Using Simulation to Compare Scale Parameter and Reliability function for two parameter's Frechet Distribution

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Abstract

This paper deals with comparing different estimators of scale parameter and reliability function for two parameters Frechet distribution (λ,θ), when (λ) is shape parameter considered known, while (θ) is scale parameter and reliability function [R(t)] is estimated by different methods. These methods are least square, L – moment, proposed Bayes estimator under general entropy loss function and finally the estimator using maximum likelihood of ordered observations. All the derivation required are explained and the results of simulation given in tables.

Keywords: shape parameter, scale parameter, $\hat{\theta}_{MLEO}$ under quadratic loss function, $\hat{\theta}_{Bayes}$ under general entropy loss function.

المستخلص

تناول البحث مقارنة مقدرات مختلفة لمعلمة القياس ودالة المعولية لنموذج (Frechet) ذي المعلمتين (θ وλ) حيث أن (λ) تمثل معلمة الشكل والتي أعتبرت معلومة، بينما تم تقدير معلمة القياس (θ) ودالة المعولية بعدة طرق مختلفة، وهذه الطرق هي: طريقة المربعات الصغرى، وطريقة (L - Moment)، وطريقة (Proposed Bayes) بأستخدام طريقة الأمكان الأعظم، وتم وضع هذه المناقشات وتحليلاتها ضمن جداول.

1. Introduction

The reliability of any machine or device, is the possibility of work continuity of the machine without breaking down of failure during a period $[o, t)$ without reducing reliability applications in industrial fields. Reliability have been studied under various applications by [Moore, A. H. and Bilikam (1978) and Chare, D. J. (1985)]. Rytgaard (1990) estimated reliability function of Pareto. Many other researchers like [Hossain, A. M. Zimmer, W.J. (2000), and Nadaraj, S. and Kotz (2003)], Erefaie, A. and Parsian (2005), estimated parameters and reliability of Pareto, also Saleh, S.M. (2006), estimated reliability function of Pareto also. Farhan, Hla Salman (2007), estimated survival function of Lomax distribution in case of censored data (II). Also Pandy H. and Raol A. K. (2009), gives different Bayes estimator. Singh G. B.P., S.K.V. and R.D. (2011) estimated shape and reliability function when the shape parameter is unknown. Setiya Parul, Kumar Vinol (2013) estimated shape parameter reliability and hazard function of Pareto type (I) model. Here we continue the work under reliability field by comparing different Bayes estimators of scale parameter (θ) and reliability function for two parameters Frechet distribution. The comparison between likelihood and Bayes estimators using squared error loss function, and general entropy function has been done through simulation.

Definition

The $p.d.f$ of the random variable (x) that have two parameters Frchet;

$x \sim Frechet\ (\lambda;\theta)$

Defined by;

$$f(x;\lambda,\theta) = \frac{\lambda}{\theta}\left(\frac{\theta}{x}\right)^{(\lambda+١)} e^{-\left(\frac{\theta}{x}\right)^{\lambda}} \qquad x,\lambda,\theta > ٠ \tag{1}$$

While the C. D. F is;

$$F(x) = e^{-\left(\frac{\theta}{x}\right)^{\lambda}} \qquad x,\lambda,\theta > ٠ \tag{2}$$

Definition of Reliability Function

The reliability function $[\ R(t)]$ defined as;

$$R(t) = pr(T > t) \tag{3}$$

Which means that the item is still working at time (t), it is defined on the domain $\{t \in [٠,\infty)\}$, so $\{R(t) \in [٠.١)\}$ and some of its properties are;

a. $R(٠) = ١$

b. $\lim\limits_{t\to\infty} R(t) = ٠$

c. $R(t_a) \geq R(t_b) \quad \Leftrightarrow \quad t_a \leq t_b$

2. Methods of Estimation

We consider the estimation of parameter (θ) by using the following methods;

2.1 Least Square Method

This method is well known in mathematics, and statistics, it depend on estimating parameters by minimizing the total sum square of deviation between true values and estimated values. In our subject (i.e frechet distribution, we depend on the C.D.F;

$$F(x) = e^{-\left(\frac{\theta}{x}\right)^{\lambda}}$$

And the minimize;

$$T = \sum_{i=١}^{n} \left(e^{-\left(\frac{\theta}{x}\right)^{\lambda}} - \frac{i}{n+١} \right)^{٢} \qquad (٤)$$

Which is the difference between observed and estimated as follows;

$$\because F(x) = e^{-\left(\frac{\theta}{x}\right)^{\lambda}}$$

$$\ln F^{-١}(x) = \left(\frac{\theta}{x}\right)^{\lambda}$$

$$\ln\left[\ln F^{-١}(x)\right] = \lambda \ln \theta - \lambda \ln x_i$$

$$y_i = \lambda \ln \theta - \lambda \ln x_i$$

$$\overline{x} = \frac{\sum_{i=١}^{n} \ln x_i}{n}$$

$$\overline{y} = \frac{١}{n} \sum_{i=١}^{n} \ln\left[\ln F^{-١}(x_i)\right] \qquad (٥)$$

Since the shape parameter (λ), is considered known, then we estimate only (θ) by least square as;

$$\hat{\theta}_{OLS} = e^{\left(\overline{y} - \frac{\overline{x}}{\lambda}\right)} \qquad (6)$$

2.2 L – Moment

This method depends on equating sample moment with population moment as;

$$\beta_r = \int_{٠}^{\infty} x \; F^{r}(x) f(x) dx \qquad (7)$$

And sample moment;

$$b_r = \frac{١}{n C_r^{n-١}} \sum_{i=١}^{n} C_r^{i-١} x_{(i)} \qquad (٨)$$

Solve (β_r) using the $p.d.f$ and C.D.F of Frechet, we obtain;

$$\beta_r = \frac{\theta}{(r+1)^{\frac{1}{\lambda}}} \; \Gamma\left(\frac{\lambda-1}{\lambda}\right) \tag{9}$$

Solve $[\beta_1 = b_1]$;

$$\frac{\theta}{(2)^{\frac{1}{\lambda}}} \; \Gamma\left(\frac{\lambda-1}{\lambda}\right) = \frac{1}{n(n-1)}\sum_{i=1}^{n}(i-1)x_{(i)}$$

$$\Rightarrow \; \hat{\theta}_{LMOM} = \frac{(2)^{\frac{1}{\lambda}}\sum_{i=1}^{n}(i-1)\,x_{(i)}}{n(n-1)\,\Gamma\left(\frac{\lambda-1}{\lambda}\right)} \tag{10}$$

Where; $x_{(1)} \le x_{(2)} \le \cdots \le x_{(n)}$ are the ordered observation.

