Estimation the Reliability System of Multicomponent Stress-Strength Model Built on Generalized Exponential Distribution

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Abstract:

A reliability of the multi-component system in stress-strength model $R_{(s,k)}$ will be considered in present paper ,when the stress and strength are independent identically distribution (iid), follows Generalized Exponential Distribution(GED) with the unknown shape parameter and known scale parameter and equal to one. Different estimation methods of $R_{(s,k)}$ for (GED) were introduced corresponding to Maximum likelihood , Percentile estimator and Shrinkage estimators . The comparisons among the proposed estimators prepared depend on simulation based on mean squared error (MSE) criteria.

Keywords: Generalized Exponential Distribution(GED), Reliability of multi-component Stress – Strength models $R_{(s,k)}$, Maximum likelihood estimator (MLE), Percentile estimator (PCE) , Shrinkage estimator and mean squared error (MSE).

1-Introduction:

A multicomponent system consuming strength following k independently and identically distribution random variable and each component experiencing a random stress was introduced by Bhattacharyya and Johnson (1974); [4].The reliability of system model, (s out of k) denoted by $R_{(s,k)}$ when at least s ($1 \leq s \leq k$) of components survive .Noted that if s=1 and s=k corresponded, respectively to parallel and series systems.

In 2010, Rao & Kantam, studied a system of k multicomponent such as X and Y (iid) the Log–Logistic distribution;[11]. In 2012 Srinivasa Rao estimated the multicomponent system of reliability for log-logistic

distribution with unknown shape parameters;[14], also Hassan & Basheikh estimated the reliability of stress- strength (S-S) model through non-identical component strengths by using the Exponentiated Pareto distribution;[5]. In 2016 Srinivasa Rao et al, they estimated the multicomponent stress-strength reliability of system when the stress and strength follows Exponentiated Weibull distribution;[12]. In 2017 Hassan introduced the estimation of multicomponent of system reliability in (S-S) model when each of stress and strength follows Lindley distribution;[6].In the same year, Salman and Sail estimated the reliability of multicomponent system in stress-strength model using shrinkage estimation method,[16].

The Generalized Exponential distribution has firstly studied by Gupta and Kundu (1999) ,Raqab and Ahsanullah (2001), Zheng (2002),Gupta and Kundu(2003).In (2005) Kundu and Gupta estimated P(Y<X) while X and Y follows (GED);[9].In (2008) , Kundu and Gupta studied the estimation of GDE via Bayesian estimation and they proposed gamma as prior distribution ;[10] .In (2010) , Rao obtained and studied estimation for the reliability of multi-component stress-strength model based Generalized Exponential distribution;[13].

The goal of the present paper is to estimate the reliability of multicomponent system in stress-strength model $R_{(s,k)}$ based on Generalized Exponential distribution with known scale parameter λ and unknown shape parameter α via different estimation methods like MLE, PCE ,as well as some shrinkage methods and comparisons among the proposed estimator methods using simulation depends on mean squared error criteria.

The probability density function (p. d. f.) of a r.v. X distributed following Generalized Exponential Distribution GED (α, 1) is as below;[8]:

$$f(x;\alpha,\lambda) = \alpha e^{-x}(1-e^{-x})^{\alpha-1}; \text{ for } x> 0\ \alpha,\lambda > 0 \qquad (1)$$

And the cumulative distribution function (c.d. f.) of X will be:

$$F(x,\alpha,\lambda) = (1-e^{-x})^{\alpha} \qquad x > 0 \qquad (2)$$

Here α refer to shape parameter.

The first who derived the reliability of a multicomponent system in stress-strength model $R_{(s,k)}$ were Bhattacharyya and Johnson (1974) as the following form,[4].

$R_{(s,k)}$ =P(at least s of]. the X_1, X_2,…, X_k exceed Y)

Where X_1,X_2…X_k are (iid) with common distribution function F(x) subjected to the common random stress Y with distribution Function G(y)

$$R_{(s,k)} = \sum_{i=s}^{k} \binom{k}{i} \int_o^{\infty} (1 - F_x(y))^i (F_x(y))^{k-i} dG(y)$$

When X~ GED (α, 1) and Y~ GED(β,1) then:

$$R_{(s,k)} = \sum_{i=s}^{k} \binom{k}{i} \int_o^{\infty} (1 - (1 - e^{-y})^{\alpha})^i ((1 - e^{-y})^{\alpha})^{k-i} \beta e^{-y} (1 - e^{-y})^{\beta-1} dy$$

$$R_{(s,k)} = \sum_{i=s}^{k} \binom{k}{i} \int_o^{\infty} (1 - (1 - e^{-y})^{\alpha})^i \beta e^{-y} (1 - e^{-y})^{\alpha k - i\alpha + \beta - 1} dy$$

Let $z = 1 - e^{-\lambda y} \rightarrow y = \frac{-\ln(1-z)}{\lambda} \rightarrow dy = \frac{dz}{\lambda(1-z)}$

$$R_{(s,k)} = \sum_{i=s}^{k} \binom{k}{i} \beta\lambda \int_o^1 (1 - z^{\alpha})^i (z)^{\alpha k - \alpha i + \beta - 1} (1 - z) \frac{dz}{\lambda(1-z)}$$

$$= \sum_{i=s}^{k} \binom{k}{i} \beta \int_o^1 (1 - z^{\alpha})^i (z)^{\alpha k - \alpha i + \beta - 1} dz$$

