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Process Control Chart Using Bayesian Mewma On Steel Manufacturing

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Abstract: Exponentially weighted moving average (EWMA) control chart is a statistical method of monitoring the process stability and applies exponentially decreasing weights to prior data observations. Compared with traditional Shewhart control charts, this approach is more sensitive in detecting small shifts in the process mean. When you really need to detect small deviations as soon as possible, the EWMA control chart is ideal. An extended version of the EWMA has been developed to simultaneously monitor multiple linked quality characteristics (Multivariate EWMA (MEWMA) control chart). Such univariate charts might miss the detection of multivariate shifts and this can be referred to MEWMA, since it is based on considering process behavior as a whole by taking into account the relationships among their elements. Further research led to the development of the Bayesian MEWMA control chart, which incorporates historical data or expert views into the monitoring process. By changing the process parameter probability distributions as fresh data grows available, the Bayesian method boosts the flexibility and sensitivity to changes of the chart. In the steel manufacturing sector, When there is a shortage of data or when it is desirable to add prior information, Bayesian inference is very useful. Both preserving high product quality and cutting down on errors are essential for customer satisfaction and operational efficiency. The MEWMA chart and other complementary multivariate control charts are useful for tracking several quality attributes, but they frequently lack the flexibility required for dynamic operations. This study introduces a Bayesian approach to the MEWMA control chart, which leverages Bayesian updating to improve sensitivity to small shifts in quality metrics while accommodating parameter uncertainty.

Keywords: Statistical Process Control (SPC), Process Mean, EWMA, MEWMA, Bayesian Approach.

مخطط مراقبة العمليات باستخدام MEWMA بايزي في صناعة الصلب

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المستخلص: المتوسط المتحرك الموزون أسياً (EWMA) هو مخطط تحكم إحصائي لمراقبة استقرار العملية ويطبق أوزانًا متناقصة أسيًا على الملاحظات السابقة للبيانات. مقارنة بمخططات التحكم التقليدية لشيوهارت، فإن هذا النهج أكثر حساسية في اكتشاف التغيرات الصغيرة في متوسط العملية. عندما تحتاج بالفعل إلى اكتشاف الانحرافات الصغيرة في أقرب وقت ممكن، يكون مخطط التحكم EWMA مثاليًا.. تم تطوير نسخة موسعة من EWMA لمراقبة خصائص الجودة المتعددة المرتبطة في وقت واحد (مخطط مراقبة EWMA متعدد المتغيرات (MEWMA)). قد تفوت مثل هذه المخططات أحادية المتغيرات اكتشاف التحولات متعددة المتغيرات ويمكن الإشارة إلى ذلك بـ MEWMA، لأنه يعتمد على النظر في سلوك العملية ككل من خلال الأخذ في الاعتبار العلاقات بين عناصره. أدى المزيد من البحث إلى تطوير مخطط مراقبة MEWMA بايزي، والذي يدمج البيانات التاريخية أو آراء الخبراء في عملية المراقبة. من خلال تغيير توزيعات احتمالية معلمات العملية مع نمو البيانات الجديدة المتاحة، تعزز الطريقة البايزية المرونة. في قطاع تصنيع الصلب، عندما يكون هناك نقص في البيانات أو عندما يكون من المرغوب إضافة معلومات مسبقة، فإن الاستدلال البايزي يكون مفيدًا للغاية. يعد الحفاظ على جودة المنتج العالية وتقليل الأخطاء أمرًا أساسيًا لرضا العملاء والكفاءة التشغيلية. يعد الرسم البياني MEWMA والرسوم البيانية متعددة المتغيرات المكملة الأخرى مفيدة لتتبع العديد من سمات الجودة، ولكنها غالبًا ما تفتقر إلى المرونة اللازمة للعمليات الديناميكية. تقدم هذه الدراسة نهجًا بايزيًا للرسم البياني MEWMA، والذي يستفيد من التحديث البايزي لتحسين الحساسية للتغيرات الصغيرة في مقاييس الجودة مع مراعاة عدم اليقين في المعلمات.

الكلمات المفتاحية: التحكم الإحصائي في العملية (SPC)، متوسط العملية، EWMA، MEWMA، النهج البايزي.

