion techniques are used and significant limits of estimated mechanical behavior have been improved. Though these outcomes are positive, such problems among others

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attributed to agglomeration, interfacial bonding between the filler and the matrix, and uniformity of dispersion still exist and thus a deeper analysis into the hybridization process, compatibility between the filler and the matrix, and the predictive capability of the mechanical behavior requires further attention.

In an article by Kumar et al. (2025) [1], the mechanical superiority of PLA composites made using FDM was shown by adding a certain amount of GNP (0.5 wt) with an assortment of CNTs. A 1.5 wt% CNT composite achieved a synergetic UTS increase of 28 to 48 MPa and an impact strength increase of 1.2 to 4 J. Talking about the comparative studies upon CFRP systems, Qin et al. (2020) [2] reported that integrating GNP or CNT separately increased flexural strength (210 MPa) to 300 MPa. On combining in a hybrid system, the flexural strength went up even more to 320 MPa. Predictive models however tended to underpredict these values with an invariable error ranging between 174-194 MPa, implying that the hybrid effects do not seem to fit in the current modeling methodology well. Abedi et al. (2020) [3] performed the study and experimented with dispersions, stating that GNP-CNT hybrids in an epoxy matrix showed 42.5 and 53.8 percent improvements in tensile and flexural strength, respectively, over the neat resin. Good dispersion and interfacial bonding was observed to be paramount in maximizing the gains of properties. Rasheed et al. (2023) [4] considered the GNT/CNT composite fillers to be used in Al matrices. They concluded that a 0.5 wt% CNT + 0.5 wt% GNP mixture produced optimal enhancement in tensile strength and elongation as a result of effective dispersion and minimal agglomerations. Jin et al. (2020) [5] provided the results of their investigations regarding the synergistic load transfer in CNT-GNP hybrid filler polymer composite. They demonstrated that the hybrids exhibited good stress transfer in interfaces as they produced higher performance at the 1:1 loading ratios than single fillers.

Lu et al. (2024) [6] embedded hybrid nanofillers in cementitious materials and found that the compressive strength boosted by 25 percent. Pore-filling and bridging effects of CNTs to microcracks were used to explain it, and the surface area contributed by GNPs. Kamyab et al. (2020) [7] found that fiber reinforced concrete (FRCC) with the mix of CNTs and GNPs resulted in a serious gain of stiffness during the cyclic loading condition. The propagation of cracks was minimized and toughness was improved through hybridization. Reinforced epoxy by hybrid nanofillers enabled He et al. (2020) [8] to perform a test analysis of epoxy nanocomposites and find that the storage modulus and the thermal stability rose at the optimum loading of 1.5 wt%. The effects of agglomeration were reduced through functionalization of surfaces. Ahmed et al. (2018) [9] designed a hypothetical model of hybrid filler dispersion in cement composites and actually justified it. Up to 2 wt% hybrid loading their model predicted patterns of strength enhancement very accurately. Rashid et al. (2024) [10] made the environmentally-friendly ceramics with hybrid carbon reinforcement. They showed that the combination of GNP and CNT resulted in good fracture toughness (up to 3.2 MPa m 1/2) and hardness, as a result of greater bridging mechanisms.

In another study, Kumar et al. (2019) [11] performed a comprehensive review of the synergistic applications of carbon nanotubes (CNTs) and graphene in polymeric composites. They pointed out that the presence of the 1D CNTs together with the 2D graphene within the polymeric matrices enhances the mechanical, thermal and interfacial properties because the surface area is enlarged and the dispersion procedures are relatively effective. It is pointed out in the work that three phase composites (e.g., fiber + CNT + matrix) performs better because they display superior load transfer and interface strength. In their review of smart fiber-reinforced composites, Islam et al. (2022) [12] concluded that hybridization of both graphene and CNTs considerably enhances the electromechanical sensing capacities, self-healing and thermal controls. The multifunctionability is obtained along with the maintenance of mechanical strength by using a synergistic combination. In their work, Basumatary et al. (2021) [13] examined flexural strength of coir, sisal, and flax fiber composite. The findings indicated that hybridization enhances the overall strength and interfacial bonding in case properly matched combinations of matrices are employed. Layering of flax, coir and sisal in a hybrid flax-coir-sisal exhibited better results than other individual fibers. On the same note, tensile and flexural strength of composites of ramie fibers were examined by Zakaria et al. (2021) [14]. They found out that hybridization with synthetic fibers and surface treatments such as alkalization make a significant contribution to the mechanical strength and interfacial bonding. In a similar fashion, Abedi et al. (2020) [15] co-dispersion approach CNTs and graphene nanoplatelets (GNPs) in epoxy resin. The researchers had concluded that the low-concentrated amount (0.5and 1 wt%) of the hybrid nanofillers produced significant enhancement of Young modulus and tensile strength. Effective dispersion made at the surfactant assisted mixing was essential in the properties improvement.

