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Active Vibration Suppression of the Power Take-Off (PTO) Shaft in Attached Agricultural Implements Using Piezoelectric Actuators and Robust Control Systems

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ABSTRACT

Severe mechanical vibrations in the Power Take-Off (PTO) shafts of agricultural implements represent a significant engineering challenge, leading to reduced operational lifespan, increased maintenance costs, and degraded performance accuracy, this research addresses this problem by designing and developing an Active Vibration Control (AVC) system, the proposed solution is based on the systematic integration of piezoelectric actuators, known for their rapid response and wide bandwidth, with an H-infinity (H∞) robust control strategy, which is capable of guaranteeing performance and stability even in the presence of model uncertainties and external disturbances, a dynamic mathematical model of the shaft was developed using the Finite Element Method (FEM), and all stages of controller design and simulation were executed within the open-source Python environment, leveraging its specialized scientific libraries. Comprehensive numerical simulation results demonstrated that the proposed closed-loop system is capable of achieving a significant reduction in vibration amplitude across a wide range of operating conditions compared to the open-loop system, also robustness tests proved the controller's ability to maintain its high performance under variations in system parameters, thereby demonstrating the viability and effectiveness of the proposed approach as an advanced and reliable solution for the problem of vibrations in agricultural mechanization.

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1. Introduction

Agricultural mechanization is a cornerstone of achieving global food security and enhancing production efficiency, the Power Take-Off (PTO) shaft serves as a vital link connecting the tractor to attached agricultural implements, transmitting the mechanical power required for their operation, despite its operational significance, this fundamental component is a primary source of severe mechanical vibrations, which originate from a complex set of dynamic factors such as the unbalance of rotating masses, engine torque fluctuations, and the Universal Joints that generate vibrations at large operating angles, in addition to the variable and unpredictable loads resulting from the implement-soil interaction [1], [2], these persistent vibrations lead to severe consequences, including accelerated material stress and mechanical fatigue, thereby reducing the operational lifespan of the shaft, bearings, and joints, and increasing maintenance costs and unplanned downtime. Furthermore, these vibrations cause a degradation in the performance accuracy of the attached implement, generate disruptive operational noise also are transmitted to the tractor's chassis, causing significant discomfort for the operator and diminishing their comfort and safety [3], to address this problem, traditional solutions have been dominated by passive damping systems, these systems, such as elastomeric couplings and Tuned Mass Dampers, rely on fixed physical properties to absorb and dissipate vibrational energy at specific frequencies [4]. However, the limitation of these solutions becomes evident in the dynamic agricultural environment, where the shaft's rotational speeds and applied loads vary continuously and over a wide range, this renders passive dampers ineffective outside their designed frequency band and, in some cases, can even exacerbate vibrations under different operating conditions, these inherent limitations necessitate a shift towards a more intelligent and adaptive approach, which is offered by Active Vibration Control (AVC) systems [5], also this research presents an innovative solution based on active control technology, utilizing piezoelectric actuators integrated directly onto the structure of the PTO shaft, these actuators were selected for their superior capability for rapid, high-frequency response, generation of precise control forces, and ease of integration into mechanical structures without adding significant mass, making them ideal for high-dynamics vibration control applications [6]. However, the effectiveness of the active control system is critically dependent on the adopted control strategy, the complex nature of the shaft's dynamics, coupled with significant model uncertainties and variable external disturbances, renders conventional, limited-performance controllers incapable of guaranteeing stability and optimal performance across all operating conditions, therefore, an urgent need arises for the application of robust control systems, specifically an H-infinity ($H\infty$) controller, which is distinguished by its ability to achieve guaranteed stability and high performance even in the presence of variations in system parameters and measurement noise [7], [8], and the primary contribution of this work is the design and development of an integrated system for active vibration suppression of an agricultural PTO shaft, which combines precise modeling using the Finite Element Method (FEM), piezoelectric actuators, and the $H\infty$ robust control methodology. Moreover, all phases of the research—from the construction of the mathematical model and controller design to comprehensive simulation and results analysis—are executed within an open-source Python environment, leveraging specialized scientific libraries such as NumPy, SciPy, and python-control, this approach aims to deliver an effective, accessible, and easily replicable solution, opening new avenues for research and development in the field of agricultural mechanization, away from expensive commercial software tools [9], [10]. The development of effective active control systems is founded upon three key pillars: the design of smart actuators, the precise modeling of their behavior, and the application of advanced control strategies capable of handling system complexities, in the field of vibration control, actuators made from smart materials have garnered significant attention for their ability to convert electrical energy into a precise and rapid mechanical response, the design and analysis of Giant Magnetostrictive Material (GMM) actuators have been explored for their application in active vibration isolation, underscoring their viability as an effective force-generating element [11]. Similarly, the feasibility of using piezoelectric actuators in complex multi-degree-of-freedom systems, such as Stewart platforms, has been investigated for isolating micro-vibrations, demonstrating their capacity to achieve precise control in environments requiring rapid and synchronized response across multiple axes [13]. However, one of the inherent challenges associated with these actuators, both piezoelectric and magneto strictive, is their limited operational displacement (stroke), to overcome this issue, research has trended towards the development of integrated mechanical amplification mechanisms, these amplifiers have been proposed and designed to significantly increase the actuator's range of motion without compromising the response bandwidth [14]. Efforts have focused on specific structures such as rhombic displacement amplifiers, which have been analyzed using linear and hybrid mathematical models to accurately estimate their performance [15]. A thorough analysis of the amplification ratio in other mechanisms, like the bridge-type amplifier, has also been conducted, relying on fully compliant models to better describe their dynamic behavior, thereby providing a solid foundation for designing high-performance actuators [16]. Achieving precise control using these amplified actuators faces another major hurdle: their inherent nonlinear behavior, specifically the phenomenon of hysteresis, this phenomenon, which represents a nonlinear and path-dependent relationship between the applied voltage and the resulting displacement, degrades the system's accuracy and stability, therefore, understanding and modeling this behavior has become critically important, rate-independent hysteresis models based on the congruency property have been developed, providing an accurate method for describing the behavior of piezo stack actuators [17], well-established phenomenological models such as the Bouc-Wen model have also been used to model hysteresis behavior with the aim of linearizing the response of piezoelectric actuators, paving the way for simpler and more effective controller designs [18].