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Introduction

The Shewart chart, EWMA chart and CUSUM chart are three control charts that are frequently used to identify changes in a series of independent normal data with a similar variance originating from a specific process. Control of statistical quality. There is a wide range of research on approaches. It is in its own right. Dr. Walter Shewhart developed statistical quality control in 1924, which is essential to success in contemporary industry, which places a strong focus on cutting costs while simultaneously enhancing quality. He understood that there would always be variance in the final items produced throughout a manufacturing process. Hotelling worked extensively on multivariate statistical control systems between 1930 and 1940. The Hotelling's T^2control chart is one of the most widely used tools in multivariate statistical process control [5][13] .

Multivariate control charts, such as the MEWMA chart or the Hotelling T^2 chart, are frequently used to monitor processes that in corporate many quality characteristics. Multivariate process monitoring is made more challenging by the addition of several unknown variables that require computation. Because it lends more weight to recent data, the MEWMA control chart is a useful tool for spotting subtle changes. sensitivity to even the smallest changes. Nevertheless, MEWMA depends on preset parameters, which could not be sufficiently flexible for processes with variable fluctuations, especially in intricate industrial environments like steel manufacturing. Additionally, a Bayesian approach enhances the MEWMA framework and makes the model more sensitive to process variations by including prior information and dynamically changing parameters in reaction to incoming data [3][12]

A Bayesian MEWMA control chart is used in this work to monitor and evaluate key quality parameters in steel manufacturing. Through the integration of Bayesian updating with the conventional MEWMA architecture, the proposed method improves detection accuracy and reduces false alarms while offering real-time reaction to process changes. By offering more reliable multivariate data monitoring, the case study illustrates how the Bayesian MEWMA approach may enhance quality control in complex industrial environments [18]

Gathering data, estimating parameters via Bayesian updating, computing MEWMA statistics, and determining adaptive control limits are the essential processes in implementing a Bayesian MEWMA control chart in the steel industry. The system can adapt dynamically to multivariate process variations, which are frequently seen in the steel industry, by adding Bayesian inference to traditional MEWMA control charts [14].

1st: Method:

Suppose a (multivariate) normally distributed process's output contains an in-control mean vector (μ_0) and a covariance matrix (Σ_0). Although we often take them for granted, as we will see in the next part, this is a significant assumption that isn't always accurate. A size-based subgroup k, is created by placing the vector-valued output at time t, with observed values $\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_k$, If we calculate the sample mean vector [3][4] .

$$\bar{\mathbf{x}}_t = \frac{1}{k}\sum_{t=1}^{k}\mathbf{x}_t \quad (1)$$

$$Z_t = \lambda X_t + (1-\lambda)Z_{t-1} \qquad t = 1,2,3,4, \ldots \ldots .., n$$

(٢)

Where, λ : Is a constant and weight $0 < \lambda \leq 1$

$$Z_0 = \mu_0$$

(٣)

Occasionally, the initial value of the EWMA is the mean of the preliminary data, such that

$$Z_0 = \bar{X}$$

(٤)

The EWMA Z_t may be shown to be a weighted average of all prior sample means by replacing w_{t-1} on the right-hand side of equation (2) with Z_t, Z_{t-1}: Are exponentially weighted moving averages at time t , t-1 .

X_t: is the t^{th} observation value $\sim N(0, \sigma^2)$

For showing that EWMA Z_t is weighted average of all previous sample means, therefore by replacing of the Z_{t-1} of equation (2) to get

$$Z_t = \lambda X_t + (1-\lambda)Z_{t-1} \qquad (0 \leq \lambda < 1)$$

(٥) The center line and the control limits for EWMA charts are given by:

$$UCL = \mu_0 + L\sigma\sqrt{\left(\frac{\lambda}{2-\lambda}\right)[1-(1-2)^{2t}]} \quad (6)$$

$$Centrel\ Line = \mu_0$$

(7)

$$LCL = \mu_0 - L\sigma\sqrt{\left(\frac{\lambda}{2-\lambda}\right)[1-(1-2)^{2t}]}$$

(8)

The Multivariate Exponentially Weighted Moving Average (MEWMA) model has the statistic:(1)

$$T_t^2 = z_t'\Sigma_{zt}^{-1}z_t \quad (9)$$

Where

Z_t : is t^{th} EWMA vector

x_t : is i^{th} vector of observation $t = 1,2,3, \ldots ., n$

Z_0 : is a vector of variable values derived from past data.