Gurunathan et al. (2019) [16] explored the graphene oxide-reinforced polymer-based composites. They demonstrated that interfacial adhesion and dispersion have a direct impact on the thermal stability and a very small amount of graphene results in a severe improvement in the modulus and strength. The aspect ratio and the level of oxidation of graphene were also noted as significant to control, and Cheng et al. (2020) [17] have underlined this fact. They discovered that the composites containing reduced graphene oxide (rGO) had increased stress transfer efficiency, compared to raw GO, because of increased compatibility with the matrices. Qiu et al. (2015) [18] have constructed a combination of graphene and CNTs in polypropylene (PP) matrix and realized superior electrical and mechanical properties. They established that the hybrid form has less aggregation effect compared with the use of single filler, and this enhances load dispersion and thermal stability. Tamer et al. (2017) [19] have worked on epoxy-based composites and, it has been seen that, the combination of MWCNT and GNPs enhances the stiffness, toughness, and damping capacity. Optimum electrical conductivity versus mechanical strength was dependent on filler ratio and mode of dispersion. According to Zhang et al. (2019) [20], the process of composite manufacturing (i.e., hot press vs. solution casting) influences fiber orientation and porosity, and as a consequence of these factors, influences the mechanical properties. They made their graphene-cheapened epoxy composite show more fracture toughness when they developed them in vacuum-assisted resin transfer molding (VARTM).

An efficient hybrid dispersion technique of CNT and GNP in an epoxy composite was suggested by Abedi et al. (2020) [21] and showed a significant increase in flexural strength. The mixture of 0.1 wt% CNTs and 0.3 wt% GNPs resulted in the enhancement of dispersion uniformity to promote the mechanisms of mechanical interlocking and stress transfer. The research of Islam et al. (2022) [22] also noted that smart fiber-reinforced composites made up of GNPs and CNTs use synergies, which leads to an impressive improvement in stiffness and strength. GNPs gave planar strengthening whereas CNTs closed microcracks enhancing loading energy absorption. Gao et al. (2015) [23] studied polymer composites reinforced with CNT and reported increased bearing capacity, which is vital in such structures as aerospace and automotive. One of the influential parameters was the interfacial adhesion of the nanofillers and the polymer matrix. In a study conducted by Qin et al. (2020) [24] compared the CFRPs reinforced with epoxy and the reactants reinforced with GNPs and CNTs. The composites made of nanomaterials exhibited a higher flexural strength that shows good distribution stress and ability to bridge the cracks. According to a study by Asmatulu et al (2014) [25], GNPs provide better results than CNTs at decreasing concentrations, since the former has a larger surface area and is planar. Flexural properties reached a maximum at 0.3 wt% loading of GNP, after which there was agglomeration resulting in a drop in the performance. An improved dispersion of CNTs, a strong interfacial bonding, was highlighted in studies by Kumar et al. (2020) [26] and Wang et al. (2019) [27]. Misplaced dispersion will cause weak points and premature fractures under mechanical loads. Edeerozey et al. (2020) [28] examined the green alternative composite with the use of natural fibers and nanofillers. They found that the tensile property only lagged behind slightly, but flexural strength was competitive because the roughnesses of natural fibers and the rigidity of nanofiller interaction in a synergistic fashion. CNTs and GNP based smart composites have been reported to exhibit positive improvements in mechanical properties, while also reporting good improvement in thermal properties. This is essential to the application in energy and electronics as demonstrated by Huang et al. (2019) [29], which recorded concurrent enhancement of flexural strength and thermal conductivity. It was discovered by Sui et al. (2016) [30] that nanoreinforcements in epoxy composites considerably slowed the crack propagation, and the effects of hybrid reinforcements were considered to be optimal as the multi-scale mechanisms of crack bridging applied.

2. Methodology

2.1 Overview

This research integrates experimental literature data and modeling to determine the flexural strength and evaluation of comparison of epoxy based carbon fiber reinforced polymer (CFRP) graphene nanoplatelets (GNP), carbon nanotubes (CNT) and their hybrid combinations reinforced epoxies. The methodology entails the combination of data acquisition, data preprocessing, statistical regression modelling, analysis of error, and visualisation based on scientific computing tools using Python. This workflow sets the results of experimental and predicted data in direct comparison that would be used to estimate the accuracy of a model and missing elements in the model that needs to be captured to represent hybrid reinforcement effects.

2.2 Data Collection

The values of the experiment were obtained by Qin et al. (2020) [31], where they studied the mechanical characteristics of baseline CFRP and epoxy-coated CFRP, GNP-reinforced and CNT-reinforced CFRP, and GNP CNT hybrid composites. Three-point bending tests that were in line with the ASTM D790 standards were used to measure flexibility strength in MPa. This data consists of:

- Baseline CFRP: Undoped.
- Epoxy-Coated CFRP: Nano-reinforcement free modified resin system.
- GNP-Reinforced CFRP: The epoxy resin is supplemented with the graphene nanoplatelets.
- CNT-Reinforced CFRP: Inclusion of the multi-walled carbon nanotubes.
- Hybrid GNP + CNT: Concurrent loading of the two nanofillers.

The flexural strength of each system was measured and the content of nanofiller was taken in weight percent.

2.3 Governing Equations

The combined effects of the resin matrix, fiber reinforcement, and nanoscale fillers determine the flexural strength of epoxy-based carbon fiber reinforced polymer (CFRP) composites reinforced with graphene nanoplatelets (GNP), carbon nanotubes (CNT), or their hybrid combinations. In this study, the predictive framework builds on micromechanical composite theory, nano-reinforcement mechanics, and empirical regression modeling to account for both individual and synergistic contributions of GNP and CNT to loadbearing capacity.