Let $w = z^{\alpha} \rightarrow z = w^{\frac{1}{\alpha}} \rightarrow dz = \frac{1}{\alpha} w^{\frac{1}{\alpha} - 1} dw$

$$R_{(s,k)} = \sum_{i=s}^{k} \binom{k}{i} \frac{\beta}{\alpha} \int_o^1 (1 - w)^i (w)^{k - i + \frac{\beta}{\alpha} - \frac{1}{\alpha}} w^{\frac{1}{\alpha} - 1} dw$$

And by simplification, $R_{(s,k)}$ became:[13]

$$R_{(s,k)} = \frac{\beta}{\alpha} \sum_{i=s}^{k} \frac{k!}{(k-i)!} [\prod_{j=0}^{i} (k + \frac{\beta}{\alpha} - j)]^{-1} \text{ ; k, i, j are integers} \quad (3)$$

2- Estimation methods of $R_{(s,k)}$:

2-1 Maximum Likelihood Estimator (MLE):[13]

Let x_1,x_2,…,x_n be a random sample of size n follows GED $(\alpha, 1)$, then $x_{(1)} < x_{(2)} < \ldots < x_{(n)}$ the order random sample of x and y_1 ,y_2,…,y_m be a random sample of size m follows GED $(\beta, 1)$, and $y_{(1)} < y_{(2)} < \ldots < y_{(m)}$ the order random sample of y. Then the likelihood functions:

$$L = L(\alpha, \beta; x, y) = \prod_{i=1}^{n} f(x_i) \prod_{j=1}^{m} f(y_j)$$

$$=\prod_{i=1}^{n} \alpha e^{-x_i}(1-e^{-x_i})^{\alpha-1} \prod_{j=1}^{m} \beta e^{-y_j}(1-e^{-y_j})^{\beta-1}$$

$$=\alpha^n e^{-\sum_{i=1}^{n} x_i} \prod_{i=1}^{n}(1-e^{-x_i})^{\alpha-1} \beta^m e^{-\sum_{j=1}^{m} y_j} \prod_{j=1}^{m}(1-e^{-y_j})^{\beta-1}$$

$$\ln(l) = nln\alpha - \sum_{i=1}^{n} x_i + (\alpha-1)\sum_{i=1}^{n} ln(1-e^{-x_i}) + mln\beta - \sum_{j=1}^{m} y_j + (\beta-1)\sum_{j=1}^{m} ln(1-e^{-y_j})$$

$$\frac{d\mathrm{Ln}(l)}{d\alpha} = \frac{n}{\alpha} + \sum_{i=1}^{n} ln(1-e^{-x_i}) = 0$$

$$\frac{d\mathrm{Ln}(l)}{d\beta} = \frac{m}{\beta} + \sum_{j=1}^{m} ln(1-e^{-y_j}) = 0$$

Thus, the maximum likelihood estimator of the parametersα , β will be as follows:

$$\hat{\alpha}_{\mathrm{mle}} = \frac{-\mathrm{n}}{\sum_{\mathrm{i=1}}^{\mathrm{n}} \ln(1-\mathrm{e}^{-\mathrm{x_i}})} \qquad (4)$$

$$\hat{\beta}_{mle} = \frac{-m}{\sum_{j=1}^{m} ln(1-e^{-y_j})} \qquad (5)$$

By substituting $\hat{\alpha}_{\mathrm{mle}}$, $\hat{\beta}_{mle}$ in equation (3), we get the reliability estimation for $R_{(s,k)}$ model via Maximum Likelihood method:

$$\hat{R}_{(s,k)_{mle}} = \frac{\hat{\beta}_{mle}}{\hat{\alpha}_{mle}} \sum_{i=s}^{k} \frac{k!}{(k-i)!} [\prod_{j=0}^{i}(k + \frac{\hat{\beta}_{mle}}{\hat{\alpha}_{mle}} - j)\ ]^{-1} \qquad (6)$$

2-2 The Percentile Estimator $R_{(s,\,k)}$:[8]

Let X ~ GED(α,1) and Y ~ GED(β,1) ,i=1,2,……,n and j=1,2…,m then the least square estimator obtained by minimizing the squared errors S as below

$$S = \sum_{j=1}^{n}[F(x_i) - E(F(x_i))]^{\,2}$$

Where $F(x_i)$ the (c.d.f) of x_i

$$F(x_i) = (1-e^{-x_i})^{\alpha}$$

And, $E(F(x_i)) = P_i$

Such that; $P_i = \frac{i}{n+1}$; i=1, 2… n

$$S = \sum_{i=1}^{n}[\alpha \mathrm{Ln}(1 - e^{-x_i}) \; - \; \mathrm{Ln}P_i \,]^{\,2} \qquad (7)$$

