Comparing Different Fuzzy Estimators Of Hazard Rate Function for Mixed Distribution from Rayleigh and Maxwell

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Abstract :

This paper deals with building mixed model from one scale (Θ) parameter Rayleigh and also one scale (Θ) parameter Maxwell distribution , using mixing proportions are $(\frac{\alpha}{\alpha+1}, \frac{1}{\alpha+1})$.

The mixed p.d.f is obtained and then mixed (C D F) , and also mixed reliability function and mixed hazard rate function are obtained ,and then the two parameters (α, Θ) are estimated by different Methods ,moments and Maximum Likelihood Method , and proposed one , then fuzzy hazard Rate is compared by simulation., After that we work on applying simulation procedure to Estimate fuzzy hazard rate function .

Keywords: compound distribution , fuzzy hazard rate function , Maximum Likelihood Method (MLE), Moments Estimate, (MOM).

Introduction:

There are many applications that depend on statistical tools and probability distributions, in case of non-precise data , and also the data may obtained from source of uncertainty , and also due to different factors some of these are random and another are affected due to disease Like blood leuk enema different type of cancer , so fuzzy data may be yield from imprecise observation , which lead to fuzzy data (Zadeh (1965)), who proposed fuzzy set , and then introduce first some definition of fuzzy number , with its membership function . the theory of fuzzy Reliability was proposed and developed by several authors(Like cai etal)(1993) introduce fuzzy states for theory fuzzy Reliability , and also (Cai) (1996) introduce system of failure engineering and fuzzy methodology.

Many other researcher like utkin , L. V. and Gurov, S.V.(1995),introduce fuzzy reliability analysis in case of possibility context. also Aliev and Kara,7. (2004) introduce fuzzy system reliability analysis using dependent fuzzy set. And Guo , R . Zhao , R.Q . and Li,X .(2007) Reliability analysis based on scalar fuzzy variables while Yao.J.S . and shih (2008) . introduce fuzzy reliability for system under triangular fuzzy number , based on statistical data, Also Karpisek,Z., stepanek , weibull fuzzy probability distribution for reliability of Concrete structures.

Theoretical Parts:

first of all we construct mixed p.d.f and mixed C.D.F from Maxwell – Rayleigh –with mixing proportions are ($\frac{\alpha}{\alpha+1}$) and ($\frac{1}{\alpha+1}$) the one scale parameter Rayleigh (Θ) is defined by in equation (1) as:

$$f(t,\Theta) = \frac{2}{\Theta} t\, e^{-\frac{t^2}{\theta}} \qquad t > 0 \qquad \dots (1)$$

$$0 \qquad o/w$$

And its cumulative distribution function (C.D.F) is given in equation (2):

$$F_T(t,\theta) = (1 - e^{\frac{-t^2}{\theta}}) \qquad \dots .(2)$$

And Reliability function is :

$$R_T(t,\theta) = 1 - F_T(t,\theta) = e^{\frac{-t^2}{\theta}} \qquad \dots .(3)$$

Also we can show that the mean of one parameter Rayleigh (θ) is

E(T)= mean = $\frac{\sqrt{\pi(\theta)}}{4}$

And the variance is

$$\sigma^2 = V(t) = \theta(1 - \frac{\pi}{4}) \qquad \dots .(4)$$

While the p.d.f of one scale parameter (θ) , Maxwell is :

$$g(t,\theta) = \frac{4}{\sqrt{\pi(\theta)^3}} t^2 e^{-(\frac{t}{\theta})^2} \qquad \dots .(5)$$

$$\theta > 0 \quad , \quad t > 0$$

$$G(t,\theta) = \left(1 - \frac{4}{3\sqrt{\pi}} e^{-(\frac{t}{\theta})^3}\right) \qquad \dots .(6)$$

$$\theta > 0 \quad , \quad t > 0$$

$$R(t,\theta) = \left(\frac{4}{3\sqrt{\pi}} e^{-(\frac{t}{\theta})^3}\right) \qquad \dots .(7)$$

