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Some properties of Pre-locally closed set and Pre D-set and relation between them

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ABSTRACT
The aim of this work presented locally closed set and D-set generated by pre-open set . Also we introduced basic properties of these sets and relations among them in this paper.

Key words : pre-open set , pre*-continuous function , pre*-open function , locally closed , pre-locally closed , D-set , pre-D-set and pre-locally D-set .

1 - INTRODUCTION

Bourbaki in 1966 [1] defined locally closed set . In 1981 [4] Mashhour presented the concepts of pre-open set , pre-closure and pre-interior . Tong in 1982[6] studied D-set . Popa introduced pre-separated sets in 1987 [5] . In 2001[3] Jafari investigated definition of pre-D-set . In this paper we introduced pre-locally closed , locally pre-closed set , pre-D-set and pre-locally D-set . Properties of these new concepts are studied as well as the relationships among them .

2 - Pre-locally closed set

Definition(2.1)[4]

Let X be a space A subset G of X is said to be pre-open set in X if $G \subseteq \overline{G}^{\circ}$. The complement of *pre-open* set is said to be *pre -closed* that is if $\overline{G^{\circ}} \subseteq G$

Remark (2.1)[2]

Every open is pre-open . But the apposite is not true

Remark (2.2)

1- The union of two pre-open sets is pre-open . [6]
2- The intersection of two pre-closed sets is pre-closed

Definition(2.2)

i) The pre-closure of A denoted by p-CL(A) or $\overline{A}^{p} = \cap\{F : F \text{ pre-closed} ; A \subseteq F\}$ **[3]**

ii)The pre-interior of A denoted by p-int(A) or A^{op} =∪{U:U pre-open ; U ⊆ A}. **[4]**

iii)The pre-Exterior of A denoted by p-Ext(A) or $A^{c^{op}}$=∪{U :U pre-open ; U ⊆ A^c}.

Remark (2.3)

Let X be a space and $A \subseteq X$ we have ($\overline{A}^p)^c = (A^c)^{op}$ = p-Ext (A) .

Definition(2.3)

Suppose X is a space and G ⊆ X is said to be

i) locally closed **[2]** if G = K ∩ H, where K open and H is closed in X .

ii) locally pre closed if G = K ∩ H , where K open and H is pre closed in X .

iii) pre-locally closed if G = K ∩ H , where K pre-open and H is pre closed in X .

Theorem (2.2)

i- Every locally closed is pre-locally closed .

ii- Every locally pre-closed is pre-locally closed .

Proof:

i- Suppose G is locally closed then G = K ∩ H where K ∈ τ , H is closed .Thus K pre-open and F pre-closed by Remark (2.1) .Therefore G pre-locally closed

ii- Suppose G locally pre-closed then G = K∩H , Such that K∈ τ , H is pre-closed .Thus K pre-open Remark (2.1) . Therefore G pre-locally closed

But pre-locally closed is need not pre-open for the following example .

Example (1.1)

Let X={m ,n , p , q} and $\tau=\{\emptyset, X,$ {p , q}}topology on X .If A={m}. Since A ={m, p}∩{m, n} and {m ,p} is pre-open , {m , n} is pre-closed thus A is pre-locally closed . But A not pre-open

Theorem (2.3)

The intersection of pre-closed and pre-locally closed is pre-locally closed .

Proof:

Let A pre-closed and B pre-locally closed then B= U ∩ F such that U pre-open and F pre-closed .Thus A ∩ B = A ∩ U ∩ F = U ∩ H . Such that H = A ∩ F is pre-closed and U pre-open . Therefore A ∩ B pre-locally closed .