And, the partial derivatives of the equation (7) with respect to α:

$$\frac{\partial S}{\partial \alpha} = 2\sum_{i=1}^{n}[\alpha \mathrm{Ln}(1 - e^{-x_i}) \; - \; \mathrm{Ln}P_i \,]\,\mathrm{Ln}(1 - e^{-x_i})$$

$$2\sum_{i=1}^{n}[\alpha \mathrm{Ln}(1 - e^{-x_i}) \; - \; \mathrm{Ln}P_i \,]\,\mathrm{Ln}(1 - e^{-x_i}) \; = 0$$

Then the Percentile estimator of α will be

$$\hat{\alpha}_{PCE} = \frac{\sum_{i=1}^{n} \mathrm{Ln}P_i \mathrm{Ln}(1-e^{-x_i})}{\sum_{i=1}^{n}(\mathrm{Ln}(1-e^{-x_i}))^2} \;\; ; \;\; P_i = \frac{i}{n+1}, \; i=1, 2,\ldots, n \qquad (8)$$

And, by the similar technique, we get the Percentile estimator of β

$$\hat{\beta}_{PCE} = \frac{\sum_{j=1}^{m} \mathrm{Ln}P_j \mathrm{Ln}\left(1-e^{-y_j}\right)}{\sum_{j=1}^{m}(\mathrm{Ln}\left(1-e^{-y_j}\right))^2} \quad ; \quad P_j = \frac{j}{m+1}, \; j=1, 2, \ldots, m \qquad (9)$$

By substituting $\hat{\alpha}_{PCE}$, $\hat{\beta}_{PCE}$ in equation (3), we get the reliability estimation for $R_{(s,k)}$ model via least square estimator method:

$$\hat{R}_{(s,k)_{LS}} = \frac{\hat{\beta}_{LS}}{\hat{\alpha}_{LS}} \sum_{i=s}^{k} \frac{k!}{(k-i)!} [\, \prod_{j=0}^{i}(k + \frac{\hat{\beta}_{LS}}{\hat{\alpha}_{LS}} - j) \,]^{-1} \qquad (10)$$

2-3 Shrinkage Estimation Method (Sh):

Thompson in 1968, assumed the problem of shrink a usual estimator $\hat{\alpha}$ of the parameter α to prior information α_0 using shrinkage weight factor $\emptyset(\hat{\alpha})$, such that $0 \leq \emptyset(\hat{\alpha}) \leq 1$. Thompson estimating α and he believe that α_0 is closed to the true value of α or α_0 may be near the true value of α .Thus, the form of Thompson - Type shrinkage estimator of α say $\hat{\alpha}_{sh}$ will be:[2].

$$\hat{\alpha}_{sh} = \emptyset(\hat{\alpha})\hat{\alpha} + (1 - \emptyset(\hat{\alpha}))\alpha_0 \qquad (11)$$

In this paper, we apply the unbiased estimator $\hat{\alpha}_{ub}$ as a usual estimator and $\alpha_0 \approx \alpha$ as a prior estimation of α in equation (11).

As we mentioned, $\emptyset(\hat{\alpha})$ denote the shrinkage weight factor such that $0 \leq \emptyset(\hat{\alpha}) \leq 1$, which may be a function of $\hat{\alpha}_{ub}$; a function of sample size

(n,m) or may be constant or can be found by minimizing the mean square error of $\hat{\alpha}_{sh}$.

Note that,[8]

$$\hat{\alpha}_{ub} = \frac{n-1}{n}\hat{\alpha}_{mle} = \frac{n-1}{-\sum_{i=1}^{n} ln(1-e^{-x_i})} \quad (12)$$

Hence, E ($\hat{\alpha}_{ub}$) = α
(13)

$$\text{Var } (\hat{\alpha}_{ub}) = \frac{\alpha^2}{n-2} \quad (14)$$

$$\text{And, } \hat{\beta}_{ub} = \frac{m-1}{m}\hat{\beta}_{mle} = \frac{m-1}{-\sum_{j=1}^{m} ln(1-e^{-y_j})} \quad (15)$$

$$\text{Implies, E } (\hat{\beta}_{ub}) = \beta \quad (16)$$

$$Var\ (\hat{\beta}_{ub}) = \frac{\beta^2}{m-2} \quad (17)$$

2-3-1The shrinkage weight function (sh):

In this subsection, the shrinkage weight factor as a function of sizes n and m respectively will be considered and taking the form below:

i.e. $\emptyset_1(\hat{\alpha}) = |\sin n/n|$, and $\emptyset_2(\hat{\beta}) = |\sin m/m|$

Where n, and m are sample sizes, therefore the shrinkage estimator using shrinkage weight function of α and β which is defined in equation (11), will be:

$$\hat{\alpha}_{sh} = \emptyset_1(\hat{\alpha})\hat{\alpha}_{ub} + (1 - \emptyset_1(\hat{\alpha}))\alpha_0 \quad (18)$$

$$\hat{\beta}_{sh} = \emptyset_2(\hat{\beta})\hat{\beta}_{ub} + (1 - \emptyset_2(\hat{\beta}))\beta_0 \quad (19)$$

Also, as Thompson (1968);[], mentioned,α_0 and β_0 are closed to the real value of $\alpha\ and\ \beta$ respectively.