Now we obtained the the new mixed probability from Maxwell and Rayleigh from :

$$f_T(t) = \frac{\alpha}{(\alpha+1)} f_{maxwell}(t,\theta) + \frac{1}{(\alpha+1)} f_{Ray}(t,\theta)$$

$$= \frac{\alpha}{(\alpha+1)} \frac{4}{\sqrt{\pi(\theta)^3}} t^2 \ e^{-(\frac{t}{\theta})^2} + \frac{1}{(\alpha+1)} \frac{2}{\theta} \ t \ e^{\frac{-t^2}{\theta}} \qquad \ldots .(8)$$

While the mixed C.D.F is

$$F_T(t) = \frac{\alpha}{(\alpha+1)} \left(1 - \frac{4}{3\sqrt{\pi}} \ e^{-(\frac{t}{\theta})^3} \right) + \left(\frac{1}{\alpha+1}\right) (1 - e^{\frac{-t^2}{\theta}}) \qquad \ldots .(9)$$

And mixed Reliability function is

$$R_T(t) = \left(\frac{\alpha}{\alpha+1}\right)\left(\frac{4}{3\sqrt{\pi}} \ e^{-(\frac{t}{\theta})^3}\right) + \left(\frac{1}{\alpha+1}\right) \ e^{\frac{-t^2}{\theta}} \qquad \ldots .(10)$$

2-Estimation of parameters Methods:

The parameter (θ, α) of mixed (Maxwell – Rayleigh) are estimated by Maximum Likelihood and Moments and proposed in .

1-Moments Estimators :

This method required to find $E(t), E(t^2)$ and then solve

$E(t) = \left(\frac{\sum_{i=1}^{n} t_i}{n}\right)$ and

$E(t^2) = \left(\frac{\sum_{i=1}^{n} t^2}{n}\right)$, to generate two equations and find $(\hat{\theta}_{mom}, \hat{\alpha}_{mom})$

$$(\alpha + 1)\sqrt{\pi} \ \ \bar{t} = \sqrt{\theta} \ \left(\frac{\alpha\pi}{2} + 2\sqrt{\theta}\right) \qquad \ldots ..(11)$$

$$(\alpha + 1)^2 \left(\frac{\sum_{i=1}^{n} t^2}{n}\right) = \left[\alpha^2\theta \ (1 - \frac{\pi}{4}\right] + \frac{5\theta^2}{22}$$

These two equations are obtained from equaling

$$E(t) = \frac{\sum_{i=1}^{n} ti}{n}$$

And $E(t^2) = \left(\frac{\sum_{i=1}^{n} t^2}{n}\right)$

Then solving equation (11) and (12) gives moments estimators of θ *and* α

2-Estimation by Maximum likelihood method (M L E):

Let $t_1, t_2, \ldots \ldots, t_n$, be random sample from p.d.f. equation (8) , then

$$L = (L_1) + (L_2)$$

$$L_1 = \alpha^n \ (\alpha+1)^{-n} \ (\frac{4}{\sqrt{\pi}})^n \ \theta^{-3} \prod_{i=1}^{n} ti^2 \ e^{-(\frac{\sum_{i=1}^{n} ti}{\theta})^2}$$

$$L_1 = \ (\alpha+1)^{-n} \ 2^n \ \theta^{-n} \prod_{i=1}^{n} ti^2 \ e^{-\left(\frac{\sum_{i=1}^{n} ti}{\theta}\right)^2}$$

Then

$$\begin{aligned} L = \ & n \log \alpha - n \log \ (\alpha+1) + n \log k \\ & - 3 \log \theta \\ & + \sum_{i=1}^{n} \log ti^2 - \left(\frac{\sum_{i=1}^{n} ti}{\theta}\right)^2 - n \log(\alpha+1) + n \log 2 - n \log \theta + \sum_{i=1}^{n} \log ti \\ & - \frac{\sum_{i=1}^{n} t^2}{\theta} \end{aligned}$$

$$\frac{\sum_{i=1}^{n} ti}{n} = \frac{1}{(\alpha-1)} \left\{ \frac{\alpha}{2} \sqrt{\pi}(\theta) + \frac{2\theta}{\sqrt{\theta}} \right\} \qquad \ldots..(12)$$

$$\frac{\sum_{i=1}^{n} ti}{n} = v(t) + (\ E(t))^2$$

$$= (\frac{\alpha}{\alpha+1})^2 \ \theta(1 - \frac{\pi}{4}) + \frac{1}{(\alpha+1)^2} \frac{5\theta^2}{22}$$

$$\frac{\sum_{i=1}^{n} ti^2}{n} = \frac{1}{(\alpha+1)^2} \left\{ \alpha^2(\theta)(1 - \frac{\pi}{4} + \frac{5\theta^2}{22} \right\} \qquad \ldots..(13)$$

