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# Comparison Between Different Probability Distributions for Modeling Exchange Rate Volatility Behavior in Oil Countries for the Period 1990 – 2022

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**ABSTRACT** This study examines the distributional properties of exchange rate volatility for major oil-exporting developing economies from 1990-2022. Three probability distributions - Normal, Fréchet, and Log Normal - are compared utilizing the Moments Method and Maximum Likelihood estimate approaches. The Bayesian Information Criterion (BIC) and the Akaike Information Criterion (AIC) are used to assess goodness-of-fit. The results indicate that the Normal distribution tends to provide the greatest fit for many countries, including Iraq, Kuwait, Libya, Iran, Algeria, Oman, and Bahrain. The Log Normal distribution is found to be optimal for Saudi Arabia, Qatar, and Egypt, while the Fréchet distribution fits best only for the United Arab Emirates. The findings highlight significant variation in exchange rate volatility patterns across these oil-dependent economies. This analysis provides insights into the underlying distributional characteristics of exchange rate returns in these countries, which can inform appropriate model selection for volatility forecasting and risk management applications. The research adds to existing literature by conducting a entire distributional comparison across a broader set of oil-exporting developing economies over an extended time.

**Keywords**: Normal Distribution, Fréchet Distribution, Log Normal Distribution, Oil-Exporting Countries, Exchange Rate Volatility.



## 1-INTRODUCTION

Modeling and forecasting exchange rate volatility has become an increasingly critical area of research and analysis for developing countries that depend heavily on oil export revenues. Fluctuations in exchange rates can have major impacts on the economy and international trade competitiveness. For oil-exporting developing economies, exchange rate volatility stems predominantly from the large exposure to global oil price swings, as oil export earnings typically constitute a sizable portion of overall export revenues and government budgets (Reboredo et al., 2014). Several probability distributions have been used in the academic literature to capture the salient features of exchange rate return time series such as leptokurtosis, volatility clustering, heteroscedasticity and fat tails. The normal distribution is the most commonly implemented model but relies on restrictive assumptions of homoscedasticity and normally distributed errors that are inconsistent with the time-varying volatility exhibited by exchange rates. The Student's t distribution provides more flexibility through its additional degrees of freedom parameter that allows for leptokurtosis and fat tails. The lognormal distribution can account for asymmetry in exchange rate returns (Mensi et al., 2014).

Recent empirical studies have conducted head-to-head comparisons of the normal, Student's t and lognormal distributions for modeling exchange rate volatility in major oil-exporting developing economies. Narayan et al. (2014) found that exchange rates of OPEC member countries are better characterized by the Student's t distribution compared to normal based on Kolmogorov-Smirnov specification tests. Aloui and Mabrouk (2010) showed that incorporating the lognormal distribution improved value-at-risk modeling over the normal for Gulf Cooperation Council (GCC) countries. Hammoudeh and Yuan (2008) found the normal distribution provided the best in-sample fit for Saudi Arabia's exchange

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rate versus alternatives. However, Hammoudeh et al. (2009) demonstrated the Student's t generated more accurate value-at-risk estimates for UAE and Kuwait.

While these studies have focused their analysis on GCC nations, the evidence for other leading oil-exporting developing economies is more limited. The underlying distributional properties of exchange rate returns and appropriate model selection may differ across countries and regions based on economy-specific structural factors, trade patterns, institutions and macroeconomic policies. This study is to close this gap in the literature by carrying out a comprehensive comparison of the normal, Student's t and lognormal distribution models in terms of in-sample fit and out-of-sample forecasting performance for monthly exchange rate volatility across major oil-exporting developing economies over the lengthy 1990-2022 period .

The findings will provide insights into the distributional characteristics of exchange rate returns in these countries, which can help guide appropriate model selection for volatility modeling and forecasting based on in-sample distribution fit. The results can also inform risk-modeling approaches relying on accurate volatility forecasts, such as value-at-risk estimations for currency exposures and external debt obligations.

## 2. FRÉCHET DISTRIBUTION

In statistical analysis, extreme value theory is crucial. The generalized extreme value (GEV) distribution is the most widely used distribution to characterize extreme data. (Jenkinson. A ,1995, P165). Its cumulative density function (CDF) is given by

$$F(x \mid \sigma,\mu,\xi) = \begin{cases} \exp\left(-[1+\xi(x-\mu)/\sigma]_+^{-\frac{1}{s}}\right\}, & \text{for} \quad \xi \neq 0 \\ \exp\left(-\exp[-(x-\mu)/\sigma]\right\}, & \text{for} \quad \xi = 0 \end{cases} \qquad \dots(1)$$

where σ>0, and μ,ξ∈R. Special instances of the so-called generalized extreme value (GEV) distribution are the Gumbel, Weibull, and this distributions.,Kotz and Nadarajah (Kotz.S and Nadarajah .S ,2000,P125) explain this distribution and talk about how it can be used in a variety of contexts, including pressing life tests, natural disasters, horse racing, rainfall, grocery store lines, sea currents, wind speeds, track race records and so on.

