QJPS
Al-Qadisiyah Journal of Pure Science
Al-Qadisiyah Journal of Pure Science
ISSN(Printed): 1997-2490 ISSN(Online): 2411-3514
DOI: /10.29350/jops.
HTTP://QU.EDU.IQ/JOURNALSC/I

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COMPARISON OF TWO NUMERICAL METHODS FOR SOLVING THE NEWELL-WHITEHEAD-SEGEL EQUATION

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Abstract

In this paper, the purpose is to take advantage of using the finite difference method to interpret the approximate solution of the Newell-Whitehead-Segel. A mathematical model is constructed to display the solution of the Newell-Whitehead-Segel equation. This involves, illustrating the formula, considering the computational analysis, and evaluating the approximate solution using both explicit forward time-centered space and implicit Crank-Nicolson methods. The two methods calculate the approximate solution based on the initial and boundary conditions. It has been recognized the stability of the solution by a certain condition and that is shown in figures and tables. Additionally, two different formulas of the Newell-Whitehead-Segel equations have been examined. Finally, we show that the implicit Crank-Nicolson method is more dependable than the explicit forward time-centered space method. Furthermore, the implicit Crank-Nicolson technique of solving problems contributes assertion that solutions can be obtained easily. MATLAB has been used to generate the algorithm of these methods.

Key words: partial differential equation, explicit forward time-centered space method, implicit generalized finite difference method.

1 Introduction

Physicists and mathematicians have spent an extensive amount of time and effort over the years for the purpose of deriving exact and explicit solutions to the semi-linear partial differential equations (SLPDEs) in a wide range of related fields. As examples, in physics, describe continuity in fluids, elastic solids, shock wave propagation in a gas and temperature [1]–[4]. Chemistry uses the SLPDEs to describe how molecules are moved in a cell with compartmentalization, while in nature, roll patterns are seen all over the place in a variety of spatially extended systems. Those patterns include seashells, ripples in the sand, as well as those that appear in human fingerprints [5]. It is well known that science and technology use SLPDEs to describe numerous scientific phenomena because of their ability to express multiple variables [6][7]. Analytical methods are generally the most dependable method for finding exact solutions to these types of equations, but they become increasingly difficult to apply to more complicated equations as they become more difficult to solve [8], [9]. However, despite the fact that numerical methods provide approximate solutions similar to analytical solutions, they have gained popularity due to the advancement of computing capabilities. It is important to note that many systems in a variety of phenomena are unable to be tracked by formulae because of their complex functionality, which contributes to their inability to predict many systems' behavior. This requires simulating and analyzing the system to estimate the behavior of the solution using computational approaches [10], [11]. Among the numerical and semi-analytical techniques commonly applied iteration, variational, He's Polynomial method, and a finite difference scheme (numerical differentiation: the most commonly utilized schemes include forward, backward, and central difference methods) [12]–[16]. The finite difference method is a numerical technique which is employed to overcome the difficulties associated with the determination of PDEs. The basic idea behind the finite difference scheme is to replace derivatives with difference quotients. This allows us to convert a continuous problem into a discrete problem that can be solved through algebraic equations. The strategy of this scheme is extensively used in various fields, including physics, engineering, and computational mathematics because they often used when high precision is required or when dealing with difficult problems [15], [16]. Among the equations that are solved by finite difference method is the semi-linear Newell Whitehead Segel (SLNWS) equation. The solution of this equation has also been derived by applying other techniques which have been investigated by researchers interested in determining the solution [17]–[19]. Other ideas derived from decomposition methods presented an approach to achieve the solution of the SLNWS model such as the Laplace-Adomian transform scheme [20]. As well as that, there have been a number of studies done in recent years on the solution to the SLNWS equation, including one in which the homotopy perturbation method was used by [21], and many others can be seen in [22]–[26]. In this paper, we will apply the forward time-centered space (FTCS) and Crank-Nicolson (CN) methods to solve the SLNWS equation [27], [28]. We consider that the out put has two possibilities, either stable or unstable solution. The stability of the solution of the SLNWS equation is checked by assuming different cases depending on certain information. Based on these information, both methods are used, when the solution is unstable, we will change the assuming information in order to reach the stable case for the SLNWS equation. By solving three different examples of the SLNWS, we will demonstrate the worth of using these methods. The FTCS and CN methods are used to discretize the SLNWS equation each method have its own forms. As a result of this, the researchers will be able to solve problems faster and understand the mathematics involved in the problem facilely. Throughout the article, you will find the following structure: In Section (2), our mathematical methods are displayed using the FTCS and CN methods for the SLNWS equation. Section (3) demonstrates two different examples of the SLNWS equations with different cases. Finally, Section (4) is devoted to the conclusions.

2 Mathematical methods

This paper presents the explicit FTCS and the implicit CN methods, respectively to solve the SLNWS equation which has the following general form:

$$\mathcal{U}_t(r,t) = n\,\mathcal{U}_{rr}(r,t) + m\,\mathcal{U}(r,t) - w\,\mathcal{U}^q(r,t), \qquad (1)$$

