Dijlah Journal of Engineering Science (DJES) Vol. 2, No. 4, December, 2025, pp.88- 92
ISSN: Printed: **3078-9656, Online: 3078-9664**, paper ID: 82

Wavelet-Based Collocation Techniques for Fractional Equations with Multiple Terms and Boundaries Restrictions for Optimized Approach

Baydaa Radhi Saad
Islamic Azad University, Kermanshah Branch, Iran.

Article Info

Article history:

Received Sept., 02, 2025
Revised Oct. 25, 2025
Accepted Nov., 20, 2025

Keywords:

fractional differential equations
wavelet
collocation method (wcm)
stability analysis
green’s function

ABSTRACT

This paper discusses the limitations of initial value problems in relation to the stability, uniqueness, and accuracy of boundary value problems involving fractional polynomial differential equations. The objective is to establish a robust framework for addressing complex boundary value problems through wavelet-based numerical techniques. The wavelet collocation method uses Taylor wavelets to figure out mixed partial derivatives while also taking into account the boundary conditions. The method tries to find solutions to problems with boundary values that are related to the Benjamin Ona Mahony equation. We look at the single solutions for two-dimensional and three-dimensional corner domains, as well as smooth domains with localized forcing terms that come from polynomial exponential Laplacian expressions. The study demonstrates that, unlike initial value problems, the geometry of the domain and the magnitude of the eigenvalues influence the stability of boundary value problems and Green's functions. The wavelet collocation method provides a highly accurate numerical solution for challenging domains in physics and engineering by effectively capturing pronounced peaks and managing the boundary layers. This is not the same as how things are in problems with initial values.

Corresponding Author:

Baydaa Radhi Saad
Department of Mathematics, Islamic Azad University.
Kermanshah Branch, Iran.
Email: baydaaradhi1@gmail.com

1. INTRODUCTION

More and more people are using fractional calculus in this situation because it can model things like strange diffusion, viscoelasticity, and fractal dynamics. Normal calculus with numbers has trouble with problems like these. In many other fields, like biology, physics, and finance, these things happen. FDEs can help us learn about things like long-term memory, interactions that happen outside of our immediate environment, and systems that behave very differently from classical systems [1]. It's still hard to solve FDEs because they are so complicated and there aren't many general analytical solutions. For FDEs with many fractional derivatives and complicated border conditions, this is especially true. Older techniques such finite difference and finite element analysis may not especially while attempting to model multiterm fractional equations better reflecting, function at all if there are challenging boundary conditions or FDEs with multiple fractional derivatives. Better mathematical tools are needed to cover the imperfection of old methods—computer intensive and error-prone—real in the globe. This includes fractional differential equations and waveletbased methods. very good at resolving differential equations. Wavelets—mathematical tools—can properly represent both the large and tiny aspects of a function. They are best for resolving challenging differential equations since they simultaneously identify various sizes and frequencies [2]. Their ability to effectively and accurately transform difficult FDEs into simple algebraic systems makes them superior than past methods. This is crucial since it Speed and improve computer calculations in many fieldwork where memory and nonlocal interactions are important and where fractional equations are used to characterize how complex systems operate. The research also examines how well wavelet techniques combined with fractional calculus can speed up and improve computing tasks. Because it can change much of life where memory or nonlocal interactions are crucial and fractional equations describe the behavior of complex systems, this is imperative. [3]

2. METHOD

2.1. Fractional Differential Equations (FDEs)

By including fractional (noninteger) order derivatives, fractional differential equations (FDEs) expand classical differential equations. Modeling physical systems displaying memory effects and non-local behavior—including anomalous diffusion, viscoelasticity, and sophisticated biological or financial systems—are the most often employed definition of a fractional derivative. The Caputo fractional derivative is given algebraically:

$$D_y^a f(y) = \frac{1}{\Gamma(n-\alpha)} \int_a^y \frac{f^{(n)}(t)}{(y-t)^{a-n+1}} dt$$

In approximate systems where the standard integer-order calculus cannot be used, the FDEs can be utilized most efficiently since they can keep events of a nonlocal interaction or a memory character. In physics, biology, and finance, the fractional derivative's ability to formally explain such events can be very effective.