2.3 Proposed Bayes Estimator

The second estimator of (θ) is the proposed Bayes estimator of (θ) under general entropy loss function which is defined by;

$$L_{GE}(\hat{\theta},\theta) = w\left[\left(\frac{\hat{\theta}}{\theta}\right)^{c} - c\;\ln\left(\frac{\hat{\theta}}{\theta}\right) - 1\right] \quad w > 0 \tag{11}$$

Since the risk function is expected loss;

$$R(\hat{\theta},\theta) = E\left[L_{GE}(\hat{\theta},\theta)\right] = \int_0^{\infty}\left[\left(\frac{\hat{\theta}}{\theta}\right)^{c} - c\;\ln\left(\frac{\hat{\theta}}{\theta}\right) - 1\right]h(\theta|x)\,d\theta \qquad w = 1$$

$$\frac{\partial R(\hat{\theta},\theta)}{\partial\hat{\theta}} = \int_0^{\infty}\left[\frac{c\,(\hat{\theta})^{c-1}}{\theta^{c}} - \frac{c}{\hat{\theta}}\right]h(\theta|x)\,d\theta$$

That means $(\hat{\theta})$ is Bayes estimator under general entropy loss function;

$$\hat{\theta}_{GE} = \left[\frac{1}{E(\theta^{c}|x)}\right]^{\frac{1}{c}} \tag{12}$$

Therefore under prior distribution of (θ) i.e;

$$[g(\theta) = k\theta^{c}] \tag{13}$$

We find the posterior distribution of $h(\theta|x)$;

$$h_1(\theta|x) = \frac{\lambda\theta^{n\lambda+c}\; e^{-\theta^{\lambda}T}\; T^{\left(n+\frac{c+1}{\lambda}\right)}}{\Gamma\left(n+\frac{c+1}{\lambda}\right)} \tag{14}$$

To find the formula $(\hat{\theta}_{GE})$, we solve the expectation;

$$E\left(\frac{١}{\theta^c}\middle| x\right) = \int_{٠}^{\infty}\left(\frac{١}{\theta^c}\right) h_{١}(\theta|x)d\theta$$

$$= \frac{T^{\left(\frac{c}{\lambda}\right)}\,\Gamma\left(n+\frac{١}{\lambda}\right)}{\Gamma\left(n+\frac{c+١}{\lambda}\right)} \qquad (15)$$

$$T = \sum_{i=١}^{n}\left(\frac{١}{x_i}\right)^{\lambda}$$

The Bayes estimator of (θ) under entropy loss function is;

$$\hat{\theta}_{Bayes\,(GE)} = \left[\frac{\Gamma\left(n+\frac{c+١}{\lambda}\right)}{T^{\left(\frac{c}{\lambda}\right)}\,\Gamma\left(n+\frac{١}{\lambda}\right)}\right]^{\frac{١}{c}} \qquad (16)$$

Where (λ) known constant and (c) also, known.

$$T = \sum_{i=١}^{n}\left(\frac{١}{x_i}\right)^{\lambda}$$

2.4 Maximum Likelihood Method of Ordered Observation

This method depend on ordered observation considering assumed the estimator (θ) lies between $(x_m, x_{m+١})$ observation and the likelihood function obtained from ordered observation $x_{(١)} \le x_{(٢)} \le \cdots \le x_{(m)} \le \cdots \le x_{(n)}$ as;

$$L(x_{١}, x_{٢}, \dots, x_{٢}) = \prod_{i=١}^{n} f(x_i) \qquad (١٧)$$

$$= \prod_{i=١}^{n} f(x_i) \prod_{i=m+١}^{n} f(x_i)$$

$$= \lambda^m \theta^{m\lambda} \prod_{i=١}^{n} x_i^{-(\lambda+١)} \prod_{i=m+١}^{n} e^{-\left(\frac{\theta}{x_i}\right)^{\lambda}}$$

$$\log L = m\ln\lambda + (m\lambda)\ln\theta - (\lambda+١)\sum_{i=١}^{m}\ln x_i - \sum_{i=m+١}^{n}\left(\frac{\theta}{x_i}\right)^{\lambda}$$

$$\frac{\partial \ln L}{\partial\theta} = \frac{m\lambda}{\theta} - (n-m)\lambda\left[\theta^{(n-m)\lambda-١}\right]\sum_{i=m+١}^{n} x_i^{-\lambda}$$

$$m\lambda - (n-m)\lambda\theta^{(n-m)\lambda} \sum_{i=m+١}^{n} x_i^{-\lambda} = ٠$$

$$m - (n-m)\theta^{(n-m)\lambda} \sum_{i=m+١}^{n} x_i^{-\lambda} = ٠$$

$$\hat{\theta}_{mleor} = \left[\frac{m}{(n-m)\sum_{i=m+١}^{n} x_i^{-\lambda}}\right]^{\frac{١}{(n-m)\lambda}} \quad (18)$$

Which is an implicit function of (λ) can be solved numerically to obtain $[\hat{\theta}_{mleor}]$.

3. Simulation

To compare the results of estimation procedures and estimators of reliability function, we apply simulation procedures using Matlab 18, and generating values from the C.D.F and applying the four method at different set of initial values are;

We use the simulation procedure to compare different estimators of (θ) and $[R(t_i)]$, using different sample size $(n = ٢٠, ٤٠, ٦٠, ٨٠, ١٠٠)$, and different known values of (c, B, λ). And the initial values for (λ) and (θ) are;

<table>
<tr><td>λ</td><td colspan="2">2</td><td colspan="2">3</td><td>4</td></tr>
<tr><td>θ</td><td colspan="2">0.5</td><td colspan="2">1</td><td>1.5</td></tr>
<tr><td>c</td><td colspan="2">1</td><td colspan="3">2</td></tr>
</table>

The replicate of each experiment $(R = ٥٠٠)$, the results $(\hat{\theta})$ then $[\hat{R}(t_i)]$ are compared using statistical measure (MSE), and applying methods illustrated above, all results explained in tables.

Estimation of Scale Parameters (θ) when (λ) is known as follows;

Table (1): $\lambda = ٢,\ \theta = ٠.٥,\ c=2$

n		*LS*	*L − Mom*	*Proposed Bayes*	*MLEO*	*Best*
20	$\hat{\theta}$	0.365532	0.350532	0.510775	0.504571	
	$MSE(\hat{\theta})$	0.016135	0.041058	0.002215	0.002328	Prop.Bayes
40	$\hat{\theta}$	0.360361	0.345793	0.504468	0.502374	
	$MSE(\hat{\theta})$	0.016636	0.020225	0.001085	0.001072	*MLEO*
60	$\hat{\theta}$	0.357723	0.331814	0.502498	0.502105	
	$MSE(\hat{\theta})$	0.016822	0.025821	0.000654	0.000452	*MLEO*
80	$\hat{\theta}$	0.357398	0.348129	0.501108	0.501065	
	$MSE(\hat{\theta})$	0.016557	0.024802	0.000435	0.000432	*MLEO*
100	$\hat{\theta}$	0.354971	0.341253	0.501068	0.501132	
	$MSE(\hat{\theta})$	0.01733	0.024267	0.000437	0.000442	*MLEO*