where, $\mathcal{U}(r,t)$ is a function with variables r and t. Also, n, m and w are constant and real. The nonlinear part of Eq. (1) shown in $\mathcal{U}^q(r,t)$, where q is an integer and $q > 1$. As well as by Eq. (1), we can know that the constants m and w are the coefficients of linear and nonlinear reaction, respectively. In addition to that, the constant n is the coefficient of diffusion. Here, r and t are defined in general as $a_1 \leq r \leq b_1$ and $a_2 \leq t \leq b_2$, respectively, where $t \geq 0$ and a_1, a_2, b_1 and b_2 are constants. We also defined the number of points as $N_1 = \frac{b_1 - a_1}{n_1}$ and $N_2 = \frac{b_2 - a_2}{n_2}$, where n_1 and n_2 are integers represents the number of intervals of r and t respectively. In this case, for $c = 0, 1, \ldots, n_1 + 1$ and $z = 0, 1, \ldots, n_2 + 1$, then, the values of $\mathcal{U}_0^1, \ldots, \mathcal{U}_0^z$, are demonstrate the values of the time variables in the left boundary condition, and nodes $\mathcal{U}_c^1, \ldots, \mathcal{U}_c^z$ are state the values of the time variables in the right boundary condition. The spatial values for of the net is when $z = 0$ and $c = 0, 1, \ldots, n_1 + 1$, then $\mathcal{U}_0^0, \mathcal{U}_1^0, \ldots, \mathcal{U}_c^0$ represents the values at the initial condition. Additionally, we need to find the values of the internal nodes.

2.1 FTCS Method

This method is one of the popular methods used to find the solution of the linear PDEs approximately. In this paper, we apply it on the nonlinear PDEs equation. This is a method that depends on the value of the unknown functions are $\mathcal{U}_c^{z+1}$ at $t = z + 1$, where the values of the known functions at the points $\mathcal{U}_{c-1}^z$, $\mathcal{U}_c^z$, and $\mathcal{U}_{c+1}^z$ when $t = z$. Recall, Eq. (1):

$$\mathcal{U}_t(r,t) = n\,\mathcal{U}_{rr}(r,t) + m\,\mathcal{U}(r,t) - w\,\mathcal{U}^q(r,t),$$

the explicit scheme is used on both side of Eq. (1) to convert the continuous PDE to a discrete type as follows:

$$\mathcal{U}_c^{z+1} - \mathcal{U}_c^z = \frac{nN_2}{(N_1)^2}(\mathcal{U}_{c+1}^z - 2\mathcal{U}_c^z + \mathcal{U}_{c-1}^z) + m\,N_2\mathcal{U}_c^z - w\,N_2(\mathcal{U}_c^z)^q, \qquad (2)$$

where, N_1 is the space steps and N_2 is the time steps. If we let $d = \frac{nN_2}{(N1)^2}$ to be a convergence constant, then Eq. (2) becomes discrete Newell–Whitehead–Segel (DNWS-FTCS) equation,

$$\mathcal{U}_c^{z+1} = d\mathcal{U}_{c+1}^z + (1 - 2d + mN_2)\mathcal{U}_c^z + d\mathcal{U}_{c-1}^z - wN_2(\mathcal{U}_c^z)^q, \qquad (3)$$

when $c = 1, \ldots, n_1 - 1$ in Eq. (3), we will obtain system of equations:

$$\begin{aligned}
\mathcal{U}_1^{z+1} &= d\mathcal{U}_2^z + (1 - 2d + mN_2)\mathcal{U}_1^z + d\mathcal{U}_0^z - wN_2(\mathcal{U}_1^z)^q\,, \\
\mathcal{U}_2^{z+1} &= d\mathcal{U}_3^z + \big(1 - 2d + m(N_2)\big)\mathcal{U}_2^z + d\mathcal{U}_1^z - wN_2(\mathcal{U}_2^z)^q\,, \\
&\vdots \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (4) \\
\mathcal{U}_{n_1-1}^{z+1} &= d\mathcal{U}_{n_1}^z + (1 - 2d + mN_2)\mathcal{U}_{n_1-1}^z + d\mathcal{U}_{n_1-2}^z - wN_2\big(\mathcal{U}_{n_1-1}^z\big)^q\,,
\end{aligned}$$

Next step will be substituting $z = 0, \ldots n_2 - 1$ in Eq. (4) to find the other unknown values of $\mathcal{U}_{n_1-1}^{z+1}$. For instance, when $z = 0$, Eq. (4) becomes:

$$\begin{aligned}
\mathcal{U}_1^1 &= d\mathcal{U}_2^0 + (1 - 2d + mN_2)\mathcal{U}_1^0 + d\mathcal{U}_0^0 - wN_2(\mathcal{U}_1^0)^q\,, \\
\mathcal{U}_2^1 &= d\mathcal{U}_3^0 + (1 - 2d + mN_2)\mathcal{U}_2^0 + d\mathcal{U}_1^0 - wN_2(\mathcal{U}_2^0)^q\,, \\
&\vdots \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (5) \\
\mathcal{U}_{n_1-1}^1 &= d\mathcal{U}_{n_1}^0 + (1 - 2d + mN_2)\mathcal{U}_{n_1-1}^0 + d\mathcal{U}_{n_1-2}^0 - wN_2\big(\mathcal{U}_{n_1-1}^0\big)^q\,,
\end{aligned}$$

Similarly, for other values of z.

2.2 CN Method

The implicit CN method is a numerical method for solving PDEs. It solves the equation by approximating the derivatives with finite differences. This method is suitable for problems involving large time steps. To start with this method, recall Eq. (1):

$$\mathcal{U}_t(r,t) = n\,\mathcal{U}_{rr}(r,t) + m\,\mathcal{U}(r,t) - w\,\mathcal{U}^q(r,t),$$

applying implicit scheme on Eq. (1) to obtain:

$$(\mathcal{U}_t)_c^{z+\frac{1}{2}} = n\,(\mathcal{U}_{rr})_c^{z+\frac{1}{2}} + m\,\mathcal{U}_c^z - w\,(\mathcal{U}_c^z)^q, \qquad (6)$$

In Eq. (6), expressed $\mathcal{U}_{rr}$ at $z + \frac{1}{2}$ time level by the average of the time values at z and $z + 1$ respectively.