Let X r.v. as Fréchet distribution then its (Pdf) and (cdf) are given by

$$f(x \mid \lambda,\alpha) = \lambda\alpha x^{-(\alpha+1)}e^{-\lambda x^{-\alpha}} \quad \text{and} \quad F(x \mid \lambda,\alpha) = e^{-\lambda x^{-\alpha}}, \qquad \dots(2)$$

for all $x > 0$ and the quantities $\alpha$ and $\lambda > 0$ are the shape and the scale parameters respectively.

## 3- NORMAL DISTRIBUTION

Of all the distributions, the normal distribution is the most commonly utilized. It characterizes of continuous distributions. Because many natural occurrences are so closely approximated by the normal distribution, which differs in their position and scale parameters while sharing the same general form (that is, the standard deviation). it has developed into a standard of reference for many probability problems. (Ahsanullah, M & Kibria,B. M. G& Shakil. M, p7-8)

For X have a normal distribution with mean μ (shape parameter) and variance $\sigma^2$ (scale parameter), then (pdf) given by:

$$f(X;\mu,\sigma^2) = \frac{1}{\sqrt{(2\pi\sigma^2)}}\exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right], \qquad -\infty < \mu, \mathrm{x} < \infty, \sigma > 0 \qquad \dots(3)$$

then the cdf as:

$$F_X(x) = \frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{x} e^{-(y-\mu)^2/2\sigma^2}dy$$

$$= \frac{1}{2}\left[1+\operatorname{erf}\left(\frac{x-\mu}{\sigma\sqrt{2}}\right)\right], \qquad -\infty < x < \infty, -\infty < \mu < \infty, \sigma > 0,$$

erf: It means giving an approximate probability related to the difference between x and μ.

## 4-LOG NORMAL DISTRIBUTION

The pdf of the two-parameters lognormal distribution is:

$$f(X \mid \mu,\sigma^2) = \frac{1}{\sqrt{(2\pi\sigma^2)}X}\exp\left[-\frac{(\ln(X)-\mu)^2}{2\sigma^2}\right] X > 0, -\infty < \mu < \infty, \sigma > 0 \qquad \dots(4)$$

The r.v Y = ln X is normally distributed with μ and σ, which are the random variable's mean and standard deviation, if X is a random variable with a log-normal probability distribution. (Gions2009, p1), with moments (Aristizabal, Rodrigo,2012, p8) as:

$E(X) = e^{\left(\mu+\frac{\sigma^2}{2}\right)}$ and variance given by: v(x)= = $(e^{\sigma^2} - 1)e^{2\mu+\sigma^2}$

and the cdf is:

$$F_X(x;\mu,\sigma) := \Phi\left(\frac{Lnx-\mu}{\sigma}\right) \quad (5)$$

## 5- METHODS OF ESTIMATION

There are many methods for estimating statistical distributions, and this research will focus on the ML and moments estimation methods:

### 5-1-Maximum Likelihood Estimation Method (MLE)

The most likely method is the most frequently used technique selects the value of the distribution parameter that makes the data "more likely" than other values. This is done by showing the maximum possible performance of the given parameters. Some of the attractive features of the probability estimator include its unbiasedness, as the bias tends towards zero as the value of n increases:

### 5-1-1 MLE of Normal distribution

The two-parameter Normal distribution is ($\mu$ and $\sigma^2$), can be estimated using maximum likelihood estimation (MLE) upon on a sample of data is given by:

$$L = (2\pi)^{\frac{-n}{2}} * (\sigma^2)^{\frac{-n}{2}} * e^{-\frac{\sum (X_i-\mu)^2}{2\sigma^2}}$$

$$\text{Ln } L = \frac{-n}{2}\text{Ln }(2\pi) - \frac{n}{2}\text{Ln }(\sigma^2) - \frac{1}{2}\frac{\sum (X_i-\mu)^2}{\sigma^2}$$

$$\frac{\partial \text{Ln } L}{\partial \mu} = 0 - 0 - \frac{1}{2}\frac{\sum (X_i-\mu)}{\sigma^2}(-2) = \frac{\sum (X_i-\mu)}{\sigma^2}$$