λ : Is a constant and weight $0 < \lambda \leq 1$.

then we would compute the Hotelling T^2 statistic as

$$T_t^2 = (\bar{\mathbf{x}}_t - \mu_0)'\Sigma_0^{-1}(\bar{\mathbf{x}}_t - \mu_0)$$

(10)

Which takes into consideration the covariances among the elements of $\bar{\mathbf{x}}$, is measured by the statistic T_t^2. When Hotelling T_t^2 is large, it means that the process mean vector is not $\boldsymbol{\mu}_0$. For the Hotelling T^2 chart, we may select the upper control limit h in order to achieve a desirable in-control average run length (ARL), which is the predicted number of subgroups until a signal is raised. Because successive Hotelling T_t^2 values have a fixed probability of surpassing h and are stochastically independent, the run duration has a geometric distribution. The Hotelling T^2 chart's ARL is hence,

$ARL = \frac{1}{P(T^2>h)}$.

Using the χ^2distribution (when the process is in-control,$\boldsymbol{\mu} = \boldsymbol{\mu}_0$) or the noncentral χ^2distribution (when the process is not in-control, $\mu \neq \mu_0$), it is easy to calculate the probability in the denominator.The probability in the denominator can be readily computed using the χ^2 distprocess is in-control, $\boldsymbol{\mu} = \boldsymbol{\mu}_0$) or the noncentral χ^2 distribution (when the process is not in-control, $\mu \neq \mu_0$) [15].

(MEWMA) chart as an alternative to the Hotelling T^2 chart. This entails calculating a weighted average of the previous MEWMA statistic and the current $\overline{\mathbf{x}}_t$ at each time point; that is $Z_t = \lambda X_t + (1-\lambda)Z_{t-1}$

where $\mathbf{Z}_0$ is defined arbitrarily (usually $\mathbf{Z}_0 = \mu_0$) and λ is a smoothing constant that satisfies $0 < \lambda \leq 1$. Note that if $\lambda = 1$, then $\mathbf{Z}_t = \overline{\mathbf{x}}_t, t = 1,2, \dots$ and the chart becomes the Hotelling T^2 chart. A signal is raised on the MEWMA chart whenever we have [15]

$$T^2_{\text{MEWMA},t} = \frac{(2-\lambda)}{\lambda}(\mathbf{Z}_t - \mu_0)^T \Sigma_0^{-1}(\mathbf{Z}_t - \mu_0) > UCL = \chi^2_{k,1-\alpha} \quad (11)$$

where the upper control limit $UCL = \chi^2_{k,1-\alpha}$ is resolved to provide the intended in-control ARL. The MEWMA statistic, which is a weighted average of recent and historical observations, accumulates data on the mean vector's value [15].

$$UCL = \chi^2_{k,1-\alpha} \quad (12)$$

Where $\chi^2_{k,1-\alpha}$ is the upper 100(1−α) %100 percentile of the chi-square distribution with k degrees of freedom. The choice of $\boldsymbol{\alpha}$ alpha determines the false alarm rate; a common choice is α=0.0027, corresponding to a 99.73% confidence level [13].

For process control, Bayesian method. For the in-control mean and covariance matrix, he used the following assumptions: normal and inverse Wishart priors [15]

$$\Sigma_0^{-1} \sim \text{Wishart}_d(v_0, \mathbf{h}_0) \quad \text{and} \quad \boldsymbol{\mu}_0 \Big| \Sigma_0 \sim \text{N}\left(\mathbf{m}_0, \frac{1}{n_0}\Sigma_0\right).$$

If we have a sample $\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_n$ from the process when it is assumed to be in control, that is, with a $\text{N}(\boldsymbol{\mu}_0, \Sigma_0)$ distribution, and if we let $\overline{\mathbf{x}} = \sum_{t=1}^{n} \mathbf{x}_t / n$ and

$$\mathbf{v}_x = \frac{1}{n-1}\sum_{t=1}^{n}(\mathbf{x}_t - \overline{\mathbf{x}})(\mathbf{x}_t - \overline{\mathbf{x}})^T, \quad (13)$$

then the posterior distributions are Σ_0^{-1} | data ~ Wishart $_d(v_1, \mathbf{h}_1)$

And data, where the posterior parameters are

$$v_1 = v_0 + (n-1),$$

$$\mathbf{h}_1 = \mathbf{h}_0 + (n-1)\mathbf{v}_x + \frac{n_0 n}{n_0 + n}(\overline{\mathbf{x}} - \mathbf{m}_0)(\overline{\mathbf{x}} - \mathbf{m}_0)^T,$$

$$n_1 = n_0 + n,$$

And $\mathbf{m}_1 = \frac{n_0\mathbf{m}_0 + n\overline{\mathbf{x}}}{n_0 + n}$.