$$(\mathcal{U}_t)_c^{z+\frac{1}{2}} = \frac{n}{2}\ [(\mathcal{U}_{rr})_c^z + (\mathcal{U}_{rr})_c^{z+1}] + m\,\mathcal{U}_c^z - w\,(\mathcal{U}_c^z)^q, \qquad (7)$$

The time derivative at $z + \frac{1}{2}$ time level and the space derivatives may now be approximated by second order central difference approximations, respectively:

$$\frac{\mathcal{U}_c^{z+1} - \mathcal{U}_c^z}{N_2} = \frac{n}{2}\left(\frac{\mathcal{U}_{c+1}^z - 2\mathcal{U}_c^z + \mathcal{U}_{c-1}^z}{(N_1)^2} + \frac{\mathcal{U}_{c+1}^{z+1} - 2\mathcal{U}_c^{z+1} + \mathcal{U}_{c-1}^{z+1}}{(N_1)^2}\right) + m\mathcal{U}_c^z - w\,N_2(\mathcal{U}_c^z)^q, \qquad (8)$$

after some simplifications, we have the discrete Newell–Whitehead–Segel (DNWS-CN) equation:

$$-\hat{d}\mathcal{U}_{c+1}^{z+1} + (1 + 2\hat{d})\mathcal{U}_c^{z+1} - \hat{d}\mathcal{U}_{c-1}^{z+1} = \hat{d}\mathcal{U}_{c+1}^z + (1 - 2\hat{d} + mN_2)\mathcal{U}_c^z + \hat{d}\mathcal{U}_{c-1}^z - wN_2(\mathcal{U}_c^z)^q, \qquad (9)$$

where, $\hat{d} = \frac{nN_2}{2(N1)^2}$ is the convergence constant and $c = 1, \dots n_1 - 1$. We can write Eq. (9) in a general form as:

$$A\mathcal{U}^{z+1} = W^z, \qquad (10)$$

where, $A, \mathcal{U}$ and W are matrices that defined as:

$$A = \begin{bmatrix} 1+2\hat{d} & -\hat{d} & 0 & 0 & & 0 & 0 \\ -\hat{d} & 1+2\hat{d} & -\hat{d} & 0 & \cdots & 0 & 0 \\ 0 & -\hat{d} & 1+2\hat{d} & -\hat{d} & & 0 & 0 \\ 0 & 0 & -\hat{d} & 1+2\hat{d} & & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & & -\hat{d} & 0 \\ 0 & 0 & 0 & 0 & \cdots & 1+2\hat{d} & -\hat{d} \\ 0 & 0 & 0 & 0 & & -\hat{d} & 1+2\hat{d} \end{bmatrix}, \qquad (11)$$

$$\mathcal{U}^{z+1} = \begin{bmatrix} \mathcal{U}_1^{z+1} \\ \mathcal{U}_2^{z+1} \\ \mathcal{U}_3^{z+1} \\ \mathcal{U}_4^{z+1} \\ \vdots \\ \mathcal{U}_{n_1-3}^{z+1} \\ \mathcal{U}_{n_1-2}^{z+1} \\ \mathcal{U}_{n_1-1}^{z+1} \end{bmatrix}, \qquad (12)$$

and

$$\mathcal{W}^z = \begin{bmatrix} \hat{d}(\mathcal{U}_0^{z+1} + \mathcal{U}_0^z) + (1 - 2\hat{d} + mN_2)\mathcal{U}_1^z + \hat{d}\mathcal{U}_2^z - wN_2(\mathcal{U}_1^z)^q \\ \hat{d}\mathcal{U}_1^z + (1 - 2\hat{d} + mN_2)\mathcal{U}_2^z + \hat{d}\mathcal{U}_3^z - wN_2(\mathcal{U}_2^z)^q \\ \hat{d}\mathcal{U}_2^z + (1 - 2\hat{d} + mN_2)\mathcal{U}_3^z + \hat{d}\mathcal{U}_4^z - wN_2(\mathcal{U}_3^z)^q \\ \hat{d}\mathcal{U}_3^z + (1 - 2\hat{d} + mN_2)\mathcal{U}_4^z + \hat{d}\mathcal{U}_5^z - wN_2(\mathcal{U}_4^z)^q \\ \hat{d}\mathcal{U}_4^z + (1 - 2\hat{d} + mN_2)\mathcal{U}_5^z + \hat{d}\mathcal{U}_6^z - wN_2(\mathcal{U}_5^z)^q \\ \hat{d}\mathcal{U}_5^z + (1 - 2\hat{d} + mN_2)\mathcal{U}_6^z + \hat{d}\mathcal{U}_7^z - wN_2(\mathcal{U}_6^z)^q \\ \vdots \\ \hat{d}\mathcal{U}_{n_1-2}^z + (1 - 2\hat{d} + mN_2)\mathcal{U}_{n_1-1}^z + \hat{d}(\mathcal{U}_{n_1}^{z+1} + \mathcal{U}_{n_1}^z) - wN_2(\mathcal{U}_{n_1-1}^z)^q \end{bmatrix} \quad (13)$$

The elements of the main diagonal in the matrix A are equal to $(1 + 2\hat{d})$, and the elements above and below it are equal to $(-\hat{d})$. In this method we will also substitute the values of z from $0 \rightarrow n_2 - 1$. In this case we will obtain system of equations and after solving this system, we will get the internal nodes $\mathcal{U}_1^1, \mathcal{U}_2^1, \ldots, \mathcal{U}_{n1-1}^1$ and $\mathcal{U}_1^2, \mathcal{U}_2^2, \ldots, \mathcal{U}_{n1-1}^2$ and so on for other values of z.