And

$$\frac{\sum_{i=1}^{n} ti}{n} = \frac{1}{(\alpha+1)} \left\{ \frac{\alpha}{2} \sqrt{\pi\theta} + (\frac{2\theta}{\sqrt{\pi}} \right\}$$

$$\bar{t} = \frac{1}{(\alpha+1)} \frac{1}{\sqrt{\pi}} \left[\frac{\alpha\sqrt{\theta\,\pi}}{2} + 2\theta \right] \qquad \ldots.(14)$$

$$(\alpha+1) = \frac{1}{\sqrt{\pi}\ \bar{t}} \ \left[\frac{\alpha\sqrt{\theta\,\pi}}{2} + 2\theta \right]$$

$$(\alpha+1)\ E(t^2) = \alpha \left[\theta \left(1 - \frac{\pi}{4}\right) + \frac{\pi(\theta)}{4} + \frac{33}{\theta^2} \right]$$

$$(\alpha+1) \frac{\sum_{i=1}^{n} ti^2}{n} = \alpha\theta + \frac{33}{\theta^2}$$

$$(\alpha+1)\sqrt{\pi}\ \bar{t} = \frac{\sqrt{\theta}}{2} \left[\alpha\sqrt{\pi} + \frac{2\sqrt{\theta}}{\sqrt{\pi}} \right]$$

$$(\alpha+1)\sqrt{\pi}\,\bar{t} = \frac{\sqrt{\theta}}{2}\sqrt{\pi}\left[\alpha+\frac{2\sqrt{\theta}}{\sqrt{\pi}}\right]$$

$$(\alpha+1)\,\bar{t} = \frac{\sqrt{\theta}}{2}\left[\alpha+\frac{2\sqrt{\theta}}{\sqrt{\pi}}\right] \qquad \ldots..(15)$$

$$(\alpha+1)\frac{\sum_{i=1}^{n} ti^2}{n} = \alpha\left(\theta+\frac{33}{\theta^2}\right) \qquad \ldots..(16)$$

Equation (15) and (16) , solved numerically to find ($\hat{\alpha}_{mom}$, $\hat{\theta}_{mom}$)

$$\frac{\partial \log L}{\partial \alpha} = \frac{n}{\alpha} - \frac{n}{\alpha+1} - \frac{n}{\alpha+1}$$

$$\frac{\partial \log L}{\partial \theta} = \frac{-3}{\alpha} + 2\frac{\sum_{i=1}^{n} ti}{\theta^3} - \frac{n}{\theta} - \frac{\sum_{i=1}^{n} ti^2}{\theta^2}$$

$$\frac{n}{\alpha} - \frac{2n}{\alpha+1} = 0$$

$$n(\alpha+1) - 2n\alpha = 0$$

$$n\alpha + n - 2n\alpha = 0$$

$$n = n\alpha$$

$$\alpha = 1$$

$$\frac{2(\sum_{i=1}^{n} ti)^2 - \theta^2 - n\theta^2 - \theta\sum_{i=1}^{n} ti^2}{\theta^3} = 0$$

$$2(\sum_{i=1}^{n} ti)^2 - \theta^2(n+3) - \theta\sum_{i=1}^{n} ti^2 = 0$$

$$2(\sum_{i=1}^{n} ti)^2 = \theta^2(n+3) - \theta\sum_{i=1}^{n} ti^2$$

Solving this equation numerically to find ($\hat{\theta}_{MLE}$) and ($\hat{\alpha}_{MLE}$).

3-Estimation by percentiles estimators:

The estimators of ($\hat{\theta}_{PEC}$) obtain from minimizing .

For simplicity we assume one parameter is known (α *is known*) and estimate (θ) as follos from Reliability function as .