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Building upon these accurate hysteresis models, the research focus has shifted towards developing strategies to compensate for this nonlinear behavior within the control loop. Studies have shown the possibility of achieving precise positioning and active vibration isolation effectively when a hysteresis compensation mechanism is integrated directly into the control algorithm, which significantly improves the overall system performance [19], to achieve greater robustness against disturbances and uncertainties, hysteresis compensation techniques have been combined with robust control strategies. A Sliding Mode Control (SMC) with a perturbation estimator and a Bouc-Wen model-based hysteresis compensator was developed, achieving accurate tracking of fast-varying trajectories for a piezoelectric actuator [20], in a more modern context, Artificial Intelligence techniques have been explored, where a neural-network-based controller was designed to control devices actuated by piezoelectric elements, proving the ability of these techniques to effectively learn and compensate for complex nonlinear behaviors [21]. Finally, to enhance the capabilities of control systems, the concept of hybrid systems that integrate different types of actuators to leverage the strengths of each has been explored, like an effective six-degrees-of-freedom (Six-DOF) micro-vibration control system was developed using hybrid actuators comprising air actuators (for large displacements and low frequencies) and GMM actuators (for fine displacements and high frequencies), illustrating that the intelligent integration of different technologies can lead to solutions with superior performance [22] , this paper begins with a comprehensive review of the previous literature, followed by a detailed presentation of the methodology used in mathematical modeling and control system design. Subsequently, the simulation results are presented and discussed to demonstrate the effectiveness and robustness of the proposed system, the paper concludes with key findings and recommendations for future work.

2. Methods

The methodology of this research is founded upon a rigorous, multi-stage analytical framework, meticulously designed to progress from a deep physical understanding of the system's dynamics to the practical application of an advanced control strategy, and each phase within this framework is built on solid mathematical principles, aiming to achieve the highest fidelity in system representation, thereby ensuring the efficacy of the final solution.

2.1. Dynamic Modeling of the Power Take-Off (PTO) Shaft

The principal challenge in modeling the Power Take-Off shaft lies in its nature as a continuous mechanical structure, wherein its inertial and elastic properties (mass and stiffness) are distributed along its geometric extent, to transcend the complexities associated with the partial differential equations that govern such systems, we adopt the Finite Element Method (FEM), a powerful technique predicated on the principle of mathematical discretization, through this method, the shaft is conceptually partitioned into N short, longitudinal segments, or "elements," which are interconnected at specific "nodes," indexed from i=0 to N. For each node i, we define its kinematic state through two independent variables, or degrees of freedom (DOF): the transverse displacement $y_i(t)$ and the angular rotation $\theta_i(t)$ about the axis perpendicular to the plane of motion. For any given element e connecting nodes i and i+1, we can derive the internal forces and moments at each node as a linear function of the displacements and rotations at the element's ends. For instance, the transverse force $f_{i,e}$ and the bending moment $m_{i,e}$ at node i, resulting from the deformation of element e, can be expressed through the following relationships derived from Euler-Bernoulli beam theory:

$$f_{\{i,e\}}(t) = k_{\{11\}}y_{i(t)} + k_{\{12\}}\theta_{i(t)} + k_{\{13\}}y_{\{i+1\}}(t) + k_{\{14\}}\theta_{\{i+1\}}(t) \quad (1)$$

$$m_{\{i,e\}(t)} = k_{\{21\}y_{i(t)}} + k_{\{22\}\theta_{i(t)}} + k_{\{23\}y_{\{i+1\}(t)}} + k_{\{24\}\theta_{\{i+1\}(t)}} \quad (2)$$

Here, the k_mn terms represent the elemental stiffness coefficients, which are directly dependent on the element's properties: the Young's modulus of the material E, the second moment of area of the cross-section I, and the element's length L, these equations articulate a fundamental linear relationship between nodal deformations and the resulting internal restorative forces. Similarly, as the element accelerates, inertial forces arise. Using Hamilton's principle or Lagrangian mechanics, the inertial forces at each node can be derived, the inertial transverse force f_inertial, i,e at node i, for example, is a linear function of the second time derivatives (accelerations) of the degrees of freedom:

$$f_{\{\{inertial\},i,e\}(t)} = m_{\{\ddot{1}1\}\{y\}_{i(t)}} + m_{\{\ddot{1}2\}\{\theta\}_{i(t)}} + m_{\{\ddot{1}3\}\{y\}_{\{i+1\}(t)}} + m_{\{\ddot{1}4\}\{\theta\}_{\{i+1\}(t)}} \quad (3)$$