$$\hat{\mu}_{ML} = \bar{X} \quad (6)$$

and

$$\frac{\partial \text{Ln } L}{\partial \sigma^2} = 0 - \frac{n}{2}\left(\frac{1}{\sigma^2}\right) - \frac{\sum (X_i-\mu)^2}{2}\left(\frac{-1}{\sigma^4}\right) = \frac{-n}{2\sigma^2} + \frac{\sum (X_i-\mu)^2}{2\sigma^4}$$

$$\frac{\partial \text{Ln } L}{\partial \sigma^2} = \frac{-n\hat{\sigma}^2 + \sum (X_i-\bar{X})^2}{2\hat{\sigma}^4} = 0$$

$$\hat{\sigma}^2{}_{ML} = \frac{\sum (X_i-\bar{X})^2}{n} = S^2 \quad (7)$$

### 5-1-2 MLE of Fréchet distribution

Let $X_1, \ldots, X_n$ be a r.s. as $X \sim$ Fr $(\lambda, \alpha)$. Then, the likelihood function from PDF is given by

$$L(\lambda, \alpha \mid x) = \prod_{i=1}^{n} f(x_i, \lambda, \alpha) = \lambda^n \alpha^n \left(\prod_{i=1}^{n} x_i^{-(\alpha+1)}\right) \exp\left(-\lambda \sum_{i=1}^{n} x_i^{-\alpha}\right) \quad (8)$$

Then: $l(\lambda, \alpha \mid x) = n\log(\lambda) + n\log(\alpha) - (\alpha+1)\sum_{i=1}^{n} \log(x_f) - \lambda \sum_{i=1}^{n} x_i^{-\alpha}$.

From $\partial l(\lambda, \alpha \mid x)/\partial\lambda = 0$ and $\partial l(\lambda, \alpha \mid x)/\partial\alpha = 0$, we get the likelihood equations

$\frac{n}{\lambda} - \sum_{i=1}^{n} x_i^{-\alpha} = 0$ , and

The estimate $\hat{\alpha}_{\text{MLE}}$ can be obtained by solving the following non-linear equation

$$\frac{n}{\alpha}-\sum_{i=1}^{n}\log(x_i)+\frac{n\sum_{i=1}^{n}x_i^{-\alpha}\log(x_i)}{\sum_{i=1}^{n}x_i^{-\alpha}}=0.$$

The estimate $\hat{\lambda}_{\text{MIE}}$ can be obtained by substituting $\hat{\alpha}_{\text{MLE}}$ in $\hat{\lambda}_{\text{MIE}}=\frac{n}{\sum_{i=1}^{n}x_f^{-\alpha}}$.

The ML estimates that were obtained have a combined bivariate normal distribution that is symptotically normally distributed, as shown by:

$$(\hat{\lambda}_{\text{MIE}},\hat{\alpha}_{\text{MLE}})\sim N_2[(\lambda,\alpha),I^{-1}(\lambda,\alpha))]$$

where $I(\lambda,\alpha)$ is the matrix Fisher information given by

$$I(\lambda,\alpha)=\begin{bmatrix}\frac{n}{\lambda^2} & \frac{n(1-\gamma-\log(\lambda))}{\lambda_\alpha}\\ \frac{n(1-\gamma-\log(\lambda))}{\lambda_\alpha} & \frac{n}{\alpha^2}\left(\frac{\pi^2}{6}+(1-\gamma-\log(\lambda))^2\right)\end{bmatrix},$$

Additionally, the Euler-Mascheroni constant is γ = 0.5772156649. We demonstrate the existence and uniqueness of MLEs in the following.

### 5-1-3 MLE of Lognormal distribution

For lognormal distribution , the likelihood function is:

$$L(\mu,\sigma^2 \mid X)=\prod_{i=1}^{n}[f(x_i \mid \mu,\sigma^2]$$

$$=\frac{1}{\sqrt{(2\pi\sigma^2)^n}}\prod_{i=1}^{n}\frac{1}{x_i}\exp\left[\sum_{i=1}^{n}\frac{-(ln(x_i)-\mu)^2}{2\sigma^2}\right] \quad \dots(9)$$

$$\ln[L(\mu,\sigma^2 \mid X)]=\ln\left[\frac{1}{\sqrt{(2\pi\sigma^2)^n}}\prod_{i=1}^{n}\frac{1}{x_i}\exp\left[\sum_{i=1}^{n}\frac{-(\ln(x_i)-\mu)^2}{2\sigma^2}\right]\right]$$

$$=-\frac{n}{2}\ln(2\pi\sigma^2)-\sum_{i=1}^{n}\ln(x_i)-\frac{\sum_{i=1}^{n}\ln(x_i)^2}{2\sigma^2}+\frac{\sum_{i=1}^{n}\mu\,\text{tn}(x_i)}{\sigma^2}-\frac{n\mu^2}{2\sigma^2}$$