Let $\hat{P}_i = \hat{R}_T(ti)$

$$\hat{P}_i \frac{1}{(\alpha+1)} = \left(\frac{4\alpha}{3\sqrt{\pi}}\; e^{-(\frac{t}{\theta})^3} + \; e^{-\frac{t^2}{\theta}}\right)$$

$$(\alpha+1)\ \hat{P}_i = \left(\frac{4\alpha}{3\sqrt{\pi}}\ e^{-(\frac{t}{\theta})^3} + \ e^{-\frac{t^2}{\theta}}\right)$$

$$T \quad = \sum_{i=1}^{n} \left[\hat{P} - \hat{R}_T(ti)\right]^2$$

$$T \quad = \sum_{i=1}^{n} \left[\frac{i-\frac{3}{8}}{n+\frac{1}{4}} - \frac{1}{(\alpha+1)}\left(\frac{4\alpha}{3\sqrt{\pi}}\ e^{-(\frac{t}{\theta})^3} + \ e^{-\frac{t^2}{\theta}}\right)\right]^2$$

$$\frac{\partial T}{\partial \alpha} = \left[\frac{i-\frac{3}{8}}{n+\frac{1}{4}} - \frac{1}{(\alpha+1)}\left(\frac{4\alpha}{3\sqrt{\pi}}\ e^{-\left(\frac{t}{\theta}\right)^3} + \ e^{-\frac{t^2}{\theta}}\right)\right]$$

$$\left[0 - \frac{1}{(\alpha+1)}\left(\frac{4}{3\sqrt{\pi}}\ e^{-\left(\frac{t}{\theta}\right)^3} + \frac{4\alpha}{3\sqrt{\pi}}\ e^{-\left(\frac{t}{\theta}\right)^3}\ e^{-\frac{t^2}{\theta}}\right)\left(\frac{-1}{(\alpha+1)^2}\right)\right]$$
$$- \frac{1}{(\alpha+1)^2}\left[(\alpha+1)\frac{4}{3\sqrt{\pi}}\ e^{-(\frac{t}{\theta})^3} + \frac{4}{3\sqrt{\pi}}\ e^{-\frac{t^2}{\theta}}\right] = 0$$

$$\frac{4}{3\sqrt{\pi}}\ e^{-\left(\frac{t}{\theta}\right)^3}(\alpha+1) = 0$$

$$e^{-\left(\frac{t}{\theta}\right)^3}(\alpha+2) = 0$$

Solving this equation numerically to find $(\hat{\alpha}_{PEC} = -2)$.

Simulation procedures:

Description of steps of simulation to compare different methods

1-To find estimate fuzzy hazard Rate function for mixed (Rayleigh and Maxwell)

First off all we choose Sample size n=(20, 40, 60) taking different sets of initials values $(\alpha > -1)$, like $(\theta = 0.8, 1.2)$, $(\alpha = 2,4)$, $\hat{k}i = 0.4, 0.7$

And we choose hazard values of (t_i) from the region of random variable $(0 < ti < \infty)$

2-Generate random variable (T) with two parameters (θ): scale parameter , $(\theta > 0)$, $(\alpha > -1)$ is location parameter distribution as mixed in C D F (equation (9)) by method of Rejection and acceptation.

And Generate random variable distribution uniform $u_i \in (0,1)$.

3-Generate two random variables random variable Z_i ~ Rayleigh (θ) and W_i ~ Maxwell (α).

4-if $= U_i \le p\left(\frac{\alpha}{\alpha+1}\right)$ then $x_i = zi$ other wise $x_i = wi$

Table 1: Fuzzy hazard Rate function when $\alpha = 2, \theta = 0.8, \widetilde{k\iota} = 0.4$