3 Numerical Results

In this paragraph, we will present three different examples solving them using the FTCS and CN methods. The results between two methods will be presented in different cases. In addition, the appropriate condition was found for the method to be more stable, as we can see in the tables and figures.

Example 1 [17]

Given a NWSE :

$$\mathcal{U}_t(r,t) = \mathcal{U}_{rr} - \mathcal{U}^3, \quad (14)$$

with left boundary condition

$$\mathcal{U}(0,t) = 0,$$

and right boundary condition

$$\mathcal{U}(1,t) = \sqrt{2}\ \left(\frac{2}{2+6t}\right),$$

and initial boundary condition

$$\mathcal{U}(r,0) = \sqrt{2}\ \left(\frac{2r}{r^2+1}\right).$$

In this example, our aim to find the solution of Eq. (14) $\mathcal{U}(r,t)$, using both methods FTCS and CN, where $0 < r < 1$ and $0 < t < 0.06$. By comparing Eq. (1) with Eq. (14) of Example 1, we get: $n = 1, m = 0,\ w = 1$, and $q = 3$. The results of using the FTCS and CN methods are presented in the following figures and tables in different cases.

Case 1

Let the number of intervals for r axis is $n_1 = 12$, and the time intervals $n_2 = 8$. Then, $N_1 = 0.0833$ and $N_2 = 0.0075$ and the convergence constant $d = \frac{n\,N_2}{(N_1)^2} = 1.08$ in the FTCS method and $\hat{d} = \frac{n\,N_2}{2(N_1)^2} = 0.54$ in the CN method. In Figure (1), (a) is the exact solution, (b) is the approximation solution by the FTCS method and (c) is the approximate solution by the CN method. From Figure (1), the FTCS method has big errors in some points. Table (1), displays the lease square error for both methods at $t = 0.0225$. In the next cases, we will change the step size for r or t, or both of them in case we have better solution for the FTCS method.

Table 1: The lease square error by the FTCS and CN methods when $d = 1.08$ and $\hat{d} = 0.54$ at $t = 0.0225$.

r	t	Error (FTCS method)	Error (CN method)
0.000000	0.022500	0.000000	0.000000
0.083333	0.022500	6.577124e-07	6.139224e-09
0.166667	0.022500	2.449146e-06	5.805162e-09
0.250000	0.022500	4.886823e-06	3.033825e-09
0.333333	0.022500	7.336204e-06	9.998357e-08
0.416667	0.022500	9.222405e-06	4.427449e-07
0.500000	0.022500	1.019883e-05	1.076687e-06
0.583333	0.022500	1.020852e-05	1.862470e-06
0.666667	0.022500	9.430682e-06	2.512295e-06
0.750000	0.022500	8.162522e-06	2.685503e-06
0.833333	0.022500	2.990805e-06	2.115020e-06
0.916667	0.022500	3.014988e-06	8.933219e-07
1.000000	0.022500	0.000000	0.000000
Least square error		6.855864e-05	1.170300e-05

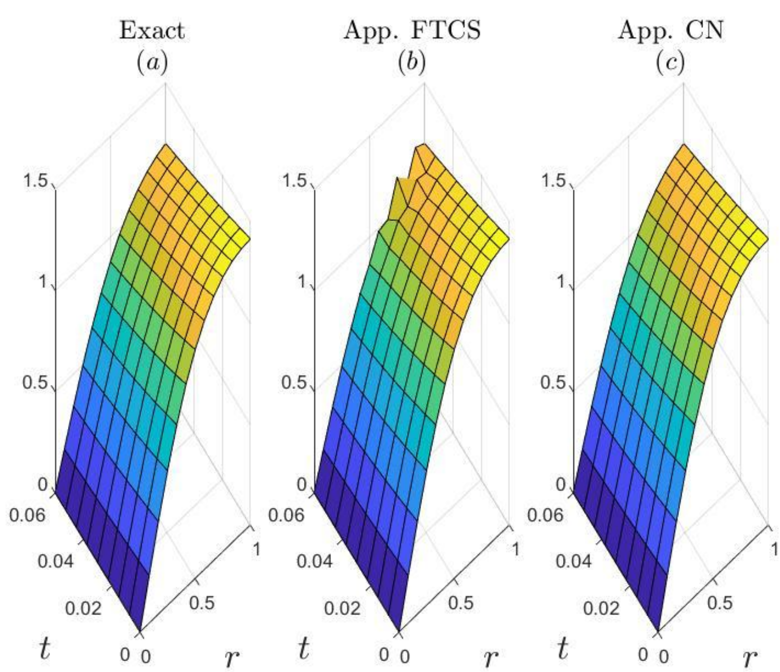

Figure 1: The exact and the numerical solution for Case (1) of Eq. (14) by the FTCS and CN methods.

Case 2: In this case, the r step size only. Let $n_1 = 8$, then $N_1 = 0.125$. The values of $n_2 = 8$, $N_2 = 0.0075$, are the same in Case (1). This gives us $d = 0.48$ and $\hat{d} = 0.24$.