Theorem (2.4)

For a space X the following statements are equivalent

1- A pre-locally closed
2- A = U∩ $\overline{A}^p$ for some U pre-open
3- $A = U - p\_Ext(A)$ for some U pre-open

4- $X-A = F \cup p\_Ext(A)$ for some F pre-closed

Proof :

1) → 2) Let A pre-locally closed then A= U∩ H such that U pre-open and H pre-closed thus A⊆ U and A ⊆ H then A ⊆ $\overline{A}^p \subseteq \overline{H}^p = H$. Hence A ⊆ U∩ $\overline{A}^p$. Also U∩ $\overline{A}^p$ ⊆ U∩U$-$ $p\_Ext(A)$ for some U pre-open . $\overline{H}^p$ = U∩ $H = A$. We have A = U∩ $\overline{A}^p$.

2) → 3) A = U∩ $\overline{A}^p$ for some U pre-open then

= U$-(\overline{A}^p)^c$, $(\overline{A}^p)^c = (A^c)^{op} = p\_Ext(A)$ hence

= U$-$ $p\_Ext(A)$ for some U pre-open .

3) → 4) Since A = U$-$ $p\_Ext(A)$ for some U pre-open .

= U ∩ $(p\_Ext(A))^c$

$A^c = (U \cap (p\_Ext(A))^c)^c$

$= U^c \cup p\_Ext(A)$. Let $F = U^c$

= F ∪ $p\_Ext(A)$. Since U pre-open thus F pre-closed .

4) → 1) Since $X-A = F \cup p\_Ext(A)$ for some F pre-closed .

Therefore A = $F^c \cap (p\_Ext(A))^c$

= U ∩ $((A^c)^{op})^c$. Such that U= F^c is pre-open

= U ∩ $\overline{A}^p$.

Since $\overline{A}^p$ is pre-closed . We have A is pre-locally closed .

Theorem (2.5)

If A pre-locally closed in X then

1) $\overline{A}^p - A = (\overline{A}^p \cup U^c)$ for some U pre-open
2) $\overline{A}^p - A = (\overline{A}^p \cup F)$ for some F pre-closed
3) A ∪ p-$Ext(A)$ is pre-open
4) A ⊆ $(A \cup p\_Ext(A))^{op}$

Proof

1) Since A pre-locally closed then there is U pre-open where A the intersection of U and p-closure of A

$\overline{A}^p - A = \overline{A}^p \cap A^c$

$= \overline{A}^p \cap (U \cap \overline{A}^p)^c$

$= \overline{A}^p \cap (U^c \cup (\overline{A}^p)^c)$

$= (\overline{A}^p \cup U^c) \cap (\overline{A}^p \cup (\overline{A}^p)^c)$

$= (\overline{A}^{p} \cup U^{c}) \cap X$

$= (\overline{A}^{p} \cup U^{c})$

2) Since $\overline{A}^{p} - A =(\overline{A}^{p} \cup U^{c})$ such that U^{c} = F pre-closed hence $\overline{A}^{p} - A = \overline{A}^{p} \cup F$

3) A is pre-locally closed then there is U pre-open and A = U$\cap \overline{A}^{p}$

A$\cup$ p-$Ext(A)$ = (U$\cap \overline{A}^{p}$) $\cup$ A^{cop}

= (U$\cap \overline{A}^{p}$) $\cup$ $\overline{A}^{pc}$

= (U $\cup \overline{A}^{pc}$) $\cap$ ($\overline{A}^{p} \cup \overline{A}^{pc}$)

= V $\cap X$ = V .

U and $\overline{A}^{pc}$ are pre-open we have the intersection of U and the complement of p-closure of A is pre-open by Remark (2.2) hence A $\cup$ p-$Ext(A)$ is pre-open .

4) Since the union of A and p-$Ext(A)$ is pre-open then A$\cup$ p-$Ext(A)$ = (A $\cup$ p_$Ext(A)$)$^{\boldsymbol{op}}$ **.**

Since A subset of A$\cup$ p-$Ext(A)$) thus A $\subseteq$ (A $\cup$ p_$Ext(A)$)$^{\boldsymbol{op}}$.

