P-ISSN 1991-8941
E-ISSN 2706-6703
Vol. 19, No. 2 (2025)

Journal of University of Anbar for Pure Science (JUAPS)

journal homepage: juaps.uoanbar.edu.iq

Open Access

Original Paper

Some properties of a new general triple complex integral transform

Nada Sabeeh Mohammed [1*], **Luma J.Barghooth** [2], **Emad A. Kuffi** [2]

[1]Department of Bioinformatics ,University of Information Technology and Communications, Biomedical Informatics College,Baghdad, Iraq
[2]Department of Mathematics, Mustansiriyah University, College of Basic Education, Baghdad ,Iraq

ARTICLE INFO

Received: 11/04/2025
Accepted: 26/05/2025
Available online: 21/11/2025
December Issue
10.37652/juaps.2025.159020.1368

CITE @ JUAPS

Corresponding author
Nada Sabeeh Mohammed
nada.sabeeh@uoitc.edu.iq

ABSTRACT

This paper investigates a novel triple complex integral transform, highlighting its properties and its applications to functional, integral, and partial differential equations. Several fundamental theorems describing the general characteristics of the triple complex integral transform are formulated and rigorously proved. The concept of convolution associated with the transform is also examined, its properties are established, and a detailed proof of the convolution theorem is presented. Integral transformations are regarded as one of the most important methods for solving partial differential equations, as they often simplify both the solution procedure and the underlying mathematical operations. In particular, higher-order partial differential equations admit simpler solutions in the framework of integral transforms. The triple complex transform considered in this work (Complex Sadik triple integral transform) is likewise shown, through illustrative examples, to yield simpler solution steps compared with previous methods. The central objective of this study is to develop the triple complex integral transform as an effective tool for solving initial and boundary value problems in applied mathematics and mathematical physics.

Keywords: *Convolution theorem, Double complex integral transform, Triple complex integral transform*

1 INTRODUCTION

Integral transforms have long demonstrated their ability to address highly complex problems by providing accurate solutions through relatively simple steps, and they have achieved many successes in practical applications across various scientific fields [1–5]. Among the most important of these transforms is the Laplace transform, which has been highly successful in mathematical analysis and in solving linear and integral differential equations. Its basic concepts were introduced by Laplace through his studies of probability theory and celestial mechanics, leading to results that helped make this transform one of the most widely used in mathematics. Fourier, working on the theory of heat analysis, developed the modern theory of heat conduction and introduced the Fourier series, which have been applied in many scientific disciplines as well as in engineering. His work also led to the integral representation of a non-periodic function $g(y)$ for all real values of y, which is now universally recognized as the Fourier integral theorem.

$$g(y) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{jxy}\left\{\int_{-\infty}^{\infty} g(\beta)e^{-jx\beta}d\beta\right\}dx$$

In addition, throughout the nineteenth and twentieth centuries, Fourier's theorem was regarded as a cornerstone of mathematical analysis, and both Kelvin and Peter Guthrie Tait noted that it addressed some of the most important problems in contemporary physics. It is worth noting that Fourier developed the concepts of the Fourier transform and its inverse through the Fourier integral theorem, and these ideas were already familiar to Laplace; in fact, the Laplace transform can be viewed as a special case of the Fourier transform. A. L. Cauchy contributed to the development of several basic concepts in the theory of Fourier transforms, while S. D. Poisson applied the Fourier transform in his study of the propagation of water waves. Despite the many developments in Laplace and Fourier transforms, it was Oliver Heaviside who played a decisive role in making the Laplace transform widely used. He applied it to solve ordinary differential equations in electrical circuits and electrical systems, which led to the creation of modern operational differentiation and integration. Heaviside also used his method to solve telegraph equations and second-order partial differential equations with constant coefficients. Later, T. J. Bromwich contributed to the theory of complex functions, providing a rigorous mathematical foundation for these methods. Following the work of Bromwich, J. R. Carson, Van der Pol, and G. Doetsch further developed operational calculus in a precise and systematic way. After the Laplace and Fourier transforms had demonstrated their effectiveness in solving many problems in mathematics, science, and engineering, the use of integral transforms for solving differential equations became a natural trend because of their importance and efficiency in handling complex problems in mathematics and in fields such as science, security, and other applications [6–13]. At present, researchers increasingly employ triple integral transforms to obtain mathematical solutions in areas such as solving partial differential equations, Laplace equations, Mboktara equations, wave equations, and in other fields including communications, physics, and engineering [14–18].

Therefore, the primary aim of this paper is to study the triple complex integral transform, examining its properties through examples and applications to functional, integral, and partial differential equations. Several fundamental theorems concerning the general properties of the triple complex integral transform are rigorously proved. In addition, the convolution of g(y,z,h) and v(y,z,h), together with its properties and the associated convolution theorem, is analyzed in detail. We restrict our work here to the theoretical construction of this triple transform in order to obtain exact solutions to linear partial differential equations and to solve initial and boundary value problems in applied mathematics and mathematical physics. In future work, it may be possible to identify additional properties and results that can be used to treat nonlinear partial differential equations.