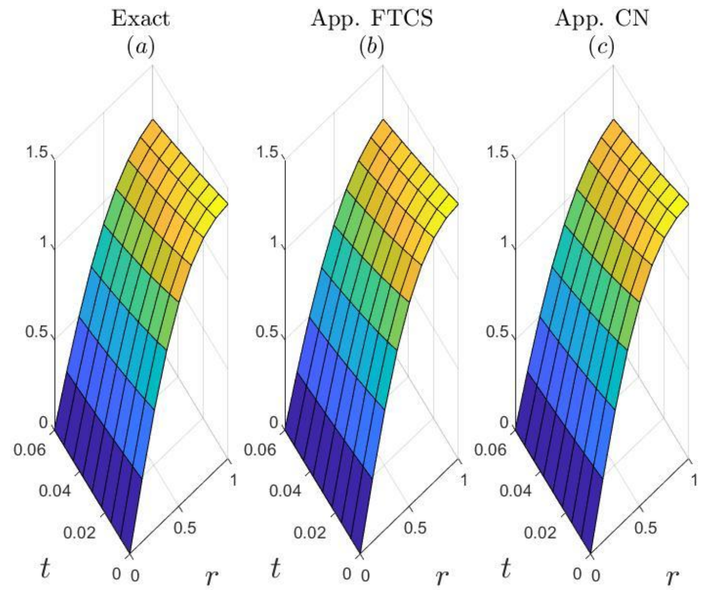

Figure 2: The exact and the numerical solution for Case (2) of Eq. (14) by the FTCS and CN methods.

Case 3: In this case, we are going to change the t step size only. Let $n_2 = 18$, then, $N_2 = 0.0033$. The values of $n_1 = 12, N_1 = 0.0833$ are the same as in Case 1. This gives us $d = 0.476$ and $\hat{d} = 0.238$. Figure (3) shows the difference when we changed n_2 and compering Figure (1) and Figure (3), the solution by the FTCS method (b) becomes stable for all r and t values when $d = 0.476$. The solution by the CN method (c) gives a stable solution too when $\hat{d} = 0.238$ (see Figure (3)).

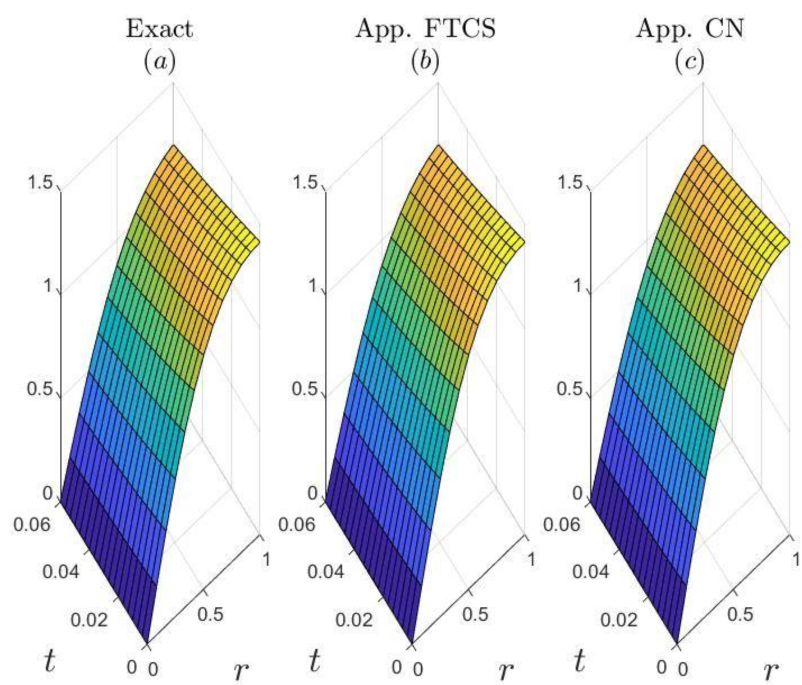

Figure 3: The stable solution for Case (3) of Eq. (14) by the FTCS and CN methods.

Case 4: In this case, we are going to change the t and r steps sizes. Let $n_1 = 5, n_2 = 3$, then $N_1 = 0.2$ and $N_2 = 0.02$, this gives us $d = 0.5$ and $\hat{d} = 0.25$. Figure (4) shows the solution of the Eq. (14). In Figure (4) the solution by the FTCS (b) and the CN methods (c) gives stable solutions for all values of r and t when $d = 0.5$ and $\hat{d} = 0.25$.

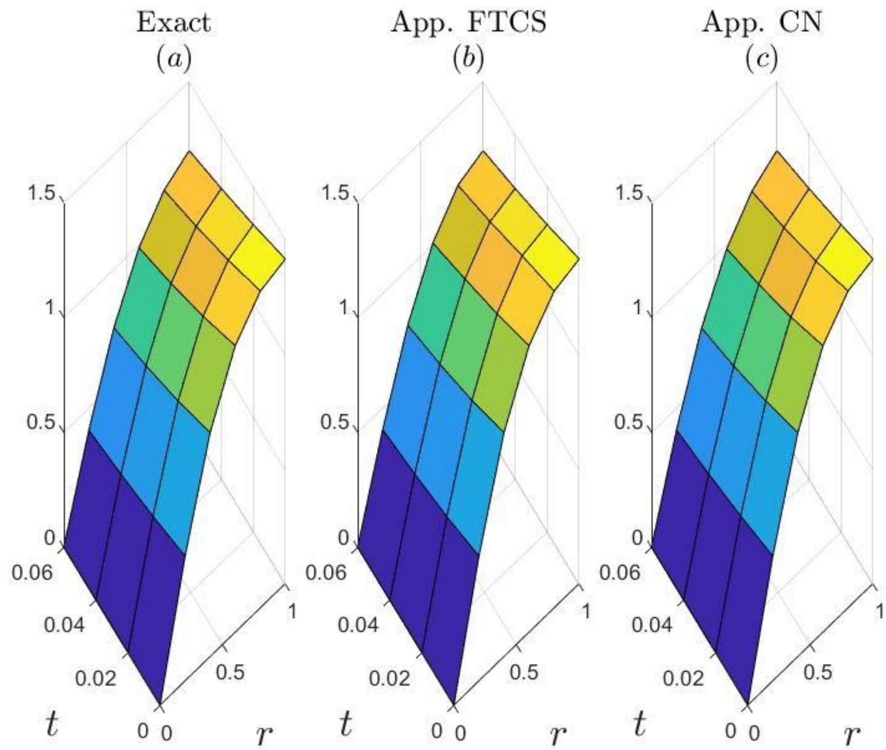

Figure 4: The stable solution for Case (4) of Eq. (14) by the FTCS and CN methods.