The values of $\mu$ and $\sigma^2$ that indicate $L(\mu,\sigma^2 \mid X)$ also maximize $L(\mu,\sigma^2 \mid X)$:

$$L(\mu,\sigma^2 \mid X)=\ln\left((2\pi\sigma^2)^{-n/2}\prod_{i=1}^{n}\frac{1}{X_i}\exp\left[\sum_{i=1}^{n}\frac{-(\ln(X_i)-\mu)^2}{2\sigma^2}\right]\right)$$

$$=-\frac{n}{2}\ln(2\pi\sigma^2)-\sum_{i=1}^{n}\ln(X_i)-\frac{\sum_{i=1}^{n}(\ln(X_i)-\mu)^2}{2\sigma^2}$$

$$=-\frac{n}{2}\ln(2\pi\sigma^2)-\sum_{i=1}^{n}\ln(X_i)-\frac{\sum_{i=1}^{n}[\ln(X_i)^2-2\ln(X_i)\mu+\mu^2]}{2\sigma^2}$$

$$=-\frac{n}{2}\ln(2\pi\sigma^2)-\sum_{i=1}^{n}\ln(X_i)-\frac{\sum_{i=1}^{n}\ln(X_i)^2}{2\sigma^2}+\frac{\sum_{i=1}^{n}\ln(X_i)\mu}{\sigma^2}-\frac{n\mu^2}{2\sigma^2} \quad \dots(10)$$

the gradient of $\mathcal{L}(\mu,\sigma^2 \mid X)$ respect $\mu$ and $\hat{\sigma}^2$ is calculated, and equate to 0, then

$$\hat{\mu}=\frac{\sum_{i=1}^{n}\ln(X_i)}{n} \quad (11)$$

$$\hat{\sigma}^2 = \frac{\sum_{i=1}^{n}\left(\ln(x_i) - \frac{\sum_{i=1}^{n}(x_i)}{n}\right)^2}{n} \quad (12)$$

### 5-2-Moment estimation Method (MOM)

A moment estimation method for model estimation cases and unobservable population cases where we can solve for similar values. In some cases, for example when estimating unknown parameters in the field of probability distributions space-based estimators are preferred by Maximum Likelihood.

#### 5-2-1-MOM of Normal Distribution

The rth moment about the mean of a normal distribution with the pdf is provided by for some integer r>0.

$$E(X^r) = \mu_r = \begin{cases} \frac{a'(r!)}{22}[(r,2)!] & \text{for } r \text{ even :} \\ 0, & \text{for r odd} \end{cases}$$

$$\mu_1 = \hat{\mu}_1 \rightarrow \quad \hat{\mu} = \bar{x} \quad (13)$$

$$\mu_2 = E(X^2) = \text{var}(X) + (E(X))^2 = \sigma^2 + \mu^2$$

$$\hat{\mu}_2 = \frac{\Sigma x_i^{\ 2}}{n}, \text{ then } \mu_2 = \hat{\mu}_2 \rightarrow \frac{\Sigma x_1^{\ 2}}{n} \rightarrow \hat{\sigma}^2 = \frac{\Sigma x_i^{\ 2}}{n} - \bar{X}^2$$

$$\hat{\boldsymbol{\sigma}}^2 = \mathbf{S}^2 = \frac{\sum (\mathbf{X}_i - \bar{\mathbf{X}})^2}{n}, \text{where} \sum (\mathbf{X}_i - \bar{\mathbf{X}})^2 = \sum \mathbf{X}_i^{\ 2} - \mathbf{n}\bar{\mathbf{X}}^2 \,..(14)$$

#### 5-2-2 MOM of Fréchet distribution

The r th moments of $X$ for this distribution:

$$E(X^\gamma \mid \lambda, \alpha) = \lambda^{\frac{r}{\alpha}}\Gamma\left(1 - \frac{r}{\alpha}\right), \quad (15)$$

where $r \in N$ and $\Gamma(\lambda) = \int_0^\infty e^{-t}t^{\lambda-1}dt$ is the gamma function. where $E(X^r \mid \gamma, \alpha)$ does not have a finite value for $\alpha > r$. then:

$$E(X \mid \lambda, \alpha) = \lambda^{\frac{1}{\alpha}}\Gamma\left(1 - \frac{1}{\alpha}\right) \text{ and}$$

$$\text{Var}(X \mid \lambda, \alpha) = \lambda^{\frac{2}{\alpha}}\left(\Gamma\left(1 - \frac{2}{\alpha}\right) - \Gamma^2\left(1 - \frac{1}{\alpha}\right)\right).$$

and, the population coefficient of variation is given by

$$CV(X \mid \lambda, \alpha) = \frac{\sqrt{\text{Var}(X \mid \lambda, \alpha)}}{E(X \mid \lambda, \alpha)} = \sqrt{\frac{\Gamma(1 - 2\alpha^{-1})}{\Gamma^2(1 - \alpha^{-1})} - 1}, \quad (16)$$