Definition (2.4)[5]

For a space X the sets A and B are pre-separated sets if $\overline{A}^{p} \cap B = A \cap \overline{B}^{p} = \emptyset$.

Theorem (2.6)

If A_1 and A_2 are pre-locally closed and pre-separated then union is pre-locally closed

Proof:

Suppose that A_1 and A_2 are pre-locally closed .Thus there is G and H are pre-open such that A= G$\cap$ $\overline{A_1}^{p}$, B=H$\cap$ $\overline{A_2}^{p}$ by Theorem(2.4)**.**

Put U= G $\cap$ $(\overline{A_2}^{p})^{c}$ and V= H $\cap$ $(\overline{A_1}^{p})^{c}$. Then U $\cap \overline{A_1}^{p}$= G $\cap (\overline{A_2}^{p})^{c} \cap \overline{A_1}^{p} = A_1 \cap (\overline{A_2}^{p})^{c} = A_1$. (since A_1 subset of $(\overline{A_2}^{p})^{c}$)

Similarity V $\cap \overline{A_2}^{p} = A_2$

Since U $\cap \overline{A_2}^{p} = \emptyset$ and V$\cap \overline{A_1}^{p} = \emptyset$. Since U and V pre-open by Remark(2.2) thus

(U$\cup V) \cap (\overline{A_1 \cup A_2}^{p}) = ($ U $\cup V) \cap (\overline{A_1}^{p} \cup \overline{A_2}^{p})$

= (U$\cap \overline{A_1}^{p}) \cup (U \cup \overline{A_2}^{p}) \cup \left(V \cap \overline{A_1}^{p}\right) \cup \left(V \cap \overline{A_2}^{p}\right)$

= A$\cup$B . Therefore A$\cup$B is pre locally closed

Theorem (2.7)

If G and H are pre-locally closed then there exist K pre-close set such that G$-$ H = (G $\cap$ K) $\cup$ (G $\cap$ $p\_Ext(H)$)

Proof:

Let G and H are pre-locally closed sets .Thus G= S $\cap \overline{G}^p$ and H = L $\cap \overline{B}^p$ such that S and L are pre-open then G − H = (S $\cap \overline{G}^p$) − (L $\cap \overline{H}^p$)

$$= (S \cap \overline{G}^p) \cap (L \cap \overline{H}^p)^c$$
$$= (S \cap \overline{G}^p) \cap (L^c \cup (\overline{H}^p)^c)$$
$$= (S \cap \overline{G}^p \cap L^c) \cup (S \cap \overline{G}^p \cap (\overline{H}^p)^c)$$
$$= (G \cap L^c) \cup (G \cap p\_Ext(H))$$

Since L pre-open thus L^c pre-closed. Let K= L^c hence G − H=(G ∩ K) ∪ (G ∩ $p\_Ext(H)$)

Theorem (2.8)

If A_1 and A_2 are pre-locally closed and $A_1 \cap p\_Ext(A_2) = A_2 \cap p\_Ext(A_1) = \emptyset$ then there exist H and F pre closed sets such that $A_1 \Delta A_2$= (A $\cap H$) $\cup$ ($B \cap F$) .

Proof:

Let A_1 and A_2 are pre-locally closed thus ∃ U and V are pre-open where

A_1=U$\cap \overline{A_1}^p$, A_2=V$\cap \overline{A_2}^p$ by Theorem(2.5)