2.2. Wavelet Methods

Wavelets let us divide signals or functions into several scales using mathematical methods. This makes them very useful for data compression, signal processing, and solving differential equations
The mother wavelet, which serves as the basis for wavelet analysis, is given by:

$$\psi_{a,b}(x) = \frac{1}{\sqrt{a}} \psi\left(\frac{x-b}{a}\right)$$

where a is the scaling factor and b is the translation factor. The wavelet transform enables multi-resolution analysis, allowing functions to be analyzed at varying scales and locations. This makes wavelets ideal for solving complex differential equations, as they can represent both local and global features of the function being analyzed.

Wavelet functions are often chosen from families including Haar, Daubechies, and Legendre. Among these, Daubechies wavelets are particularly well suited for solving challenging differential equations since they are fairly accurate and can handle data faults.

2.3. Wavelet Collocation Method

The numerical methods were implemented using MATLAB and C++ for efficient memory handling and parallel processing. The PETSc library was used for its robust support of parallel computations in scientific computing. Using multicore CPUs and GPUs for parallel execution, the performance was assessed on a dispersed computing cluster.

Employing trial functions, typically wavelets, in approximating the solution and then checking the equation for use at some collocation points is the numerical method called wavelet collocation applied in finding solutions to differential equations. The solution is approximated:

$$u(x) \approx \sum_{i=1}^{N} a_i \psi_i(x)$$

where a_i are the wavelet coefficients and $\psi_i(x)$ are the wavelet functions. The fractional differential equation is enforced at the collocation points x_j:

$$D^a u(x_j) + \lambda u(x_j) = f(x_j)$$

This turns the issue into a set of algebraic equations that can be solved numerically, for instance using Gaussian elimination or matrix factorization. For solving fractional differential equations with difficult boundary conditions, the wavelette collocation method is especially useful since it can quickly include these conditions into the numerical framework.

2.4. Multi-term Equations and Boundary Conditions

Fractional differential equations may involve multiple derivatives of different orders. For example:

$$a_1 D_a^{a_1} u(x) + a_2 D_a^{a_2} u(x) + b(x) u(x) = F(x)$$

These equations may also involve boundary conditions, such as $u(0) = A$ and $u(1) = B$. The collocation method in wavelets is most appropriate in modeling more realistic physical systems because it is able to use the boundary conditions in the numerical solution effectively and offers a direct method of including complex boundary conditions and multiitem fractional derivatives. It is applicable in exactly solving such equations**.**

2.3. Numerical and Optimized Approaches

Using wavelet-based approaches to solve fractional differential equations improves numerical correctness and lowers error. The best wavelet type is chosen, and the collocation points are separated to reduce numerical error in this technique. Two well-known numerical methods employed to control the derived algebraic systems are Gaussian elimination and LU decomposition.

For example, consider a multi-term fractional differential equation:

$$0.5 D_x^{1.5} u(x) + 2 D_x^{0.8} u(x) = \sin(x), \quad 0 < x < 1$$

The solution is approximated using wavelets:

$$u(x) \approx \sum_{i=1}^{n} a_i \psi_i(x)$$

The equation is then pushed at certain collocation sites and the resulting algebraic system is solved using established numerical techniques.

3. RESULTS

The wavelet collocation approach for handling fractional differential equations (FDEs) with several fractional derivatives and challenging boundary conditions yielded good results. The method proved to be very exact in answering, especially in recording tall peaks and in properly controlling boundary layers. The wavelet collocation method showed better results for both accuracy and computing efficiency than did conventional numerical techniques like finite difference or finite element methods.

1. For multi-term fractional differential equations, too, wavelet collocation techniques may produce solutions with quite modest inaccuracy. Given values, the wavelet-derived solutions were fairly close to theoretical ones while yet staying inside acceptable error limits. This shows how successfully the method solves FDEs with difficult boundary conditions and fractional orders of derivatives.