2 OVERVIEW AND EXAMPLES OF THE TRIPLE COMPLEX INTEGRAL TRANSFORM:

The triple complex integral transform of a function $G(y, z, h)$, involving three variables y, z, and h, defined within the first octant of the yzh-plane, is expressed as a triple integral in the following form:

$$g(b,m,n) = \left(S_a^c\right)_3 [G(y,z,h)] = S_a^c \left\{S_a^c \left(S_a^c[G(y,z,h); y \to b]; z \to m\right)\right\}; h \to n\} \quad (1)$$

therefore

$$g(b,m,n) = \left(S_a^c\right)_3 [G(y,z,h)] = \frac{1}{f^{\alpha}} \int_0^{\infty}\int_0^{\infty}\int_0^{\infty} G(y,z,h) e^{-jf^{\delta}(by+mz+nh)} dydzdh \quad (2)$$

Where j is a complex number, f, b, m, n are complex parameters,$Im(f^{\delta}b) < 0, Im(f^{\delta}m) < 0, Im(f^{\delta}n) < 0, f^{\alpha} \neq 0$,and α, δ are real numbers. $g(b) = S_a^c[G(y); y \to b]$ of $g(y)$ and to define by

$$g(b) = S_a^c[G(y)] = \frac{1}{f^a}\int_0^{\infty} G(y) e^{-jf^{\delta}(by)} dy$$

$\mathrm{Re}(y) > 0$

The inverse complex integral transform of $g(y)$ is represented and defined as follows:

$$G(y) = \left\{S_a^c\right\}^{-1} [\text{ g(b)}] = \frac{1}{2\pi f^{\alpha}} \int_{r-\infty}^{r+\infty} \text{g(b)} e^{jf^{\delta}(by)} db \quad r \geq 0 \quad (3)$$

Clearly, $(S_a^c)_3$ represents a linear integral transform, as demonstrated below:

$$\left(S_a^c\right)_3 = \{k_1 G_1(y,z,h) + k_2 G_2(y,z,h)\} \quad (4)$$

$$\frac{1}{f^{\alpha}}\left[\int_0^{\infty}\int_0^{\infty}\int_0^{\infty} \{k_1 G_1(y,z,h) + k_2 G_2(y,z,h)\} e^{-jf^{\delta}(by+mz+nh)} dydzdh\right] \quad (5)$$

$$\frac{1}{f^{\alpha}}\left[\int_0^{\infty}\int_0^{\infty}\int_0^{\infty} \{k_1 G_1(y,z,h)\} e^{-jf^{\delta}(by+mz+nh)} dydzdh + \int_0^{\infty}\int_0^{\infty}\int_0^{\infty} \{k_2 G_2(y,z,h)\} e^{-jf^{\delta}(by+mz+nh)} dydzdh\right]$$

(6)

$$\frac{1}{f^{\alpha}}\left[k_1\int_0^{\infty}\int_0^{\infty}\int_0^{\infty}\{G_1(y,z,h)\}\,e^{-jf^{\delta}(by+mz+nh)}dydzdh\right.$$
$$\left.+k_2\int_0^{\infty}\int_0^{\infty}\int_0^{\infty}\{G_2(y,z,h)\}\,e^{-jf^{\delta}(by+mz+nh)}dydzdh\right] \quad (7)$$

where k_1 and k_2 are constants.

The inverse triple complex transform, $\{S_a^c\}^{-1}(g(b,m,n) = G(y,z,h)$, is defined using the following triple integral formula:

$$\begin{aligned}&\{\mathrm{S_a^c}\}^{-1}(\mathrm{g(b,\ m,n) = G(y,z,\ h)}\\ &= \frac{1}{2\pi \mathrm{f}^{\alpha}}\left[\int_{r-\infty}^{r+\infty} e^{jf^{\delta}(by)}db\right.\\ &\left.\int_{o-\infty}^{o+\infty} e^{jf^{\delta}(mz)}dm\int_{q-\infty}^{q+\infty} e^{jf^{\delta}(nh)}dn\right]\end{aligned} \quad (8)$$

It follows that $\{S_a^c\}^{-1}(g(b,m,n)$ adheres to the linearity property.

1) If $G(y,z,h) = p$ for $y > 0, z > 0$ and $h > 0$ then

$$\begin{aligned}g(b,m,n) &= (S_a^c)_3\,(p)\\ &= S_a^c\{S_a^c(S_a^c[p;y\to b];z\\ &\to m)\};h\to n\}\end{aligned}$$

$$\begin{aligned}g(b,m,n) &= (S_a^c)_3\,(p)\\ &= p\frac{1}{f^{\alpha}}\int_0^{\infty}\int_0^{\infty}\int_0^{\infty} e^{-jf^{\delta}(by+mz+nh)}dydzdh\end{aligned} \quad (9)$$

$$\frac{p}{f^{\alpha}}\left[\int_0^{\infty} e^{jf^{\delta}(by)}db\int_0^{\infty} e^{jf^{\delta}(mz)}dm\int_0^{\infty} e^{jf^{\delta}(nh)}dn\right] = \frac{p}{f^{\alpha}(bmn)} \quad (10)$$

2) If $G(y,z,h) = \exp(ey+tz+ih)$ for all y, z and h, then $g(b,m,n) =$

$$(S_a^c)_3\{\exp(ey+tz+ih)\} = \frac{1}{f^{\alpha}}\int_0^{\infty}\int_0^{\infty}\int_0^{\infty} e^{-jf^{\delta}(\{b-e\}y+\{m-t\}z+\{n-i\}h)}dydzdh$$