$$A_1 \Delta A_2 = [(A_1 - A_2) \cup (A_2 - A_1)$$
$$= [(A_1 \cap {A_2}^c) \cup (A_2 \cap {A_1}^c)]$$
$$= [(U \cap \overline{A_1}^p) \cap (V \cap \overline{A_2}^p)^c] \cup [(V \cap \overline{A_2}^p) \cap (U \cap \overline{A_1}^p)^c]$$
$$= [(U \cap \overline{A_1}^p) \cap (V^c \cup (\overline{A_2}^p)^c)] \cup [(V \cap \overline{A_2}^p) \cap (U^c \cup \left(\overline{A_2}^p\right)^c)]$$
$$=[U \cap \overline{A_1}^p \cap V^c) \cup (U \cap \overline{A_1}^p \cap \left(\overline{A_2}^p\right)^c) \cup (V \cap \overline{A_2}^p \cap U^c) \cup (V \cap \overline{A_2}^p) \cap \left(\overline{A_1}^p\right)^c)$$
$$= (A_1 \cap V^c) \cup (A_1 \cap \left(\overline{A_2}^p\right)^c) \cup (A_2 \cap U^c) \cup \left(A_2 \cap \left(\overline{A_1}^p\right)^c \right)$$
$$= (A_1 \cap V^c) \cup (A_1 \cap {A_2}^{cop}) \cup (A_2 \cap U^c) \cup (A_2 \cap {A_1}^{cop})$$
$$= (A_1 \cap H) \cup (A_1 \cap p\_Ext(A_2)) \cup (A_2 \cap F) \cup (A_2 \cap p\_Ext(A_1))$$

Since V and U pre open then $V^c = H$ and $U^c = F$ are pre-closed

Since $A_1 \cap p\_Ext(A_2) = A_2 \cap p\_Ext(A_1) = \emptyset$ thus $A_1 \Delta A_2$=($A_1 \cap H$) $\cup$ ($A_2 \cap F$)

Definition (2.5)

A function $h : X_1 \to X_2$ is called

i) pre* -continuous if $h^{-1}(A)$ is pre-open in X_1 for all A pre-open in X_2 .

ii) pre* -open if $h(A)$ is pre-open in X_2 for all A pre-open in X_1 .

iii) pre* -closed if $h(A)$ is pre-closed in X_2 for all A pre-closed in X_1 .

Remark(2.3)

If $h: X_1 \to X_2$ is bjective and pre*–continuous then the inverse image is pre-closed in X_1 for all A pre-closed in X_2 .

Theorem (2.9)

If $h: X_1 \rightarrow X_2$ is bjective then

i) If h pre*–continuous then the inverse image for all pre-locally closed in X_2 is pre-locally closed in X_1 .

ii) If h pre*-open then the image for all pre-locally closed in X_1 is pre-locally closed in X_2 .

Proof :

i) Let G pre-locally closed in X_2 .Thus G = V ∩ H such that V pre-open and H pre-closed sets in X_2 . Since h pre*–continuous then $h^{-1}(U\ )$ is pre-open in X . Since h bijective then $h^{-1}(F)$ is pre-closed in X_1. Hence $h^{-1}(U\ ) \cap h^{-1}(F) = h^{-1}(U\ \cap F) = h^{-1}(A\ )$ = B . There fore B pre-locally closed in X .

ii) Clear .

3 - Pre D-set

Definition (3.1)

If X be space and A subset of X . Then S is said

1- difference set (D-set) **[6]** if ∃ open sets U and V in X so that $U \neq X$ and S = $U - V$.

2- Pre difference set (pre D-set) **[3]** if ∃ pre-open sets U and V in X so that $U \neq X$ and S = $U - V$.

Theorem (3.1)

1- Every pre-open (not equal to X) is pre-D-set

2- Every D-set is pre-D-set

Proof :

1- Let S is pre-open and S ≠ X . Since S = S $-\emptyset$ and $\emptyset$ pre-open we have S is pre-D-set .

2- Let A is D-set then there are open sets A_1 and A_2 in X so that $A_1 \neq X$ and A =$A_1 - A_2$ .Thus A pre-D-set .

But the converse of (2) is not true for the following example .

Example(3.1)

Let X= { 1, 2, 3} and τ indiscrete topology on X . It is clear that for every S subset of X is pre-open thus S pre-D-set but there is no open sets open . Hence S not D-set .

Theorem (3. 2)

Every pre-D-set is pre-locally closed .