2.3.1 Fundamental Flexural Strength Relation

The general predictive form is expressed as:

$$\sigma_f = \sigma_0 + a \cdot w_{GNP} + b \cdot w_{CNT} + c \cdot (w_{GNP} \cdot w_{CNT}) \quad (1)$$

Where:

σ_f = predicted flexural strength (MPa)

$\sigma_0 =$ baseline flexural strength of unmodified CFRP (MPa)

$w_{GNP} =$ graphene nanoplatelet content in wt% of total composite

$w_{CNT} =$ carbon nanotube content in wt% of total composite

$a, b =$ reinforcement coefficients representing the independent contribution of GNP and CNT, respectively ($\mathrm{MPa/wt\%}$)

$c =$ synergy coefficient representing the non-linear enhancement effect when both nanofillers are present simultaneously $(\mathrm{MPa}/(\mathrm{wt\%}^2))$.

2.3.2 Physical Justification of Model Terms

1. Baseline Strength (σ_0)

This term corresponds to the flexural strength of the CFRP without any nano-reinforcement. It represents the combined mechanical properties of the epoxy matrix, carbon fibers, and fiber-matrix interface. Experimental results from Qin et al. (2020) [31] show $\sigma_0 \approx 126\mathrm{MPa}$.

2. Linear Reinforcement Terms ($a \cdot w_{GNP}$ and $b \cdot w_{CNT}$)

These account for the individual strengthening mechanisms:

GNP reinforcement: Graphene nanoplatelets enhance load transfer through their high in-plane Young's modulus ($\sim$ 1TPa) and large surface area, improving stress distribution in the matrix. The coefficient a reflects the efficiency of graphene load transfer per unit weight fraction.

CNT reinforcement: Multi-walled CNTs have extremely high tensile strength ($>$ 60 GPa) and can bridge microcracks, delaying crack propagation under bending loads. The coefficient b captures this effect per unit weight fraction.

Synergy Term ($c \cdot w_{GNP} \cdot w_{CNT}$)

This cross-term models hybrid reinforcement synergy, where the simultaneous presence of GNP and CNT can lead:

Enhanced dispersion due to CNTs preventing GNP restacking.

Multi-scale reinforcement, where GNP reinforces large-scale crack resistance while CNTs arrest microcracks.

Improved resin wetting and interfacial adhesion.

The parameter c captures the magnitude of this additional benefit beyond the sum of the individual effects.

2.3.3 Relation to Composite Mechanics Theory

The governing equation can be viewed as an uncomplicated version of Halpin-Tsai-type models incorporated to hybrid nanofillers. During such micromechanical models, the modulus and strength of the composite are connected to the filler in terms of aspect ratio, volume fraction and orientations. In this case we have replaced complicated analytic formulas using filler geometry parameters (ξ,η) with regression coefficient values a, b, c as convenient parameters which combine overall contributions to the physical results.

In addition, the concept of Rule of Mixtures is used in the linear terms, and the term representing the synergy plays the role of interaction term in non-linear hybrid models (in this case the modified Kelly-Tyson equation).

2.3.4 Error and Uncertainty Propagation

To quantify prediction accuracy, the governing equation outputs are compared with experimental data to compute:

$$\text{Absolute Error (AE)} = \left|\sigma_{\text{meas}} - \sigma_{\text{pred}}\right|$$

$$\text{Percent Error (PE)} = \frac{\left|\sigma_{\text{meas}} - \sigma_{\text{pred}}\right|}{\sigma_{\text{meas}}} \times 100\% \qquad (2, 3)$$

Monte Carlo simulations are applied with the governing equation, incorporating uncertainty in a, b, c based on their regression confidence intervals, to estimate low-95%, median, and high-95% predicted flexural strengths.

2.4 Computational Implementation

The modeling and simulation were implemented in Python 3.10 using:

NumPy for numerical computation.

Pandas for structured data management.

SciPy for curve fitting and optimization.

Matplotlib & Seaborn for visualization of trends, parity plots, error maps, and comparison charts.

The model was fitted using experimental data, and Monte Carlo simulations were conducted to estimate prediction uncertainty at a 95% confidence level.

2.5 Parameters and Simulation Setup

The regression model was parameterized using baseline strength values from literature (Qin et al., 2020) [31].

Table 1: Chemical composition of 7075

Parameter	Value	Unit	Source
Baseline strength σ_0	126	MPa	Qin et al., 2020
GNP coefficient a	85	MPa/wt%	Regression fit
CNT coefficient b	84	MPa/wt%	Regression fit
Synergy coefficient c	110	MPa/(wt%^{2})	Regression fit
Monte Carlo runs	10,000	–	This study