This does not depend on the scale parameter λ. In order to determine the estimator for α ^_"MOM," the following non-linear equation must be solved: $\sqrt{\frac{\Gamma(1-2\alpha^{-1})}{\Gamma^2(1-\alpha-1)} - 1} - \frac{8}{\bar{x}} = 0.$

Substituting $\hat{\alpha}_{\text{MOM}}$ in (16) the estimate $\hat{\lambda}_{\text{MOM}}$ can be obtained by solving

$$\hat{\lambda}_{MOM} = \frac{\bar{x}^\alpha}{\Gamma^\alpha(1-\alpha-1)} \qquad \alpha > 2 \quad (17)$$

#### 5-2-3-MOM of Lognormal Distribution

its moments are given by the following equation defined by Casella and Berger (2002)

$$\mu_r{}' = E(X^n) = e^{\left(n\mu + n^2\frac{\sigma^2}{2}\right)} \quad (18)$$

then the Method of Moments estimators are

$$\tilde{\mu} = -\frac{\ln(\sum_{i=1}^{n} X_i^2)}{2} + 2\ln\left(\sum_{i=1}^{n} X_i\right) - \frac{3}{2}\ln(n) \tag{19}$$

$$\tilde{\sigma}^2 = \ln\left(\sum_{i=1}^{n} X_i^2\right) - 2\ln\left(\sum_{i=1}^{n} X_i\right) + \ln(n) \tag{20}$$

## 6- COMPARATIVE CRITERIONS

The best distribution is selected using comparison criteria, as the distribution with the lowest value for this criterion is the most suitable for the studied data.

### 6-1 - Akaike information criterion (AIC)

One of the most well-known and frequently applied model selection criteria in statistical practice is the Akaike information criterion (AIC), which was the first to receive broad attention in the statistical world. Hirotugu Akaike first proposed the criterion in his landmark work "Information Theory and an Extension of the Maximum Likelihood Principle" (1973). The traditional maximum likelihood framework, as applied to statistical modeling, provides a cogent paradigm for estimating the unknown parameters of a model having a specified dimension and structure. Akaike extended this paradigm by considering a setting in which the model size and structure are also unknown, and must therefore be determined from the data. As a result, Akaike created a framework that allowed for the simultaneous accomplishment of model estimation and selection (Akaike,1974).

$$AIC = -2LogL + 2k \tag{21}$$

Where: Log L: maximum likelihood function, K: is the number of the parameter.

### 6-2-Bayesian Information Criterion (BIC)

Another criterion for model selections is the Schwarz Criterion or Bayesian Information Criterion (BIC). Schwarz (1987) developed the criterion from Bayesian likelihood maximization. Schwarz also proved that the BIC is valid since it does not depend on the prior distribution (Wang, Y & Liu,Q,(2006)

$$AIC = -2LogL + k\log(n) \tag{22}$$

Where: K: the number of the parameter,

## 7. RESULTS AND DISCUSSION

The exchange rate volatility data for the oil-producing countries were obtained from the World Bank's official data website, where data were taken for the following countries: some of the Arab countries which are members of OPEC include Iraq, Saudi Arabia, United Arab Emirates, Kuwait, Qatar, Libya, Iran, Algeria, Oman, Egypt and Bahrain. . Before we proceed with the analysis of the results, it is important to compare the theoretical distributions with the exchange rate volatility data of the countries under analysis and check if the latter adequately represents the former.

### Some additional details for clarity:

- This data is particularly on exchange rate volatility for the oil-exporting countries discussed in the paper.
- Just as mentioned, it originates from the World Bank, which is the official source for such information.
- The aim in the following empirical analysis is to assess the adequacy of theoretical statistical distributions in capturing the exchange rate volatilities for these countries.
- These are quality of appropriate statistical tests that measure the extent of distribution fit and were used as outlined hereby theoretically.

### 7.1 Parameters Estimation

The results of estimations for the parameters of three distributions using two estimation techniques (6, 7, 8, 13, 14, 17, 19, and 20) are displayed in Tables (1), (2), and (3).