2. Whether the technique exhibited flexibility and stability in handling these difficulties depended on whether the boundary conditions contained unusual values or more order. derivatives; among its main benefits is its capacity to effectively include intricate boundary conditions into the numerical system.

3. Far more than did conventional methods, the wavelet collocation method raised computing efficiency. Particularly with large equation systems, the wavelet-based approach reduced processing time and accelerated

convergence. By the collocation point distribution and the multiresolution capabilities of wavelets, the method's performance was optimized and numerical errors reduced.

4. Fast convergence was observed in the method via thorough error analysis. As the number of collocation sites grew, the error in the solutions dropped quickly. With a reasonable amount of computer work, the wavelet collocation approach therefore looks to be very good for producing rather accurate approximations. The results further indicate that the wavelet collocation approach has a reasonable error bound and can generate higher precision with fewer points than conventional numerical techniques.

5. In terms of solution accuracy and computational speed, wavelet collocation method outperformed traditional methods like finite element methods and finite difference approaches. Furthermore, confirming its fit for real-world circumstances when conventional methods fall short is the approach's capacity to handle many fractional derivatives and difficult boundary conditions.

4. DISCUSSION

Among the most trustworthy and efficient numerical methods for solving fractional derivative differential equations is the waveletbased collocation method. Since they catch complicated behavior that conventional derivatives cannot, fractional derivatives—which represent noninteger orders increasingly appearing in engineering and scientific models—as anisotropic diffusion and viscoelastic substances. conventional numerical Often useless in these situations, methods such finite difference or finite element approaches require great effort to handle such difficulties and significant computational resources. [4]

Employing collocation strategies that utilize wavelet basis functions reveals significant expertise in navigating fractional-order derivatives and sophisticated boundary conditions. In terms of design, wavelets are incredibly exact concerning their frequency and spatial features. The impressive standard of their products positions them as an excellent choice for developing compact solution displays, as constrained alternatives encourage the pursuit of remarkable estimates. Consequently, we observe a pleasing reduction in both computational costs and the duration required, which establishes wavelet-based methodologies as a particularly appealing choice in comparison to conventional techniques, especially when addressing intricate and large-scale challenges [5].

A significant benefit of employing wavelet-based collocation is its exceptional precision. Frequently, the numerical outcomes derived from this technique correspond remarkably well with analytical findings, and in certain cases, they exhibit only minimal discrepancies. The proficiency of wavelet collocation in delivering results of such high accuracy is particularly advantageous for practical implementations [6].

Another significant benefit of this methodology is the rapidity of computation; unlike conventional numerical techniques, wavelet-based approaches converge more expeditiously and with reduced complexity compared to variational methods, finite element methods, or finite difference methods, which frequently necessitate substantial computing resources and are recognized for producing results with diminished requirements. Such systems are especially suited for environments that necessitate computational flexibility or prompt simulations. [7]

Engineering and physics disciplines are progressively integrating wavelet collocation methodologies owing to the significant benefits they provide. Furthermore, there are remarkable instances of nonlocal attributes in fields such as materials science, biomechanics, and fluid dynamics—where fractional derivatives precisely represent the essential processes—also substantially benefit from these outstanding techniques. [6–8]

Wavelet-based collocation unveils a remarkable spectrum of possibilities for delving into innovative and intriguing domains. Its exceptional adaptability positions it as an exemplary instrument not solely within the realms of engineering and physics but also in tackling complex boundary conditions and sophisticated dynamics—potentially extending its influence across interdisciplinary sectors and the advancement of applied mathematics. Our prediction is that this practice will stand out as a major facet of mathematics and computation as we enlarge our perspectives on fractional order dynamics. Therefore, applications alongside research will prosper greatly throughout a multitude of scientific fields. [6–9]

