Solving Linear Programming with Fuzzy Variables and Fuzzy Constraints

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Abstract

This paper deals with constructing fuzzy linear programming model (FLPM), we explain how to solve fuzzy triangular were all the coefficients of variables, right hand side of constraints, and coefficients of objective function are also fuzzy number. Further we explain all algebraic operations required of fuzzy number, also on membership function are explained.

Keywords: Fuzzy triangular number, Fuzzy trapezoidal numbers, membership functions.

1. Introduction:

Linear programming is one of the most problems, that has many applications in production and financial planning, which work on finding optimal allocation of resources to satisfy certain target, it is one of most frequently applied in operations research technique[4],Indicate to this subject, in many other researchers works on developing (LP). Fuzzy linear programming was first proposed by Tananka et al (1974), using the concepts of fuzzy decision introduced by Bellman and Zadeh (1965)[6], but the first formulation of (FLP) was introduced by Zimmermann (1978). The ranking function which maps each fuzzy number into real line, is considered as efficient method to transform (FLP)[7,8]. The paper is organized as;

Section (1): deals with abstract and introduction.

Section (2): some basic definitions, and formulation of fuzzy linear programming, then we explain by fuzzy LP fuzzy model with solution.

2. Some Basic Definitions :

We know that the fuzzy environment was proposed by Bellman and Zadeh (1970)[16], Tanaka (1984), and Zimmermann (1985)[17].

The fuzzy number is a number $(\tilde{a})$ belong to fuzzy set $(\tilde{a}: R \rightarrow [0,1])$, the membership function $[\mu_{\tilde{a}}(x)]$ is piecewise continuous and $(\tilde{a})$ is fuzzy convex that is:

$$\mu_{\tilde{a}}[\lambda x + (1-\lambda)y] \geq \min[\mu_{\tilde{a}}(x), \mu_{\tilde{a}}(y)] \; \forall x, y \in R \; and \; \lambda \in [0,1]$$

For each fuzzy number;

$$\tilde{a} = (a^l, a^u, \alpha, \beta)$$

Such that;

$$a^l \leq a^u \quad \alpha, \beta > 0 \; and \; a^l, a^u, \alpha, \beta \in R$$

We explain some arithmetic operations for fuzzy number;

If $x > 0 \; and \; x \in R$ then;

$$x_{\tilde{a}} = (xa^l, xa^u, \alpha, \beta) < R$$

3. Definition of membership Function:

The membership function is important concept of fuzzy number, and it can be represented by numerical approach or functional approach, we explain triangular function which represents values of belonging elements to fuzzy set, as a line and it have different types:

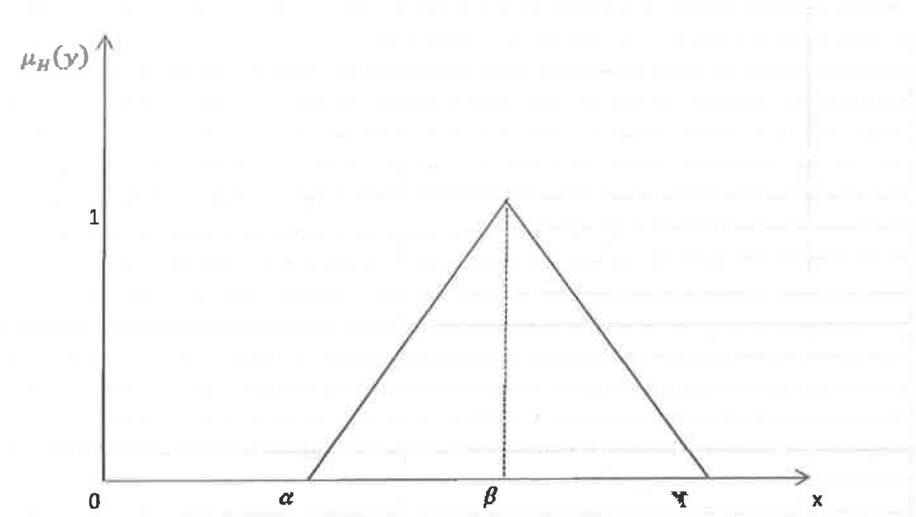
❖ Triangular Function

Type of membership function the membership function taken different shapes, which may Triangular function, having three parameters (α, β, γ), and can be represented as;

$$\mu_H: x \rightarrow [0,1]$$

$$\mu_H(x,\alpha,\beta,\gamma) = \begin{cases} 0 & if \quad x < \alpha \\ \frac{x-\alpha}{\beta-\alpha} & if\ \alpha \le x \le \beta \\ \frac{\gamma-x}{\gamma-\beta} & if\ \beta \le x \le \gamma \\ 1 & x > \gamma \end{cases} \qquad \ldots\ldots (1)$$