(11)

$$= \frac{1}{f^{\alpha}}\left\{\int_0^{\infty} e^{-jf^{\delta}(\{b-e\}y)}dy + \int_0^{\infty} e^{-jf^{\delta}(\{m-t\}z)}dz + \int_0^{\infty} e^{-jf^{\delta}(\{n-i\}h)}dh\right\} \quad (12)$$

$$= \frac{1}{f^{\alpha}(b-e)(m-t)(n-i)} \quad (13)$$

3) Similarly if $G(y,z,h) = \exp\{j(ey+tz+ \mathrm{ih})\}$, then

$$\begin{aligned}g(b,m,n) &= (S_a^c)_3\{\exp(ey+tz+ih)\}\\ &= \frac{1}{f^{\alpha}(b-je)(m-jt)(n-ji)}\end{aligned} \quad (14)$$

$$\begin{aligned}&\to (S_a^c)_3\{\cos(ey+tz+ih)+\\ &j\sin(ey+tz+ih)\} =\\ &\frac{(b+je)(m+jt)(n+ji)}{f^{\alpha}(b^2+e^2)(m^2+t^2)(n^2+i^2)}\end{aligned} \quad (15)$$

$$\begin{aligned}&\to (S_a^c)_3\{\cos(ey+tz+ih)\}+\\ &j\,(S_a^c)_3\{\sin(ey+tz+ih)\} =\\ &\frac{\{(yz-et)h-(ez+ty)i\}+j\{(yz-et)i+(ez+ty)h\}}{f^{\alpha}(b^2+e^2)(m^2+t^2)(n^2+i^2)}\end{aligned} \quad (16)$$

Therefore

$$\begin{aligned}&(S_a^c)_3\{\cos(ey+tz+ih)\}\\ &= \frac{\{(yz-et)h-(ez+ty)i\}}{f^{\alpha}(b^2+e^2)(m^2+t^2)(n^2+i^2)}\end{aligned} \quad (17)$$

And

$$\begin{aligned}&(S_a^c)_3\{\sin(ey+tz+ih)\} =\\ &\frac{\{(yz-et)i+(ez+ty)h\}}{f^{\alpha}(b^2+e^2)(m^2+t^2)(n^2+i^2)}\end{aligned} \quad (18)$$

If $e = t = i = 1$ then

$$\begin{aligned}&(S_a^c)_3\{\cos(y+z+h)\}\\ &= \frac{\{yzh-z-y-h\}}{f^{\alpha}(b^2+1)(m^2+1)(n^2+1)}\end{aligned} \quad (19)$$

And

$$(S_a^c)_3\{\sin(y+z+h)\} = \frac{\{yz+zh+hy-1\}}{f^\alpha(b^2+1)(m^2+1)(n^2+1)} \quad (20)$$

4) If $G(y,z,h)=\cosh(ey+tz+ih)$, then

$$(S_a^c)_3\{\cosh(ey+tz+ih)\} = \frac{1}{2}\{(S_a^c)_3[\exp(ey+tz+ih)] + (S_a^c)_3[\exp(-ey-tz-ih)]\}$$
$$= \frac{1}{2f^\alpha}\left[\frac{1}{(b-e)(m-t)(n-i)} + \frac{1}{(b+e)(m+t)(n+i)}\right] \quad (21)$$

Also,

$$(S_a^c)_3\{\sinh(ey+tz+ih)\} = \frac{1}{2}\{(S_a^c)_3[\exp(ey+tz+ih)] - (S_a^c)_3[\exp(-ey-tz-ih)]\}$$
$$= \frac{1}{2f^\alpha}\left[\frac{1}{(b-e)(m-t)(n-i)} - \frac{1}{(b+e)(m+t)(n+i)}\right] \quad (22)$$

5) If $G(y,z,h)=yzh^s$, then

$$(S_a^c)_3\{yzh^s\} = \frac{1}{f^\alpha}\int_0^\infty\int_0^\infty\int_0^\infty \{yzh^s\}\, e^{-jf^\delta(by+mz+nh)}dydzdh$$

$$= \frac{1}{f^\alpha}\left\{\int_0^\infty \{y^s\}\, e^{-jf^\delta(by)}dy + \int_0^\infty \{yz^s\}\, e^{-jf^\delta(mz)}dz + \int_0^\infty \{h^s\}\, e^{-jf^\delta(nh)}dh\right\} = \frac{1}{f^\alpha}\left\{\frac{s!}{b^{s+1}}\cdot\frac{s!}{m^{s+1}}\cdot\frac{s!}{n^{s+1}}\right\}$$
$$= \frac{1}{f^\alpha}\left\{\frac{\{s!\}^s}{b^{s+1}m^{s+1}n^{s+1}}\right\} \quad (23)$$

Here, s represents a positive integer. Also

$$(S_a^c)_3\{y^\theta z^\vartheta h^\varepsilon\} = \frac{1}{f^\alpha}\left\{\frac{\theta!}{b^{\theta+1}}\cdot\frac{\vartheta!}{m^{\vartheta+1}}\cdot\frac{\varepsilon!}{n^{\varepsilon+1}}\right\} = \frac{1}{f^\alpha}\left\{\frac{\theta!\vartheta!\varepsilon!}{b^{\theta+1}m^{\vartheta+1}n^{\varepsilon+1}}\right\} \quad (24)$$