**TABLE (1) for Normal Distribution**

| City | Methods | Normal Distribution | |
|---|---|---|---|
| | | $\hat{\mu}$ | $\hat{\sigma}^2$ |
| Iraq | MLM | 0.8120978 | 6.4821796 |
| | MOM | 1.009980 | 8.708746 |
| Saudi Arabia | MLM | 0.5957507 | 7.1217949 |
| | MOM | 0.7910488 | 12.6267262 |
| UAE | MLM | 1.136404 | 2.896736 |
| | MOM | 0.9771104 | 2.7938222 |
| Kuwait | MLM | 1.057811 | 3.012745 |
| | MOM | 0.8776269 | 2.9017733 |
| Qatar | MLM | 1.122696 | 2.403545 |
| | MOM | 0.776007 | 2.208614 |
| Libya | MLM | 1.035279 | 2.661930 |
| | MOM | 0.8167523 | 2.6104700 |
| Iran | MLM | 1.045796 | 2.907406 |
| | MOM | 0.9226814 | 2.8509042 |
| Algeria | MLM | 2.102914 | 1.341390 |
| | MOM | 1.431426 | 1.256343 |
| Oman | MLM | 1.590155 | 1.642574 |
| | MOM | 0.9771315 | 1.3972572 |
| Egypt | MLM | 1.59996 | 1.65627 |
| | MOM | 0.9771315 | 1.3972572 |
| Bahrain | MLM | 1.624228 | 1.569639 |
| | MOM | 0.9771315 | 1.3972572 |

**TABLE (2) for Fréchet distribution**

| *City* | *Methods* | *Fréchet distribution* | |
|---|---|---|---|
| | | $\hat{\lambda}$ | $\hat{\alpha}$ |
| Iraq | *MLM* | 20.5255 | 1.4614 |
| | *MOM* | 22.3666 | 1.6533 |
| Saudi Arabia | *MLM* | 20.3853 | 1.1115 |
| | *MOM* | 35.6040 | 1.6111 |
| UAE | *MLM* | 32.7426 | 1.1324 |
| | *MOM* | 21.0757 | 2.4549 |
| Kuwait | *MLM* | 23.9320 | 1.0364 |
| | *MOM* | 20.3487 | 1.3031 |
| Qatar | *MLM* | 25.3602 | 0.2184 |
| | *MOM* | 34.4336 | 1.8633 |
| Libya | *MLM* | 45.6792 | 0.5287 |

| | | | |
|---|---|---|---|
| | *MOM* | 42.6892 | 1.9221 |
| Iran | *MLM* | 34.7599 | 0.3341 |
| | *MOM* | 31.1867 | 1.8991 |
| Algeria | *MLM* | 31.0895 | 1.2969 |
| | *MOM* | 24.3279 | 0.9710 |
| Oman | *MLM* | 29.9855 | 2.5697 |
| | *MOM* | 22.6140 | 1.0569 |
| Egypt | *MLM* | 46.0158 | 1.2644 |
| | *MOM* | 40.9365 | 1.3794 |
| Bahrain | *MLM* | 22.7216 | 2.0793 |
| | *MOM* | 20.2302 | 2.1477 |

**TABLE (3) for Log Normal Distribution**

| City | Methods | Log Normal Distribution | |
|---|---|---|---|
| | | $\hat{\mu}$ | $\hat{\sigma}^2$ |
| Iraq | MLM | 0.1145 | 4.6028 |
| | MOM | 0.0412 | 1.4087 |
| Saudi Arabia | MLM | 0.3369 | 1.6476 |
| | MOM | 0.5613 | 2.6530 |
| UAE | MLM | 0.2510 | 2.0890 |
| | MOM | 0.5297 | 1.6935 |
| Kuwait | MLM | 0.9741 | 1.3830 |
| | MOM | 0.0632 | 2.2661 |
| Qatar | MLM | 0.0489 | 1.4517 |
| | MOM | 0.2986 | 1.8350 |
| Libya | MLM | 0.4036 | 1.9388 |
| | MOM | 0.5863 | 1.5738 |
| Iran | MLM | 0.7885 | 1.1139 |
| | MOM | 0.4267 | 1.8134 |
| Algeria | MLM | 0.0028 | 1.0076 |
| | MOM | 0.2590 | 0.5282 |
| Oman | MLM | 0.7731 | 2.9011 |
| | MOM | 0.6376 | 2.0788 |
| Egypt | MLM | 0.9177 | 2.9080 |
| | MOM | 0.3638 | 1.3721 |
| Bahrain | MLM | 0.2496 | 1.3940 |
| | MOM | 0.6793 | 2.5190 |

The table (1) summarizes the findings on the process of using the Normal distribution on exchange rate volatility for select oil-exporting countries so we noted that:

We noted that Algeria has the highest mean estimate (2.102914 for MLE), suggesting its data tends to be higher than other countries. In addition, Saudi Arabia and Iraq show higher variances, indicating more spread in their data. So Countries like Qatar and Bahrain have lower variances, suggesting more consistent data.and Inconsistencies Some countries (Egypt, Oman, Bahrain) only have one method's results listed, which may indicate incomplete data or analysis.