Choosing the incorrect and no informative priors by taking $n_0 \to 0, v_0 \to 0, \mathbf{m}_0 = \mathbf{0}$, and $\mathbf{h}_0 = \mathbf{0}$,

then the posteriors are proper and are of the form (12) and (13) with

$$v_1 = n - 1$$

$$\mathbf{h}_1 = (n-1)\mathbf{v}_x$$

$$n_1 = n$$

$$\mathbf{m}_1 = \overline{\mathbf{x}}.$$

Additionally, we explained how to deal with the situation in which the preliminary sample is composed of a collection of N subgroups, each of size K.

The MEWMA statistic, which is specified in (11), has a conditional distribution with k units in each subgroup [15].

$$\mathbf{Z}_t \Big| (\mu_0, \Sigma_0) \sim N_d\left((1-\lambda)^t \mathbf{Z}_0 + [1 - (1[\lambda)^t]\boldsymbol{\mu}_0, \frac{\lambda[1-(1-\lambda)^{2t}]}{k(2-\lambda)}\Sigma_0\right).$$

For large values of t this distribution is $\mathbf{Z}_t \,\Big|\, (\mu_0, \Sigma_0) \sim \mathrm{N}_d\left(\mu_0, \frac{\lambda}{k(2-\lambda)}\Sigma_0\right)$.

Let $\mathcal{D} = \{n_0, \mathbf{m}_0, v_0, \mathbf{h}_0; k, \bar{x}, \mathbf{v}_\chi\}$ reflect the data from both the sample and the previous.

The predictive distribution of $\mathbf{e}_t$ for large value of t is then $\mathbf{Z}_t \,\Big|\, \mathcal{D} \sim \mathrm{St}_d\left(v_1 - d + 1, E_{z_t}, \frac{v_1-d-1}{v_1-d+1}V_{\mathbf{z}_t}\right)$,

where

$E_{\mathbf{e}_t} = E(\mathbf{e}_t \mid \mathcal{D})$,

and

$$\mathbf{V}_{\mathbf{z}_t} = V(\mathbf{z}_t \mid \mathcal{D}) = \left(\frac{1}{n_1} + \frac{\lambda}{k(2-\lambda)}\right)\hat{\Sigma}, \tag{14}$$

and $\mathrm{St}_d(v, \mathbf{m}, \mathbf{v})$ is the d-dimensional t distribution with v degrees of freedom, center $\mathbf{m}$ and covariance matrix $\mathbf{v}$. Here $\hat{\Sigma}$ is the posterior mean of Σ_0, that is, $\hat{\Sigma} = E(\Sigma_0 \mid \mathcal{D}) = \frac{1}{v_1-d-1}\mathbf{h}_1$.

The purpose of the MEWMA chart is to track a multivariate process's mean vector and determine when a change has taken place. The data

$$w_t = \frac{1}{d}\frac{v_1 - d + 1}{v_1 - d - 1}\left(\mathbf{z}_t - E_{\mathbf{e}_t}\right)^T \mathbf{V}_{\mathbf{z}_t}^{-1}\left(\mathbf{z}_t - E_{\mathbf{z}_t}\right) \tag{15}$$

has an $F(d, v_1 - d + 1)$ distribution given the information contained in $\mathcal{D}$. If we take the noninformative and improper priors given in (14) then the statistic from (15) becomes

$$w_t = \frac{\frac{n-d}{d(n-1)}}{\frac{1}{n} + \frac{\lambda}{k(2-\lambda)}}(\mathbf{z}_t - \overline{\mathbf{x}})^T \mathbf{v}_\chi^{-1}(\mathbf{z}_t - \overline{\mathbf{x}}) \tag{16}$$

which has an $F_{d,n-d}$ distribution. The Bayesian MEWMA control chart based on the statistic in (15) or (16) then signals when [10]. $w_t > F_{1-\alpha;d,n-d}$ or $w_t > \chi^2_{k,1-\alpha}$ or equivalently, when

$$T^2_{\mathrm{MEWMA},t} = \frac{(2-\lambda)}{\lambda}(\mathbf{z}_t - \overline{\mathbf{x}})^T \mathbf{v}_x^{-1}(\mathbf{z}_t - \overline{\mathbf{x}}) > \frac{\frac{k(2-\lambda)}{n\lambda}}{\frac{n-d}{d(n-1)}} F_{1-\alpha;d,n-d} \cdot = \chi^2_{k,1-\alpha} \tag{17}$$

For a MEWMA chart monitoring a process with k variables, the UCL is determined based on the Hotelling's T^2 statistic, which, under the assumption of multivariate normality, follows a chi-square distribution with k degrees of freedom. The UCL is then set as:

$$UCL = \chi^2_{k,1-\alpha}$$

Where $\chi^2_{k,1-\alpha}$ is the upper 100(1−α) %100 percentile of the chi-square distribution with k degrees of freedom. The choice of $\boldsymbol{\alpha}$ alpha determines the false alarm rate; a common choice is α=0.0027, corresponding to a 99.73% confidence level [10].