n	t_i	Rreal h(ti)	$\hat{h}_{MLE}$	$\hat{h}_{MOM}$	$\hat{h}_{PEC}$	Best
20	1.5	0.5500	0.3762	0.4226	0.3886	MLE
	2.5	0.600	0.4566	0.4901	0.4722	MLE
	3.5	0.6500	0.5028	0.5326	0.5268	MLE
	4.5	0.7000	0.5536	0.5604	0.5463	PEC
	5.5	0.7500	0.5702	0.5812	0.5755	MLE
	6.5	0.8000	0.5772	0.5672	0.5866	MOM
	7.5	0.8500	0.6029	0.5971	0.6088	MOM
	8.5 .	0.9000	0.6091	0.6166	0.6174	MLE
40	1.5	0.5500	0.3864	0.4006	0.3862	PEC
	2.5	0.600	0.4588	0.4626	0.4672	MLE
	3.5	0.6500	0.5032	0.5168	0.5142	MLE
	4.5	0.7000	0.5346	0.5468	0.5434	MLE
	5.5	0.7500	0.5562	0.5688	0.5820	MLE
	6.5	0.8000	0.5732	0.5852	0.5862	MLE
	7.5	0.8500	0.5864	0.5981	0.6066	MLE
	8.5	0.9000	0.5954	0.6087	0.6132	MLE
60	1.5	0.5500	0.3886	0.3762	0.3933	MOM
	2.5	0.600	0.4629	0.4609	0.4667	MOM
	3.5	0.6500	0.5072	0.5062	0.5117	MOM
	4.5	0.7000	0.5376	0.5374	0.5423	MOM
	5.5	0.7500	0.5578	0.5622	0.5632	MLE
	6.5	0.8000	0.5764	0.5788	0.5811	MLE
	7.5	0.8500	0.5897	0.6024	0.5942	MLE
	8.5	0.9000	0.6004	0.6110	0.6067	MLE

Table 2: Fuzzy hazard Rate function when $\alpha = 2, \theta = 0.8, \widetilde{k\iota} = 0.7$

n	t_i	Rreal h(ti)	$\hat{h}_{MLE}$	$\hat{h}_{MOM}$	$\hat{h}_{PEC}$	Best
20	1.5	0.5000	0.5721	0.6064	0.5434	PEC
	2.5	0.5500	0.6326	0.6527	0.6123	PEC
	3.5	0.6000	0.6567	0.6806	0.6542	PEC
	4.5	0.6500	0.6927	0.7022	0.6828	PEC
	5.5	0.7000	0.7087	0.7192	0.7037	PEC
	6.5	0.7500	0.7326	0.7319	0.7169	PEC
	7.5	0.8000	0.7347	0.7424	0.7321	PEC
	8.5	0.8500	0.7515	0.7504	0.7421	PEC
40	1.5	0.5726	0.5699	0.5428	0.6952	MOM
	2.5	0.6233	0.6188	0.6029	0.6671	MOM
	3.5	0.6564	0.6523	0.6399	0.6132	PEC
	4.5	0.6821	0.6745	0.6644	0.5434	PEC
	5.5	0.6881	0.6918	0.6847	0.5820	PEC
	6.5	0.7241	0.7043	0.7004	0.6055	PEC
	7.5	0.7333	0.7242	0.7124	0.6165	PEC
	8.5	0.7409	0.7372	0.7219	0.6246	PEC
60	1.5	0.6525	0.5569	0.7299	0.4887	PEC
	2.5	0.6133	0.6062	0.7364	0.4619	PEC
	3.5	0.6468	0.6386	0.5441	0.5071	PEC
	4.5	0.6609	0.6645	0.6212	0.5366	PEC
	5.5	0.6884	0.6824	0.6360	0.5599	PEC
	6.5	0.7017	0.6954	0.6612	0.5766	PEC
	7.5	0.7142	0.7063	0.6788	0.5898	PEC
	8.5	0.7306	0.7232	0.7056	0.6003	PEC

Table 3: Fuzzy hazard Rate function when $\alpha = 2, \theta = 1.2, \tilde{k_i} = 0.4$