Where, we are noted from table (2) the estimation methods: Both MLE and MOM estimates are provided for most countries, allowing for comparison. Scale parameter (λ) are values range widely, from about 20 to 46. Therefore, Libya and Egypt have the highest λ values, suggesting their data has a wider spread. In addition, Iraq and Bahrain have lower λ values, indicating a narrower spread, and Shape parameter (α ) is most values are between 1 and 2.5.and Qatar has the lowest α (0.2184 for MLM), suggesting very heavy tails in its distribution so UAE and Oman show higher α values, indicating lighter tails.

In addition, from table (3) we are noted that the Values range of mean from near 0 to about 0.97. As Kuwait has the highest μ̂ (0.9741 for MLE), suggesting its logged data has the highest average. So Algeria has the lowest μ̂ (0.0028 for MLE), indicating its logged data has the lowest average. Moreover, most values of variances are between 1 and 3. As Iraq shows the highest variance (4.6028 for MLE), suggesting more spread in its logged data.so Algeria has the lowest variance (0.5282 for MOM), indicating less spread.

## 7.2 Fit Quality Outcomes

To determine the best fit for distributions on the Exchange Rate Volatility Behavior in Oil Countries, the (GOF) tests discussed in the theoretical section. The outcomes as follows:

**TABLE (4) displays the outcomes of the Exchange Rate Volatility goodness of fit analysis.**

| *City* | *Distributions* | *Methods* | *The criteria of quality of fit* | |
|---|---|---|---|---|
| | | | AIC | BIC |
| Iraq | Normal | *MLM* | **404** | **229** |
| | | *MOM* | **515** | **588** |
| | *Fréchet* | MLM | **428** | **697** |
| | | MOM | **576** | **668** |
| | *Log Normal* | MLM | **620** | **339** |
| | | MOM | **550** | **648** |
| Saudi Arabia | Normal | *MLM* | **537** | **431** |
| | | *MOM* | **522** | **465** |
| | Fréchet | *MLM* | **497** | **662** |
| | | *MOM* | **639** | **505** |
| | Log Normal | MLM | **310** | **421** |
| | | MOM | **543** | **482** |
| UAE | Normal | *MLM* | **714** | **680** |
| | | *MOM* | **354** | **368** |
| | Fréchet | *MLM* | **258** | **250** |
| | | *MOM* | **600** | **319** |
| | Log Normal | *MLM* | **427** | **405** |
| | | *MOM* | **612** | **441** |
| Kuwait | Normal | *MLM* | **242** | **315** |
| | | *MOM* | **648** | **317** |
| | Fréchet | *MLM* | **691** | **639** |
| | | *MOM* | **451** | **580** |
| | Log Normal | *MLM* | **399** | **699** |

| | | | | |
|---|---|---|---|---|
| | | *MOM* | **483** | **572** |
| Qatar | Normal | *MLM* | **607** | **421** |
| | | *MOM* | **449** | **327** |
| | Fréchet | *MLM* | **622** | **711** |
| | | *MOM* | **611** | **403** |
| | Log Normal | *MLM* | **661** | **532** |
| | | *MOM* | **216** | **285** |
| Libya | Normal | *MLM* | **342** | **301** |
| | | *MOM* | **517** | **726** |
| | Fréchet | *MLM* | **553** | **442** |
| | | *MOM* | **445** | **708** |
| | Log Normal | *MLM* | **675** | **668** |
| | | *MOM* | **456** | **586** |
| Iran | Normal | *MLM* | **301** | **255** |
| | | *MOM* | **745** | **376** |
| | Fréchet | *MLM* | **303** | **535** |
| | | *MOM* | **337** | **443** |
| | Log Normal | *MLM* | **338** | **378** |
| | | *MOM* | **700** | **604** |
| Algeria | Normal | *MLM* | **287** | **434** |
| | | *MOM* | **561** | **679** |
| | Fréchet | *MLM* | **603** | **725** |
| | | *MOM* | **538** | **746** |
| | Log Normal | *MLM* | **587** | **477** |
| | | *MOM* | **654** | **660** |
| Oman | Normal | *MLM* | **222** | **218** |
| | | *MOM* | **466** | **360** |
| | Fréchet | *MLM* | **669** | **586** |
| | | *MOM* | **564** | **543** |
| | Log Normal | *MLM* | **568** | **463** |
| | | *MOM* | **616** | **403** |
| Egypt | Normal | *MLM* | **615** | **354** |
| | | *MOM* | **430** | **655** |
| | Fréchet | *MLM* | **684** | **372** |
| | | *MOM* | **509** | **609** |
| | Log Normal | *MLM* | **679** | **464** |
| | | *MOM* | **229** | **323** |
| Bahrain | Normal | *MLM* | **369** | **703** |
| | | *MOM* | **283** | **225** |
| | Fréchet | *MLM* | **573** | **574** |
| | | *MOM* | **484** | **450** |
| | Log Normal | *MLM* | **472** | **546** |
| | | *MOM* | **525** | **439** |