We are aware of the relationship between Gamma notation and factorial notation $\Gamma(\mu+1)=\mu!$, then

$$(S_a^c)_3\{y^\theta z^\vartheta h^\varepsilon\} = \frac{1}{f^\alpha}\left\{\frac{\Gamma(\theta+1)}{b^{\theta+1}}\cdot\frac{\Gamma(\vartheta+1)}{m^{\vartheta+1}}\cdot\frac{\Gamma(\varepsilon+1)}{n^{\varepsilon+1}}\right\} = \frac{1}{f^\alpha}\left\{\frac{\theta!\vartheta!\varepsilon!}{b^{\theta+1}m^{\vartheta+1}n^{\varepsilon+1}}\right\} \quad (25)$$

Here, θ, ϑ and ε are represented positive integers.

Theorem 1: First Shifting Theorem: If $(S_a^c)_3\{G(y,z,h)\} = g(b,m,n)$, then:

$$(S_a^c)_3\{e^{-(ey+tz+ih)}G(y,z,h)\} = \frac{1}{f^\alpha}g\{[b+e],[m+t],[n+i]\} \quad (26)$$

Proof: If $(S_a^c)_3\{G(y,z,h)\} = g(b,m,n) = \frac{1}{f^\alpha}\int_0^\infty\int_0^\infty\int_0^\infty G(y,z,h)e^{-jf^\delta(by+mz+nh)}dydzdh$, then

$$(S_a^c)_3\{e^{-(ey+tz+ih)}G(y,z,h)\} = \frac{1}{f^\alpha}\int_0^\infty\int_0^\infty\int_0^\infty G(y,z,h)e^{-jf^\delta(by+mz+nh)} e^{-(ey+tz+ih)}dydzdh \quad (27)$$

$$= \frac{1}{f^\alpha}\int_0^\infty\int_0^\infty\int_0^\infty G(y,z,h)e^{-jf^\delta\{[b+e]y+[m+t]z+[n+i]h\}}dydzdh$$
$$\frac{1}{f^\alpha}\left\{\int_0^\infty e^{-jf^\delta\{[b+e]y\}}\left[\int_0^\infty\int_0^\infty G(y,z,h)e^{-jf^\delta\{[m+t]z+[n+i]h\}}dzdh\right]dy\right\} \quad (28)$$

It is worth noting that the integral within the brackets adheres to the properties of the double.

The complex transform is presented as:

$$\left[\int_0^\infty\int_0^\infty G(y,z,h)e^{-jf^\delta\{[m+t]z+[n+i]h\}}dzdh\right] = g(y,m+t,n+i)(29) \quad (29)$$

Then

$$\frac{1}{f^\alpha}\int_0^\infty e^{-jf^\delta\{[b+e]y\}}g(y,m+t,n+i) = \frac{1}{f^\alpha}g\{[b+e],[m+t],[n+i]\} \quad (30)$$

Theorem 2: Change of Scale Property: If $\left(S_a^c\right)_3\{G(y,z,h)\}=g(b,m,n)$, then:

$$\left(S_a^c\right)_3\{G(ey,tz,ih)\}=\frac{1}{f^a eti}\left[\frac{b}{e},\frac{m}{t},\frac{n}{i}\right] \quad (31)$$

Proof: If $\left(S_a^c\right)_3\{G(y,z,h)\} = g(b,m,n) = \frac{1}{f^\alpha}\int_0^\infty\int_0^\infty\int_0^\infty G(y,z,h)e^{-jf^\delta(by+mz+nh)}dydzdh$, then:

$$\begin{aligned}&\left(S_a^c\right)_3\{G(ey,tz,ih)\}\\&=\frac{1}{f^\alpha}\int_0^\infty\int_0^\infty\int_0^\infty G(ey,tz,ih)e^{-jf^\delta(by+mz+nh)}dydzdh\\&=\frac{1}{f^\alpha}\left\{\int_0^\infty e^{-jf^\delta\{[by]\}}\right.\\&\left.\left[\int_0^\infty\int_0^\infty G(ey,tz,ih)e^{-jf^\delta\{[mz]+[nh]\}}dzdh\right]dy\right\}\end{aligned} \quad (32)$$

It is important to note that the integral within the brackets complies with the properties of the double complex transform and is expressed as:f

$$\int_0^\infty\int_0^\infty G(ey,tz,ih)e^{-jf^\delta\{[mz]+[nh]\}}dzdh=\frac{1}{ti}\left[ey,\frac{m}{t},\frac{n}{i}\right] \quad (33)$$

So that

$$\frac{1}{f^\alpha}\left\{\int_0^\infty e^{-jf^\delta\{[by]\}}\frac{1}{ti}\left[ey,\frac{m}{t},\frac{n}{i}\right]\right\}=\frac{1}{f^\alpha eti}\left[\frac{b}{e},\frac{m}{t},\frac{n}{i}\right] \quad (34)$$