Example 2 [26]

Given a NWSE :

$$\mathcal{U}_t(r,t) = \mathcal{U}_{rr} + \mathcal{U} - \mathcal{U}^2, \tag{15}$$

with left boundary condition

$$\mathcal{U}(1,t) = \frac{1}{\left(1 + e^{\frac{1}{\sqrt{6}} - \frac{5}{6}t}\right)^2},$$

and right boundary condition

$$\mathcal{U}(1.8,t) = \frac{1}{\left(1 + e^{\frac{1.8}{\sqrt{6}} - \frac{5}{6}t}\right)^2},$$

and initial boundary condition

$$\mathcal{U}(r,0) = \frac{1}{\left(1 + e^{\frac{1}{\sqrt{6}}r}\right)^2}.$$

In this example, our aim is to find the solution of Eq. (15) $\mathcal{U}(r,t)$, using both methods FTCS and CN, where $1 < r < 1.8$ and $0 < t < 0.04$. The results of using the FTCS and CN methods are presented in the following figures and tables in different cases. By comparing Eq. (1) with Eq. (15) of Example 2, we get: $n = 1, \quad m = 1, w = 1, q = 2$.

Case 1: The number of spatial intervals $n_1 = 16$ and the time intervals $n_2 = 4$. Then, $N_1 = 0.05$ and $N_2 = 0.01$ and the convergence constant $d = 4$ in the FTCS method and $\hat{d} = 2$ in the CN method. According to Figure (5), the error by using the CN method is smaller than the error by the FTCS method for the most of the points. Table (4), shows the least square error for the two methods at $t = 0.04$.

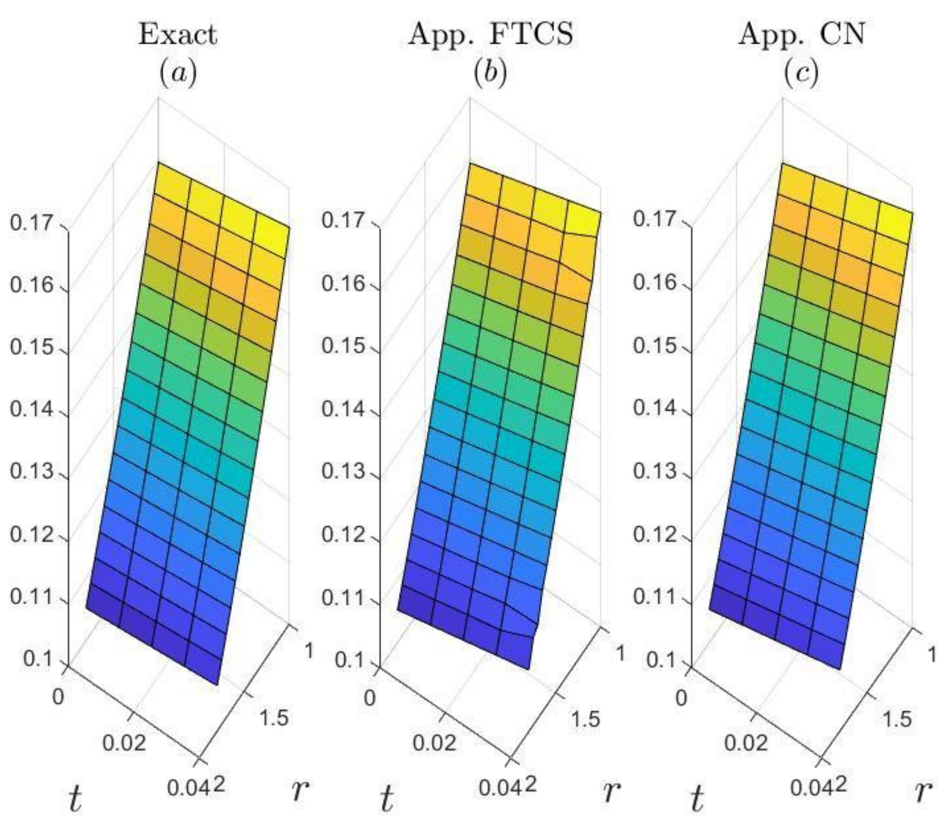

Figure 5: The exact and the numerical solution for Case (1) of Eq. (15) by the FTCS and CN methods.

Table 2: The lease square error by the FTCS and CN methods when $d = 4$ and $\hat{d} = 2$ at $t = 0.04$.

r	t	Error (FTCS method)	Error (CN method)
1.000000	0.040000	0.000000	0.000000
1.050000	0.040000	1.280754e-06	2.902537e-11
1.100000	0.040000	1.096269e-06	9.321359e-11
1.150000	0.040000	1.028542e-07	1.653718e-10
1.200000	0.040000	4.662396e-10	2.310063e-10
1.250000	0.040000	4.657841e-10	2.832958e-10
1.300000	0.040000	4.647188e-10	3.195264e-10
1.350000	0.040000	4.630589e-10	3.392108e-10
1.400000	0.040000	4.608210e-10	3.428518e-10
1.450000	0.040000	4.580240e-10	3.311460e-10
1.500000	0.040000	4.546874e-10	3.046770e-10
1.550000	0.040000	4.508329e-10	2.641132e-10
1.600000	0.040000	4.464826e-10	2.108495e-10
1.650000	0.040000	9.256789e-08	1.480118e-10
1.700000	0.040000	9.893914e-07	8.196773e-11
1.750000	0.040000	1.154282e-06	2.515311e-11
1.800000	0.040000	0.000000	0.000000
Least square error		4.720249e-06	3.169420e-09

Looking on Figure (5), there are unstable parts in some points when we use the FTCS method (see Figure 5), (b)) where $d = 4$. On other hand, the CN method (see Figure (5), (c)) is smoother and more stable. In order to have stable solution by the FTCS method, different cases are illustrated in the following paragraph by changing the step size of r or t or both.