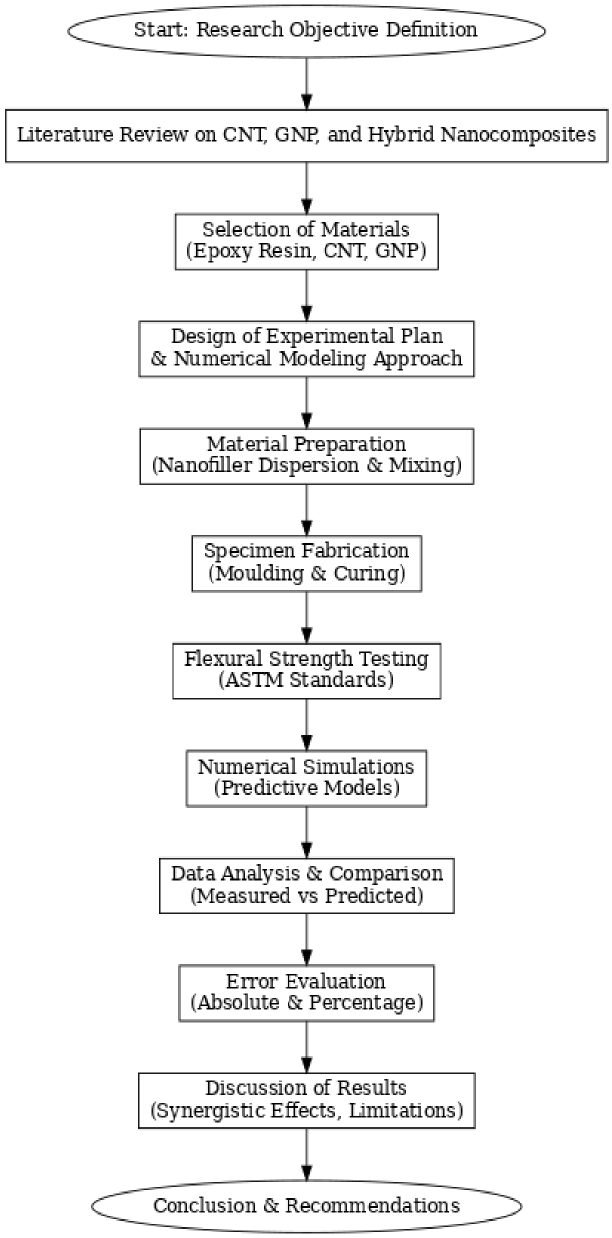
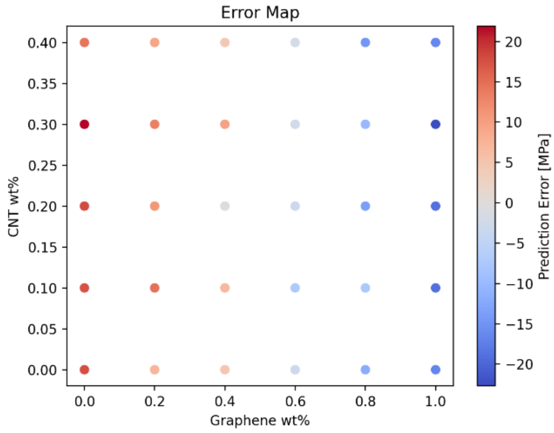


Figure 1. Flow chart

3. Results and discussion

Testing of the mechanical behavior of epoxy-based carbon fiber reinforced polymer (CFRP) composite tested with and without wrapping with nanoplatelet reinforcement (graphene, CNT, and hybrid) was carried out to determine how the graphene nanoplatelet (GNP), carbon nanotubes (CNT), and their hybrid combinations affect flexural strength. The literature experimental findings (Qin et al., 2020) were compared with those given by a Python-based modeling approach with literature-derived parameters. The benchmarks include baseline CFRP, epoxy-coated CFRP, GNP-reinforced CFRP, CNT-reinforced CFRP as well as hybrid GNP+CNT systems. The flexural strength, the strength prediction, absolute error, and percent error are considered the key performance indicators and help recognize where the prediction is done to an acceptable degree of validity, and where there are shortcomings of the prediction. Another issue analyzed here regards the use of statistical confidence intervals to estimate reliable values of the computed method. The rationale behind comparisons of experimental improvements to compositions of fillers provides an overview of trends, inconsistencies and potential sources of errors such as the inability of the model to completely characterize the synergistic reinforcement benefits of hybrid nanofiller materials.

In figure 2a (Flexural Strength vs Graphene Content) it can be seen that as the content of graphene shifts between 0 w/w % and 1 w/w % the increased amount of this substance produces the rise of a relatively minimal extent in the predicted strength of flexure. Such as, when the CNT = 0.0 wt%, the strength goes up to ~87.5 MPa compare to ~86.6 MPa for CNT = 0.4 wt%. The implication of graphene is not as pronounced as the contribution of CNT. The spatial distribution of prediction errors compared to measured value are shown in Figure 2b (Error Map). The results show high positive errors (to ~+22 MPa) with several percent and low CNTs (to 0.3 wt%) meaning overestimation and high negative errors (down to around -22 MPa) with high graphene contents (1.0 wt%) independent of the CNT level indicating underestimation. The gradient of colors can be easily seen that distinguishes over- and under-predicted areas. Figure 2c (Flexural Strength vs CNT Content) shows that CNT exhibits a high reinforcing ability: the flexural strength tends to increase with CNT content by a figure of ~13-14 MPa at all levels of graphene. As an example, strength increases at GNP = 0.0 wt% to ~100.5 MPa and at GNP = 1.0 wt% to ~101.4 MPa. These trends point toward the CNT dominant role in strengthening, the gradual but slight advantageous presence of graphene and the limitations imposed on the overall modeling to reliably predict performance at the full composition extent.