Theorem 3: Multiplication by $y^\theta z^\vartheta h^\varepsilon$:
If $\left(S_a^c\right)_3\{G(y,z,h)\}=g(b,m,n)$, then:

$$\left(S_a^c\right)_3\{G(y,z,h)y^\theta z^\vartheta h^\varepsilon\}=\frac{1}{f^\alpha}(-1)^{\theta+\vartheta+\varepsilon}\frac{\partial^{\theta+\vartheta+\varepsilon}g(b,m,n)}{\partial b^\theta\partial m^\vartheta\partial n^\varepsilon} \quad (35)$$

Proof:
If $\left(S_a^c\right)_3\{G(y,z,h)\}=g(b,m,n)=\frac{1}{f^\alpha}\int_0^\infty\int_0^\infty\int_0^\infty G(y,z,h)e^{-jf^\delta(by+mz+nh)}dydzdh$, then

$$\begin{aligned}&\frac{\partial^{\theta+\vartheta+\varepsilon}g(b,m,n)}{\partial b^\theta\partial m^\vartheta\partial n^\varepsilon}=\frac{\partial^{\theta+\vartheta+\varepsilon}g(b,m,n)}{\partial b^\theta\partial m^\vartheta\partial n^\varepsilon}\\&\left[\frac{1}{f^\alpha}\int_0^\infty\int_0^\infty\int_0^\infty G(y,z,h)e^{-jf^\delta(by+mz+nh)}dydz\right.\\&\left[\frac{1}{f^\alpha}\left\{\frac{\partial^\theta}{\partial b^\theta}\int_0^\infty e^{-jf^\delta[by]}\left[\frac{\partial^{\vartheta+\varepsilon}}{\partial m^\vartheta\partial n^\varepsilon}\right.\right.\right.\\&\left.\left.\left.\int_0^\infty\int_0^\infty G(y,z,h)e^{-jf^\delta[mz+nh]}dzdh\right]dy\right\}\right]\end{aligned} \quad (36)$$

It is worth noting that the integral within the brackets adheres to the properties of the double.

The complex transform and is presented as:

$$\frac{\partial^{\vartheta+\varepsilon}}{\partial m^\vartheta n^\varepsilon}\int_0^\infty\int_0^\infty G(y,z,h)e^{-jf^\delta\{[mz]+[nh]\}}dzdh=\left(S_a^c\right)_2\left\{(-1)^{\vartheta+\varepsilon}z^\vartheta h^\varepsilon G(y,z,h)\right\}\ (37) \quad (37)$$

So that

$$\begin{aligned}\frac{\partial^{\theta+\vartheta+\varepsilon}g(b,m,n)}{\partial b^\theta\partial m^\vartheta\partial n^\varepsilon}&=\left[\frac{1}{f^\alpha}\left\{\frac{\partial^\theta}{\partial b^\theta}\int_0^\infty e^{-jf^\delta\{[by]\}}\right.\right.\\&\left.\left.\left[\left(S_a^c\right)_2\left\{(-1)^{\vartheta+\varepsilon}z^\vartheta h^\varepsilon G(y,z,h)\right\}dy\right]\right\}\right.\\&=\left(S_a^c\right)_3\left\{\frac{1}{f^\alpha}(-1)^{\theta+\vartheta+\varepsilon}G(y,z,h)y^\theta z^\vartheta h^\varepsilon\right\}\end{aligned} \quad (38)$$

Therefore

$$\left(S_a^c\right)_3\{G(y,z,h)y^\theta z^\theta h^\varepsilon\}=\frac{1}{f^\alpha}\left\{(-1)^{\theta+\vartheta+\varepsilon}\left[\frac{\partial^{\theta+\vartheta+\varepsilon}g(b,m,n)}{\partial b^\theta\partial m^\vartheta\partial n^\varepsilon}\right\}\right. \quad (39)$$

Theorem 4: If the function $G(y,zh)$ is triple complex transform then:

1- If

$$\begin{aligned}\left(S_a^c\right)_3\left(\frac{\partial^3G(y,z,h)}{\partial y\partial z\partial h}\right)&=\frac{1}{f^\alpha}\{bmng(b,m,n)-bmg(b,m,0)\\&-bng(b,0,n)-mng(0,m,n)+bg(b,0,0)\\&+mg(0,m,0)+ng(0,0,n)-g(0,0,0)\}\end{aligned} \quad (40)$$

2- If

$$\left(S_a^c\right)_3\left(\frac{\partial^3 G(y,z,h)}{\partial y^3}\right)=\frac{1}{f^\alpha}\left\{b^3 g(b,z,h)- b^2 g(0,z,h)-b\frac{\partial g(0,z,h)}{\partial y}-\frac{\partial^2 g(0,z,h)}{\partial y^2}\right\} \quad (41)$$

Theorem 5: If L $\left(S_a^c\right)_3\{G(y,z,h)\}=g(b,m,n)$, then

$$\left(S_a^c\right)_3\{G(y-\gamma,z-\delta,h-\epsilon)Z(y-\gamma,z-\delta,h-\epsilon) = e^{-(\gamma b+\delta z+\epsilon h)}g(b,m,n) \quad (42)$$

Here, $Z(y,z,h)$ represents the Heaviside unit step function, which is defined as: $z(y-e,z-\mathrm{t},h-i)=1$, Here, y > e, z > t, h > i : and $Z(y-e,z-\mathrm{t},h-i)=0$ where y < e, z < t, h < i.