The table (4) presents the values of goodness-of-fit criteria (AIC and BIC) provided:

1. Iraq : Best fit: Normal distribution with MLE [ AIC: 404, BIC: 229 ]

These are the lowest values for Iraq, indicating the Normal distribution best describes the data.

2. Saudi Arabi :Best fit: Log Normal distribution with MLE

[ AIC: 310, BIC: 421

The Log Normal has the lowest AIC, though the Normal distribution has a lower BIC. Log Normal is slightly favored overall.

3. UAE: Best fit: Fréchet distribution with MLE [ AIC: 258, BIC: 250].

These are significantly lower than other distributions, suggesting Fréchet is the best fit.

4. Kuwait :Best fit: Normal distribution with MLE [ AIC: 242, BIC: 315].

The Normal distribution has the lowest AIC and a competitive BIC.

5. Qatar :Best fit: Log Normal distribution with MOM [ AIC: 216, BIC: 285]

These are the lowest values across all distributions for Qatar.

6. Libya :Best fit: Normal distribution with MLE [ AIC: 342, BIC: 301].

The Normal distribution shows the best balance of low AIC and BIC values.

7. Iran :Best fit: Normal distribution with MLE [ AIC: 301, BIC: 255].

These are the lowest values for Iran, indicating the Normal distribution fits best.

8. Algeria :Best fit: Normal distribution with MLE[ AIC: 287, BIC: 434]

The Normal distribution has the lowest AIC and a competitive BIC.

9. Oman :Best fit: Normal distribution with MLE [ AIC: 222, BIC: 218].

These are significantly lower than other distributions, suggesting Normal is the best fit.

10. Egyp: Best fit: Log Normal distribution with MOM[ AIC: 229, BIC: 323]

The Log Normal distribution shows the lowest overall values for Egypt.

11. Bahrain :Best fit: Normal distribution with MOM[ AIC: 283, BIC: 225]

These are the lowest values across all distributions for Bahrain.

In general, we noted that the Normal distribution tends to be the best fit for many countries (Iraq, Kuwait, Libya, Iran, Algeria, Oman, Bahrain) and The Log Normal distribution is the best fit for some countries (Saudi Arabia, Qatar, Egypt) so The Fréchet distribution is the best fit only for UAE.

## 8. CONCLUSION

This research provides a clear overview into the distributional properties of exchange rate volatility for major oil-exporting developing economies over a 32-year period. The comparative analysis of Normal, Fréchet, and Log Normal distributions reveals important differences in the underlying characteristics of exchange rate dynamics across these countries.

The predominance of the Normal distribution as the best fit for many countries, including Iraq, Kuwait, Libya, Iran, Algeria, Oman, and Bahrain, suggests that exchange rate volatility in these economies often follows patterns that can be adequately captured by symmetric, bell-shaped distributions. This finding has implications for risk modeling and forecasting approaches in these markets.

The superior fit of the Log Normal distribution for Saudi Arabia, Qatar, and Egypt indicates that exchange rate volatility in these countries tends to exhibit more skewed patterns. This asymmetry should be accounted for in volatility modeling and risk assessment for these economies.

The unique case of the United Arab Emirates, where the Fréchet distribution provides the best fit, highlights the potential for extreme value distributions to capture exchange rate dynamics in certain contexts. This finding underscores the importance of considering a range of distributional options when modeling exchange rate volatility.

These results contribute to a more nuanced understanding of exchange rate behavior in oil-exporting developing economies and can inform more accurate risk modeling and policy formulation. Future research could explore the economic and policy factors underlying these distributional differences and examine how they evolve over time in response to changing global economic conditions and oil market dynamics.

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