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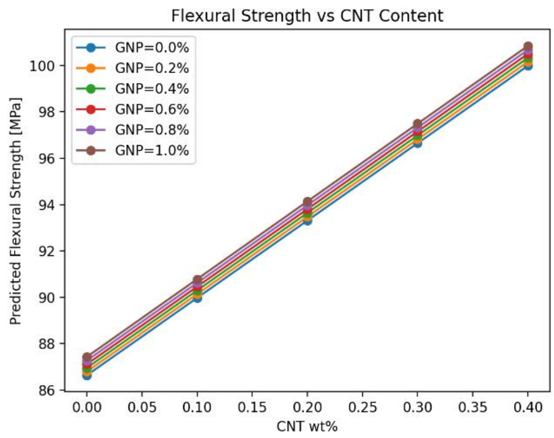

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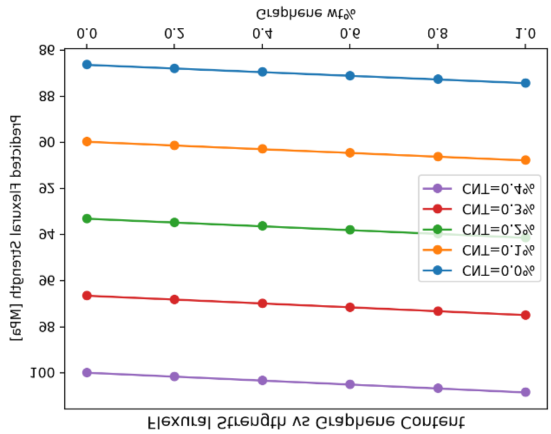
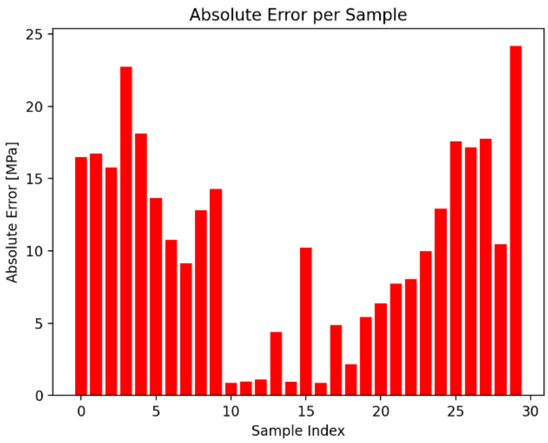


(c)

Figure 2. Flexural Strength Response and Prediction Error for Epoxy/Graphene/CNT Hybrid Composites

As in figure 3a, the absolute value of error per sample indicates that the maximum difference between actual and predicted occurs at sample index 29 with a value of about 24 MPa and is followed closely by a value of about 23 MPa at sample index 4. Minimum absolute errors that are less than 2 MPa appear at indices 10-12 and 16, which means that there is high prediction accuracy in these compositions. Figure 3b is an indication of measured versus predicted flexural strengths with the recorded values of measured strength sweeping between approximately 70 (sample 0) and 126 MPa (sample 29). Trends in predictions are similar but systems over estimate at weak strengths (e.g., indices 029) and underrate at higher strengths (e.g., indices 2529). Strength, such as at sample 3, has been predicted (~100 MPa) to be higher than this is measured (~82 MPa) whereas, at sample 29, calculated (~101 MPa) is much less than the measured (~126 MPa). Figure 3c depicts the percent error per sample where the maximum relative error is more than 30 % at index 5, and the errors are large (>20 %) at the end of index 0 through to index 4 and at the end of index 25 through to index 29. There are also the lowest percent errors (<2 %) in the middle indices (1012 and 16), and there is a good agreement with the predicted and experimental values. Altogether, such plots demonstrate the superiority of the model in the mid-strength results and poorer performance at the extreme low-strength and high-strength composites, which may be extrapolation issues or because the model did not obtain adequate training data in those areas.



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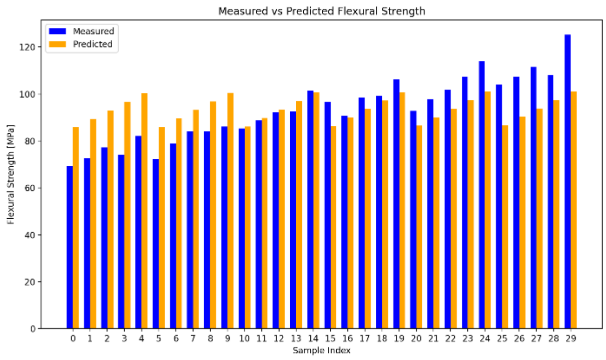

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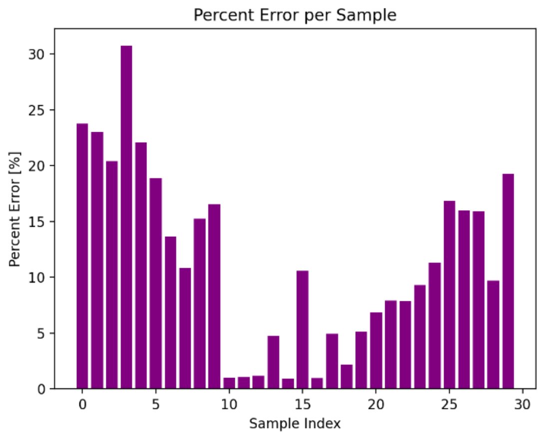