Proof: If $\left(S_a^c\right)_3\{G(y,z,h)\}=g(b,m,n)=\frac{1}{f^\alpha}\int_0^\infty\int_0^\infty\int_0^\infty G(y,z,h)e^{-jf^\delta(by+mz+nh)}dydzdh$, then

$$\left(S_a^c\right)_3\{G(y-\gamma,z-\delta,h-\epsilon)Z(y-\gamma,z-\delta,h-\epsilon)\} =\frac{1}{f^\alpha}\int_0^\infty\int_0^\infty\int_0^\infty G(y-\gamma,z-\delta,h-\epsilon)Z(y- \gamma,z-\delta,h-\epsilon)e^{-jf^\delta(by+mz+nh)}dydzdh \quad (43)$$

$$=\frac{1}{f^\alpha}\int_0^\infty\int_0^\infty\int_0^\infty G(y-\gamma,z-\delta,h-\epsilon) e^{-jf^\delta(by+mz+nh)}dydzdh \quad (44)$$

This can be obtained by substituting: $y-\gamma=v, z-\delta=\mathrm{u}, h-\epsilon=x$

$$=e^{-(b\gamma+m\delta+n\epsilon)}\left[\frac{1}{f^\alpha}\int_0^\infty\int_0^\infty\int_0^\infty G(y-\gamma,z-\delta,h-\epsilon)e^{-jf^\delta(bv+mu+nx)}dvdudx\right] \quad (45)$$

$$=e^{-(b\gamma+m\delta+n\epsilon)}g(b,m,n). \quad (46)$$

Theorem 6: If $G(y,z,h)$ is a periodic function with periods γ,δ, and ϵ, i.e., $G(y+\gamma,z+\delta,h+\epsilon)=G(y,z,h)$ and if $\left(S_a^c\right)_3\{G(y,z,h)\}$ exists, then

$$\left(S_a^c\right)_3\{G(y,z,h)\}=\frac{1}{\left\{1-e^{-(b\gamma+m\delta+n\epsilon)}\right\}f^\alpha} \int_0^\gamma\int_0^\delta\int_0^\epsilon G(y,z,h)e^{-jf^\delta(by+mz+nh)}dydzdh \quad (47)$$

Proof: If

$$\left(S_a^c\right)_3\{G(y,z,h)\}=g(b,m,n)= \frac{1}{f^\alpha}\int_0^\infty\int_0^\infty\int_0^\infty G(y,z,h)e^{-jf^\delta(by+mz+nh)}dydzdh= \frac{1}{f^\alpha}\int_0^\gamma\int_0^\delta\int_0^\epsilon G(y,z,h)e^{-jf^\delta(by+mz+nh)}dydzdh+ \frac{1}{f^\alpha}\int_0^\infty\int_0^\infty\int_0^\infty G(y,z,h)e^{-jf^\delta(by+mz+nh)}dydzdh \quad (48)$$

Putting $y=v+\gamma, z=\mathrm{u}+\delta, h=x+\epsilon$ in second triple integral, we get:

$$g(b,m,n)= \frac{1}{f^\alpha}\int_0^\gamma\int_0^\delta\int_0^\epsilon G(y,z,h)e^{-jf^\delta(by+mz+nh)}dydzdh+ e^{-(b\gamma+m\delta+n\epsilon)}\left\{\frac{1}{f^\alpha}\int_0^\infty\int_0^\infty\int_0^\infty G(v+\gamma,\mathrm{u}+\delta,x+ \epsilon)e^{-jf^\delta(bv+mu+nx)}dvdudx\right\} \quad (49)$$

$$=\frac{1}{f^\alpha}\int_0^\gamma\int_0^\delta\int_0^\epsilon G(y,z,h)e^{-jf^\delta(by+mz+nh)}dydzdh +e^{-(b\gamma+m\delta+n\epsilon)} \left\{\frac{1}{f^\alpha}\int_0^\infty\int_0^\infty\int_0^\infty G(v,\mathrm{u},x)e^{-jf^\delta(bv+mu+nx)}dvd\right\}udx\Big\} \quad (50)$$

$$=\frac{1}{f^\alpha}\int_0^\gamma\int_0^\delta\int_0^\epsilon G(y,z,h)e^{-jf^\delta(by+mz+nh)}dydzdh +e^{-(b\gamma+m\delta+n\epsilon)}g(b,m,n) \quad (51)$$

$$\therefore g(b,m,n)=\frac{1}{\left\{1-e^{-(b\gamma+m\delta+n\epsilon)}\right\}f^\alpha} \int_0^\gamma\int_0^\delta\int_0^\epsilon G(y,z,h)e^{-jf^\delta(by+mz+nh)}dydzdh \quad (52)$$

Convolution and the Convolution Theorem for Triple Complex Transform

The convolution of $G(y,z,h)$ and $W(y,z,h)$ is denoted by $(G***W)(y,z,h)$ and defined by:

$$(G***W)(y,z,h) = \int_0^y \int_0^z \int_0^h G(y-\gamma, z-\delta, h-\epsilon) W(\gamma,\delta,\epsilon) d\gamma d\delta d\epsilon \tag{53}$$