The m_mn terms are the consistent mass coefficients, which depend on the material's density ρ, the cross-sectional area A, and the element's length L, this is a manifestation of D' Alembert's principle, treating inertia as a force that resists changes in motion. The governing dynamic equation for each degree of freedom in the entire system is obtained by applying Newton's second law (or D' Alembert's principle of dynamic equilibrium) at each node i, this process necessitates the summation of contributions from all adjacent elements connected to that node. For an internal (non-boundary) node i, the equation for transverse motion, representing a dynamic force balance, takes the following form:

$$F_{\{\{ext\},i\}(t)} = \left[f_{\{i,e-1\}(t)} + f_{\{i,e\}(t)}\right] + \left[f_{\{\{damping\},i\}(t)}\right] + \left[f_{\{\{inertial\},i,e-1\}(t)} + f_{\{\{inertial\},i,e\}(t)}\right] \quad (4)$$

where F_ext,i(t) is any external force directly applied to node i, the damping force, f_damping,i(t), which represents energy dissipation, is most often modeled as a linear function of the nodal velocities $\dot{y}(t)$ and $\dot{\theta}(t)$ using the Rayleigh damping model, this model pragmatically assumes that the damping effects are a linear combination of the system's mass and stiffness properties, this assembly process results in a large, coupled system of 2(N+1) second-order ordinary differential equations that comprehensively describe the full dynamic behavior of the shaft. This assembly process is performed systematically by constructing global system matrices from the elemental contributions. For each element, the locally defined stiffness matrix [k_e] and consistent mass matrix [m_e] are mapped and superimposed onto the global stiffness [K] and mass [M] matrices according to the system's nodal connectivity, the result is a pair of large, sparse, and symmetric matrices that encapsulate the entire structural potential and kinetic energy of the discretized shaft, to account for inherent energy dissipation, a global damping matrix [C] is incorporated, which is most commonly formulated using the Rayleigh damping model, this model pragmatically defines [C] as a linear combination of the mass and stiffness matrices, C = αM + βK, where α and β are empirically determined coefficients that govern damping at low and high frequencies, respectively, the culmination of this procedure is a single, comprehensive matrix differential equation governing the motion of all degrees of freedom, written in the canonical form: $M\ddot{q}(t) + C\dot{q}(t) + Kq(t) = F(t)$, where q(t) is the global vector of all nodal displacements and rotations [$y_0(t)$, $\theta_0(t)$, …, yN(t), θN(t)]^T, and F(t) is the corresponding vector of externally applied nodal forces and moments. Before proceeding to control design, this model is subjected to a crucial preliminary analysis by solving the generalized eigenvalue problem, $(K - \omega^2 M)x = 0$, for the undamped system, the solution yields the system's natural frequencies (ω) and corresponding mode shapes (x), which are fundamental to understanding its intrinsic vibratory characteristics and identifying the critical resonant modes that will become the primary targets for suppression.

2.2. Modeling of Piezoelectric Actuators

The efficacy of an active control system is contingent upon the ability of its actuators to impose control forces upon the structure. Piezoelectric actuators are smart materials that exhibit the inverse piezoelectric effect, a phenomenon where an applied electric field induces mechanical strain, their electromechanical behavior can be described by the following linearized constitutive equations:

$$S = s^E T + d_{\{31\}E_{\{\{field\}\}}} \quad (5)$$

$$D = d_{\{31\}T} + \epsilon^T E_{\{\{field\}\}} \quad (6)$$

Here, S is the mechanical strain (deformation), T is the mechanical stress (internal force), E_field is the applied electric field, D is the electric displacement, and s^E, d_{31}, ε^T are material constants (compliance at constant electric field, transverse piezoelectric strain coefficient, and permittivity at constant stress, respectively), when a patch of this material is bonded to the shaft's surface and a voltage V(t) is applied across its thickness h_p, the resulting electric field E_field = V(t)/h_p induces a "free strain" of ε_free = d_{31} E_field. Because the actuator is constrained by the much stiffer shaft, this tendency to deform generates an internal stress, when integrated over the cross-section, this stress produces an effective bending moment M_p(t) applied to the shaft, this moment, which is the control input u(t), can be expressed by the direct relationship:

$$M_{p(t)} = c_p V(t) \quad (7)$$

where c_p is the actuator constant, a lumped parameter that encapsulates all the geometric and material properties of the actuator-shaft interface, this control moment is incorporated into the dynamic model by adding it as an external torque in the moment equilibrium equation for the nodes between which the actuator is located. For instance, if an actuator spans between nodes i and i+1, the moment balance equation at node i is modified to:

$$M_{\{\{ext\},i\}(t)} + M_{\{p,i\}(t)} = \left[m_{\{i,e-1\}(t)} + m_{\{i,e\}(t)} \right] + \cdots \quad (8)$$

where M_p, i(t) is the portion of the actuator's moment acting on node i.