Table (2): $\lambda = ٣,\ \theta = ٠.٥,\ c = ٢$

n		*LS*	*L − Mom*	Prop.Bayes	*MLEO*	*Best*
20	$\hat{\theta}$	0.456257	0.384318	0.503873	0.504026	
	$MSE(\hat{\theta})$	0.001106	0.012708	0.001047	0.001034	*MLEO*
40	$\hat{\theta}$	0.458766	0.384585	0.502046	0.501481	
	$MSE(\hat{\theta})$	0.00158	0.012308	0.000409	0.000405	*MLEO*
60	$\hat{\theta}$	0.454819	0.384337	0.501103	0.500884	
	$MSE(\hat{\theta})$	0.001802	0.012315	0.000315	0.000313	*MLEO*
80	$\hat{\theta}$	0.443651	0.376026	0.500842	0.500700	
	$MSE(\hat{\theta})$	0.002085	0.011263	0.000231	0.000240	*MLEO*
100	$\hat{\theta}$	0.452627	0.385122	0.501237	0.501052	
	$MSE(\hat{\theta})$	0.002151	0.011274	0.000176	0.000188	Prop.Bayes

Table (3): $\lambda = ٤,\ \theta = ٠.٥,\ c = ٢$

n		LS	$L - Mom$	Prop.Bayes	$MLEO$	$Best$
20	$\hat{\theta}$	0.501436	0.421749	0.503368	0.502373	
	$MSE(\hat{\theta})$	4.21E-05	0.007559	0.000453	0.000454	Ls
40	$\hat{\theta}$	0.483008	0.420657	0.501585	0.501584	
	$MSE(\hat{\theta})$	5.14E-05	0.005837	0.000242	0.000243	LS
60	$\hat{\theta}$	0.490712	0.418703	0.500202	0.500202	
	$\mathrm{MSE}(\hat{\theta})$	9.28E-05	0.005805	0.000169	0.000178	LS
80	$\hat{\theta}$	0.478019	0.420514	0.500434	0.500443	
	$MSE(\hat{\theta})$	1.30E-04	0.005752	0.000120	0.000120	LS
100	$\hat{\theta}$	0.354394	0.343234	0.501429	0.500684	
	$MSE(\hat{\theta})$	1.85E-02	0.024570	0.000432	0.000438	LS

Table (4): $\lambda = ٢\ ,\ \theta = ١\quad c=2$

n		LS	$L - Mom$	Prop.Bayes	$MLEO$	$Best$
20	$\hat{\theta}$	0.236964	0.70139	1.020689	1.012287	
	$MSE(\hat{\theta})$	5.65E-01	0.132085	0.006896	0.006477	$MLEO$
40	$\hat{\theta}$	0.230603	0.71048	1.011421	1.005232	
	$MSE(\hat{\theta})$	5.77E-01	0.117218	0.004351	0.004237	$MLEO$
60	$\hat{\theta}$	0.227428	0.702346	1.005155	1.003272	
	$\mathrm{MSE}(\hat{\theta})$	5.71E-01	0.102516	0.002665	0.002624	$MLEO$
80	$\hat{\theta}$	0.227121	0.701011	1.00458	1.003392	
	$MSE(\hat{\theta})$	5.71E-01	0.100711	0.00203	0.002013	$MLEO$
100	$\hat{\theta}$	0.235456	0.702727	1.004316	1.002746	
	$MSE(\hat{\theta})$	5.82E-01	0.085626	0.001582	0.001574	$MLEO$

Table (5): $\lambda = ٣,\ \theta = ١,\ c = ٢$

n		LS	$L-Mom$	$Prop.Bayes$	$MLEO$	$Best$
20	$\hat{\theta}$	0.361851	0.782894	1.008851	1.007808	
	$MSE(\hat{\theta})$	0.382674	0.054429	0.003872	0.003971	**Prop.Bayes**
40	$\hat{\theta}$	0.356172	0.794780	1.002559	1.001720	
	$MSE(\hat{\theta})$	0.400640	0.047435	0.001692	0.001683	$MLEO$
60	$\hat{\theta}$	0.363725	0.780714	1.00333	1.002711	
	$MSE(\hat{\theta})$	0.404970	0.045828	0.001319	0.001312	$MLEO$
80	$\hat{\theta}$	0.363354	0.780346	1.001284	1.000802	
	$MSE(\hat{\theta})$	0.405404	0.045536	0.001002	0.001013	**Bayes**
100	$\hat{\theta}$	0.352001	0.783867	1.001716	1.001435	
	$MSE(\hat{\theta})$	0.406114	0.044758	0.000741	0.000740	$MLEO$

Table (6): $\lambda = ٤,\ \theta = ١,\ c = ٢$

n		LS	$L-Mom$	Prop.Bayes	$MLEO$	$Best$
20	$\hat{\theta}$	0.420436	0.83139	1.004236	1.005227	
	$MSE(\hat{\theta})$	3.24E-01	0.030407	0.002109	0.002119	**Prop.Bayes**
40	$\hat{\theta}$	0.423177	0.82779	1.002249	1.002335	
	$MSE(\hat{\theta})$	3.31E-01	0.028234	0.001048	0.001068	**Prop.Bayes**
60	$\hat{\theta}$	0.411215	0.829739	1.001653	1.001742	
	$MSE(\hat{\theta})$	3.34E-01	0.026605	0.000716	0.000715	$MLEO$
80	$\hat{\theta}$	0.418678	0.838229	1.001162	1.001171	
	$MSE(\hat{\theta})$	3.36E-01	0.026143	0.000488	0.000487	$MLEO$
100	$\hat{\theta}$	0.416768	0.838136	1.00048	1.00048	
	$MSE(\hat{\theta})$	3.37E-01	0.026866	0.000413	0.000412	$MLEO$

Table (7): $\lambda = ٢,\ \theta = ١.٥,\ c = ١$

n		LS	$L-Mom$	Prop.Bayes	$MLEO$	$Best$
20	$\hat{\theta}$	0.154262	1.075442	1.521439	1.514067	
	$MSE(\hat{\theta})$	1.78E+00	1.068284	0.021245	0.02112	$MLEO$
40	$\hat{\theta}$	0.145358	1.061828	1.510078	1.505856	
	$MSE(\hat{\theta})$	1.71E+00	0.310259	0.010314	0.01021	$MLEO$
60	$\hat{\theta}$	0.143192	1.041071	1.506866	1.505764	
	$MSE(\hat{\theta})$	1.71E+00	0.220109	0.005124	0.00607	$MLEO$
80	$\hat{\theta}$	0.142372	1.043065	1.504101	1.502534	
	$MSE(\hat{\theta})$	1.71E+00	0.225126	0.004435	0.004316	$MLEO$
100	$\hat{\theta}$	0.142089	1.041358	1.503650	1.501308	
	$MSE(\hat{\theta})$	1.71E+00	0.220051	0.003845	0.003841	$MLEO$