n	t_i	Rreal h(ti)	$\tilde{h}_{MLE}$	$\tilde{h}_{MOM}$	$\tilde{h}_{PEC}$	Best
20	1.5	0.3102	0.3068	0.3376	0.3057	PEC
	2.5	0.3762	0.3698	0.3875	0.3702	MLE
	3.5	0.4165	0.4096	0.3449	0.442	MOM
	4.5	0.4462	0.4472	0.4601	0.4604	MLE
	5.5	0.4807	0.4562	0.4702	0.4566	PEC
	6.5	0.5043	0.4727	0.4853	0.4885	MLE
	7.5	0.5127	0.4850	0.5067	0.4687	PEC
	8.5	0.5199	0.5020	0.5242	0.5032	PEC
40	1.5	0.3116	0.3326	0.3304	0.3096	PEC
	2.5	0.3762	0.3847	0.3918	0.3742	PEC
	3.5	0.4167	0.4245	0.4326	0.4152	PEC
	4.5	0.4452	0.4525	0.4581	0.4432	PEC
	5.5	0.4656	0.4728	0.4562	0.4641	MOM
	6.5	0.4723	0.4883	0.4433	0.4798	MOM
	7.5	0.4037	0.5005	0.50504	0.4923	PEC
	8.5	0.5122	0.5104	0.5233	0.5024	MLE
60	1.5	0.3114	0.3168	0.3226	0.3662	MLE
	2.5	0.3756	0.3804	0.3844	0.3758	PEC
	3.5	0.4172	0.4217	0.4249	0.4172	MLE
	4.5	0.4456	0.4494	0.4525	0.4454	MLE
	5.5	0.4623	0.4699	0.4726	0.4662	PEC
	6.5	0.4822	0.4982	0.4884	0.4820	PEC
	7.5	0.4946	0.5080	05006	0.5047	MOM
	8.5	0.5046	0.5162	0.5186	0.5130	PEC

Table 4: Fuzzy hazard Rate function when $\alpha = 2, \theta = 1.2, \tilde{k_l} = 0.7$

n	t_i	Rreal h(ti)	$\hat{h}_{MLE}$	$\hat{h}_{MOM}$	$\hat{h}_{PEC}$	Best
20	1.5	0.3202	0.3068	0.3367	0.3178	MLE
	2.5	0.3753	0.3687	0.3972	0.3921	MLE
	3.5	0.4156	0.4096	0.4345	0.4224	MLE
	4.5	0.4462	0.4472	0.6611	0.4513	MLE
	5.5	0.4658	0.4578	0.4802	0.4726	MLE
	6.5	0.4817	0.4726	0.4851	0.4872	MLE
	7.5	0.4843	0.4952	0.5064	0.4885	PEC
	8.5	0.5043	0.5032	0.5242	0.5176	MLE
40	1.5	0.3202	0.3202	0.3304	0.3168	PEC
	2.5	0.3753	0.3642	0.3928	0.3835	MLE
	3.5	0.4156	0.4176	0.4312	0.4242	MLE
	4.5	0.4462	0.4427	0.4562	0.4524	MLE
	5.5	0.4658	0.4632	0.4771	0.4729	MLE
	6.5	0.4817	0.4788	0.4933	0.4885	MLE
	7.5	0.4843	0.4912	0.5042	0.5012	MLE
	8.5	0.5043	0.5094	0.5152	0.5193	MLE
60	1.5	0.3202	0.3116	0.5233	0.5264	MLE
	2.5	0.3753	0.3762	0.3416	0.3147	PEC
	3.5	0.4156	0.4167	0.3846	0.3687	PEC
	4.5	0.4462	0.4452	0.4246	0.4197	PEC
	5.5	0.4658	0.4823	0.4525	0.4477	PEC
	6.5	0.4817	0.5037	0.4884	0.4684	PEC
	7.5	0.4843	0.5120	0.4796	0.4842	MOM
	8.5	0.5043	0.7212	0.7026	0.4812	PEC

Table 5: Fuzzy hazard Rate function when $\alpha = 4, \theta = 0.8, \tilde{k_i} = 0.4$

n	t_i	Rreal h(ti)	$\tilde{h}_{MLE}$	$\tilde{h}_{MOM}$	$\tilde{h}_{PEC}$	Best
20	1.5	0.2779	0.2968	0.3125	0.2966	MLE
	2.5	0.3182	0.3374	0.3506	0.3262	PEC
	3.5	0.3466	0.3662	0.3769	0.3532	PEC
	4.5	0.3824	0.3875	0.3962	0.3746	PEC
	5.5	0.4048	0.4032	0.4107	0.4004	PEC
	6.5	0.4132	0.4165	0.4225	0.4223	MLE
	7.5	0.4226	0.4337	0.4319	0.4305	PEC
	8.5	0.4269	0.4409	0.4397	0.4265	PEC
40	1.5	0.2779	0.2869	0.2521	0.2744	MOM
	2.5	0.3182	0.3282	0.3364	0.3264	PEC
	3.5	0.3466	0.3564	0.3643	0.3532	PEC
	4.5	0.3824	0.3769	0.3827	0.3721	PEC
	5.5	0.4048	0.3937	0.3977	0.3876	PEC
	6.5	0.4132	0.4052	0.4086	0.4057	MLE
	7.5	0.4226	0.4156	0.4192	0.4166	MLE
	8.5	0.4269	0.4235	0.4272	0.4256	MLE
60	1.5	0.2779	0.2825	0.5339	0.2826	MLE
	2.5	0.3182	0.3232	0.3718	0.3234	MLE
	3.5	0.3466	0.3511	0.3772	0.3512	MLE
	4.5	0.3824	0.3719	0.3674	0.3716	MOM
	5.5	0.4048	0.3877	0.3694	0.3876	MOM
	6.5	0.4132	0.4006	0.3858	0.4002	MOM
	7.5	0.4226	0.4175	0.3644	0.4102	MOM
	8.5	0.4269	0.4314	0.3704	0.4232	MOM