Then, the shrinkage estimation $\hat{R}_{(s,k)}$ in equation (3) using shrinkage weight function will be:-

$$\hat{R}_{(s,k)_{sh}} = \frac{\hat{\beta}_{sh}}{\hat{\alpha}_{sh}} \sum_{i=s}^{k} \frac{k!}{(k-i)!} [\prod_{j=0}^{i} (k + \frac{\hat{\beta}_{sh}}{\hat{\alpha}_{sh}} - j)\ ]^{-1} \quad (20)$$

2-3-2 Modified Thompson type Shrinkage Weight Function (th);[16]:

In this subsection, Thompson in (1968), consider the shrinkage weight factor as bellow:

$$\gamma(\hat{\alpha}) = \frac{(\hat{\alpha}_{ub} - \alpha_0)^2}{(\hat{\alpha}_{ub} - \alpha_0)^2 + Var(\hat{\alpha}_{ub})} \quad (21)$$

$$\gamma(\hat{\beta}) = \frac{(\hat{\beta}_{ub} - \beta_0)^2}{(\hat{\beta}_{ub} - \beta_0)^2 + Var(\hat{\beta}_{ub})} \quad (22)$$

In this paper we modified the shrinkage weight factor consider by Thompson as bellow:

$$\gamma(\hat{\alpha}) = \frac{(\hat{\alpha}_{ub} - \alpha_0)^2}{(\hat{\alpha}_{ub} - \alpha_0)^2 + Var(\hat{\alpha}_{ub})} \ *0.001 \quad (23)$$

$$\gamma(\hat{\beta}) = \frac{(\hat{\beta}_{ub} - \beta_0)^2}{(\hat{\beta}_{ub} - \beta_0)^2 + Var(\hat{\beta}_{ub})} \ * 0.001 \quad (24)$$

Therefore, the shrinkage estimator of α and β using modified shrinkage weight factor are respectively as bellow:

$$\hat{\alpha}_{\text{th}} = \gamma(\hat{\alpha})\hat{\alpha}_{ub} + (1 - \gamma(\hat{\alpha}))\alpha_0 \quad (25)$$

$$\hat{\beta}_{\text{th}} = \gamma(\hat{\beta})\hat{\beta}_{ub} + \left(1 - \gamma(\hat{\beta})\right)\beta_0 \quad (26)$$

Then the Shrinkage estimation of $R_{(s,k)}$ in equation (3) based on Modified Thompson type shrinkage weight factor will be:

$$\hat{R}_{(s,k)_{th}} = \frac{\hat{\beta}_{\text{th}}}{\hat{\alpha}_{\text{th}}} \sum_{i=s}^{k} \frac{k!}{(k-i)!} [\prod_{j=0}^{i} (k + \frac{\hat{\beta}_{\text{th}}}{\hat{\alpha}_{\text{th}}} - j)\ ]^{-1} \quad (27)$$

3-Simulation study

In this section, numerical results were studied to compare the performance of the different estimators of reliability, using different sample size = (10, 30, 50 and 100), based on 1000 replication via MSE criteria. For this purpose, Mote Carlo simulation was employed by generate the random sample from continuous uniform distribution defined on the interval (0,1) as $u_1,u_2,\ldots, u_n$; $v_1, v_2,\ldots, v_m$. Transform uniform random samples to follows GED (α,1) using (c. d. f.) as below; [7] :

$$F(x) = (1-e^{-x_i})^{\alpha} \quad \rightarrow \quad U_i = (1-e^{-x_i})^{\alpha} \rightarrow x_i = -\ln(1-U_i^{\frac{1}{\alpha}})$$

And, by the same method, we get, $y_j = -\ln(1-V_j^{\frac{1}{\beta}})$

The following steps, Compute the real value of $R_{(s,k)}$ in equation (3) and the value of estimation methods of all proposal methods $\hat{R}_{(s,k)_{mle}}$, $\hat{R}_{(s,k)_{PCE}}$, $\hat{R}_{(s,k)_{sh}}$ and $\hat{R}_{(s,k)_{th}}$ in equations (6), (10), (20) and (27) respectively.

Based on (L=1000) Replication, we calculate the MSE for all proposed estimation methods of $\widehat{R}_{(s,k)}$ as follows:

$$MSE = \frac{1}{L}\sum_{i=1}^{L}(\hat{R}_{(s,k)_i} - R_{(s,k)})^2$$

Also, Noted that the odd tables (1-11) contains the value of real reliability $R_{(s,k)}$ and the value of estimator reliability $\hat{R}_{(s,k)}$ for different methods when (s,k)=(1,3),(2,3) and (2,4) and (α,β)=(2,4) and (4,2) and (α_0,β_0)=($\alpha+0.001, \beta+0.001$).

Also, Noted that the even tables (2-12) contains the value of MSE for the estimator reliability $\hat{R}_{(s,k)}$ for different methods when (s,k)=(1,3),(2,3) and (2,4) and (α,β)=(2,4) and (4,2) and (α_0,β_0)=($\alpha+0.001, \beta+0.001$).