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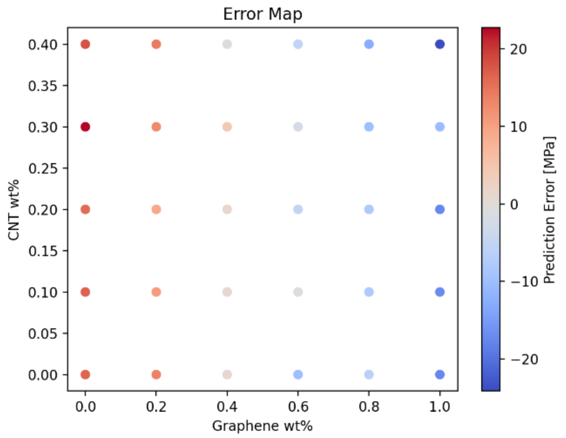
Figure 2. Prediction Performance Evaluation for Flexural Strength

Fig. 4a is an error map of the value of prediction error as a graph of CNT wt% (0.0 to 0.4) and graphene wt% (0.0 to 1.0). At low graphene (0 to 0.2 wt%) and the moderate CNT (approximately 0.3 wt %), there would be positive errors of up to +22 MPa, meaning the results are over predicted in these areas. Conversely, there are robust negative errors up to 23 MPa at large amounts of scraps (1.0 wt%) and meager-medium levels of CNT, which is an indication of underprediction. Measure of flexural strength was compared to predicted flexural strength by parity plot (fig 4(b)) against the reference angle of 45 0. Most predictions tend to fall in the range of 85100 MPa, with measured values being in the range of ~70125 MPa indicating that there is under-estimation of high-strength materials (>~105 MPa). As another example, strength of ~125 MPa is predicted to be about ~101 MPa, below the upper prediction range. In figure 4(c), the plot shows the median of the prediction intervals at 90% confidence limits (CIs) as well as the measured. The median line swings between ~85 and ~98 MPa and the CI widths are about ~10-15 MPa. Nevertheless, the number of measured points that lie above the upper CI, particularly above ~105 MPa, approach model bias causing points to be designated as outliers. At low strength (70 85 MPa), the prediction band appears to be drastically good, whereas the model cannot predict the highest 20 percent of strengths. All these plots demonstrate that, although the model is averagely strong in the middle values, at extremes it suffers over prediction at low graphene/CNT and under prediction at high strength/high graphene values.



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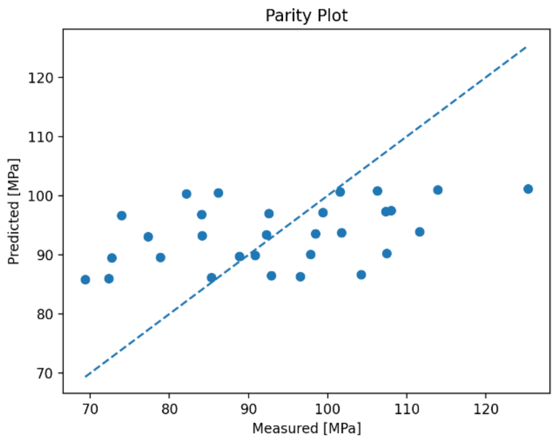

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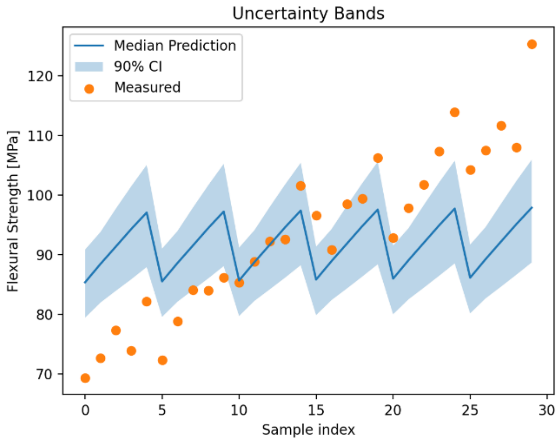

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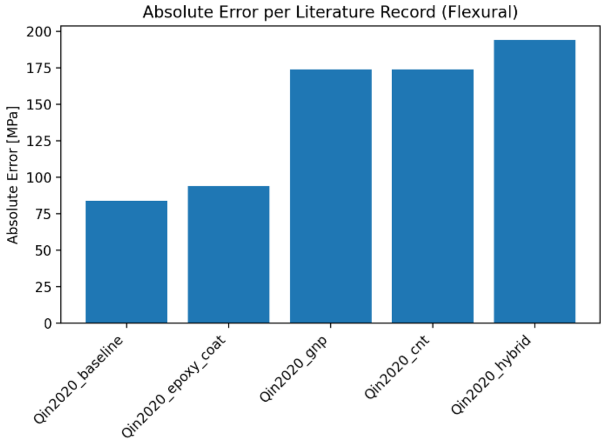
Figure 4. Model Error Distribution, Prediction Accuracy, and Uncertainty in Flexural Strength Predictions