Table (8): $\lambda = ٤,\ \theta = ١.٥,\ c = ١$

n		LS	$L-Mom$	Prop.Bayes	$MLEO$	$Best$
20	$\hat{\theta}$	0.368738	1.250877	1.504293	1.505849	
	$MSE(\hat{\theta})$	1.27E+00	0.067802	0.005193	0.005222	Prop.Bayes
40	$\hat{\theta}$	0.35347	1.25317	1.501937	1.502717	Prop.Bayes
	$MSE(\hat{\theta})$	1.27E+00	0.062422	0.002269	0.002277	Prop.Bayes
60	$\hat{\theta}$	0.351354	1.251728	1.501491	1.502011	Prop.Bayes
	$MSE(\hat{\theta})$	1.30E+00	0.060549	0.001446	0.00145	Prop.Bayes
80	$\hat{\theta}$	0.358662	1.260571	1.50211	1.502301	Prop.Bayes
	$MSE(\hat{\theta})$	1.30E+00	0.060207	0.001142	0.001143	Prop.Bayes
100	$\hat{\theta}$	0.357839	1.250793	1.502813	1.503115	Prop.Bayes
	$MSE(\hat{\theta})$	1.30E+00	0.058568	0.000907	0.00091	Prop.Bayes

Estimators of Reliability function as follows;

Table (9): $\lambda = ٢,\ \theta = ٠.٥,\ \ c = ٢$

n	t_i	*Real*	$\hat{R}_{LS}$	$MSE(\hat{R}_{LS})$	$\hat{R}_{LM}$	$MSE(\hat{R}_{LM})$	$\hat{R}_{Bayes}$	$MSE(\hat{R}_{Bay}$	$\hat{R}_{Mle}$	$MSE(\hat{R}_{Ml}$	*Best*
		0.622121	0.423465	0.034113	0.37947	0.081289	0.64548	4.87E-03	0.639514	4.76E-03	**MLE**
		0.211187	0.124762	0.007094	0.121002	0.016021	0.228296	0.001457	0.225025	0.001387	**MLE**
		0.103161	0.051838	0.001012	0.050309	0.006873	0.10988	0.000384	0.108184	0.000361	**MLE**
		0.040587	0.025699	0.000558	0.031752	0.001567	0.063968	1.43E-04	0.062954	1.33E-04	**MLE**
		0.028211	0.012731	0.000291	0.022775	0.002133	0.041226	6.36E-05	0.040564	5.95E-05	**MLE**
		0.522121	0.413343	0.036371	0.370445	0.073727	0.637372	2.31E-03	0.634346	1.41E-03	**MLE**
		0.221156	0.119407	0.007744	0.120181	0.013913	0.225034	0.000676	0.223403	0.000656	**MLE**
		0.104161	0.050055	0.002113	0.054267	0.003354	0.107294	0.000177	0.106457	0.000171	**MLE**
		0.050587	0.024000	0.000629	0.031033	0.001104	0.062178	6.50E-05	0.061681	6.26E-05	**MLE**
		0.038211	0.011863	0.000311	0.02148	0.000676	0.040225	2.76E-05	0.039899	2.66E-05	**MLE**
		0.532121	0.411358	0.03616	0.385915	0.07054	0.634364	1.51E-03	0.632338	1.42E-03	**MLE**
		0.221195	0.117854	0.007938	0.115602	0.012699	0.224292	0.000423	0.223204	0.000415	**MLE**
		0.104161	0.048891	0.002103	0.055772	0.003793	0.106591	0.000103	0.106035	0.000101	**MLE**
		0.050587	0.023674	0.000641	0.030949	0.001058	0.061314	3.81E-05	0.060986	3.72E-05	**MLE**
		0.038211	0.011622	0.000217	0.020489	0.000479	0.039871	1.87E-05	0.039656	1.82E-05	**MLE**
		0.622121	0.405776	0.047573	0.393586	0.066564	0.633933	1.24E-03	0.632411	1.22E-03	**MLE**
		0.221194	0.116513	0.00816	0.118701	0.012672	0.223981	0.000337	0.223165	0.000321	**MLE**
		0.104161	0.047808	0.002189	0.056272	0.003342	0.106298	8.74E-05	0.105882	8.50E-05	**MLE**
		0.050587	0.02311	0.000569	0.031789	0.001018	0.061388	3.05E-05	0.061141	3.00E-05	**MLE**
		0.038211	0.011547	0.000316	0.019789	0.00044	0.039613	1.32E-05	0.039452	1.28E-05	**MLE**
		0.532121	0.404541	0.047847	0.39015	0.066604	0.633021	9.06E-04	0.631801	9.04E-04	**MLE**
		0.221195	0.115987	0.008219	0.118765	0.012293	0.22327	0.000263	0.222618	0.00025	**MLE**
		0.104161	0.047984	0.00225	0.0545	0.002896	0.106496	6.80E-05	0.106162	6.67E-05	**MLE**
		0.050587	0.022958	0.000766	0.031709	0.001076	0.060961	2.41E-05	0.060765	2.37E-05	**MLE**
		0.036211	0.021271	0.000326	0.020036	0.000434	0.039592	1.08E-05	0.039463	1.06E-05	**MLE**