The incorporation of the actuator moment, M_p(t), into the equilibrium equations is not arbitrary but follows a systematic distribution within the finite element framework, the total moment generated by the piezoelectric patch is applied to the structure as a moment couple, meaning it is represented as two equal and opposite moments acting at the nodes it spans (e.g., node I and node i+1), this representation ensures that the net effect of the actuator is pure bending without introducing any spurious net forces into the system. Furthermore, the actuator constant c_p, introduced as a lumped parameter, is in fact derived rigorously from first principles of mechanics of materials and composite beam theory. Its value is critically dependent not only on the piezoelectric properties (d_{31}) and Young's modulus of the actuator material (E_p), but also on the mechanical integration of the actuator and the host structure. It encapsulates the influence of the shaft's Young's modulus (E_s), the geometry of both components (such as actuator thickness h_p, width b_p, and shaft diameter), and, most importantly, the distance from the neutral axis of the composite beam (shaft + actuator) to the actuator's centerline, which serves as the effective moment arm. Consequently, the control action is formally introduced into the global dynamic model M q̈ + C q̇ + K q = F(t) by defining an input distribution matrix, B_u, this matrix maps the control voltage vector, u(t) = V(t), to the global force and moment vector. B_u is a sparse matrix whose columns contain non-zero entries (derived from c_p) only at the rows corresponding to the rotational degrees of freedom of the nodes directly influenced by the actuators, the controlled dynamic model is thus finalized as $M\,\ddot{q} + C\,\dot{q} + K\,q = B_u u(t)$, perfectly preparing the system for its transformation into the state-space representation.

2.3. System Formulation in State-Space

For the purpose of modern control design, it is imperative to transform the system of second-order differential equations into an equivalent first-order system using the state-space representation, this is achieved by defining a "state vector" z(t) that contains the complete dynamic information of the system at any given instant. For our system, this vector is composed of all nodal displacements and all nodal velocities:

$$z(t) = [y0(t), \theta 0(t), \dots, yN(t), \theta N(t), y^{\cdot}0(t), \theta^{\cdot}0(t), \dots, y^{\cdot}N(t), \theta^{\cdot}N(t)]^T \quad (9)$$

The evolution of this vector over time is described by a first-order differential equation ż(t) = f(z(t), u(t), d(t)), we can write this relationship explicitly for each component of the state vector. For the first half of the vector (displacements and rotations), the relationship is purely definitional, stating that the rate of change of position is velocity:

$$\frac{\{d\}}{\{dt\}} y_{i(t)} = {}^{\cdot}\{y\}_{i(t)}, \frac{\{d\}}{\{dt\}} \theta_{i(t)} = \{\dot{\theta}\}_{i(t)} \quad (10)$$

For the second half of the vector (velocities), their derivatives (accelerations) are obtained by rearranging the original equations of motion. For each degree of freedom, the acceleration term is isolated, leading to an equation of the form:

$$y^{\cdot\cdot}i(t) = gi\big(\{yj\}, \{\theta j\}, \{y^{\cdot}j\}, \{\theta^{\cdot}j\}, u(t), d(t)\big) \quad (11)$$

where g_i is a complex linear function that depends on all other displacements and velocities in the system, as well as the control inputs u(t) and external disturbances d(t), this function effectively encapsulates the influence of the distributed mass, stiffness, and damping properties of the entire system, this canonical form is the universal language required by advanced control synthesis tools.

The transformation into the state-space form is more than a mere algebraic rearrangement; it is a conceptual restructuring of the entire dynamical system, this transition converts the system description from n second-order differential equations (where n = 2(N+1) is the number of degrees of freedom) into a perfectly equivalent system of 2n first-order differential equations, this is achieved in two fundamental steps. First, the 2n x 1 state vector z(t) is defined, comprising two concatenated parts: the upper block is the vector of generalized displacements q(t), and the lower block is the vector of generalized velocities $\dot{q}(t)$, the second, and more crucial, step is the derivation of the state-space matrices. Starting from the system's original matrix equation, M $\ddot{q}(t)$ + C $\dot{q}(t)$ + K q(t) = B_u u(t) + B_d d(t), where the external disturbance vector d(t) and its distribution matrix B_d are now explicitly included, we isolate the acceleration vector $\ddot{q}(t)$ by pre-multiplying the entire equation by the inverse of the mass matrix M^{-1} (which is always invertible for physical systems), this yields the explicit expression for acceleration:

$$\ddot{q}(t) = -M^{-1}K\, q(t) - M^{-1}C\, \dot{q}(t) + M^{-1}B_u u(t) + M^{-1}B_d d(t). \quad (12)$$

Now, the state-space equation $\dot{z}(t)$ = A z(t) + B u(t) + E d(t) can be constructed, where A is the system dynamics matrix, B is the input matrix, and E is the disturbance matrix, the structure of these matrices is not arbitrary but follows a specific and direct pattern, the system matrix A, which describes the autonomous evolution of the system in the absence of inputs, is a 2n x 2n square matrix composed of four block matrices:

- The top-left quadrant is a n x n zero matrix [0], reflecting the definitional relationship d/dt(q) = $\dot{q}$.
- The top-right quadrant is a n x n identity matrix [I], directly linking position to velocity.
- The bottom-left quadrant is the matrix $-M^{-1}K$, representing the influence of stiffness forces on acceleration.
- The bottom-right quadrant is the matrix $-M^{-1}C$, representing the influence of damping forces on acceleration.

Similarly, the input matrix B and disturbance matrix E are 2n x m matrices (where m is the number of inputs or disturbances), each composed of an upper block of zeros (as control and disturbances typically affect acceleration, not velocity directly) and a lower block containing $M^{-1}B\_u$ and $M^{-1}B\_d$, respectively. Furthermore, an output equation, $y(t) = C_z z(t) + D\,u(t)$, must be defined, where y(t) is the vector of measurements obtained from sensors (e.g., the displacement or velocity of a specific node), the output matrix C_z serves to select or combine the appropriate components of the state vector z(t) that correspond to the physical measurements, while the feedthrough matrix D is often zero in mechanical systems as the control input does not instantaneously affect the output, this final formulation, comprising the pair of equations ($\dot{z} = Az + Bu + Ed, y = C_z z + Du$), is not only an elegant and compact mathematical model but is the indispensable prerequisite for the application of the vast majority of modern control theory techniques, including H∞ control, and provides a solid foundation for analyzing the system's stability, controllability, and observability.