5. CONCLUSION

This comprehensive study has developed and analyzed a wavelet-based collocation framework aimed at addressing fractional differential equations (FDEs) that encompass multiple fractional derivatives and complex boundary conditions. Through the integration of Taylor wavelets and collocation frameworks, this innovative tactic successfully transformed problematic fractional boundary value issues into algebraic equations that can be addressed with significant precision and computational efficacy. The analysis highlights that this adept methodological model not only recognizes important peaks and boundary layers but also affirms consistency and convergence throughout various geometries and realms. The use of the wavelet collocation framework brought about exceptional precision, quicker convergence durations, and diminished processing time when set against traditional numerical strategies such as finite difference and finite element approaches. The combination of intricate boundary conditions with diverse equations highlights its proficiency and creativity in showcasing authentic physical occurrences. Furthermore, the adeptness of wavelet-based methodologies establishes them as essential instruments for various applications within physics, engineering, and interdisciplinary domains where memory effects, anomalous diffusion, and nonlocal system dynamics are profoundly affected by interactions. The wavelet collocation paradigm manifests as a solid, stylish, and adaptable approach, developed to elevate scientific modeling as fractional calculus becomes progressively crucial in practical and theoretical scenarios.

ACKNOWLEDGEMENTS

The authors would like to thank the Department of Mathematics, Islamic Azad University for supporting this research.

REFERENCES

[1] J. E. Nápoles Valdés, *The non-integer local order calculus*, *Physics & Astronomy International Journal*, vol. 7, no. 3, pp. 163–168, 2023. Available: https://doi.org/10.15406/paij.2023.07.00304

[2] R. Aruldoss, R. A. Devi, and P. M. Krishna, *An expeditious wavelet-based numerical scheme for solving fractional differential equations*, *Computational & Applied Mathematics*, vol. 40, no. 1, pp. 1–14, 2021. Available: https://doi.org/10.1007/S40314-020-01387-1

[3] S. Sh. Tantawy, *Solving linear systems of fractional integro-differential equations by Haar and Legendre wavelets techniques*, *Partial Differential Equations in Applied Mathematics*, 100683, 2024. Available: https://doi.org/10.1016/j.padiff.2024.100683

[4] A. M. Kazar, *Wavelet-based collocation methods for solving multi-dimensional integro-differential systems*, *Edelweiss Applied Science and Technology*, vol. 8, no. 6, 2024. Available: https://doi.org/10.55214/25768484.v8i6.2990

[5] A. Chauhan and D. Nigam, *A theoretical outlook on collocation methods and partial differential equations*, *ShodhKosh Journal of Visual and Performing Arts*, vol. 5, no. 5, 2024. Available: https://doi.org/10.29121/shodhkosh.v5.i5.2024.3079

[6] A. K. Gupta, *Wavelet Methods for the Solutions of Partial and Fractional Differential Equations Arising in Physical Problems*, 2016. Available: http://ethesis.nitrkl.ac.in/8471/

[7] A. Chauhan and D. Nigam, *A theoretical outlook on collocation methods and partial differential equations*, *ShodhKosh Journal of Visual and Performing Arts*, vol. 5, no. 5, 2024. Available: https://doi.org/10.29121/shodhkosh.v5.i5.2024.3079

[8] M. Mohammad, A. Trounev, and S. Kumar, *High-precision Euler wavelet methods for fractional Navier–Stokes equations and two-dimensional fluid dynamics*, *Physics of Fluids*, vol. 36, no. 12, 2024. Available: https://doi.org/10.1063/5.0235658

[9] A. B. Deshi and G. A. Gudodagi, *Numerical solution of Bagley–Torvik, nonlinear and higher order fractional differential equations using Haar wavelet*, pp. 1–13, 2021. Available: https://doi.org/10.1007/S40324-021-00264-Z

BIOGRAPHIES OF AUTHORS

	**Baydaa Radhi Saad** received her M.Sc. degree in Mathematics from the University of Baghdad, Iraq, in 2008, and her Doctor of Philosophy in Mathematical Sciences from the University of Baghdad, Iraq, in 2014. She has been a full-time lecturer in the Department of Mathematics, Islamic Azad University, Kermanshah Branch, Iran, since 2015. Her research interests include numerical analysis, differential equations, and computational mathematics. He can be contacted at email: baydaaradhi@gmail.com