Table 6: Fuzzy hazard Rate function when $\alpha = 4, \theta = 1.2, \tilde{k_i} = 0.4$

n	t_i	Rreal h(ti)	$\tilde{h}_{MLE}$	$\tilde{h}_{MOM}$	$\tilde{h}_{PEC}$	Best
20	1.5	0.2779	0.2966	0.3126	0.2966	PEC
	2.5	0.3162	0.3365	0.3507	0.3353	PEC
	3.5	0.3462	0.3662	0.3766	0.3642	PEC
	4.5	0.3665	0.3772	0.3962	0.3845	MLE
	5.5	0.3847	0.4032	0.4108	0.4002	PEC
	6.5	0.4049	0.4155	0.4226	0.4125	PEC
	7.5	0.4132	0.4256	0.4319	0.4224	PEC
	8.5	0.4200	0.4228	0.4398	0.4306	MLE
40	1.5	0.2779	0.2866	0.2462	0.4375	MOM
	2.5	0.3162	0.3284	0.2522	0.4434	MOM
	3.5	0.3462	0.3562	0.3364	0.2854	PEC
	4.5	0.3665	0.3668	0.3633	0.3264	PEC
	5.5	0.3847	0.3892	0.3977	0.3533	PEC
	6.5	0.4049	0.4051	0.4096	0.3732	PEC
	7.5	0.4132	0.4235	0.4192	0.3886	PEC
	8.5	0.4200	0.4364	0.4272	0.4007	PEC
60	1.5	0.2779	0.2824	0.4338	0.3106	MLE
	2.5	0.3162	0.3233	0.2866	0.3257	MOM
	3.5	0.3462	0.3513	0.3266	0.3702	MOM
	4.5	0.3665	0.3719	0.3542	0.3856	MOM
	5.5	0.3847	0.3866	0.3743	0.3876	MOM
	6.5	0.4049	0.4002	0.3897	0.39841	MOM
	7.5	0.4132	0.4102	0.4118	0.4016	PEC
	8.5	0.4200	0.4266	0.4169	0.4230	MOM

Table 7: Fuzzy hazard Rate function when $\alpha = 4, \theta = 1.2, \widetilde{ki} = 0.7$

n	t_i	Rreal h(ti)	$\widetilde{h}_{MLE}$	$\widetilde{h}_{MOM}$	$\widetilde{h}_{PEC}$	Best
20	1.5	0.550	0.5822	0.5423	0.5437	MOM
	2.5	0.600	0.6337	0.6124	0.6541	MOM
	3.5	0.6333	0.6676	0.6542	0.6632	MOM
	4.5	0.6572	0.6927	0.6827	0.6771	PEC
	5.5	0.6762	0.7087	0.7037	0.6946	PEC
	6.5	0.6887	0.7235	0.7197	0.7081	PEC
	7.5	0.7092	0.7347	0.7273	0.7184	PEC
	8.5	0.7167	0.7525	0.7424	0.7273	PEC
40	1.5	0.550	0.5618	0.5356	0.5316	PEC
	2.5	0.600	0.6231	0.6032	0.6232	MOM
	3.5	0.6333	0.6468	0.6443	0.6573	MOM
	4.5	0.6572	0.6820	0.6525	0.6816	MOM
	5.5	0.6762	0.6994	0.7077	0.6997	MLE
	6.5	0.6887	0.7232	0.7211	0.7138	PEC
	7.5	0.7092	0.7252	0.7312	0.7252	PEC
	8.5	0.7167	0.7334	0.7382	0.7343	PEC
60	1.5	0.550	0.5625	0.5416	0.5625	MOM
	2.5	0.600	0.6122	0.6014	0.6042	PEC
	3.5	0.6333	0.6369	0.6299	0.6367	MOM
	4.5	0.6572	0.6708	0.6355	0.6602	MOM
	5.5	0.6762	0.7028	0.6857	0.6884	MOM
	6.5	0.6887	0.7142	0.7006	0.7029	MOM
	7.5	0.7092	0.7241	0.7124	0.4141	MOM
	8.5	0.7167	0.7306	0.7219	0.7309	MLE