The function $\mu_H(x,\alpha,\beta,\gamma)$ can be represented by graphic (1) ;



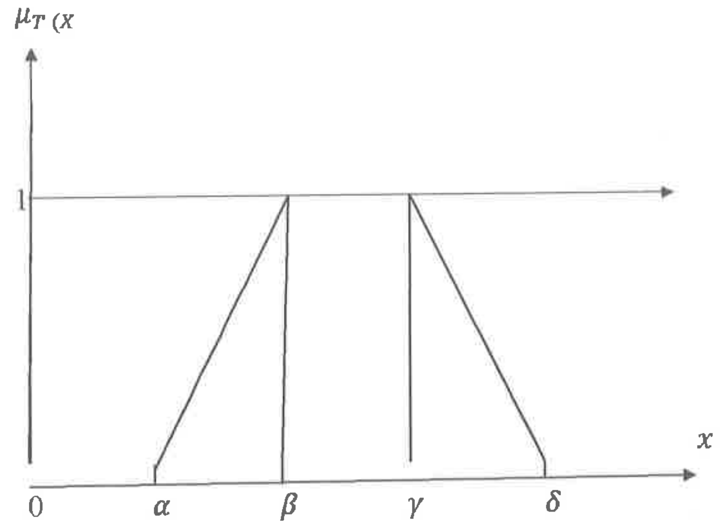
Figer 1: Triangular membership function

While the second type is called trapezoidal membership function which have four parameters $(\alpha,\beta,\gamma,\delta)$ and it is represented by;

$$\mu_T : x \rightarrow [0,1]$$

$$\mu_T(x,\alpha,\beta,\gamma,\delta) = \begin{cases} 0 & if \quad x < \alpha \\ \frac{x-\alpha}{\beta-\alpha} & if\ \alpha \le x \le \beta \\ 1 & \beta \le x \le \delta \\ \frac{\delta-x}{\delta-\gamma} & \gamma \le x \le \delta \\ 1 & x > \delta \end{cases} \qquad \ldots\ldots (2)$$

Then Graphic (2) represent trapezoidal membership function;



Figer 2: trapezoidal membership function

4. Definition of Fuzzy Linear Programming (L.P) Model :

There are different type of fuzzy linear programming , which are;

Type one:

In this type the coefficients of objective function , and coefficients of constraints as well as the values of Right hand side also are fuzzy , due to this we introduce these type ;

Type 1: $Max\ \tilde{Z} = \sum_{j=1}^{n} \tilde{C}_j x_j$ .... (3)

s.to;

$$\sum_{j=1}^{n} \tilde{a}_{ij} x_j \leq \tilde{b}_i \qquad i = 1,2, \dots, m \quad , \quad j = 1,2, \dots, n$$

$$x_j \geq 0$$

Type 2:

in this type the right hand side are fuzzy type ;

$$max\tilde{Z} = \sum_{j=1}^{n} \tilde{C}_j x_j \qquad \text{.....(4)}$$

s.to;

$$\sum_{j=1}^{n} a_{ij} x_j \le b_i \qquad i = 1,2,\dots,m \quad ,j = 1,2,..,n$$

$$x_j \ge 0$$

Type 3:

in this type the technological coefficient are fuzzy type as;

$$max\ \tilde{Z} = \sum_{j=1}^{n} C_j x_j \qquad \text{....(5)}$$

s.to;

$$\sum_{j=1}^{n} a_{ij} x_j \le \tilde{b}_i \qquad i = 1,2,\dots,m \quad ,j = 1,2,..,n$$

$$x_j \ge 0$$

Another type have two combination of fuzzy Reliability as;

$$max\ \tilde{Z} = \sum_{j=1}^{n} C_j x_j \qquad \text{......(6)}$$

s.to;

$$\sum_{j=1}^{n} \tilde{a}_{ij} x_j \le b_i \qquad i = 1,2,\dots,m \quad ,j = 1,2,..,n$$

$$x_j \geq 0$$

or ;

$$max\tilde{Z} = \sum_{j=1}^{n} \tilde{C}_j x_j \qquad \ldots\ldots(7)$$

s.to;

$$\sum_{j=1}^{n} \tilde{a}_{ij} x_j \leq \tilde{b}_i \qquad i = 1,2,\ldots,m \quad ,j = 1,2,\ldots,n$$

$$x_j \geq 0$$

In our research we discuss symmetric fuzzy linear programming model, in this model the decision maker determine the level value of objective function (Z_0), were

$$\left.\begin{array}{l} Z_0 \lesseqgtr C^T x \\ Ax \lesseqgtr b \end{array}\right\} \qquad \underline{X} \geq 0$$

