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RESEARCH ARTICLE – ENGINEERING (MISCELLANEOUS)

MOGWO-FOPID: Multi-Objective Gray Wolf Optimization Fractional Order PID Controllers in Quadruple Tank Process

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Abstract

The Quadruple Tank Process (QTP) is a laboratory model consisting of four interconnected water tanks, commonly used for control training and research purposes. The proficient regulation of the QTP is imperative, and a variety of controllers, including decentralized proportional-integral controllers, proportional controllers, integral controllers, derivative controllers, and integral-derivative-proportional controllers, are extensively utilized. A Fractional Order PID (FOPID) controller's effectiveness is largely determined by the precise organization of its parameters. This manuscript presents a Multi-Objective Gray Wolf Optimization Fractional Order PID (MOGWO-FOPID) framework, which employs the MOGWO algorithm to enhance the optimization of FOPID controller parameters for the QTP. The suggested methodology was executed and evaluated utilizing MATLAB, with its performance benchmarked against pre-existing, well-established tuning methodologies. Simulation findings indicated that the MOGWO-FOPID technique surpassed conventional methods. This superiority was corroborated through a variety of evaluation metrics, particularly emphasizing the Spacing Metric (SP) and Non-uniformity of Pareto Front (NPF) analysis. The outcomes reveal considerable advancements in both system stability and performance when employing the proposed methodology, thereby offering a more efficacious strategy for the optimization of FOPID parameters in the QTP.

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Keywords: Quadruple Tank Process; Fractional Order Proportional Integral Derivative; Multi-Objective Optimization Algorithm.

1. Introduction

The area of optimization is gaining traction as a key research focus for experts in diverse domains, with its strategies proving to be significant assets for elevating performance while effectively lowering expenditures. The creation of optimization algorithms is done with precision to discover the most effective solutions for assigned problems, aiming to either decrease or increase the respective objective function based on the specific issue addressed. These types of algorithms can usually be grouped into local and global segments. Local algorithms are distinguished by their pronounced exploration capabilities and expedited execution durations; however, they frequently exhibit a propensity to converge upon local optima rather than the global optimum, thereby presenting a notable limitation inherent to such approaches [1].

A specific problem results from trying to improve the Quadruple Tank Process (QTP), a nonlinear construction comprised of four linked water storage tanks and two hydraulic pumps. The dynamics of the system are controlled by Bernoulli's principle and the laws of mass conservation; the primary objective is optimization of the through pumps; water levels in the lower tanks (see Fig. 1) [2]. The simultaneous engagement with multiple, often contradictory objectives within this context poses a significant challenge in the field of control engineering: the management of the QTP. The challenge of optimizing controller parameters amidst several, at times inconsistent, goals is indeed substantial.

Applauded for their significant efficiency and rapid exchanges, fractional order proportional integral derivative (FOPID) controllers have materialized as a captivating substitute for modern control frameworks. The skill they exhibit in duplicating and coordinating numerous industrial and non-industrial activities has ignited considerable scholarly curiosity [3-5]. Selecting appropriate parameters for the FOPID controller is crucial, as even slight misconfigurations can lead to suboptimal performance [6, 7]. Standard techniques, notably Ziegler-Nichols and geometric root locus approaches, are utilized to find fitting parameter values; still, they do not achieve the most effective results. In contemporary research, evolutionary algorithms have gained prominence to enhance the efficacy of FOPID controllers [8-10].

Recent studies have shown that FOPID controllers can be optimized using various advanced techniques. For example, multi-objective Pareto evolutionary algorithms have been applied to boost converters, demonstrating strong dynamic performance [11]. In other works, multi-objective genetic algorithms have been utilized to optimize FOPID controllers for semi-active seat suspension systems [13, 12], while cuckoo search algorithms have been employed for robotic manipulators [14]. Additionally, optimization algorithms such as the Modified Hunger Game Search (MHGS) and Whale Optimization Algorithm (WOA) have been used to fine-tune FOPID parameters for load frequency control in power systems [15, 16]. These advancements highlight the growing potential of optimization techniques in enhancing FOPID controller performance across various applications.

Nomenclature & Symbols

FOPID	Fractional Order Proportional Integral Derivative	MOGWO	Multi-Objective Gray Wolf Optimization
QTP	Quadruple Tank Process	MHGS	Modified Hunger Game Search
PID	Proportional Integral Derivative	SISO	Single-Input and Single-Output
NPF	Non-uniformity of Pareto Front	SP	Spacing Metric
ICA	Imperialist Competitive Algorithm	PSO	Particle Swarm Optimization
ASO	Atom Search Optimization	GA	Genetic Algorithms
WOA	Whale Optimization Algorithm	COA	Cuckoo Optimization Algorithm
CFOPID	Continuous Fractional Order Proportional Integral Derivative		

The FOPID controller is increasingly used in numerous engineering domains, including DC motor control, wind energy systems, aerospace, and automatic voltage regulation [17]. The ability to fine-tune its parameters is critical to ensuring optimal performance in dynamic systems, underscoring the need for efficient control methods to optimize FOPID parameters, particularly in complex processes like the QTP.