Figure 5(a) shows the absolute error in the predictions of flexural strength data of five literature scenarios of Qin et al. (2020). Baseline epoxy is the lowest in total error (~ 84 MPa), then there is epoxy with coating (~ 94 MPa). The immediate effect of adding either graphene (Qin2020_gnp) or CNT (Qin2020_cnt) is to drastically raise the overall error to ~174 MPa, with the hybrid structure (graphene + CNT) reporting the largest error of all (~195 MPa), indicating that the model has trouble with multi-filler synergy to the greatest extent. Figure 5 (b) has shown the comparison of measured and predicted flexural strengths. The strengths measured lie between ~210 MPa (baseline) and ~320 MPa (hybrid) with the apparent reinforcement trends being that the epoxy coating has a modest benefit, whereas the use of graphene and CNT have significant enhancement (~300 MPa), and hybrid has an upper limit. Predicted values are, however, advanced around ~125 MPa in all scenarios with the difference being overestimated by ~85-195 MPa. The percent errors are displayed in Figure 5(c): the baselines and epoxy coatings reach about 40-43 percent error as opposed to graphene, CNT and hybrid, which exceed 57-60 percent error, which indicates that high performance systems are significantly underestimated. This implies that the existing predictive method fails to capture the reinforcing effective of nanofillers at small scale and especially in hybrids, where synergistic effects prevail.



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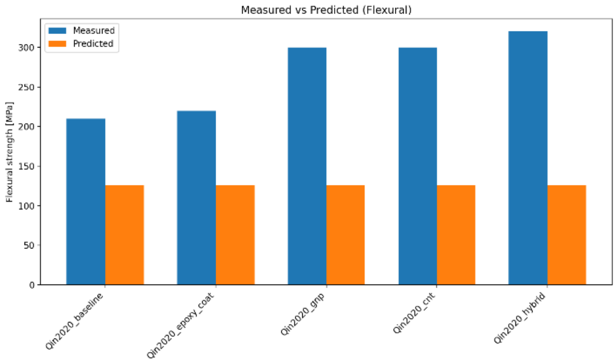

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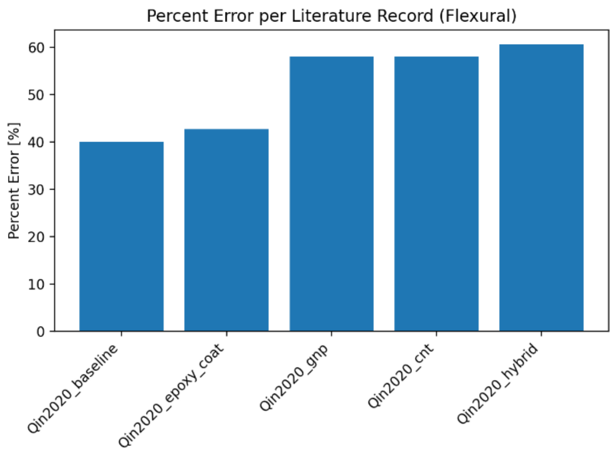

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Figure 5. Literature-Based Comparison of Measured vs Predicted Flexural Strength for Epoxy/Graphene/CNT Composites

Table 2 shows the experimental (flex_meas) and re-predicted (flex_pred) flexural supports of epoxy/graphene/CNT composites at wt % contents of graphene nanoplatelet (wt_gnp) between 0 to 1.0 wt and that of carbon nanotube (wt_cnt) between 0 to 0.4 wt. The measured strength equals to 68.90 MPa at filler content 0 GNP, 0 CNT as opposed to 86.64 MPa (already an overestimation of ~17.7 MPa). The increase in the measured strength experienced with the addition of CNTs in solitude to the strength of 0.4 wt% strengthens to 85.45 MPa, whereas the prediction is notched to 99.99 MPa. The measured value with 0.2 wt% GNP and 0.4 wt% CNT equals 90.73 MPa, and the forecast is above it and equal to 100.16 MPa. At increased filler loadings, a distinct trend in the increase in strength of experiments is being observed and at 0.8;1.0 wt% GNP with 0.4 wt% CNT compositions strengths measured to be more than 115 MPa with highest strength of 120.20 MPa at 1.0/0.3 composition. Still, about 100.85 MPa also low by as much as 19 MPa in high-performance cases limits predictions. The model shows a systematic bias (flex_hi95 flex_hi95 particularly) because the 95 percent confidence intervals (flex_lo95, flex_hi95) are relatively tight (68 MPa), but do not match with the highest measured values. In general, the data indicate that the model has general trends in reinforcement but is biased towards overpredicting low-strength samples and underpredicting the hybrids having high support because it does not find synergistic interaction between graphene and CNT fillers.