Proof :

Let S is pre-D-set there are two pre-open sets S_1 and S_2 in X so that $S_1 \neq X$ and S= $S_1 - S_2$. Thus S =$S_1 \cap\ {S_2}^c$. Since S_2 pre open thus ${S_2}^{\ c}$ pre-closed therefore S pre-locally closed

Definition (3.2)

For a space X and S □ X is said pre-dense if $\overline{S}^p = X$.

Theorem (3.3)

1) Every pre-dense and pre-locally closed is pre-D-set .
2) Every pre-dense pre-D-set is pre-open .

Proof

1) Let S pre-locally closed thus there exist U pre-open such that S =U ∩ $\overline{S}^p$. Since S pre-dense then $\overline{S}^p$ = X .Thus S=U then S is pre-open hence S is pre-D-set .

2) Let G pre-D-set thus there exist V pre-open so that V $\neq \emptyset$ and G=V ∩ $\overline{G}^p$ .Since G pre-dense then $\overline{G}^p$ = X . Thus G = V then G is pre-open .

Theorem (3.4)

For a space X the following statements are equivalent

i. S pre D-set .

ii. S = U∩ $\overline{S}^p$ for some U pre-open and U $\neq X$.

iii. $S = U - p\_Ext(S)$ for some U pre-open and U $\neq X$.

iv. $S^c = F \cup p\_Ext(S)$ for some F pre-closed and $F \neq \emptyset$.

Proof :

i→ ii Let A pre D-set thus ∃ pre-open sets U *and* V so that $U \neq X$ and S= $U - V$.
Thus S = $U \cap V^c$ then V^c pre-closed thus S pre-locally closed hence S = U∩ $\overline{S}^p$ for some U pre-open by Theorem (3.2) and Theorem (2.4) we have S = U∩ $\overline{S}^p$ for some U pre-open and U $\neq X$.

ii→ iii Let S = U∩ $\overline{S}^p$ for some U pre-open and U$\neq X$.Thus S =U−$\overline{S}^{p\,c}$ then $S = U - p\_Ext(S)$ for some U pre-open and U $\neq X$.

iii.→ iv. Let $S = U - p\_Ext(S)$ for some U pre-open and U$\neq X$ .Hence S^c=$(U - p\_Ext(S))^c$ then similar to theorem (2.5) we have $A^c = U^c \cup p\_Ext(S)$ .Whereas U pre-open therefore $U^c = F$ pre-closed . Since U $\neq X$ then $U^c = F \neq \emptyset$.

iv→ i clear

Theorem (3.5)

If A and B are pre-D-sets then there exist pre-closed set F$\neq \emptyset$ such that A− B = (A ∩ F) ∪ (A ∩ p-$Ext(A)$)

Proof

Let A and B are pre-D-sets .Thus A= K ∩ $\overline{A}^{p}$ and B = H ∩ $\overline{B}^{p}$ so that K and H are pre-open and K , H ≠ X by Theorem (2.4) then it is clear that A − B = (A∩ H^c) ∪ (A ∩ $(\overline{B}^{p})^c$) , Since H pre-open and H ≠ X thus H^c pre-closed and $H^c \neq \emptyset$. Let H^c = F . Since $(\overline{B}^{p})^c$ = p-$Ext(A)$ there fore A− B = (A∩ F) ∪ (A ∩ p-$Ext(A)$) .

Theorem (3.6)

If $h : M \to N$ is bijective thus

1) If pre*–continuous thus $h^{-1}(S)$ is pre-D-set in M for all S pre-D-set in N .
2) If pre*–open then $h(S)$ is pre-D-set in N for all S pre-D-set in M .

Proof :

1) Suppose S pre-D-set in N . Thus there exist S_1 and S_2 pre-open in N so that $S_1 \neq N$ and S = $S_1 - S_2$. Since h pre*–continuous then $h^{-1}(S_1)$ and $h^{-1}(S_2)$ pre-open in M . Since h bijective then $h^{-1}(S_1) \neq M$. Hence $h^{-1}(S_1) - h^{-1}(S_2)$ = $h^{-1}(S_1 \cap S_2)$ = $h^{-1}(S)$ = K . There fore K pre-D-set in M .
2) Clear .