This paper introduces a novel Multi-Objective Gray Wolf Optimization (MOGWO) method for adjusting the parameters of the FOPID controller within the QTP framework. The Gray Wolf Optimization (GWO) algorithm is favored for its simplicity, flexibility, and ability to solve non-differentiable optimization problems without the need for gradient-based information. The method is further enhanced by incorporating innovative objective functions designed to minimize time-domain errors, ensuring improved performance in dynamic systems.

The structure of the paper is as follows: Section II presents the materials and methods used in this study, beginning with an introduction to the QTP, followed by a detailed examination of the proposed MOGWO-FOPID methodology. Section III provides the results of the experiments conducted, while Section IV discusses the conclusions and suggests potential areas for future research.

2. Materials and Methods

This section begins with an overview of the Quadruple Tank Process (QTP) and subsequently explores the proposed MOGWO-FOPID controller method, which employs the multi-objective gray wolf algorithm in the context of the QTP.

2.1. QTP

Local optimization problems are the four-tank system, a nonlinear model with variable parameters, applicable to various industrial processes, including chemical and oil and gas operations. As illustrated in Fig. 1, the system consists of four interconnected water tanks and two pumps. The voltage provided to the pumps acts as the input, while the output is the water level in the lower tanks. The mathematical model for each tank is based on Bernoulli's principle and the law of mass balance. The objective is to control the water levels in the two lower tanks using the two pumps. Each pump's output is divided into two streams through three-way valves. In this setup, water is pumped into the top of each tank and exits through a pipe at the bottom, which is optimized.

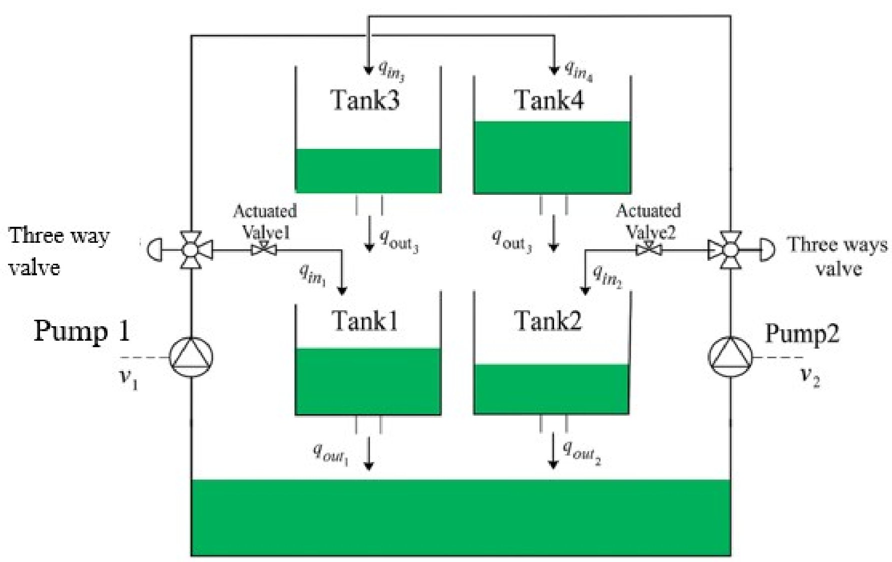


Fig. 1. QTP

$$\frac{d(\rho V)}{dt} = \rho q_{in} - \rho q_{out} \quad (1)$$

$$A_i \frac{dh_i(t)}{dt} = q_{in} - q_{out} \quad (2)$$

The input current (q_{in}) of the tank depends only on the input pump voltage according to Eq. (3). The output of the tank also depends on the gravity and height of the water in the tank according to Eq. (7).

$$q_{in_1} = \gamma_1 k_1 V_1 \tag{3}$$

$$q_{in_2} = \gamma_2 k_2 V_2 \tag{4}$$

$$q_{in_3} = k_2 V_2 (1 - \gamma_2) \tag{5}$$

$$q_{in_4} = k_1 V_1 (1 - \gamma_1) \tag{6}$$

In these relationships, k_1 and k_2 are the constants of the pumps and their positions are γ_1 and $\gamma 2$.

$$q_{out_i} = a_i \sqrt{2gh_i(t)} \tag{7}$$

In Eq. (7), a_i is the cross-section of the outlet pipes and g is the acceleration of gravity.