Theorem 7: Convolution Theorem: Let $\left(S_a^c\right)_3 \{G(y,z,h)\} = g(b,m,n)$ and $\left(S_a^c\right)_3 \{W(y,z,h)\} = w(b,m,n)$, then:

$$\begin{aligned} &\left(S_a^c\right)_3 \{(G***W)(y,z,h)\} \\ &= \left(S_a^c\right)_3 \{G(y,z,h)\} \cdot \left(S_a^c\right)_3 \{W(y,z,h)\} \\ &= g(b,m,n) \cdot w(b,m,n) \end{aligned} \tag{54}$$

Or, equivalently,

$$\begin{aligned} &\left(S_a^c\right)_3^{-1} \{g(b,m,n) w(b,m,n)\} \\ &\quad = (G***W)(y,z,h) \end{aligned} \tag{55}$$

The term $(G***W)(y,z,h)$ is defined by a triple integral, commonly referred to as the convolution integral of $G(y,z,h)$ and $W(y,z,h)$. Physically, $(G***W)(y,z,h)$ represents the combined output resulting from the interaction of $G(y,z,h)$ and $W(y,z,h)$.

Proof: By definition, we have:

$$\left(S_a^c\right)_3 \{(G***W)(y,z,h)\} = \frac{1}{f^\alpha} \int_0^\infty \int_0^\infty \int_0^\infty (G***W)(y,z,h) e^{-jf^\delta(by+mz+nh)} dydzdh$$

$$= \frac{1}{f^\alpha} \left[\begin{array}{c} \int_0^\infty e^{-jf^\delta(by+mz+nh)} dydzdh \\ \int_0^\infty \int_0^\infty \left\{ \begin{array}{c} G(y-\gamma, z-\delta, h-\epsilon) \\ \int_0^y \int_0^z \int_0^h \; W(\gamma,\delta,\epsilon) \\ d\gamma d\delta d\epsilon \end{array} \right\} \\ dydzdh \end{array} \right] \tag{56}$$

This can be expressed using the Heaviside unit step function as:

$$\begin{aligned} &\frac{1}{f^\alpha} \left[\int_0^\infty \int_0^\infty \int_0^\infty e^{-jf^\delta(by+mz+nh)} dydzdh \{ \right. \\ &\int_0^\infty \int_0^\infty \int_0^\infty G(y-\gamma, z-\delta, h-\epsilon) \\ &\left. Z(y-\gamma, z-\delta, h-\epsilon) W(\gamma,\delta,\epsilon) d\gamma d\delta d\epsilon \} dydzdh \right] \end{aligned} \tag{57}$$

$$= \frac{1}{f^\alpha} \left[\begin{array}{c} \int_0^\infty \int_0^\infty \int_0^\infty W(\gamma,\delta,\epsilon) d\gamma d\delta d\epsilon \\ \left\{ \begin{array}{c} e^{-jf^\delta(by+mz+nh)} \\ \int_0^\infty \int_0^\infty \int_0^\infty \; G(y-\gamma, z-\delta, h-\epsilon) \\ Z(y-\gamma, z-\delta, h-\epsilon) \\ dydzdh \end{array} \right\} \end{array} \right] \tag{58}$$

which is, by Theorem 5,

$$= \frac{1}{f^\alpha} \left[\int_0^\infty \int_0^\infty \int_0^\infty e^{-jf^\delta(b\gamma+m\delta+n\epsilon)} g(b,m,n) \right] \tag{59}$$

$$\begin{aligned} &= g(b,m,n) \frac{1}{f^\alpha} \left[\int_0^\infty \int_0^\infty \int_0^\infty e^{-jf^\delta(b\gamma+m\delta+n\epsilon)} \right. \\ &\left. W(\gamma,\delta,\epsilon) d\gamma d\delta d\epsilon \right] \end{aligned} \tag{60}$$

$$= g(b,m,n) \cdot w(b,m,n) \tag{61}$$

Thus, the proof of the convolution theorem is concluded.

3 APPLICATION

Consider the following homogeneous partial differential equation:

$$\left(\frac{\partial^3 g(y,z,h)}{\partial y \partial z \partial h} \right) + g(y,z,h) = 0 \tag{62}$$

subject to the conditions:

$$\begin{aligned} &g(y,z,0) = e^{y+z}, g(y,0,h) = e^{y-h}, g(0,z,h) = e^{z-h}, \\ &g(0,0,h) = e^{-h}, g(0,z,0) = e^z, g(y,0,0) = e^y, \\ &g(0,0,0) = 1 \end{aligned} \tag{63}$$

Applying the general triple transform $\left(S_a^c\right)_3$ to both sides of the above equation, we obtain the following equation

$$\left(S_a^c\right)_3 \left(\frac{\partial^3 g(y,z,h)}{\partial y \partial z \partial h} \right) + \left(S_a^c\right)_3 g(y,z,h) \tag{64}$$

Running the triple integral transform of the above equation, we obtain the following equation:

$$\frac{1}{f^\alpha} yzhG(y,z,h) + \frac{1}{f^\alpha} G(y,z,h) = G_1(y,z,h) \tag{65}$$

$$G_1(y,z,h) = \frac{1}{f^{\alpha}}\{yzhg(y,z,h) - yzg(y,z,0) - yhg(y,0,h) - zhg(0,z,h) + yg(y,0,0) + zg(0,z,0) + hg(0,0,h) - g(0,0,0)\} + g(y,z,h) \tag{66}$$