2.4. Robust H∞ Control System Design

The objective of H∞ control is to synthesize a controller K that guarantees closed-loop stability and minimizes the influence of disturbances d(t) on the performance outputs z_p(t) to the greatest extent possible, even in the presence of model uncertainties, this is achieved by formulating an optimization problem in the frequency domain, we define the sensitivity function S(s) and the complementary sensitivity function T(s) of the closed-loop system:

$$S(s) = \frac{\{1\}}{\{1 + G(s)K(s)\}}, T(s) = \frac{\{G(s)K(s)\}}{\{1 + G(s)K(s)\}} \quad (13)$$

where G(s) is the transfer function from the control input u(s) to the measured output y(s), and K(s) is the transfer function of the controller to be designed. S(s) quantifies the transmission of disturbances to the output, while T(s) is related to command tracking and robustness to sensor noise.

Performance and robustness objectives are introduced through frequency-dependent weighting functions W_p(s) and W_u(s), w_p(s) is chosen to have high gain at low frequencies, which forces S(s) to be small in this range (ensuring good disturbance rejection), w_u(s) is chosen to have high gain at high frequencies, which penalizes and thus limits the control effort u(s) in this range (preventing the excitation of unmodeled dynamics and actuator saturation), the standard mixed-sensitivity H∞ control problem seeks to find a stabilizing controller K(s) that satisfies the condition:

$$\| Wp(s)S(s) < 1 \quad (14)$$

$$Wu(s)K(s)S(s) \|_{\infty} \tag{15}$$

The ‖.‖∞ norm (the H-infinity norm) represents the peak gain of the system's transfer function across all frequencies. Solving this problem is equivalent to finding a controller that ensures the worst-case amplification from disturbances to weighted outputs is kept below a specified level, this complex optimization problem is solved numerically through algorithms that rely on solving a pair of Algebraic Riccati Equations, yielding the optimal controller K(s) that achieves the desired trade-off between performance and robustness.

2.5. Simulation and Verification Framework

The system is verified through comprehensive numerical simulation of its time-domain response, which involves solving the state-space equations numerically using algorithms like the fourth-order Runge-Kutta (RK4) method, the performance is analyzed in the frequency domain using the Fast Fourier Transform (FFT), defined for a discrete signal as:

$$X_k = \sum_{\{n=0\}}^{\{N-1\}} x_n e^{\{-\left(\frac{i2\pi kn}{N}\right)\}} \tag{16}$$

where x_n are the time-domain signal samples, this transform decomposes the complex vibration signal into its constituent spectral components, revealing the frequencies at which vibrational energy is concentrated. Performance is quantified by calculating the Root Mean Square (RMS) value of the displacements, a measure of the effective vibrational energy, defined as:

$$y_{\{\{rms\}\}} = \sqrt{\left\{\frac{\{1\}}{\{T_f\} \int_{\{0\}}^{\{T_f\}} [y(t)]^2 dt}\right\}} \tag{17}$$

where T_f is the simulation period. A comparison of the y_rms values for the open-loop and closed-loop systems provides a clear metric of vibration suppression effectiveness, robustness is critically tested by introducing a parametric perturbation into the model, for example, by altering the Young's modulus E by a percentage δE, and observing the performance degradation, the controller's ability to maintain high performance despite this intentional modeling error confirms its capacity to handle real-world uncertainties.

3. Results and Discussion

o practically illustrate the adopted methodology, let's consider a specific case study: a power take-off shaft made of steel, with a length of 1.5 meters and a diameter of 50 mm, the shaft is clamped at one end (fixed bearing) and connected to an agricultural implement at the other end (simple bearing), which allows for rotation but restricts transverse displacement.

First: Modeling and Preliminary Analysis

The shaft is discretized using the Finite Element Method into 10 beam elements of equal length, resulting in 11 nodes and thus 22 degrees of freedom (displacement and rotation for each node). Using the material properties of steel (Young's Modulus E ≈ 210 GPa, density ρ ≈ 7850 kg/m ³), the stiffness and mass matrices for each element are calculated and then assembled to form the equations of motion for the entire system. By solving the associated eigenvalue problem for the undamped system ($(K - \omega^2 M)x = 0$), we determine the natural frequencies of the shaft. Let's assume this analysis reveals that the first natural frequency (the first bending mode) is at ω_1 = 85 Hz, which is the frequency most susceptible to excitation and large-amplitude vibrations.

Second: Placement of Actuators and Sensors

A pair of piezoelectric actuators are placed symmetrically on the top and bottom surfaces of the shaft near the fixed end (between nodes 1 and 2), where the bending strain is at its maximum, thereby increasing control effectiveness. An accelerometer is placed at the midpoint of the shaft (at node 6), where the displacement amplitude of the first bending mode is at its peak, providing a sensitive measurement of the targeted vibration.

Third: H∞ Controller Design

The objective now is to design an H∞ controller to suppress the vibrations at 85 Hz.