Table (10): $\lambda = ٣,\ \theta = ٠.٥,\ c = ٢$

n	t_i	*Real*	$\hat{R}_{LS}$	$MSE(\hat{R}_{LS})$	$\hat{R}_{LM}$	$MSE(\hat{R}_{LM})$	$\hat{R}_{Bayes}$	$MSE(R_{Bayes})$	$\hat{R}_{Mle}$	$MSE(\hat{R}_{Mle})$	*Best*
	0.5	0.632121	0.556308	0.00655	0.39854	0.068923	0.640333	4.53E-03	0.638321	4.50E-03	**MLE**
	1	0.117503	0.097101	0.000468	0.064512	0.004356	0.120769	0.000444	0.120142	0.000435	**MLE**
	1.5	0.03636	0.029757	4.91E-05	0.01965	0.000417	0.03772	5.09E-05	0.037515	4.97E-05	**MLE**
	2	0.015504	0.012693	8.79E-06	0.008152	7.09E-05	0.016159	9.75E-06	0.01607	9.53E-06	**MLE**
	2.5	0.007968	0.006523	2.32E-06	0.004176	1.80E-05	0.008244	2.54E-06	0.008199	2.48E-06	**MLE**
	0.5	0.632121	0.539482	0.009006	0.395915	0.062911	0.636553	2.36E-03	0.635539	2.34E-03	**MLE**
	1	0.117503	0.092549	0.000648	0.0625	0.003512	0.119236	0.000227	0.118926	0.000224	**MLE**
	1.5	0.03636	0.028306	6.74E-05	0.018974	0.000346	0.037236	2.52E-05	0.037134	2.48E-05	**MLE**
	2	0.015504	0.012071	1.22E-05	0.008014	6.35E-05	0.015804	4.46E-06	0.01576	4.42E-06	**MLE**
	2.5	0.007968	0.006215	3.19E-06	0.004072	1.70E-05	0.008114	1.23E-06	0.008092	1.22E-06	**MLE**
	0.5	0.632121	0.533205	0.010074	0.3914	0.06303	0.634694	1.59E-03	0.634016	1.58E-03	**MLE**
	1	0.117503	0.09073	0.000732	0.061237	0.003352	0.118535	0.000135	0.118329	0.000133	**MLE**
	1.5	0.03636	0.02782	7.46E-05	0.018659	0.000338	0.036976	1.58E-05	0.036909	1.56E-05	**MLE**
	2	0.015504	0.011852	1.36E-05	0.007837	6.25E-05	0.015631	2.70E-06	0.015602	2.68E-06	**MLE**
	2.5	0.007968	0.006092	3.61E-06	0.004081	2.35E-05	0.008061	7.70E-07	0.008046	7.64E-07	**MLE**
	0.5	0.632121	0.528441	0.010969	0.395493	0.059937	0.635445	1.14E-03	0.634936	1.12E-03	**MLE**
	1	0.117503	0.089844	0.000779	0.061062	0.003352	0.118624	0.0001	0.11847	9.94E-05	**MLE**
	1.5	0.03636	0.027536	7.91E-05	0.018365	0.000341	0.036623	1.08E-05	0.036573	1.06E-05	**MLE**
	2	0.015504	0.011673	1.49E-05	0.007932	6.04E-05	0.015749	2.39E-06	0.015727	2.36E-06	**MLE**
	2.5	0.007968	0.006	3.93E-06	0.004057	1.61E-05	0.008035	5.58E-07	0.008024	5.54E-07	**MLE**
	0.5	0.632121	0.525701	0.011495	0.396298	0.059086	0.635978	9.39E-04	0.63557	9.35E-04	**MLE**
	1	0.117503	0.089155	0.000814	0.061764	0.003291	0.117922	8.56E-05	0.1178	8.52E-05	**MLE**
	1.5	0.03636	0.027289	8.32E-05	0.01853	0.000331	0.036711	9.16E-06	0.036671	9.01E-06	**MLE**
	2	0.015504	0.011605	1.54E-05	0.007916	6.13E-05	0.015666	1.64E-06	0.015649	1.62E-06	**MLE**
	2.5	0.007968	0.005967	4.05E-06	0.004016	1.62E-05	0.008045	4.39E-07	0.008036	4.36E-07	**MLE**

Table (11): $\lambda = ٤,\ \theta = ٠.٥,\ c = ٢$

n	t_i	$Real$	$\hat{R}_{LS}$	$MSE(\hat{R}_{LS})$	$\hat{R}_{LM}$	$MSE(\hat{R}_{LM})$	$\hat{R}_{Bayes}$	$MSE(\hat{R}_{Bayes})$	$\hat{R}_{Mle}$	$MSE(\hat{R}_{Mle})$	$Best$
	0.5	0.632121	0.637985	0.000326	0.391223	0.067972	0.639348	0.004721	0.639336	0.004611	**MLE**
	1	0.060587	0.061539	8.50E-06	0.032059	0.000975	0.063299	1.49E-04	0.063297	1.47E-04	**MLE**
	1.5	0.01227	0.012475	3.68E-07	0.006421	4.08E-05	0.01257	5.16E-06	0.012569	5.14E-06	**MLE**
	2	0.003899	0.003962	3.80E-08	0.002053	4.02E-06	0.004053	6.33E-07	0.004053	6.31E-07	**MLE**
	2.5	0.001599	0.00162	6.97E-09	0.000856	6.92E-07	0.001665	1.05E-07	0.001665	1.03E-07	**MLE**
	0.5	0.632121	0.614304	0.000472	0.392887	0.062893	0.635537	0.002206	0.635534	0.002105	**MLE**
	1	0.060587	0.05781	1.09E-05	0.031457	0.000915	0.06179	6.62E-05	0.061789	6.61E-05	**MLE**
	1.5	0.01227	0.011698	4.62E-07	0.006302	3.82E-05	0.012515	2.62E-06	0.012515	2.61E-06	**MLE**
	2	0.003899	0.003714	4.77E-08	0.002002	3.84E-06	0.003961	2.62E-07	0.003961	2.61E-07	**MLE**
	2.5	0.001599	0.001524	7.92E-09	0.000821	6.53E-07	0.00163	4.42E-08	0.00163	4.41E-08	**MLE**
	0.5	0.632121	0.605102	0.000829	0.391161	0.062047	0.63469	0.001488	0.634689	0.001387	**MLE**
	1	0.060587	0.056378	1.97E-05	0.031249	0.000901	0.060951	3.71E-05	0.060951	3.70E-05	**MLE**
	1.5	0.01227	0.011408	8.32E-07	0.006192	3.85E-05	0.012415	1.85E-06	0.012415	1.84E-06	**MLE**
	2	0.003899	0.003615	9.07E-08	0.002009	3.76E-06	0.003948	1.71E-07	0.003948	1.70E-07	**MLE**
	2.5	0.001599	0.001486	1.42E-08	0.000804	6.59E-07	0.001608	3.02E-08	0.001608	3.01E-08	**MLE**
	0.5	0.632121	0.59974	0.001126	0.392628	0.06053	0.633521	0.001155	0.63352	0.001054	**MLE**
	1	0.060587	0.055582	2.67E-05	0.031136	0.000901	0.061123	3.04E-05	0.061123	3.034E-05	**MLE**
	1.5	0.01227	0.011241	1.13E-06	0.006228	3.79E-05	0.012371	1.28E-06	0.012371	1.26E-06	**MLE**
	2	0.003899	0.003565	1.18E-07	0.001983	3.81E-06	0.003934	1.21E-07	0.003934	1.20E-07	**MLE**
	2.5	0.001599	0.001463	1.97E-08	0.000815	6.78E-07	0.001609	2.02E-08	0.001609	2.01E-08	**MLE**
	0.5	0.632121	0.595825	0.001375	0.39425	0.058786	0.634199	0.000887	0.634198	0.000876	**MLE**
	1	0.060587	0.055074	3.17E-05	0.031069	0.000897	0.060904	2.31E-05	0.060904	2.30E-05	**MLE**
	1.5	0.01227	0.011124	1.37E-06	0.006239	3.74E-05	0.012371	9.93E-07	0.012371	9.92E-07	**MLE**
	2	0.003899	0.003537	1.37E-07	0.001967	3.85E-06	0.003903	1.02E-07	0.003903	1.01E-07	**MLE**
	2.5	0.001599	0.001448	2.35E-08	0.000809	6.40E-07	0.001612	1.69E-08	0.001612	1.68E-08	**MLE**