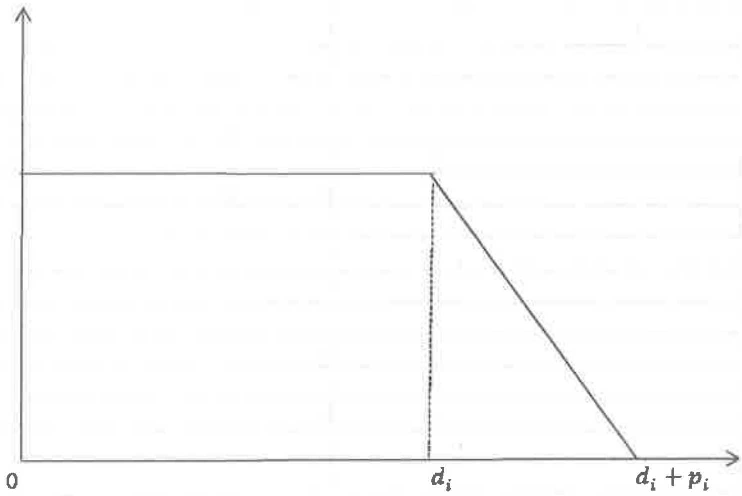
We can write above model as;

$$\left.\begin{array}{l} \begin{bmatrix} -Z \\ h \end{bmatrix} = d, \ \begin{bmatrix} -C^T \\ A \end{bmatrix} = B \\ (B\underline{X})i \lesseqgtr d_i \end{array}\right\} \qquad \underline{X} \geq 0$$

Were the number of rows in the model are $(m + 1)$ and it have $(m + 1)$ membership function which are $[M_i(x)]$;

$$M_i(x) = \begin{cases} 1 & (B\underline{X})i \le d_i \\ 1 - \frac{(B\underline{X})i - d_i}{p_i} & d_i < (B\underline{X})i \le d_i + p_i \\ 0 & (B\underline{X})i > d_i + p_i \end{cases} \quad \ldots\ldots.(8)$$

The values of member ship function $M_i(x)$ $i = 1,2,\ldots,m+1$ corresponding to rows of fuzzy linear programming model and it is defined as;



Figer 4: Graphically value of membership function

According to definition of Bellman and Zadeh, the value of fuzzy decision comes from interesting the fuzzy sets;

$$M_D(x) = \cap M_i = \min[M_i(x)] \qquad i = 1,2,\ldots,m+1$$

$$M_{max} D(x) = max \min[M_i(x)] \qquad i = 1,2,\ldots,m+1$$

To obtain feasible solution (x_0), we change $[D(x)]$ by another variable called (λ), and substitute $[M_i(x)]$ with equivalent it;

$$M_{max} D(x) = max \min[M_i(x)] \left(1 - \frac{(B\underline{X})i - d_i}{p_i}\right) \qquad i = 1,2,\ldots,m+1 \qquad \ldots\ldots (9)$$

$$\max(\lambda)$$

$\lambda p_i + (B\underline{X})i \leq d_i + p_i, \quad 0 \leq \lambda \leq 1\,,\; x \geq 0$

There are many fuzzy model of LPP like (Chana model), for symmetric model and Werner model for non – symmetric fuzzy LP model, and models of modified sub gradient approach for (non – smooth optimality problem).

Let use consider fuzzy triangular linear programming model, which is represent by three crisp numbers (S, L, r) as;

$\langle \tilde{C}x \rangle = f_i(x_j) = \max \sum_{j=1}^{n} \tilde{C}_j x_j$ ……(10)

S.to

$$\sum_{x\geq 0}^{n} (S_{ij}, L_{ij}, r_{ij}) x_{ij} \leq (t_i, u_i, v_i) \qquad v \leq i \leq m, \quad u \leq j \leq n$$

were;

$$A_{ij} = < S_{ij}, L_{ij}, r_{ij} >$$

$$B_i = \langle t_i, u_i, v_i \rangle$$

$$0 \leq i \leq m$$

$$0 \leq j \leq n$$

According to triangular fuzzy number we can rewrite linear programming model under fuzzy number as;

$$A_{ij} = < S_{ij}, L_{ij}, r_{ij} >$$

$$B_i = \langle t_i, u_i, v_i \rangle$$

Ther for

$$max \sum_{j=1}^{n} \tilde{C}_j x_j$$

s.to;

$$\left.\begin{array}{l} \sum_{j=1}^{n} S_{ij}x_j \le t_i \\ \sum_{j=1}^{n}(S_{ij}-L_{ij})x_j \le t_i - u_i \\ \sum_{j=1}^{n}(S_{ij}+r_{ij})x_j \le t_i + v_i \end{array}\right\} \quad \dots (11)$$

$$xj \ge 0$$

While membership function is;

$$\mu_T(x,\alpha,\beta,\gamma,\delta) = \begin{cases} 0 & x < \alpha_j \\ \frac{x-\alpha_j}{\beta_j-\alpha_j} & \alpha_j \le x \le \beta_j \\ 1 & \beta_j \le x \le \delta_i \\ \frac{\beta_i - x}{\beta_i-\gamma_j} & \delta_i \le x \le \gamma_j \end{cases} \quad \dots\dots (12)$$