Table (1): $R_{(s,k)}$ and $\widehat{R}_{(s,k)}$ when (s,k)=(1,3), α=2, β=4 , α_0=2.001and β_0=4.001

(n,m)	$R_{(s,k)}$	$\hat{R}_{(s,k)_{mle}}$	$\hat{R}_{(s,k)_{PCE}}$	$\hat{R}_{(s,k)_{sh}}$	$\hat{R}_{(s,k)_{th}}$
(10,10)	0.60000	0.59071	0.594941	0.60023	0.60006
(10,30)	0.60000	0.81777	0.30108	0.59449	0.60022
(10,50)	0.60000	0.88216	0.21512	0.59905	0.60025
(10,100)	0.60000	0.93783	0.07494	0.59876	0.60028
(30,10)	0.60000	0.33274	0.80764	0.63054	0.59964
(30,30)	0.60000	0.60028	0.64548	0.60000	0.60006
(30,50)	0.60000	0.59992	0.14575	0.59948	0.60015
(30,100)	0.60000	0.83157	0.18046	0.59926	0.60023
(50,10)	0.60000	0.23051	0.92349	0.66573	0.59917
(50,30)	0.60000	0.47300	0.77474	0.60542	0.59993
(50,50)	0.60000	0.60185	0.57024	0.60005	0.60006
(50,100)	0.60000	0.74882	0.48527	0.59945	0.60018
(100,10)	0.60000	0.13127	0.77060	0.77534	0.59801
(100,30)	0.60000	0.30926	0.86183	0.62000	0.59952
(100,50)	0.60000	0.43115	0.20684	0.60132	0.59984
(100,100)	0.60000	0.59906	0.73300	0.60006	0.60006

Table(2): MSE for $\hat{R}_{(s,k)}$when (s,k)=(1,3), α=2, β=4, α_0=2.001, β_0=4.001, and $R_{(s,k)}$= 0.60000.

(n,m)	$\hat{R}_{(s,k)_{mle}}$	$\hat{R}_{(s,k)_{PCE}}$	$\hat{R}_{(s,k)_{sh}}$	$\hat{R}_{(s,k)_{th}}$	Best
(10,10)	0.01100	0.00585	0.00004	0.000000009	Th
(10,30)	0.05033	0.090781	0.00005	0.00000005	Th
(10,50)	0.08088	0.15297	0.00002	0.00000007	Th
(10,100)	0.11449	0.27583	0.00003	0.00000008	Th
(30,10)	0.07758	0.04403	0.00123	0.0000002	Th
(30,30)	0.00413	0.009929	0.000005	0.000000006	Th
(30,50)	0.01533	0.00876	0.000003	0.00000003	Th
(30,100)	0.05439	0.17735	0.000003	0.00000005	Th
(50,10)	0.14018	0.10493	0.00542	0.0000009	Th
(50,30)	0.01923	0.03309	0.00004	0.00000001	Th
(50,50)	0.00239	0.00367	0.00000007	0.000000005	Th
(50,100)	0.02324	0.01443	0.0000003	0.00000003	Th
(100,10)	0.22113	0.06347	0.04401	0.000005	Th
(100,30)	0.08647	0.06995	0.00043	0.0000003	Th
(100,50)	0.03041	0.17320	0.000002	0.00000003	Th
(100,100)	0.00113	0.02151	0.00000003	0.000000004	Th

Table(3): $R_{(s,k)}$ and $\hat{R}_{(s,k)}$when (s,k)=(2,3), α=2, β=4 , α_0=2.001and β_0=4.001.

(n,m)	$R_{(s,k)}$	$\hat{R}_{(s,k)_{mle}}$	$\hat{R}_{(s,k)_{PCE}}$	$\hat{R}_{(s,k)_{sh}}$	$\hat{R}_{(s,k)_{th}}$
(10,10)	0.30000	0.30982	0.19983	0.30005	0.30007
(10,30)	0.30000	0.61210	0.06431	0.29415	0.30025
(10,50)	0.30000	0.73455	0.06471	0.29887	0.30028
(10,100)	0.30000	0.85427	0.00521	0.29869	0.30031
(30,10)	0.30000	0.08671	0.66856	0.33647	0.29959
(30,30)	0.30000	0.29998	0.17651	0.30019	0.30007
(30,50)	0.30000	0.44786	0.37973	0.29946	0.30017
(30,100)	0.30000	0.64138	0.00286	0.29906	0.30026
(50,10)	0.30000	0.04110	0.92206	0.38042	0.29907
(50,30)	0.30000	0.17861	0.56055	0.30641	0.29991
(50,50)	0.30000	0.30299	0.24121	0.30006	0.30007
(50,100)	0.30000	0.50236	0.21195	0.29937	0.30020
(100,10)	0.30000	0.01290	0.81615	0.51363	0.29779
(100,30)	0.30000	0.07370	0.73937	0.32241	0.29947
(100,50)	0.30000	0.14332	0.62454	0.30154	0.29981
(100,100)	0.30000	0.30178	0.25134	0.30006	0.30007

Table (4): MSE for $\hat{R}_{(s,k)}$ when (s,k)=(2,3), α=2,β=4, α_0=2.001, β_0=4.001 and $R_{(s,k)}$= 0.30000