Then claimed that "Given $\boldsymbol{\mu}$ and Σ and an in-control statistical process. The MEWMA chart accumulates data at every sampling step, in contrast to the Hotelling T^2 chart. If the prior result was "in control," the data are destroyed and a new sample is obtained at the specified time; the in/out of control decision is then based solely on this new sample. On the other hand, the data used in the Hotelling T^2 graphic comes from the present sample. Pignatiello and Runger (1990) and Bersimis et al. (2007) state that these statements are true for the Hotelling T^2chart but not for the MEWMA chart or any of the multivariate cumulative sum (CUSUM) charts. Regardless of whether the process is under control or not, the exceedance probability varies with time, making the entire idea of establishing α and then analyzing the exceedance probability that is, the possibility that w_t exceeds the UCL faulty. The need that the control chart not have signaled previously in order for it to signal for the first time at any given time is likewise ignored in this case [9].

2nd: Case Study:

Steel manufacturing is characterized by complex and high-dimensional quality control challenges, where multiple interrelated variables (e.g., chemical composition, tensile strength, and dimensional properties) must be monitored simultaneously. Using a Bayesian MEWMA control chart in steel manufacturing provides a number of benefits for addressing these issues. The Bayesian approach lowers the control constraints by allowing them to respond to both historical data and present process changes. providing more precise quality shift detection and the potential for false positives. Because they can respond to shifting production conditions and account for process drifts more effectively than traditional methods, Bayesian adaptive control charts are helpful in steel manufacturing applications [14].
The manufacture of steel generates a wide range of multivariate data that indicate various quality features necessary to preserve the integrity of the product and satisfy industry standards. significant parameters related to dimensional accuracy, chemical composition, and mechanical properties. Below is a detailed description of each of these data items for a typical steel manufacturing process.

3rd: Chemical Composition

- **Carbon (C):** The amount of carbon in steel affects its malleability, tensile strength, and hardness. Strength is increased by more carbon content, while weldability is decreased.
- **Silicon (Si)**: Silicon acts as a deoxidizer and strengthens the steel matrix. Excessive silicon can lead to brittleness.
- **Manganese (Mn)**: Manganese improves strength, toughness, and hardness while reducing brittleness.

These elements are measured in percentages, and strict limits are maintained to ensure the final steel product meets required specifications.

4th: Application of Bayesian MEWMA and MEWMA control chart:

The use of Hotelling's T^2 in MEWMA and Bayesian MEWMA charts is shown in Table (1). When the T^2 value above the Upper Control Limit (UCL), a possible out-of-control point is indicated by an asterisk (*). The UCL for Bayesian MEWMA is 13.93, which is a little higher and more conservative than the UCL for ordinary MEWMA, which is 13.382. As demonstrated, Bayesian MEWMA is more selective, reflecting its cautious tendency, whereas MEWMA tends to be more sensitive, highlighting more points. This comparison aids in assessing each monitoring method's robustness and efficacy.

Table (1): Apply Hotelling's T^2 MEWMA and Bayesian MEWMA

Observation	C%	Si%	Mn%	Hotelling's T^2 MEWMA	Hotelling's T^2 **Bayesian** MEWMA
1	0.349	0.321	1.1	* 21.731	*19.558
2	0.349	0.427	1.3	11.026	9.023
3	0.344	0.456	1.15	* 15.741	11.422
4	0.347	0.489	1.23	* 18.966	12.706
5	0.348	0.498	1.32	* 19.818	13.222
6	0.359	0.391	1.17	* 16.671	11.042
7	0.358	0.385	1.28	11.199	7.503
8	0.369	0.463	1.29	6.161	3.962
9	0.398	0.422	1.32	0.406	0.326
10	0.397	0.4324	1.15	2.495	2.067
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
96	0.348	0.4301	1.33	4.438	3.216
97	0.357	0.459	1.34	4.762	3.998
98	0.359	0.446	1.28	3.415	3.045