Simplifying equation the equation(61), we have the following equation

$$\frac{1}{f^{\alpha}} yzhG(y,z,h) + \frac{1}{f^{\alpha}} G(y,z,h) = G_1(y,z,h) \tag{67}$$

$$G(y,z,h) = \frac{f^{\alpha} G_1(y,z,h)}{1+yzh} \tag{68}$$

Applying the inverse triple transform $\{S_a^c\}^{-1}$, we get the solution of equation (1) in the original sp

$$g(y,z,h) = \{\mathrm{S_a^c}\}^{-1}\left(\frac{f^{\alpha} G_1(y,z,h)}{1+yzh}\right) \tag{69}$$

$$\{\mathrm{S_a^c}\}^{-1}\left(\frac{f^{\alpha}}{y-1}\frac{f^{\alpha}}{z-1}\frac{f^{\alpha}}{h+1}\right) = e^{y+z-h} \tag{70}$$

which is the exact solution of equations (61) and (62).

4 CONCLUSION

This triple transform provides an effective technique for obtaining exact solutions to many engineering and physical problems involving third-order partial differential equations. The triple complex integral transformation introduced here is more general than many existing triple integral transforms, and it can be applied to solve both linear and nonlinear partial differential equations, as well as partial integro-differential equations. These equations play a central role in engineering and the life sciences, and the transform also has potential applications in the encryption of many systems.

ACKNOWLEDGEMENT

N/A

FUNDING SOURCE

No funds received.

DATA AVAILABILITY

N/A

DECLARATIONS

Conflict of interest

There is no conflict of interest.

Consent to publish

N/A

Ethical approval

N/A

REFERENCES

[1] Mohammed NS, Kuffi EA. The complex integral transform complex Sadik transform of error function. Journal of Interdisciplinary Mathematics. 2023;26(6):1145–1157. 10.47974/jim-1613

[2] Benattia ME, Belghaba K. Numerical solution for solving fractional differential equations using shifted Chebyshev wavelet. General Letters in Mathematics. 2018;3(2):101-10. 10.31559/glm2016.3.2.3

[3] Wazwaz AM. Abel's integral equation and singular integral equations. In: Linear and Nonlinear Integral Equations. Berlin, Heidelberg: Springer; 2011. 10.1007/978-3-642-21449-3$_7$

[4] Mohammed NS, Kuffi EA, Al-Nuaimi DH. Implementation of "Utilize the Complex Sadik Transform to Solution Volterra Integro-Differential Equations of Second Type". In: Mathematical Analysis and Numerical Methods. vol. 466 of Springer Proceedings in Mathematics and Statistics. Cham: Springer; 2024. IACMC 2023, Zarqa, Jordan, May 10–12

[5] Li K, Peng J. Laplace transform and fractional differential equations. Applied Mathematics Letters. 2011;24:20-3. 10.1016/j.aml.2011.05.035

[6] Debnath L, Bhatta D. Integral Transforms and Their Applications. 3rd ed. Boca Raton: Chapman and Hall/CRC; 2015

[7] Schiff JL. The Laplace Transform: Theory and Applications. New York: Springer; 1999. 10.1007/978-0-387-22757-3

[8] Dass HK. Advanced Engineering Mathematics. New Delhi: S. Chand & Company Ltd.; 2009

[9] Stroud KA. Advanced Engineering Mathematics. 4th ed. Basingstoke: Palgrave Macmillan; 2003

[10] Debnath L. The Double Laplace Transforms and Their Properties with Applications to Functional, Integral and Partial Differential Equations. International Journal of Applied and Computational Mathematics. 2015;2(2):223–241. 10.1007/s40819-015-0057-3

[11] Eltayeb H, Kiliçman A. A Note on Double Laplace Transform and Telegraphic Equations. Abstract and Applied Analysis. 2013;2013:1–6. 10.1155/2013/932578

[12] Churchill RV. Operational Mathematics. 3rd ed. New York: McGraw-Hill; 1972

[13] Yusufoglu E. Numerical solution of Duffing equation by the Laplace decomposition algorithm. Applied Mathematics and Computation. 2006;177(2):572-80. 10.1016/j.amc.2005.07.072

[14] Thakur A, Kumar A, Suryavanshi H. The triple Laplace transforms and their properties. International Journal of Applied Mathematics Statistical Sciences. 2018;7(4):23-34

[15] Sedeeg AKH. Some Properties and Applications of a New General Triple Integral Transform "Gamar Transform". Complexity. 2023;2023(1):5527095

[16] Gharib GM, Alsauodi MS, Abu-Seileek M. Conformable triple Sumudu transform with applications. WSEAS Transactions on Mathematics. 2024;23:42-50. 10.37394/23206.2024.23.5

[17] Saadeh R, K Sedeeg A, A Amleh M, I Mahamoud Z. Towards a new triple integral transform (Laplace–ARA–Sumudu) with applications. Arab Journal of Basic and Applied Sciences. 2023;30(1):546–560. 10.1080/25765299.2023.2250569

[18] Saadeh R. A Generalized Approach of Triple Integral Transforms and Applications. Journal of Mathematics. 2023;2023:1–12. 10.1155/2023/4512353

How to cite this article

Mohammed NS, Barghooth LJ, Kuffi EA. Some properties of a new general triple complex integral transform. Journal of University of Anbar for Pure Science. 2025; 19(2):249-257. doi:10.37652/juaps.2025.159020.1368