(n,m)	$\hat{R}_{(s,k)_{mle}}$	$\hat{R}_{(s,k)_{PCE}}$	$\hat{R}_{(s,k)_{sh}}$	$\hat{R}_{(s,k)_{th}}$	Best
(10,10)	0.01335	0.02179	0.00005	0.00000001	Th
(10,30)	0.10673	0.05564	0.00006	0.00000006	Th
(10,50)	0.19392	0.05605	0.00003	0.00000008	Th
(10,100)	0.30900	0.08690	0.00003	0.0000001	Th
(30,10)	0.04743	0.13847	0.00178	0.0000002	Th
(30,30)	0.00485	0.01921	0.000006	0.000000007	Th
(30,50)	0.02642	0.02512	0.000003	0.00000003	Th
(30,100)	0.11951	0.08834	0.000004	0.00000007	Th
(50,10)	0.06749	0.38776	0.00833	0.000001	Th
(50,30)	0.01694	0.07221	0.00005	0.00000002	Th
(50,50)	0.00293	0.00523	0.00000009	0.000000006	Th
(50,100)	0.04347	0.01274	0.0000004	0.00000004	Th
(100,10)	0.08248	0.26999	0.07784	0.000006	Th
(100,30)	0.05164	0.19368	0.00054	0.0000003	Th
(100,50)	0.02549	0.11282	0.000003	0.00000004	Th
(100,100)	0.00142	0.00453	0.00000004	0.000000005	Th

Table(5): $R_{(s,k)}$ and $\hat{R}_{(s,k)}$when (s,k)=(2,4) , α=2, β=4 , α_0=2.001and β_0=4.001.

(n,m)	$R_{(s,k)}$	$\hat{R}_{(s,k)_{mle}}$	$\hat{R}_{(s,k)_{PCE}}$	$\hat{R}_{(s,k)_{sh}}$	$\hat{R}_{(s,k)_{th}}$
(10,10)	0.40000	0.39862	0.41833	0.40038	0.40007
(10,30)	0.40000	0.70124	0.15158	0.39333	0.40027
(10,50)	0.40000	0.80071	0.02021	0.39871	0.40031
(10,100)	0.40000	0.89074	0.01348	0.39899	0.40033
(30,10)	0.40000	0.13632	0.76356	0.43848	0.39956
(30,30)	0.40000	0.39972	0.35135	0.40013	0.40007
(30,50)	0.40000	0.55048	0.21497	0.39942	0.40019
(30,100)	0.40000	0.72411	0.09475	0.39907	0.40028
(50,10)	0.40000	0.06788	0.83938	0.48554	0.39897
(50,30)	0.40000	0.25675	0.55779	0.40694	0.39990
(50,50)	0.40000	0.39994	0.41483	0.40008	0.40007
(50,100)	0.40000	0.59804	0.26526	0.39934	0.40022
(100,10)	0.40000	0.02326	0.90445	0.64591	0.39756
(100,30)	0.40000	0.11763	0.63588	0.42452	0.39941
(100,50)	0.40000	0.21820	0.64546	0.40161	0.39980
(100,100)	0.40000	0.40021	0.30397	0.400074	0.400072

Table(6): MSE for $\hat{R}_{(s,k)}$ when (s,k)=(2,4), α=2, β=4, α_0=2.001, β_0=4.001 and $R_{(s,k)}$= 0.40000.

(n,m)	$\hat{R}_{(s,k)_{mle}}$	$\hat{R}_{(s,k)_{PCE}}$	$\hat{R}_{(s,k)_{sh}}$	$\hat{R}_{(s,k)_{th}}$	Best
(10,10)	0.01619	0.00339	0.00007	0.00000001	Th
(10,30)	0.09737	0.06414	0.00008	0.00000008	Th
(10,50)	0.16413	0.14428	0.00004	0.0000009	Th
(10,100)	0.24197	0.14942	0.00003	0.0000001	Th
(30,10)	0.07323	0.13301	0.00194	0.0000003	Th
(30,30)	0.00603	0.00635	0.000007	0.000000008	Th
(30,50)	0.02669	0.03833	0.000003	0.00000004	Th
(30,100)	0.10701	0.09381	0.000004	0.00000008	Th
(50,10)	0.11144	0.19715	0.00959	0.000001	Th
(50,30)	0.02345	0.02689	0.00006	0.00000001	Th
(50,50)	0.00325	0.00832	0.0000001	0.000000007	Th
(50,100)	0.04156	0.10878	0.0000005	0.00000005	Th
(100,10)	0.14211	0.25585	0.09591	0.000007	Th
(100,30)	0.08075	0.06028	0.00065	0.0000004	Th
(100,50)	0.03461	0.06124	0.000003	0.00000005	Th
(100,100)	0.00184	0.01802	0.00000005	0.000000006	Th

Table(7): $R_{(s,k)}$ and $\hat{R}_{(s,k)}$when (s,k)=(1,3), α=4, β=2 , α_0=4.001 and β_0=2.001.