Now we explain above idea through by example;

$$Max\ F_i(x_1,x_2) = \tilde{C}_1x_1 + \tilde{C}_2x_2$$

S.to;

$$(4,3,2)x_1 + (7,5,2)x_2 \le (14,6,3)$$

$$(5,2,3)x_1 + (7,6,5)x_2 \le (8,5,3)$$

And membership function of ($\tilde{C}_1$ *and* $\tilde{C}_2$) are;

$$\mu_{\tilde{C}_1}(x) = \begin{cases} 0 & x < 8 \\ \frac{x-8}{4} & 8 \le x \le 12 \\ \frac{25-x}{10} & 15 \le x \le 25 \\ 1 & 12 \le x \le 15 \\ 0 & x > 25 \end{cases}$$

$$\mu_{\bar{c}_2}(x) = \begin{cases} 0 & x < 25 \\ \frac{x-25}{5} & 25 \le x \le 30 \\ \frac{40-x}{10} & 40 \le x \le 50 \\ 1 & 30 \le x \le 40 \\ 0 & x > 50 \end{cases}$$

$$Max\ W = 35\,x_1 + 95\,x_2$$

s.to;

$$4\,x_1 + 7x_2 \le 14$$

$$5\,x_1 + 7x_2 \le 8$$

$$x_1 + 2x_2 \le 8$$

$$3\,x_1 + x_2 \le 3$$

$$6\,x_1 + 9x_2 \le 17$$

$$8\,x_1 + 12x_2 \le 13$$

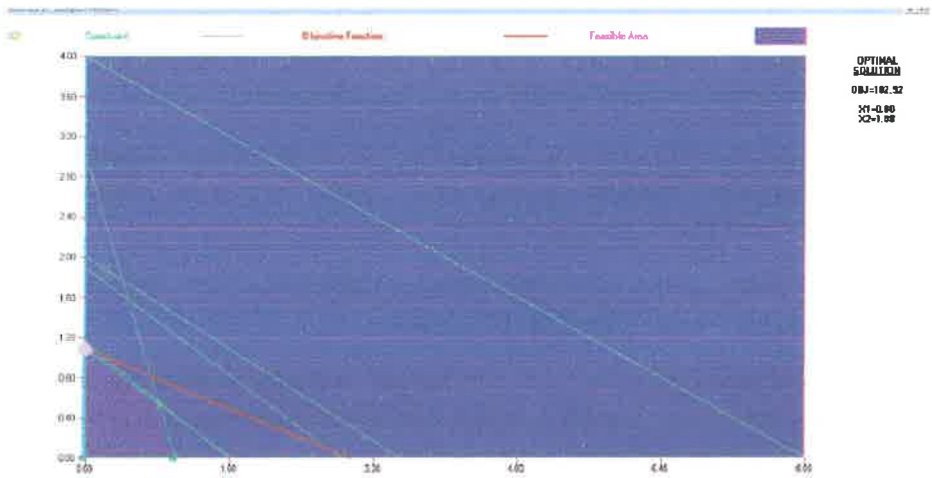
$$x_1, x_2 \ge 0$$

Solution:

Using (Win QSB) Programming to get the solution numerically and graphically as;

Table 1: The Solution Numerically

	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	X1	0	35.0000	0	-28.3333	at bound	-M	63.3333
2	X2	1.0833	95.0000	102.9167	0	basic	52.5000	M
	Objective	Function	(Max.) =	102.9167				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	C1	7.5833	<=	14.0000	6.4167	0	7.5833	M
2	C2	7.5833	<=	8.0000	0.4167	0	7.5833	M
3	C3	2.1667	<=	8.0000	5.8333	0	2.1667	M
4	C4	1.0833	<=	3.0000	1.9167	0	1.0833	M
5	C5	9.7500	<=	17.0000	7.2500	0	9.7500	M
6	C6	13.0000	<=	13.0000	0	7.9167	0	13.7143



Figer 4: Solution Graphically

The results of the variables and objective function are;

$$x_1 = 0, \ x_2 = 1.08, \ Z = 102.92$$

Conclusion

1- we solve Fuzzy Linear Programming Model graphically since it contains two variables (x_1 , x_2).
2- The results of solution graphically and (Win QSB) are identical , were $x_1 = 0, \ x_2 = 1.08, \quad and \ \ Z = 102.92$

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