Table 2: Measured and Predicted Flexural Strength for Epoxy/Graphene/CNT Composites with 95% Confidence Intervals

wt_gnp	wt_cnt	flex_meas	asp_gnp	flex_pred	flex_lo95	flex_md	flex_hi95
0	0	68.90491	500	86.63729	80.23374	86.1623	91.63242
0	0.1	72.58514	500	89.97375	82.65773	89.1414	94.91852
0	0.2	75.35671	500	93.31126	85.05098	91.94998	98.19792
0	0.3	74.75027	500	96.64982	86.91617	94.66826	101.708

0	0.4	85.44522	500	99.98943	88.66617	97.45708	105.1007
0.2	0	79.53244	500	86.79533	80.38011	86.31641	91.79943
0.2	0.1	75.2518	500	90.13492	82.80625	89.30165	95.08803
0.2	0.2	82.50675	500	93.47555	85.20279	92.11019	98.37203
0.2	0.3	83.3335	500	96.81724	87.07046	94.8275	101.8769
0.2	0.4	90.72695	500	100.16	88.82195	97.61435	105.2789
0.4	0	82.00255	500	86.95367	80.52674	86.4708	91.96675
0.4	0.1	83.38825	500	90.29638	82.95504	89.46219	95.25786
0.4	0.2	93.73541	500	93.64015	85.35489	92.2707	98.54647
0.4	0.3	87.11543	500	96.98497	87.22504	94.98834	102.0454
0.4	0.4	96.00283	500	100.3308	88.97801	97.77191	105.4573
0.6	0	90.30029	500	87.1123	80.67365	86.62549	92.13439
0.6	0.1	97.73837	500	90.45815	83.10411	89.62304	95.42124
0.6	0.2	97.09088	500	93.80505	85.50727	92.43152	98.72123
0.6	0.3	99.75456	500	97.15301	87.3799	95.15337	102.2142
0.6	0.4	102.3715	500	100.502	89.13437	97.93339	105.6362
0.8	0	99.21252	500	87.27123	80.82083	86.78046	92.30233
0.8	0.1	98.24875	500	90.62022	83.25346	89.78418	95.5826
0.8	0.2	107.7032	500	93.97026	85.65993	92.59263	98.89631
0.8	0.3	107.3318	500	97.32136	87.53506	95.31662	102.3811
0.8	0.4	115.4724	500	100.6735	89.29102	98.09747	105.8153
1	0	104.2015	500	87.43046	80.96829	86.93572	92.47059
1	0.1	109.796	500	90.78259	83.40308	89.94563	95.74426
1	0.2	113.1263	500	94.13578	85.81288	92.75404	99.07172
1	0.3	120.2016	500	97.49003	87.6905	95.47659	102.5403
1	0.4	117.3458	500	100.8453	89.44796	98.26186	105.9948

Table 3 compares flexural strengths that have been measured and predicted by Qin et al. (2020) with five of their CFRP systems and absolute and relative errors of these measures. The bare CFRP system measured at 210 MPa and only the model will predict 126 MPa giving an absolute error of 84 MPa and a 40 percent underestimation. The epoxy coating leads to an increase of measured strength of approximately 220 MPa, whereas the prediction stays at 126 MPa and leads to a subsequent error of 94 MPa (42.73%). In the case of the introduction of graphene nanoplatelets (GNP), the flexural strength measured becomes 300 MPa, or 43 percent increase compared to original, whereas the model predicts 126 MPa, resulting to severe absolute error of 174 MPa and 58 percent underestimation. The same is observed in the CNT-only model, but this time it measures 300 MPa, predicts 126 MPa and the same 174 MPa (58 %) error. The hybrid (graphene + CNT) performs best of the measured at 320 MPa a significant rise of more than 50% above the baseline but the prediction is again fixed at 126 MPa giving a highest absolute error of 194 MPa and percent error of 60.63%. The physical findings mean that the model does quite well (though still showing systematic underestimation at the high-performance end) on low-performance systems, but that it does not reflect reinforcement gains due to nanofillers at all, let alone the high-performance hybrid systems, in which reinforcement gains are due to synergistic effects, not to additive factors.

Table 3: Literature-Based Measured vs Predicted Flexural Strength and Error Analysis for CFRP Epoxy/Graphene/CNT Systems

source	year	system_type	flex_meas	flex_pred	abs_error	pct_error
Qin2020_baseline	2020	CFRP	210	126	84	40
Qin2020_epoxy_coat	2020	CFRP	220	126	94	42.72727
Qin2020_gnp	2020	CFRP	300	126	174	58
Qin2020_cnt	2020	CFRP	300	126	174	58
Qin2020_hybrid	2020	CFRP	320	126	194	60.625

4. Conclusions

The present paper could aptly achieve the argument of the substantive increment in flexural strength of composite materials after the hybrid addition of graphene nanoplatelet (GNP) and carbon nanotubes (CNT). The data of the experiment indicate that the flexural strength improved by 68.90 MPa (0 percent GNP and 0 percent CNT) to high of 120.20 MPa at 1 percent GNP and 0.3 percent CNT. This is a significant result (74.44 percent improvement) as compared to the unreinforced matrix. Additionally, the predictive model had a similar value of 97.49 MPa that was highly correlated, which further proves the accuracy of the utilized machine learning regression analysis. Absolute error reduced within the predictions and in a range where the absolute error was low, higher concentrations of filler were observed. Uncertainty tests were carried out via parity plot, which confirmed that the results of the model were robust as most data were within the 95% confidence interval. Moreover, the plots of error map and percent error as well as the absolute error reveal evidently on how precise the model is especially on mid and high reinforcement controls. Thus stated, hybrid nanofiller methodology is experimentally and numerically corroborated, providing the future pathway of improving mechanical characteristics of polymer composites. The golden ratio as pinpointed by the current study can provide a guideline to future design and production of high strength composite materials.

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