Case 2: In this case, Figure (6), the solution is stable by the FTCS and the CN methods when we changed $n_1 = 5, N_1 = 0.16, d = 0.3906$ and the values of $n_2 = 4,, N_2 = 0.01, \hat{d} = 0.1953$ are same as in Case 1.

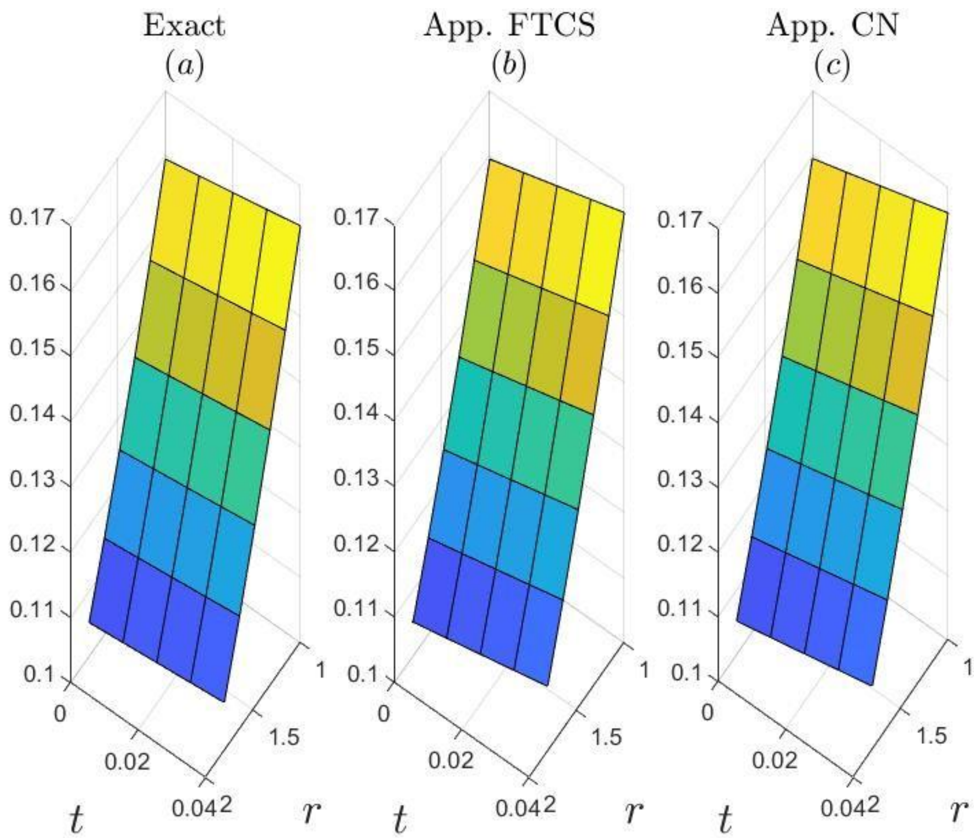

Figure 6: The exact and the approximate solution for Case 2 of Eq. (15) by the FTCS and CN methods.

Case 3: In this case the t step size and the r step size as in Case 1. In Figure (7) the solution is stable by the FTCS and CN methods when $n_1 = 16, n_2 = 32, N_1 = 0.05, N_2 = 0.0013, d = 0.52, \hat{d} = 0.26$.

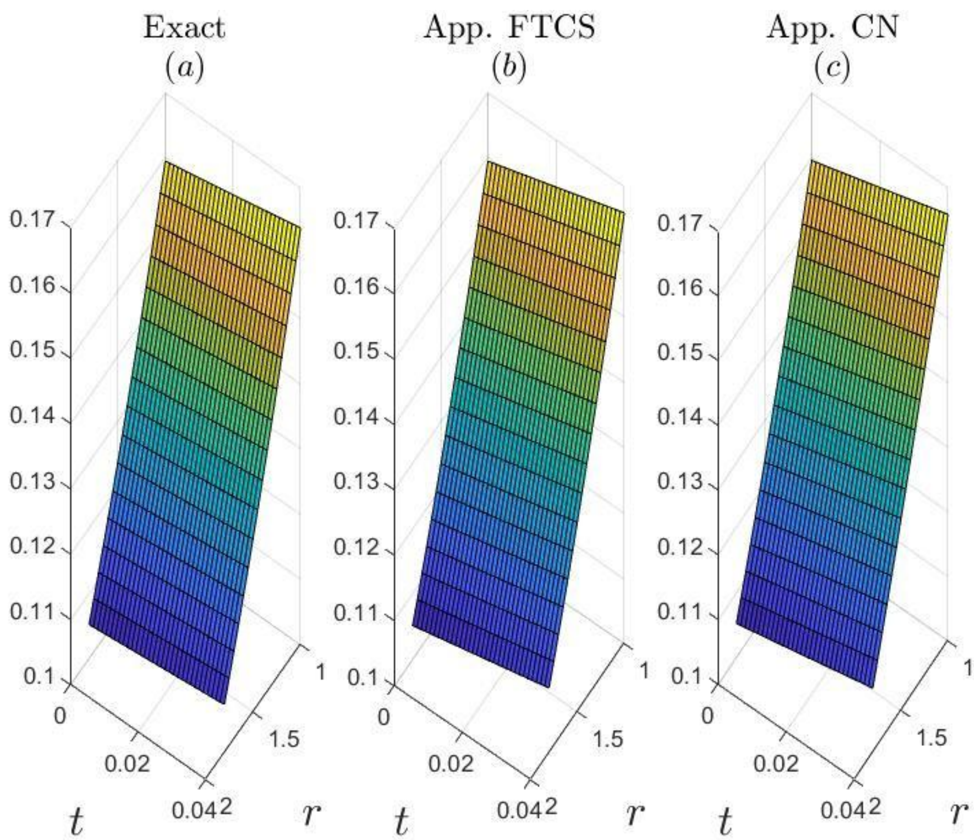

Figure 7: The exact and the approximate solution for Case 3 of Eq. (15) by the FTCS and CN methods.