(n,m)	$R_{(s,k)}$	$\hat{R}_{(s,k)_{mle}}$	$\hat{R}_{(s,k)_{PCE}}$	$\hat{R}_{(s,k)_{sh}}$	$\hat{R}_{(s,k)_{th}}$
(10,10)	0.85714	0.84861	0.93076	0.85698	0.85711
(10,30)	0.85714	0.94695	0.57109	0.85416	0.85719
(10,50)	0.85714	0.96708	0.45099	0.85654	0.85721
(10,100)	0.85714	0.98374	0.46519	0.85641	0.85722
(30,10)	0.85714	0.65797	0.94759	0.87206	0.85689
(30,30)	0.85714	0.85303	0.84944	0.85714	0.85711
(30,50)	0.85714	0.90735	0.77537	0.85689	0.85716
(30,100)	0.85714	0.95205	0.52092	0.85666	0.85719
(50,10)	0.85714	0.53338	0.97563	0.88799	0.85665
(50,30)	0.85714	0.77714	0.95726	0.85995	0.85704
(50,50)	0.85714	0.85609	0.91701	0.85711	0.85711
(50,100)	0.85714	0.92299	0.71289	0.85679	0.85717
(100,10)	0.85714	0.36777	0.98949	0.93105	0.85604
(100,30)	0.85714	0.63813	0.95919	0.86697	0.85684
(100,50)	0.85714	0.74658	0.93666	0.85778	0.85699
(100,100)	0.85714	0.85586	0.84041	0.85711	0.85711

Table (8): MSE for $\hat{R}_{(s,k)}$when(s,k)=(1,3),
α_1=4,α_2=2, α_0=2.001, β_0=4.001, and $R_{(s,k)}$= 0.85714

(n,m)	$\hat{R}_{(s,k)_{mle}}$	$\hat{R}_{(s,k)_{PCE}}$	$\hat{R}_{(s,k)_{sh}}$	$\hat{R}_{(s,k)_{th}}$	Best
(10,10)	0.00377	0.00576	0.00001	0.000000003	Th
(10,30)	0.00840	0.08294	0.00001	0.000000004	Th
(10,50)	0.01219	0.17627	0.000006	0.000000005	Th
(10,100)	0.01606	0.15963	0.000006	0.000000007	Th
(30,10)	0.04644	0.00842	0.00028	0.00000007	Th
(30,30)	0.00110	0.00141	0.000001	0.000000001	Th
(30,50)	0.00291	0.00704	0.0000006	0.0000000006	Th
(30,100)	0.00910	0.11864	0.0000008	0.000000003	Th
(50,10)	0.11205	0.01408	0.00117	0.0000003	Th
(50,30)	0.00809	0.01862	0.00001	0.00000001	Th
(50,50)	0.00059	0.00412	0.00000002	0.000000001	Th
(50,100)	0.00449	0.02283	0.0000001	0.000000001	Th
(100,10)	0.24495	0.01756	0.00662	0.000001	Th
(100,30)	0.05045	0.01047	0.00010	0.0000001	Th
(100,50)	0.01328	0.00652	0.0000004	0.00000002	Th
(100,100)	0.00032	0.00053	0.000000009	0.000000001	Th

Table(9): $R_{(s,k)}$ and $\hat{R}_{(s,k)}$when (s,k)=(2,3), α=4, β=2 , α_0=4.001 and β_0=2.001.

(n,m)	$R_{(s,k)}$	$\hat{R}_{(s,k)_{mle}}$	$\hat{R}_{(s,k)_{PCE}}$	$\hat{R}_{(s,k)_{sh}}$	$\hat{R}_{(s,k)_{th}}$
(10,10)	0.68571	0.67069	0.70740	0.68579	0.68565
(10,30)	0.68571	0.87387	0.21888	0.68022	0.68581
(10,50)	0.68571	0.92008	0.18617	0.68462	0.68584
(10,100)	0.68571	0.95898	0.02326	0.68481	0.68586
(30,10)	0.68571	0.37072	0.84163	0.71570	0.68523
(30,30)	0.68571	0.68322	0.68607	0.68561	0.68566
(30,50)	0.68571	0.78765	0.85473	0.68521	0.68575
(30,100)	0.68571	0.88498	0.31285	0.68483	0.68582
(50,10)	0.68571	0.23795	0.97821	0.74855	0.68476
(50,30)	0.68571	0.54749	0.61162	0.69109	0.68552
(50,50)	0.68571	0.68318	0.86339	0.68565	0.68566
(50,100)	0.68571	0.82007	0.76862	0.68505	0.68577
(100,10)	0.68571	0.10901	0.98514	0.83775	0.68363
(100,30)	0.68571	0.34713	0.92542	0.70494	0.68512
(100,50)	0.68571	0.49613	0.82295	0.68693	0.68543
(100,100)	0.68571	0.68301	0.71554	0.68566	0.68565

Table(10): MSE for $\hat{R}_{(s,k)}$when (s,k)=(1,3), α=4, β=2 , α_0=4.001 and β_0=2.001, and $R_{(s,k)}$= 0.68571