Table (12): $\lambda = ٢,\ \theta = ١,\ c = ٢$

n	t_i	$Real$	$\hat{R}_{LS}$	$MSE(\hat{R}_{LS})$	$\hat{R}_{LM}$	$MSE(\hat{R}_{LM})$	$\hat{R}_{Bayes}$	$MSE(\hat{R}_{Bayes})$	$\hat{R}_{Mle}$	$MSE(\hat{R}_{Mle})$	$Best$
	0.5	0.981684	0.219374	0.585654	0.820053	0.039134	0.981001	0.000168	0.979797	0.000187	**Bayes**
	1	0.632121	0.061782	3.26E-01	0.380058	0.083688	0.642127	4.21E-03	0.636145	4.12E-03	**MLE**
	1.5	0.35882	0.027426	1.10E-01	0.206	3.68E-02	0.372965	3.30E-03	0.36819	3.12E-03	**MLE**
	2	0.221199	0.015647	4.23E-02	0.122815	1.62E-02	0.231286	1.55E-03	0.22798	1.45E-03	**MLE**
	2.5	0.147856	0.010122	1.90E-02	0.080857	8.42E-03	0.154324	7.12E-04	0.152003	6.67E-04	**MLE**
	0.5	0.981684	0.210307	0.597705	0.833267	0.03003	0.981535	8.14E-05	0.980934	8.62E-05	**Bayes**
	1	0.632121	0.057829	3.30E-01	0.386637	0.074135	0.639205	2.29E-03	0.636178	2.25E-03	**MLE**
	1.5	0.35882	0.026327	1.11E-01	0.196313	3.19E-02	0.364747	1.38E-03	0.362369	1.34E-03	**MLE**
	2	0.221199	0.014681	4.27E-02	0.123216	1.53E-02	0.226453	7.11E-04	0.224813	6.87E-04	**MLE**
	2.5	0.147856	0.009456	1.92E-02	0.080772	6.92E-03	0.1514	3.43E-04	0.150249	3.31E-04	**MLE**
	0.5	0.981684	0.20695	0.602075	0.840121	0.025858	0.981096	5.44E-05	0.980685	5.67E-05	**Bayes**
	1	0.632121	0.055844	3.32E-01	0.39071	0.067486	0.637116	1.38E-03	0.635088	1.36E-03	**MLE**
	1.5	0.35882	0.02553	1.11E-01	0.200128	3.09E-02	0.363529	9.41E-04	0.361943	9.22E-04	**MLE**
	2	0.221199	0.014395	4.28E-02	0.119227	1.26E-02	0.224107	4.18E-04	0.22302	4.11E-04	**MLE**
	2.5	0.147856	0.009182	1.92E-02	0.077773	5.75E-03	0.149788	2.09E-04	0.149026	2.04E-04	**MLE**
	0.5	0.981684	0.202511	0.608554	0.849082	0.022068	0.981749	4.09E-05	0.981448	4.18E-05	**Bayes**
	1	0.632121	0.055249	3.33E-01	0.390358	0.067724	0.636011	1.09E-03	0.634487	1.07E-03	**MLE**
	1.5	0.35882	0.024984	1.11E-01	0.200875	2.97E-02	0.361383	7.87E-04	0.360196	7.78E-04	**MLE**
	2	0.221199	0.014117	4.29E-02	0.119323	1.25E-02	0.223406	3.14E-04	0.222592	3.08E-04	**MLE**
	2.5	0.147856	0.009131	1.92E-02	0.076915	5.68E-03	0.149565	1.54E-04	0.148993	1.52E-04	**MLE**
	0.5	0.981684	0.202846	0.607812	0.847306	0.021639	0.981088	3.46E-05	0.98084	3.55E-05	**MLE**
	1	0.632121	0.055208	3.33E-01	0.390436	0.065992	0.633482	9.09E-04	0.632262	9.06E-04	**MLE**
	1.5	0.35882	0.024724	1.12E-01	0.200325	2.83E-02	0.360309	5.42E-04	0.35936	5.37E-04	**MLE**
	2	0.221199	0.01404	4.29E-02	0.119387	1.31E-02	0.223626	2.90E-04	0.222973	2.85E-04	**MLE**
	2.5	0.147856	0.009007	1.93E-02	0.077084	5.50E-03	0.149243	1.28E-04	0.148786	1.25E-04	**MLE**

Table (13): $\lambda = ٣,\ \theta = ١,\ c = ٢$

n	t_i	$Real$	$\hat{R}_{LS}$	$MSE(\hat{R}_{LS})$	$\hat{R}_{LM}$	$MSE(\hat{R}_{LM})$	$\hat{R}_{Bayes}$	$MSE(\hat{R}_{Bayes})$	$\hat{R}_{Mle}$	$MSE(\hat{R}_{Mle})$	$Best$
	0.5	0.999665	0.3425	0.433663	0.963761	0.002624	0.999362	9.98E-07	0.999338	1.08E-06	**Bayes**
	1	0.632121	0.050856	0.337921	0.393945	0.069609	0.645819	0.00473	0.64381	0.004681	**MLE**
	1.5	0.256433	0.015435	0.058085	0.140067	0.01665	0.264338	0.001749	0.26309	0.001717	**MLE**
	2	0.117503	0.006497	0.012323	0.063049	0.003624	0.122662	5.35E-04	0.122026	5.25E-04	**MLE**
	2.5	0.061995	0.003337	0.003441	0.033219	0.001234	0.064428	1.45E-04	0.064082	1.41E-04	**MLE**
	0.5	0.999665	0.326167	0.454503	0.973132	0.001254	0.999549	2.22E-07	0.999539	2.21E-07	**MLE**
	1	0.632121	0.04839	0.340771	0.393219	0.065018	0.635863	0.00233	0.634849	0.002321	**MLE**
	1.5	0.256433	0.014704	0.058435	0.138464	0.015965	0.258944	0.000831	0.258329	0.000824	**MLE**
	2	0.117503	0.006199	0.012389	0.061459	0.003423	0.119852	2.29E-04	0.11954	2.26E-04	**MLE**
	2.5	0.061995	0.003175	0.00346	0.032537	0.000989	0.062767	6.03E-05	0.062599	5.97E-05	**Bayes**
	0.5	0.999665	0.321625	0.460318	0.975374	0.000905	0.999575	1.26E-07	0.999569	1.32E-07	**Bayes**
	1	0.632121	0.047233	0.342111	0.395735	0.061219	0.635508	0.001379	0.63483	0.001374	**MLE**
	1.5	0.256433	0.014297	0.058631	0.138644	0.014803	0.258821	0.000606	0.258411	0.000601	**MLE**
	2	0.117503	0.006046	0.012423	0.062221	0.003512	0.119674	1.52E-04	0.119466	1.51E-04	**MLE**
	2.5	0.061995	0.003084	0.003471	0.03259	0.00096	0.062831	4.51E-05	0.062718	4.46E-05	**MLE**
	0.5	0.999665	0.318412	0.464548	0.976907	0.000753	0.999605	8.72E-08	0.999601	8.92E-08	**Bayes**
	1	0.632121	0.0468	0.342613	0.395181	0.060306	0.632753	0.001096	0.632244	0.001094	**MLE**
	1.5	0.256433	0.014071	0.058741	0.140478	0.014454	0.257787	0.000392	0.25748	0.000381	**MLE**
	2	0.117503	0.005995	0.012434	0.061252	0.003325	0.11787	1.09E-04	0.117717	1.07E-04	**MLE**
	2.5	0.061995	0.003072	0.003472	0.03189	0.000955	0.062408	3.34E-05	0.062324	3.32E-05	**MLE**
	0.5	0.999665	0.317061	0.466299	0.977513	0.000686	0.99961	6.27E-08	0.999606	6.42E-08	**Bayes**
	1	0.632121	0.046489	0.342975	0.393747	0.060178	0.63296	0.000953	0.632552	0.000951	**MLE**
	1.5	0.256433	0.013984	0.058782	0.13939	0.014325	0.258321	0.000345	0.258075	0.000342	**MLE**
	2	0.117503	0.005901	0.012455	0.0616	0.003267	0.118554	7.87E-05	0.118431	7.82E-05	**MLE**
	2.5	0.061995	0.003036	0.003476	0.031849	0.000945	0.062444	2.48E-05	0.062377	2.46E-05	**MLE**