Case 4: In this case, both r and t are changed. Figure (8), shows the stability of the solution by the FTCS and the CN methods when $n_1 = 4, n_2 = 2, N_1 = 0.2, N_2 = 0.02, d = 0.5,\ \hat{d} = 0.25$.

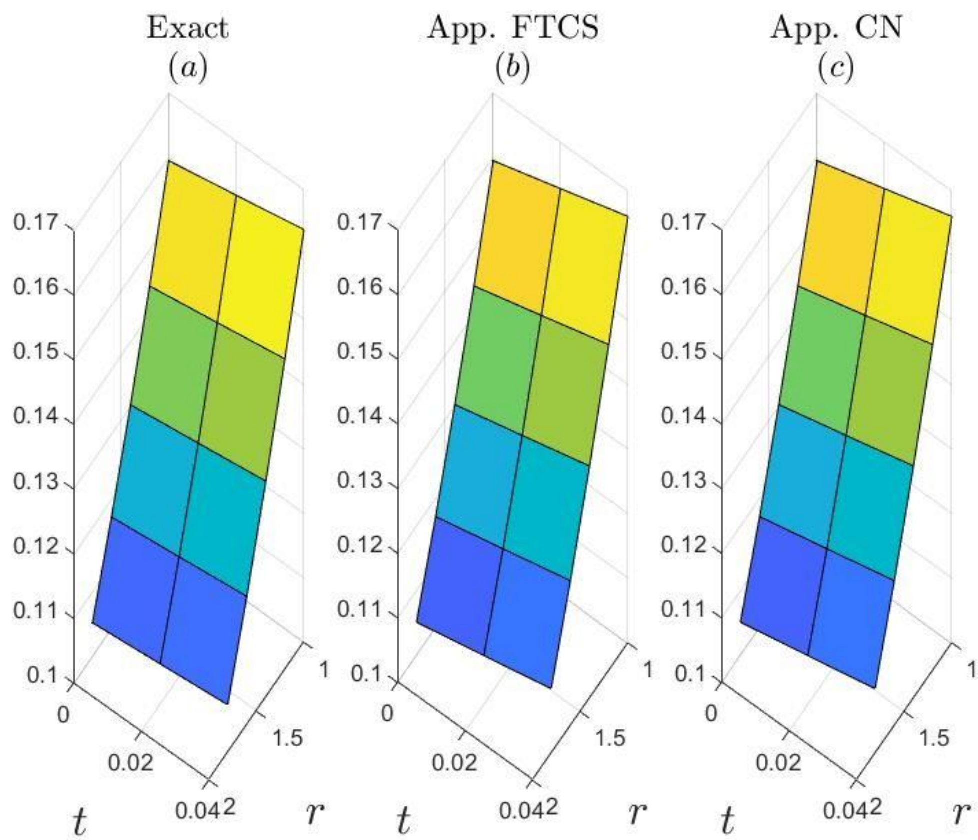

Figure 8: The exact and the approximate solution for Case 3 of Eq. (15) by the FTCS and CN methods.

4 Conclusions

The results of this article is briefly summarized in the following steps. The first step is employing the FTCS and CN methods to solve different types of the SLNWS equation. These methods transform the continuous SLNWS into a DNWS-FTCS or DNWS-CN equation depending on the special formula for each method. This transformation deals with system of algebraic equations that is easier to compute the result for two Examples which is the second step.

In Example (1), four cases are displayed, in these four cases, the solution of the CN method is stable for all points, while the FTCS method has a problem in some points. The details of these cases are: in Case 1, the convergence conditions for the FTCS and the CN methods read $d = 1.08$ and $\hat{d} = 0.54$, respectively. In this case the FTCS method has unstable solution, however, the CN method is more stable see Figure (1). Also, Table (1), shows the least square error for both methods at $t = 0.0225$. In Case 2, the solution of the FTCS method becomes stabler than in Case 1 (see Figures (2)). In this case, the step size of r is changed, this changed leads to change the convergence conditions for the FTCS and the CN methods to be $d = 0.48$ and $\hat{d} = 0.24$, respectively. In Case 3, the step size of t is changed, this leads to have stable solution for both methods with $d = 0.476$ and $\hat{d} = 0.238$ (see Figures (3)). In Case 4, the step sizes r and t are changed. This leads to have stable solution for both methods with $d = 0.5$ and $\hat{d} = 0.25$ (see Figures (4)).

In Example (2), four cases are demonstrated and we did the same technique to check the stability of the solution for both methods. In Case 1, Table (2) shows the least square error for both methods for all values of r at $t = 0.04$. The four Cases (1-4), have stable solution (see Figures (5-8) with small error.

In conclusion, the FTCS and the CN methods are used to find a suitable solution for the SLNWS equation. Through this work, we conclude, the FTCS method has a stable solution when $d < 0.5$, while, the CN method is always stable without any condition.

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