Table 8: Fuzzy hazard Rate function when $\alpha = 4, \theta = 0.8, \tilde{k_i} = 0.7$

n	t_i	Rreal h(ti)	$\hat{h}_{MLE}$	$\hat{h}_{MOM}$	$\hat{h}_{PEC}$	Best
20	1.5	0.5502	0.5820	0.6064	0.5728	PEC
	2.5	0.6133	0.6336	0.6416	0.6226	PEC
	3.5	0.6334	0.6676	0.6726	0.6542	PEC
	4.5	0.6572	0.6927	0.7032	0.6773	PEC
	5.5	0.6688	0.7097	0.7192	0.6946	PEC
	6.5	0.6753	0.7234	0.7019	0.7082	MOM
	7.5	0.7092	0.7349	0.7421	0.7488	MLE
	8.5	0.7167	0.7515	0.7506	0.7567	MOM
40	1.5	0.5502	0.5617	0.5864	0.5704	MLE
	2.5	0.6133	0.6234	0.6338	0.6187	PEC
	3.5	0.6334	0.6558	0.6654	0.6535	PEC
	4.5	0.6572	0.6801	0.6782	0.6654	PEC
	5.5	0.6688	0.6949	0.7038	0.6935	PEC
	6.5	0.6753	0.7130	0.7182	0.7073	MLE
	7.5	0.7092	0.7242	0.7374	0.7182	PEC
	8.5	0.7167	0.7332	0.7445	0.7271	PEC
60	1.5	0.5502	0.5626	0.5688	0.5354	PEC
	2.5	0.6133	0.6133	0.6177	0.6074	PEC
	3.5	0.6334	0.6468	0.6532	0.6410	MLE
	4.5	0.6572	0.6708	0.6745	0.6655	PEC
	5.5	0.6688	0.6884	0.6918	0.6820	PEC
	6.5	0.6753	0.7-29	0.7054	0.6970	PEC
	7.5	0.7092	0.7140	0.7163	0.7066	MLE
	8.5	0.7167	0.7310	0.7252	0.7166	PEC

We summarize the results of comparison in the table by percentage

Table 9. Summary of comparsion of estimators

Tables	% MLE	% PEC	% MOM	Ki
Table (1)	66.6	8.333	25	0.4
Table (2)	zero	91.666	8.333	0.7
Table(3)	62.5	33.333	4.166	0.4
Table (4)	37.5	33.333	4.166	0.7
Table(5)	62.5	41.25	25	0.4
Table(6)	33.333	54.166	33.333	0.4
Table(7)	8.333	41.666	50	0.7
Table(8)	20.333	70.8333	8.333	0.7

Conclusion:

For comparison of fuzzy hazard rate at fuzzy vague factor($\tilde{ki} = 0.4$) ,the first best one estimator is$(\hat{h}_{MOM})$.

As indicated by table (6) with percentage (33.333) , secondly for also($\tilde{ki} = 0.4$) the percentage is (25.000)for table (1) ,also for table (5) , the first best one is MOM .with percentage (25.000).

While the comparison for $(\hat{h}_{PEC})$ at ($\tilde{ki} = 0.4$) appear with percentage$(\hat{h}_{PEC} = \mathbf{54.166\%})$ in table (6) and table (5) $(\hat{h}_{PEC} = \mathbf{41.25\%})$.

secondly for fuzzy factor (0.7) the first best one is $(\hat{h}_{PEC})$ with percentage(91.666) from table (2) ,and secondly $(\hat{h}_{PEC} = \mathbf{70.8333\%})$ from table (8).

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