Table (14): $\lambda =$ ٤ , $\theta =$ ١, $c =$ ٢

n	t_i	$Real$	$\hat{R}_{LS}$	$MSE(\hat{R}_{LS})$	$\hat{R}_{LM}$	$MSE(\hat{R}_{LM})$	$\hat{R}_{Bayes}$	$MSE(\hat{R}_{Bayes})$	$\hat{R}_{Mle}$	$MSE(\hat{R}_{Mle})$	$Best$
	0.5	1	0.424605	0.331931	0.998054	1.73E-05	0.999999	5.78E-11	0.999999	5.76E-11	**MLE**
	1	0.632121	0.033974	3.58E-01	0.395768	0.067242	0.637514	4.21E-03	0.637502	4.20E-03	**MLE**
	1.5	0.179245	0.006792	2.97E-02	0.096924	8.17E-03	0.18422	9.84E-04	0.184214	9.82E-04	**MLE**
	2	0.060587	0.002158	3.41E-03	0.032235	9.56E-04	0.062948	1.37E-04	0.062945	1.35E-04	**MLE**
	2.5	0.025275	0.000881	5.95E-04	0.013341	1.69E-04	0.026123	2.48E-05	0.026122	2.46E-05	**MLE**
	0.5	1	0.403436	0.356286	0.998987	3.56E-06	1	1.43E-12	1	1.41E-12	**MLE**
	1	0.632121	0.031773	3.60E-01	0.396362	0.061497	0.636092	2.23E-03	0.636088	2.21E-03	**MLE**
	1.5	0.179245	0.006376	2.99E-02	0.095793	7.62E-03	0.181973	4.69E-04	0.181971	4.67E-04	**MLE**
	2	0.060587	0.002015	3.43E-03	0.031803	9.06E-04	0.061677	5.97E-05	0.061677	5.95E-05	**MLE**
	2.5	0.025275	0.000831	5.98E-04	0.012847	1.67E-04	0.025665	1.12E-05	0.025665	1.11E-05	**MLE**
	0.5	1	0.395117	0.366189	0.999211	1.89E-06	1	5.85E-13	1	5.83E-13	**MLE**
	1	0.632121	0.031016	3.61E-01	0.394628	0.059996	0.631864	1.52E-03	0.631862	1.51E-03	**MLE**
	1.5	0.179245	0.00619	2.99E-02	0.096054	7.31E-03	0.180595	2.92E-04	0.180594	2.91E-04	**MLE**
	2	0.060587	0.001962	3.44E-03	0.031316	9.00E-04	0.061129	3.94E-05	0.061129	3.92E-05	**MLE**
	2.5	0.025275	0.000806	5.99E-04	0.012855	1.62E-04	0.025554	7.49E-06	0.025554	7.47E-06	**MLE**
	0.5	1	0.391368	0.370615	0.999347	1.08E-06	1	2.02E-13	1	2.01E-13	**MLE**
	1	0.632121	0.030565	3.62E-01	0.393585	0.05972	0.633426	1.14E-03	0.633425	1.12E-03	**MLE**
	1.5	0.179245	0.006097	3.00E-02	0.095296	7.31E-03	0.180688	2.29E-04	0.180688	2.27E-04	**MLE**
	2	0.060587	0.001938	3.44E-03	0.031019	9.05E-04	0.061025	2.98E-05	0.061025	2.96E-05	**MLE**
	2.5	0.025275	0.000794	5.99E-04	0.012756	1.62E-04	0.025465	5.04E-06	0.025465	5.02E-06	**MLE**
	0.5	1	0.388202	0.374476	0.999379	9.43E-07	1	9.06E-14	1	9.04E-14	**MLE**
	1	0.632121	0.030271	3.62E-01	0.394508	0.058926	0.633248	1.02E-03	0.633248	1.01E-03	**MLE**
	1.5	0.179245	0.00605	3.00E-02	0.09533	7.28E-03	0.179948	1.77E-04	0.179948	1.75E-04	**MLE**
	2	0.060587	0.001923	3.44E-03	0.030797	9.11E-04	0.06077	2.43E-05	0.06077	2.41E-05	**MLE**
	2.5	0.025275	0.000787	6.00E-04	0.012729	1.62E-04	0.025384	4.21E-06	0.025384	4.20E-06	**MLE**