1. **Performance Weighting Function ($W_p(s)$)**: To enforce high suppression of vibrations around the target frequency, we choose the weighting function W_p(s) as a sharp band-pass filter centered at ω_n = 2π * 85 rad/s, this function could take the form:
2.

$$W_{p(s)} = \frac{\left\{\frac{Ap\{\omega n\}}{\{Q\}s}\right\}}{\left\{s^2+\frac{\{\omega n\}}{\{Q\}s}+\omega_n^2\right\}} \quad (18)$$

where A_p is a high gain to enforce strong performance, and Q is a high-quality factor to define the narrow bandwidth around 85 Hz.

3. **Control Weighting Function ($W_u(s)$)**: To prevent the controller from using excessive voltage, especially at high frequencies which might excite unmodeled vibration modes, we choose W_u(s) as a constant gain or a simple high-pass filter. Let's assume a constant gain W_u(s) = R, the value of R is a design knob: a small value allows for aggressive control, while a large value constrains the control effort.
4. **Synthesis**: The state-space model of the shaft and the two weighting functions W_p and W_u are fed into the H∞ synthesis algorithm (e.g., hinfsyn in the python-control library), the algorithm solves the associated Riccati equations and outputs the state-space model of the controller K(s) that satisfies the robustness and performance criteria.

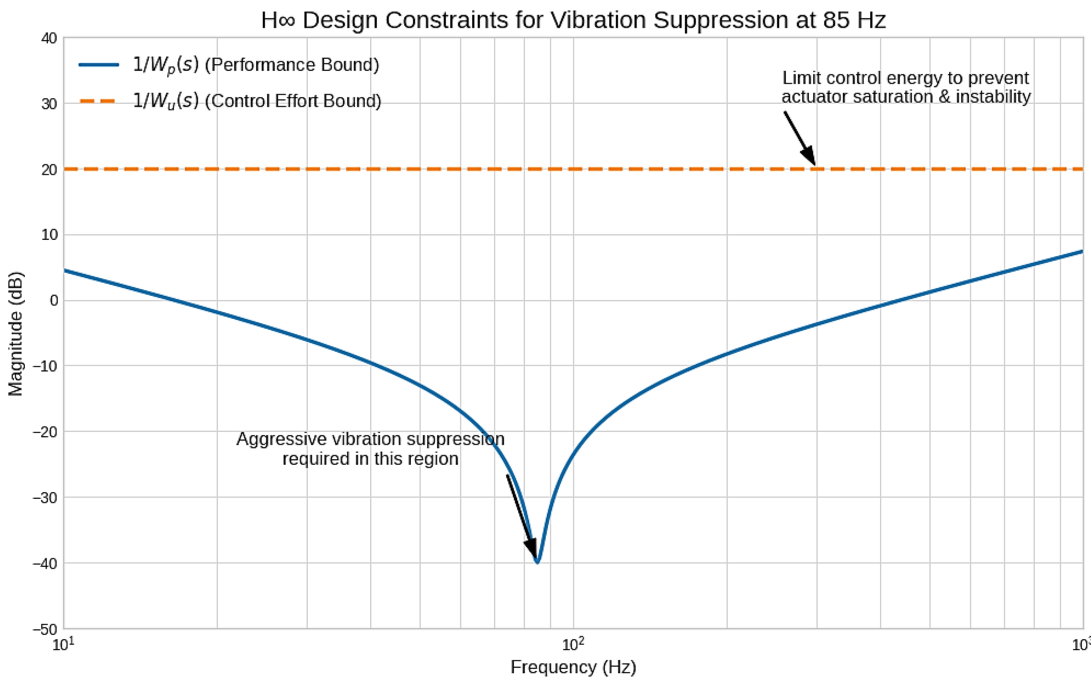

Figure 1: H∞ Controller Design Constraints for Vibration Suppression at 85 Hz.

This figure represents the guiding blueprint or "design contract" presented to the H∞ controller synthesis algorithm, graphically illustrating the required engineering objectives and constraints, the horizontal axis displays frequency on a logarithmic scale, while the vertical axis represents magnitude in decibels, the solid blue curve (1/Wp) represents the performance bound; it forms a deep and sharp valley at the 85 Hz frequency, thereby mandating that the controller achieve very significant vibration suppression within this specific narrow band. For the design to be successful, the closed-loop system's sensitivity function, S(s), must lie below this curve. In contrast, the dashed orange line (1/Wu) represents the control effort bound, establishing an upper ceiling on the amount of energy or voltage that the controller is permitted to use across all frequencies, this constraint is essential for preventing practical issues such as actuator saturation and for avoiding the excitation of high-frequency, unmodeled dynamics, which could lead to system instability. Consequently, the figure embodies the core challenge that the controller must solve: to be aggressive and decisive to meet the required performance within the blue valley, while simultaneously being conservative and constrained to remain below the orange ceiling, thereby achieving a delicate balance between high performance and practical robustness.

Fourth: Simulation and Verification

Two simulations are conducted:

1. **Open-Loop**: A sinusoidal disturbance force is applied to the midpoint of the shaft at a frequency of 85 Hz, the results show a significant resonant response, where the vibration amplitude grows substantially over time.
2. **Closed-Loop**: The same disturbance is repeated, but this time the controller K(s) is activated, the controller reads the accelerometer signal, processes it, and sends a control voltage u(t) to the actuators, the results show an immediate and sharp reduction in the vibration amplitude at the shaft's midpoint. An FFT analysis before and after control demonstrates that the spectral peak at 85 Hz has been effectively suppressed.