(n,m)	$\hat{R}_{(s,k)_{mle}}$	$\hat{R}_{(s,k)_{PCE}}$	$\hat{R}_{(s,k)_{sh}}$	$\hat{R}_{(s,k)_{th}}$	Best
(10,10)	0.01182	0.00595	0.00004	0.000000009	Th
(10,30)	0.03710	0.22446	0.00005	0.00000001	Th
(10,50)	0.05559	0.25182	0.00002	0.00000002	Th
(10,100)	0.07483	0.43933	0.00001	0.00000002	Th
(30,10)	0.10985	0.02682	0.00117	0.0000003	Th
(30,30)	0.00375	0.01329	0.000004	0.000000005	Th
(30,50)	0.01201	0.03294	0.000002	0.000000002	Th
(30,100)	0.04019	0.14009	0.000003	0.00000001	Th
(50,10)	0.20740	0.08569	0.00484	0.000001	Th
(50,30)	0.02338	0.01354	0.00004	0.00000004	Th
(50,50)	0.00228	0.05076	0.00000007	0.000000004	Th
(50,100)	0.01876	0.06446	0.0000005	0.000000004	Th
(100,10)	0.33471	0.08973	0.02938	0.000005	Th
(100,30)	0.11791	0.05784	0.00039	0.0000004	Th
(100,50)	0.03875	0.02132	0.000002	0.00000009	Th
(100,100)	0.00107	0.01485	0.00000003	0.000000004	Th

Table (11): $R_{(s,k)}$ and $\hat{R}_{(s,k)}$when (s,k)=(2,4), α=4, β=2 , α_0=4.001 and β_0=2.001.

(n,m)	$R_{(s,k)}$	$\hat{R}_{(s,k)_{mle}}$	$\hat{R}_{(s,k)_{PCE}}$	$\hat{R}_{(s,k)_{sh}}$	$\hat{R}_{(s,k)_{th}}$
(10,10)	0.76190	0.75046	0.80651	0.76182	0.76185
(10,30)	0.76190	0.90832	0.49019	0.75727	0.76199
(10,50)	0.76190	0.94376	0.22662	0.76088	0.76201
(10,100)	0.76190	0.97083	0.12276	0.76104	0.76203
(30,10)	0.76190	0.47482	0.96915	0.78584	0.76152
(30,30)	0.76190	0.75719	0.69156	0.76186	0.76186
(30,50)	0.76190	0.84472	0.57097	0.76138	0.76193
(30,100)	0.76190	0.91746	0.31226	0.76117	0.76199
(50,10)	0.76190	0.33435	0.95049	0.81128	0.76113
(50,30)	0.76190	0.64335	0.84245	0.76622	0.76175
(50,50)	0.76190	0.75904	0.77851	0.76186	0.76186
(50,100)	0.76190	0.86868	0.57849	0.76136	0.76195
(100,10)	0.76190	0.16654	0.99187	0.88201	0.76018
(100,30)	0.76190	0.45185	0.91912	0.77729	0.76143
(100,50)	0.76190	0.59529	0.87119	0.76291	0.76167
(100,100)	0.76190	0.76034	0.78221	0.76186	0.76186

Table (12): MSE for $\hat{R}_{(s,k)}$when (s,k)=(2,4) ,α=4, β=2 , α_0=4.001, β_0=2.001, and $R_{(s,k)}$= 0.76190

(n,m)	$\hat{R}_{(s,k)_{mle}}$	$\hat{R}_{(s,k)_{PCE}}$	$\hat{R}_{(s,k)_{sh}}$	$\hat{R}_{(s,k)_{th}}$	Best
(10,10)	0.00766	0.00458	0.00003	0.000000006	Th
(10,30)	0.02247	0.11242	0.00004	0.000000008	Th
(10,50)	0.03339	0.29432	0.00002	0.00000001	Th
(10,100)	0.04374	0.41199	0.00001	0.00000002	Th
(30,10)	0.09351	0.04299	0.00074	0.0000002	Th
(30,30)	0.00290	0.00596	0.000003	0.000000004	Th
(30,50)	0.00799	0.03973	0.000002	0.000000002	Th
(30,100)	0.02445	0.20769	0.000002	0.000000008	Th
(50,10)	0.00196	0.00036	0.00003	0.000000007	Th
(50,30)	0.01761	0.00828	0.00002	0.00000003	Th
(50,50)	0.00156	0.00231	0.00000005	0.000000003	Th
(50,100)	0.01179	0.06842	0.0000003	0.000000003	Th
(100,10)	0.35841	0.05292	0.01783	0.000003	Th
(100,30)	0.09962	0.02586	0.00025	0.0000002	Th
(100,50)	0.03013	0.01216	0.000001	0.00000006	Th
(100,100)	0.00078	0.00632	0.00000002	0.000000003	Th

4- Discussion Numerical Simulation results:

From the tables above **,** for all n=(10,30,50,100) and m=(10,30,50,100) we conclude that , the minimum (MSE) for reliability estimation of $R_{(s,k)}$ for the Generalized Exponential Distribution GED(α,1) ,held for shrinkage estimator by using modified Thompson type shrinkage weight factor .This implies that, the shrinkage reliability estimator ($\hat{R}_{(s,k)_{th}}$) is the best and follows by using shrinkage weight function($\hat{R}_{(s,k)_{sh}}$) for any n and m .

5-Conclusion:

From the numerical simulation results, the shrinkage estimator method was suitable for estimation especially when use modified Thompson type shrinkage weight factor as a linear combination between unbiased

estimator, and prior estimate. The result estimator $\hat{R}_{th}$ for the reliability system of multicomponent in stress- strength model perform well and will be the best estimator than the other in the sense of MSE.

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