Table (15): $\lambda = ٢,\ \theta = ١.٥,\ c = ١$

n	t_i	*Real*	$\hat{R}_{LS}$	$MSE(\hat{R}_{LS})$	$\hat{R}_{LM}$	$MSE(\hat{R}_{LM})$	$\hat{R}_{Bayes}$	$MSE(\hat{R}_{Bayes})$	$\hat{R}_{Mle}$	$MSE(\hat{R}_{Mle})$	*Best*
	0.5	0.999877	0.106974	0.79955	0.966255	2.62E-03	0.999734	2.49E-07	0.999717	2.77E-07	**Bayes**
	1	0.894601	0.028333	7.51E-01	0.63607	0.088195	0.896254	1.76E-03	0.894356	1.81E-03	**Bayes**
	1.5	0.632121	0.012506	3.84E-01	0.376482	8.43E-02	0.638787	4.15E-03	0.63577	4.11E-03	**MLE**
	2	0.430217	0.007151	1.79E-01	0.247794	5.00E-02	0.439324	3.67E-03	0.436653	3.62E-03	**MLE**
	2.5	0.302324	0.004573	8.87E-02	0.166311	2.81E-02	0.313733	2.52E-03	0.3116	2.45E-03	**MLE**
	0.5	0.999877	0.099024	0.812697	0.975955	1.16E-03	0.99982	4.90E-08	0.999814	5.24E-08	**MLE**
	1	0.894601	0.024902	7.56E-01	0.65852	0.070488	0.895357	9.40E-04	0.894389	9.48E-04	**Bayes**
	1.5	0.632121	0.011352	3.85E-01	0.387738	7.46E-02	0.634883	2.33E-03	0.633364	2.31E-03	**MLE**
	2	0.430217	0.006459	1.80E-01	0.246909	4.28E-02	0.437454	1.83E-03	0.436113	1.80E-03	**MLE**
	2.5	0.302324	0.0041	8.89E-02	0.167438	2.47E-02	0.306001	1.16E-03	0.304949	1.14E-03	**MLE**
	0.5	0.999877	0.09524	0.819205	0.978835	7.89E-04	0.999833	2.61E-08	0.999829	2.75E-08	**Bayes**
	1	0.894601	0.024439	7.57E-01	0.658297	0.067238	0.894618	6.05E-04	0.893966	6.09E-04	**Bayes**
	1.5	0.632121	0.010799	3.86E-01	0.393111	6.77E-02	0.635907	1.55E-03	0.634891	1.53E-03	**MLE**
	2	0.430217	0.006268	1.80E-01	0.243442	4.10E-02	0.432569	1.16E-03	0.43168	1.14E-03	**MLE**
	2.5	0.302324	0.003959	8.90E-02	0.164501	2.16E-02	0.305174	6.89E-04	0.304474	6.82E-04	**MLE**
	0.5	0.999877	0.093022	0.823183	0.980041	6.90E-04	0.999841	1.93E-08	0.999839	2.01E-08	**Bayes**
	1	0.894601	0.02369	7.59E-01	0.66737	0.061493	0.895143	4.77E-04	0.894655	4.78E-04	**Bayes**
	1.5	0.632121	0.0109	3.86E-01	0.385422	6.80E-02	0.632434	1.09E-03	0.631671	1.07E-03	**MLE**
	2	0.430217	0.006077	1.80E-01	0.243484	3.98E-02	0.433474	8.53E-04	0.432805	8.46E-04	**MLE**
	2.5	0.302324	0.003906	8.91E-02	0.164959	2.22E-02	0.303867	5.00E-04	0.303343	4.96E-04	**MLE**
	0.5	0.999877	0.090527	0.827515	0.982205	5.30E-04	0.99985	1.29E-08	0.999848	1.33E-08	**Bayes**
	1	0.894601	0.023862	7.58E-01	0.660535	0.062177	0.894207	3.97E-04	0.893814	3.98E-04	**Bayes**
	1.5	0.632121	0.010431	3.87E-01	0.395235	6.39E-02	0.633851	9.13E-04	0.633239	9.11E-04	**MLE**
	2	0.430217	0.005913	1.80E-01	0.247746	3.78E-02	0.432282	7.08E-04	0.431748	7.04E-04	**MLE**
	2.5	0.302324	0.003867	8.91E-02	0.163863	2.18E-02	0.30367	4.45E-04	0.303251	4.42E-04	**MLE**

Table (16): $\lambda = ٤ ,\ \theta = ١.٥ ,\ c = ١$

n	t_i	$Real$	$\hat{R}_{LS}$	$MSE(\hat{R}_{LS})$	$\hat{R}_{LM}$	$MSE(\hat{R}_{LM})$	$\hat{R}_{Bayes}$	$MSE(\hat{R}_{Bayes})$	$\hat{R}_{Mle}$	$MSE(\hat{R}_{Mle})$	$Best$
	0.5	1	0.258415	0.550843	1	1.10E-18	1	0.00E+00	1	0.00E+00	**Bayes**
	1	0.99367	0.01853	9.51E-01	0.902607	0.012774	0.992236	4.52E-05	0.992381	4.34E-05	**MLE**
	1.5	0.632121	0.003701	3.95E-01	0.400364	6.52E-02	0.637574	4.78E-03	0.639066	4.81E-03	**Bayes**
	2	0.271237	0.001169	7.29E-02	0.154772	1.72E-02	0.280014	2.15E-03	0.280986	2.17E-03	**Bayes**
	2.5	0.121553	0.000483	1.47E-02	0.065382	3.78E-03	0.124492	5.29E-04	0.124972	5.37E-04	**Bayes**
	0.5	1	0.243978	0.571992	1	1.93E-22	1	0.00E+00	1	0.01E+00	**Bayes**
	1	0.99367	0.017436	9.53E-01	0.910478	0.009371	0.992721	2.30E-05	0.992792	2.24E-05	**MLE**
	1.5	0.632121	0.003448	3.95E-01	0.397936	6.05E-02	0.636898	2.29E-03	0.637655	2.31E-03	**Bayes**
	2	0.271237	0.001094	7.30E-02	0.149405	1.63E-02	0.275024	9.92E-04	0.275506	9.99E-04	**Bayes**
	2.5	0.121553	0.000448	1.47E-02	0.064474	3.55E-03	0.123041	2.27E-04	0.123279	2.29E-04	**Bayes**
	0.5	1	0.239059	0.579311	1	4.51E-24	1	0.00E+00	1	0.01E+00	**Bayes**
	1	0.99367	0.016931	9.54E-01	0.911626	0.00838	0.993117	1.36E-05	0.993163	1.34E-05	**MLE**
	1.5	0.632121	0.003363	3.95E-01	0.397866	5.90E-02	0.634376	1.51E-03	0.634883	1.53E-03	**Bayes**
	2	0.271237	0.001065	7.30E-02	0.149455	1.56E-02	0.273962	6.03E-04	0.274284	6.07E-04	**Bayes**
	2.5	0.121553	0.000437	1.47E-02	0.063641	3.51E-03	0.122698	1.46E-04	0.122857	1.48E-04	**Bayes**
	0.5	1	0.23606	0.583805	1	5.24E-25	1	0.00E+00	1	0.02E+00	**Bayes**
	1	0.99367	0.016666	9.55E-01	0.915379	0.007321	0.993307	9.21E-06	0.993341	9.12E-06	**MLE**
	1.5	0.632121	0.003319	3.95E-01	0.394854	5.92E-02	0.634312	1.20E-03	0.634692	1.22E-03	**Bayes**
	2	0.271237	0.001048	7.30E-02	0.147985	1.59E-02	0.272184	4.66E-04	0.272424	4.68E-04	**Bayes**
	2.5	0.121553	0.00043	1.47E-02	0.063234	3.52E-03	0.122481	1.08E-04	0.1226	1.09E-04	**Bayes**
	0.5	1	0.2337	0.587385	1	2.94E-23	1	0.00E+00	1	0.03E+00	**Bayes**
	1	0.99367	0.016497	9.55E-01	0.917001	0.006825	0.993434	6.58E-06	0.993461	6.51E-06	**MLE**
	1.5	0.632121	0.003271	3.95E-01	0.398861	5.68E-02	0.63344	9.07E-04	0.633745	9.08E-04	**Bayes**
	2	0.271237	0.001042	7.30E-02	0.147334	1.58E-02	0.271978	3.55E-04	0.27217	3.56E-04	**Bayes**
	2.5	0.121553	0.000425	1.47E-02	0.063133	3.50E-03	0.122636	8.59E-05	0.122732	8.65E-05	**Bayes**

Conclusion

The research consist on estimating scale parameter (θ) and reliability function by different fourth methods, we found the first best estimator in maximum likelihood of ordered observation, then the proposed Bayes at all different sample size, also at the set of initial values, the third best estimator is least square, while the best estimator of reliability function is $[\hat{R}_{MLEO}]$, since this method depend on ordered observation which is necessary in estimation.

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