Definition (3.3)

A subset S of space X is called Pre-locally D-set if S = U ∩ G , where U pre-open and G is pre-D-set .

Theorem (3.7)

Every pre-open is pre-locally D-set

Proof : Clear

Example(3.2)

Let X={ s_1, s_2 , s_3 , s_4} and $\tau = \{\emptyset , X, \{s_3 , s_4\}\}$. Let G={s_1, s_2 , s_3} and K ={ s_1, s_4} . It is clear that G and K are pre-open then K is pre-D-set thus G∩K ={s_1} is pre-locally D-set but not pre-open .

Theorem (3.8)

For A Y then the following statements are equivalent

1) A pre-locally D-set
2) A is intersection of pre-open is not equal to Y and pre-locally closed

Proof :

1) → 2) Since A pre-locally D-set then A =U ∩ G such that U pre-open and G pre-D-set . Since G pre-D-set thus G = V−K such that V≠ Y and V, K are pre-open therefore A = U ∩ (V− K)

= U ∩ (V∩ K^c)

= V ∩ (U ∩ K^c) since K pre-open then K^c pre-closed

= V ∩ M such that M = U ∩ K^c pre locally closed

Hence A= V ∩ M such that V ≠ Y is pre-open and M = U ∩ K^c pre locally closed

2) → 1) Since A = U ∩ K so that U ≠ Y is pre-open and K pre locally closed hence K= V∩F such that V pre-open and F pre-closed therefore
A= U ∩ (V ∩ F) . Let F= H^c
= V ∩ (U ∩ H^c) =V ∩ (U− H) since U and H pre-open such that U ≠ Y thus M= U− H pre-D-set
A= V ∩ M since V pre-open we have A pre locally D-set

Remark(3.1)

If $h : X \to Y$ is bijective then

1. If pre*–continuous then the inverse image for all pre-locally D-set in Y is pre-locally D-set in X .
2. If pre*–open thus the image of all pre-locally D-set in X is pre-locally D-set in Y

The following diagram shows that the relationships of pre-D-set with locally closed sets generated by pre-open set as shown in Fig.1 .

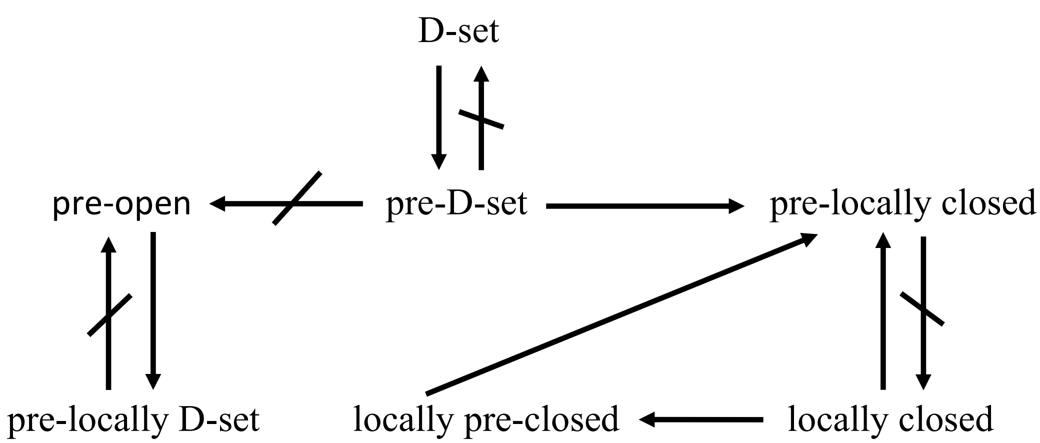

Fig . 1. Relationship of pre-D-set with locally closed set .

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