99	0.392	0.493	1.25	5.346	4.531
100	0.361	0.4163	1.33	4.116	3.493

This table and the accompanying control chart present a comparative analysis of the Multivariate Exponentially Weighted Moving Average (MEWMA) Hotelling T^2 and their Bayesian MEWMA counterparts across 100 observations. The Hotelling T^2, a measure of multivariate deviation from the process mean, is used in both control charts to monitor shifts in the underlying process. In a typical MEWMA chart, any value above the Upper Control Limit (UCL) of 13.382 suggests that things may be out of control. By modifying the control statistics based on dynamic estimations or prior information, the Bayesian MEWMA graphic produces a somewhat more conservative 13.93 UCL. Observations that exceed these bounds are marked as out-of-control points, indicated by an asterisk in the table, and highlighted on the figures. The chart clearly shows where each strategy picks up signals by graphically displaying both control lines over time. Standard MEWMA tends to detect more observations as out-of-control due to its sensitivity, but Bayesian MEWMA often responds more conservatively, reducing the likelihood of false alarms.

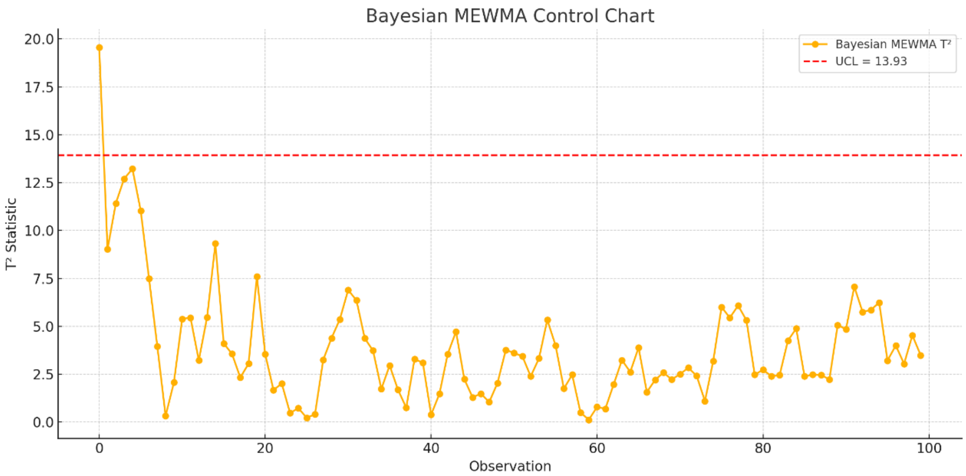

Figure (1): Bayesian MEWMA control chart

The T^2 statistic for 100 observations is shown in the Bayesian MEWMA Control Chart to track process stability. The majority of data fall below the red dashed line, which represents the upper control limit (UCL) of 13.93, suggesting a largely stable process. Some preliminary findings surpass the UCL, indicating possible uncontrollable circumstances at the beginning, but the process eventually stabilizes. The T^2 values are shown by the orange line, which exhibits fluctuation but generally stays within control limits.

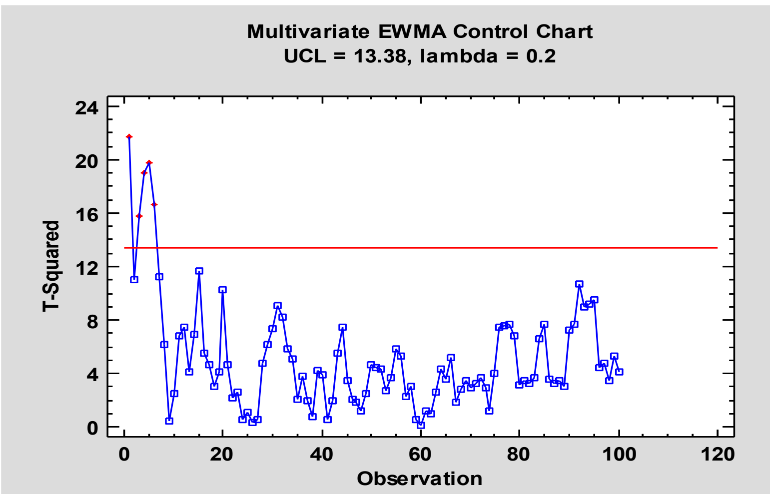

Figure (2): Multivariate EWMA control chart

The Multivariate EWMA Control Chart, which has a smoothing parameter (lambda) of 0.2 and an upper control limit (UCL) of 13.38, presents T-squared values across 100 observations to track process stability. A few early points show possible out-of-control scenarios at the start by exceeding the control limit. After then, though, the process stabilizes, and most observations stay well below the UCL, indicating continued, efficient regulation.