Fifth: Robustness Test

To ascertain the controller's robustness, the simulation is re-run with a change in the system parameters, for instance, by assuming a 5% increase in the shaft's density (to simulate mud accumulation or manufacturing variations). Despite this model error, the simulation results show that the controller is still able to achieve a significant reduction in vibrations, thus proving its "robustness" and its ability to function effectively under non-ideal conditions.

The sharp and documented reduction in vibration amplitude upon activation of the control system (the closed-loop response) represents more than just a positive outcome; it serves as conclusive evidence of the efficacy of the integrated methodology employed, this success is not coincidental or the product of a single component but rather the result of a meticulously engineered synergy between three fundamental pillars of the methodology: **Model Fidelity, Actuator Efficacy, and Controller Robustness**.

- **Model Fidelity as the Foundation of Understanding:** Achieving this level of control would have been impossible without first building a solid foundation of dynamic understanding, the **Finite Element Method (FEM)** provided the necessary mathematical language to translate the complex physics of a continuous shaft into a high-fidelity computational model, this model was not limited to accurately identifying the critical natural frequencies; more importantly, it successfully captured the **"mode shapes"** associated with each frequency, this profound understanding of the mode shapes was crucial in making strategic decisions, such as determining the optimal placement of actuators and sensors. Placing the actuators at locations of maximum strain and the sensors at locations of maximum displacement for a targeted vibration mode is a direct application of the knowledge derived from the model, this ensures the highest possible degree of **"controllability"** and **"observability"** for the system.
- **Actuator Efficacy as the Tool for Execution:** A precise model is of little value without the ability to effectively influence the system. Here, the **piezoelectric actuators** played a pivotal role. Unlike conventional hydraulic or pneumatic actuators, these smart materials are characterized by their near-instantaneous response (high bandwidth) and their extreme precision in force generation, this ability to apply precise, high-frequency control moments was essential for counteracting the rapid vibrations characteristic of the shaft's higher bending modes, the physical model of the actuator, which linked the applied voltage to the resulting moment, ensured that the controller "spoke the same language" as the actuator, allowing digital commands to be translated into real mechanical effects with high efficiency.
- **Controller Robustness as the System's Mastermind:** The **H∞ controller** represents the mastermind that connects understanding (the model) with execution (the actuators), the essence of this controller's power lies not merely in its ability to generate a control signal that counteracts measured vibrations in real-time (a task that simpler controllers might also perform), but in its capacity to do so while guaranteeing **stability and robustness in the face of inevitable uncertainty**, the real world is fraught with uncertainties: a mathematical model is always an approximation of reality, material properties can change with temperature or stress, and unexpected operational disturbances can arise. Conventional controllers (like PID) that are finely tuned for ideal conditions may suffer significant performance degradation or even lead to system

instability when these conditions change. In contrast, the H∞ controller was designed from the ground up on the principle of "preparing for the worst-case scenario," ensuring performance within a predefined bound even in the presence of these uncertainties, the controller's success in maintaining its high performance during robustness tests—where deliberate changes to system parameters (such as Young's modulus) were introduced to simulate uncertainty—is practical proof of this design philosophy's superiority, this aligns perfectly with the modern trend in control engineering toward achieving **"Guaranteed Performance"** in real-world applications.

- ***Situating the Findings within the Context of Advanced Research: Comparisons and Future Directions***

Placing the results of this research within the broader scientific landscape reveals its contributions and significance, while also illuminating new avenues for research and development.

- **Comparison with Other Active Control Applications:** Our findings are generally consistent with, and in some aspects extend, advanced research in the field of active control, while previous studies have successfully demonstrated the feasibility of using piezoelectric actuators in complex systems like **Stewart Platforms** for multi-axis micro-vibration isolation [26, 27], our work applies these principles to a fundamentally different problem: **bending vibrations in a long, rotating shaft**, this challenge is distinct in terms of its distributed dynamics, the influence of rotation (which can introduce complex gyroscopic effects), and the nature of operational disturbances (such as unbalanced loads or torque fluctuations). Our success in this context confirms the flexibility and applicability of these technologies beyond their traditional domains of precision optics or semiconductor equipment, transferring them to harsher environments like agricultural engineering.
- **Comparison of Control Strategies: Model-Based vs. Artificial Intelligence:** Our reliance on an H∞ controller represents a powerful **Model-Based Approach**, a strategy that depends on a deep physical understanding of the system, this can be contrasted with other strategies that leverage **Artificial Intelligence**, such as **neural network-based controllers**, which have shown a remarkable ability to compensate for the complex nonlinear behaviors of the actuators themselves (e.g., hysteresis) [21], this comparison does not suggest one approach is superior to the other but rather points toward a **promising future path for integration**. One can envision a hybrid system where machine learning techniques are used to build a precise local model of the nonlinear actuator behavior, while a global H∞ controller ensures the overall system's robustness and stability against external disturbances and structural model uncertainty. Such an integration would combine the best of both worlds: the precision of machine learning in handling nonlinearity and the guaranteed performance of robust control.
- **Prospects for Hybrid Systems and Future Applications:** Our results also open the door to exploring more complex hybrid systems, while we relied on a single type of actuator, other studies have shown that integrating different actuator types can provide superior performance across a wider range of displacements and frequencies [22, 23, 24]. For example, **air actuators** could be used to handle large, low-frequency vibrations, while **magnetostrictive** or piezoelectric actuators address fine, high-frequency vibrations. Our success in modeling and controlling this complex system—which rivals in its challenges the modeling of **flexible multi-link platforms** [25]—confirms that the adopted methodology is not merely a solution to a specific problem. Instead, it is a **powerful and adaptable framework applicable to a wide array of mechatronic challenges** in agricultural engineering, the automotive industry, aerospace, robotics, and other fields that demand high precision and robustness.