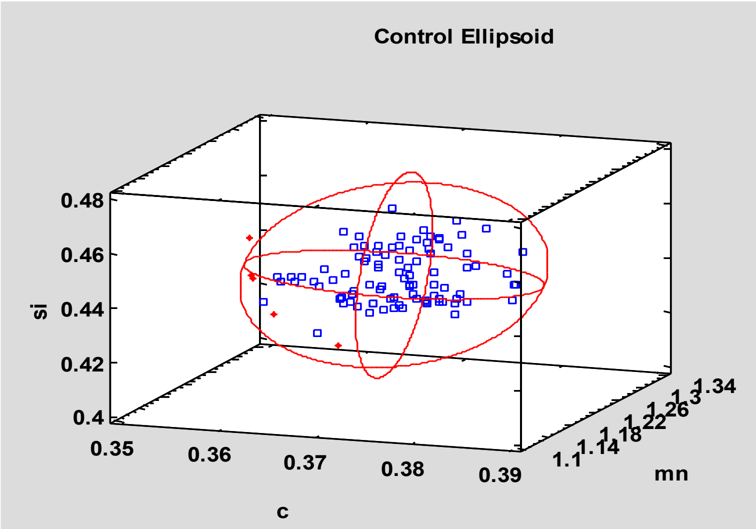

Figure (3): Control Ellipsoid for components

The link between three variables, carbon (C), silicon (Si), and manganese (Mn), is visualized by this three-dimensional control ellipsoid graphic. The multivariate control zone is shown by the red ellipsoid, which, according to statistical confidence bounds, usually shows the range of normal variation. Red stars indicate outliers that fall beyond the ellipsoid, indicating possible abnormalities or variations in the process, while blue squares represent sample data points. Plots of this kind might be used to spot unusual compositions in manufacturing or quality control procedures.

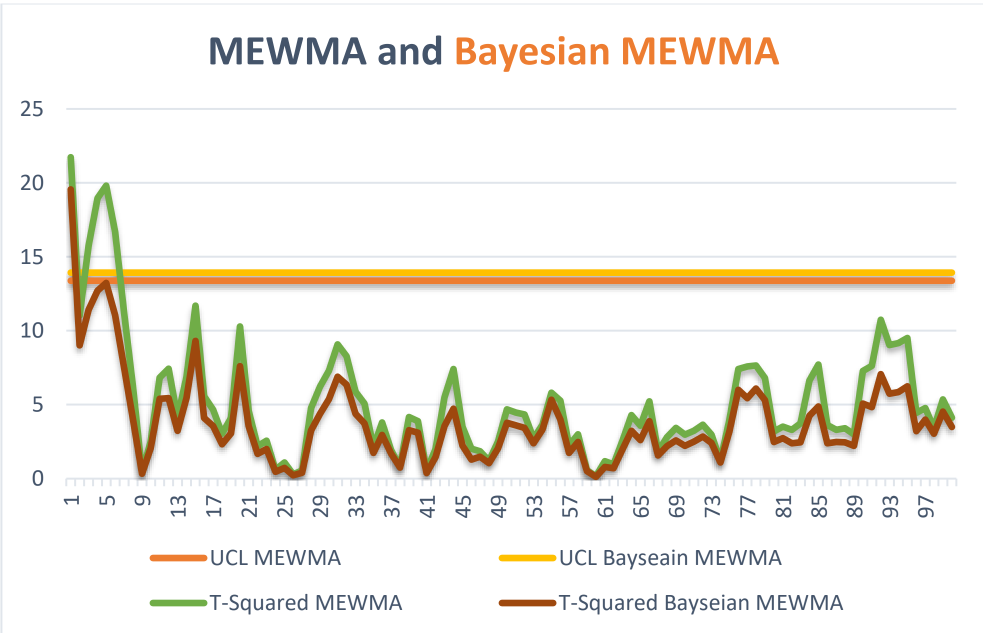

Figure (4): Compares MEWMA and Bayesian MEWMA control charts

This line graph contrasts the Bayesian MEWMA control technique with the Multivariate Exponentially Weighted Moving Average (MEWMA) control method over 100 observations. It displays two important metrics. the Upper Control Limits (UCLs) and the T-Squared statistic, which is used to track process variance. The orange and yellow lines display the matching UCLs, while the brown and green lines indicate the T-Squared values for Bayesian MEWMA and MEWMA, respectively. With Bayesian MEWMA often exhibiting smoother behavior and somewhat reduced variability, the chart aids in evaluating how well each technique identifies possible process alterations.

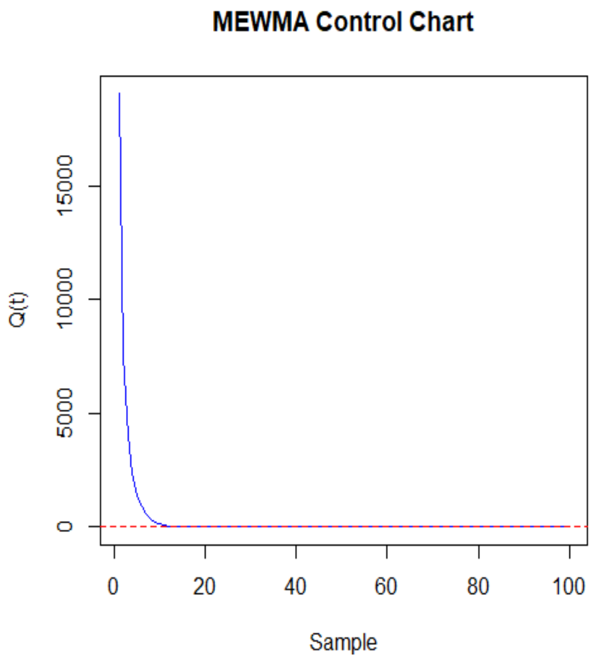

Figure (5): ARL MEWMA Control chart

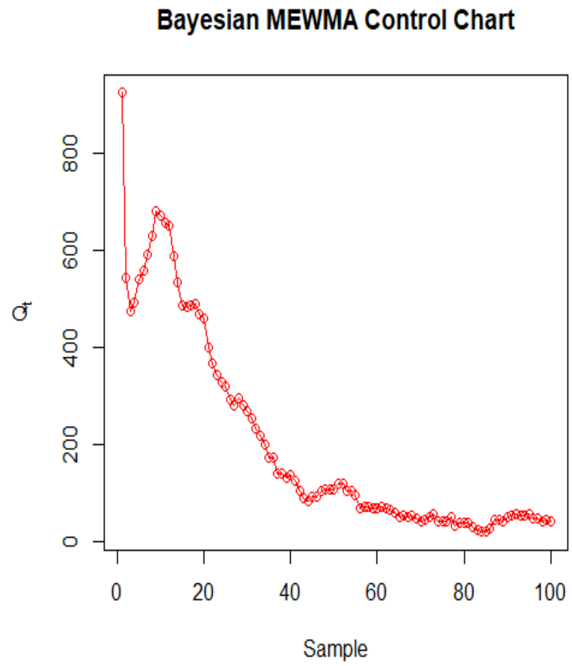

Figure (6): ARL BMEWMA Control chart

The ARL performance of Bayesian MEWMA and Classical MEWMA control charts is contrasted in the two charts. With unstable, high starting values, the Classical MEWMA exhibits a very quick decline in Q(t), signaling nearly instantly (ARL = 1-2). The Bayesian MEWMA provides faster but smoother detection, however the first signal often shows up between Samples 2–4 (ARL = 3–5). Bayesian MEWMA is a better option when previous knowledge is available or controlled false alarms are crucial since it provides a more reliable and adaptable response overall.

5th: Conclusion:

The Bayesian Multivariate Exponential Weighted Moving Average (MEWMA) control chart offers a good basis for spotting changes and maintaining process stability when applied in steel manufacturing operations. Using Bayesian integration and prior knowledge approaches to raise sensitivity to both tiny and significant changes in multivariate data, the MEWMA chart is particularly helpful in complex industrial settings such as steel production. According to the study, the Bayesian MEWMA method may effectively identify out-of-control circumstances even in cases when correlated variables are present a condition common in the steel sector. The findings show how fast and precisely it can identify issues and respond with corrections. This guarantees higher process dependability, decreases mistakes, and raises output quality. In conclusion, future studies may incorporate real-time data analysis and expand the method to dynamic process scenarios in order to guarantee adaptability to changing industrial environments. Bayesian techniques improve the effectiveness of multivariate process monitoring even when typical MEWMA control charts are utilized, particularly in intricate industrial settings like steel manufacture. The application of prior data and sophisticated sampling procedures enhances the sensitivity, adaptability, and overall process control of Bayesian MEWMA charts.

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