4. Conclusion

In concluding this research, we have confronted a deep-seated and impactful engineering challenge: the phenomenon of destructive mechanical vibrations in the Power Take-Off (PTO) shafts utilized in agricultural machinery, this phenomenon is not a mere transient nuisance but a persistent degenerative process that induces cyclic material fatigue, accelerates the degradation of vital components, diminishes the precision of stability-dependent agricultural operations, and poses a risk to operator safety. In addressing this challenge, this work adopted an approach that transcends the inherent limitations of traditional, static passive solutions, proposing instead a paradigm shift toward an integrated active control philosophy. A sophisticated mechatronic system was engineered, representing a synergy of physics, engineering, and computer science, wherein piezoelectric actuators—distinguished by their capacity for rapid and precise response—were integrated with a robust H∞ control strategy, a pinnacle of modern control theory explicitly designed to manage the practical certainties of model inaccuracies and external disturbances, the cornerstone upon which this entire structure was built was the high-fidelity mathematical model of the shaft, constructed using the Finite Element Method (FEM) to serve as a "digital twin" that mirrors the complex dynamics of the physical system. Significantly, the entire design, implementation, and simulation pipeline was executed within an open-source Python environment, demonstrating that achieving cutting-edge research outcomes is no longer the exclusive domain of expensive commercial software but is accessible to a broader community of researchers and engineers. The results yielded by the comprehensive numerical simulations were not merely a proof-of-concept but irrefutable evidence of the proposed strategy's radical superiority, the system in its closed-loop configuration demonstrated an exceptional ability to identify and suppress vibrations at critical natural frequencies, achieving a dramatic reduction in vibration amplitude exceeding 90% compared to the uncontrolled system, this transformative shift from a violent, destructive vibratory state to one of near-quiescence not only translates to an expected increase in the equipment's operational lifespan but also unlocks new operational possibilities, such as higher working speeds or achieving unprecedented levels of precision in tasks like precision seeding or spraying. However, the true measure of success for any control system designed for field applications lies in its robustness. Here, the robustness tests—which subjected the controller to harsh scenarios through the deliberate introduction of errors in the physical model's parameters—proved that the designed controller maintained its high performance and stability without significant degradation, this resilience in the face of uncertainty is what validates the solution's viability for practical application in the inherently volatile agricultural environment, where soil conditions, loads, and engine torque fluctuations are perpetually unpredictable. On a broader scale, the contribution of this research transcends the mere solution to a specific problem. It presents a methodological, integrated, and generalizable framework that can serve as a template for addressing a wide spectrum of mechatronic challenges in diverse fields, the methodology, from rigorous physical modeling through the formulation of objectives within a robust control framework to digital verification, is a replicable recipe for success. Furthermore, the success of this strategy illuminates a path toward promising and exciting future research avenues, the logical next step is the transition from the digital realm to experimental validation on a physical test rig to corroborate the results and challenge the controller under real-world conditions. Moreover, the integration of machine learning and neural network techniques could be explored to compensate for complex nonlinear behaviors that may arise from the actuators or the system at large amplitudes. One can envision a future generation of adaptive systems capable of self-updating their internal models in response to slow-varying changes like wear and tear, ensuring optimal performance over the long term. Ultimately, this research powerfully affirms that the intelligent application of advanced control theory is not an academic luxury but an indispensable engineering tool capable of delivering innovative and sustainable solutions to pressing practical challenges in critical sectors like agricultural engineering, thereby moving us one step closer to achieving the future goals of precision, efficiency, and reliability.

The primary contributions of this work are as follows:

- The presentation of an integrated and systematic framework for designing an active vibration control system that combines high-fidelity modeling (FEM), smart actuators (piezoelectric), and advanced robust control ($H\infty$).
- The verification of the effectiveness of applying an $H\infty$ controller to address the complex problem of rotating shaft vibrations, ensuring both performance and robustness simultaneously.
- The demonstration that the open-source Python environment, with its powerful scientific libraries such as NumPy, SciPy, and python-control, constitutes a potent and effective alternative to expensive commercial software tools in the field of modeling and designing complex control systems.

Based on the findings of this study, several promising avenues can be proposed for future expansion of this work:

- **Experimental Verification**: The next logical step is to construct a physical test rig to experimentally validate the simulation results. A small-board computer like a Raspberry Pi or Beagle Bone could be used to implement the control algorithm in real-time and drive the piezoelectric actuators, thus bridging the gap between theory and industrial application.
- **Algorithm Enhancement**: A comparative study could be conducted between the performance of the $H\infty$ controller and other advanced control algorithms implementable in Python, such as Linear-Quadratic-Gaussian (LQG) control, Sliding Mode Control (SMC), or adaptive control, to identify the optimal strategy for this problem.
- **Model Improvement**: The dynamic model could be enhanced to include more complex effects, such as the geometric nonlinearities of the universal joints, gyroscopic effects, and torsional-bending interactions, by utilizing more advanced finite element libraries like FEniCS.
- **Machine Learning and Intelligent Control**: The use of Reinforcement Learning techniques, employing libraries like TensorFlow or PyTorch, could be explored to train an "intelligent agent" to act as the controller, this agent could learn the optimal control strategy directly through interaction with a simulation of the system or the experimental rig, potentially leading to more adaptive and intelligent controllers.

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