The mass balance relation is rewritten according to Fig. 2.

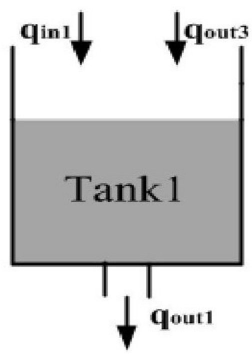


Fig. 2. Single tank diagram

$$A_1 \frac{dh_1(t)}{dt} = q_{in_1} + q_{out_3} - q_{out_1} = \gamma_1 k_1 V_1 + a_3\sqrt{2gh_3(t)} - a_1\sqrt{2gh_1(t)} \tag{8}$$

The non-linear equations of each tank for QTP are obtained similarly to Eq. (8):

$$A_1 \frac{dh_1(t)}{dt} = \gamma_1 k_1 V_1 + a_3\sqrt{2gh_3(t)} - a_1\sqrt{2gh_1(t)} \tag{9}$$

$$A_2 \frac{dh_2(t)}{dt} = \gamma_2 k_2 V_2 + a_4\sqrt{2gh_4(t)} - a_2\sqrt{2gh_2(t)} \tag{10}$$

$$A_3 \frac{dh_3(t)}{dt} = (1 - \gamma_2) k_2 V_2 - a_3\sqrt{2gh_3(t)} \tag{11}$$

$$A_4 \frac{dh_4(t)}{dt} = (1 - \gamma_1) k_1 V_1 - a_4\sqrt{2gh_4(t)} \tag{12}$$

The linearized state space equations in this case are:

$$\frac{dx}{dt} = \begin{bmatrix} \frac{-1}{T_i} & 0 & \frac{A_3}{A_1 T_3} & 0 \\ 0 & \frac{-1}{T_2} & 0 & \frac{A_4}{A_2 T_4} \\ 0 & 0 & \frac{-1}{T_3} & 0 \\ 0 & 0 & 0 & \frac{-1}{T_4} \end{bmatrix} X + \begin{bmatrix} \frac{\gamma_1}{A_1} & 0 \\ 0 & \frac{\gamma_2}{A_2} \\ 0 & \frac{(1-\gamma_2)}{A_3} \\ \frac{(1-\gamma_1)}{A_4} & 0 \end{bmatrix} u \tag{13}$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} x \tag{14}$$

Where the time constant is defined as follows:

$$T_i = \frac{A_i}{a_i}\sqrt{\frac{2h_i^0}{g}} \qquad i = 1, \ldots, 4 \tag{15}$$

The transformation function matrix is determined using equation (16).

$$G(s) = \begin{bmatrix} \frac{\gamma_1 c_1}{1+T_1} & \frac{(1-\gamma_2)c_1}{(1+T_3 s)(1+sT_1)} \\ \frac{(1-\gamma_1)c_2}{(1+sT_4)(1+sT_2)} & \frac{\gamma_2 c_2}{1+sT_2} \end{bmatrix} \tag{16}$$

By setting γ_2 and γ_1 equal to zero and executing the corresponding MATLAB coding in the working point of Table 1, the transformation function matrix is obtained as follows:

$$G(s) = \begin{bmatrix} 0 & \frac{0.4204}{4286s^2+232.1s+1} \\ \frac{0.8159}{5168s^2+423.8s+1} & 0 \end{bmatrix} \tag{17}$$

Table 1. Introducing the parameters of the QTP

Parameter	Name	Values
a_i	Cross-section of the outlet tanks	$a_1 = 0.35\ cm^2, a_2 = 0.696\ cm^2$ $a_3 = 2.25\ cm^2, a_4 = 2\ cm^2$
k_1, k_2	Constants (relationship between control voltages and the pumps' water flow)	3.33, 3.35
k_c	Fixed measuring device	0.50
h_{max}	Tank water height	$h_1^0,\ h_2^0 = 504$ $h_3^0 = 4$ $h_4^0 = 1.22$
A_i	Cross-section of the tanks	$A_1, A_2, A_3, A_4 = 504\ cm^2$
q_i	Dabi each of the pumps	$q_1^0 = 98$ $q_2^0 = 200$
g		$981\frac{cm^2}{s}$
γ_1, γ_2	The parameters of the three-way valve	0

2.2. Proposed method

The proposed MOGWO-FOPID approach is presented to design the FOPID controller in the QTP. First, the proposed MOGWO algorithm is called, and then the target functions in each iteration are called, Finally Fig. 3 displays the ideal values for the FOPID controller parameters in the QTP. This section begins by describing the basic GWO algorithm. It then discusses the improvements made to the GWO algorithm to address multi-objective challenges associated with the FOPID controller. Lastly, the proposed cost functions are analyzed.

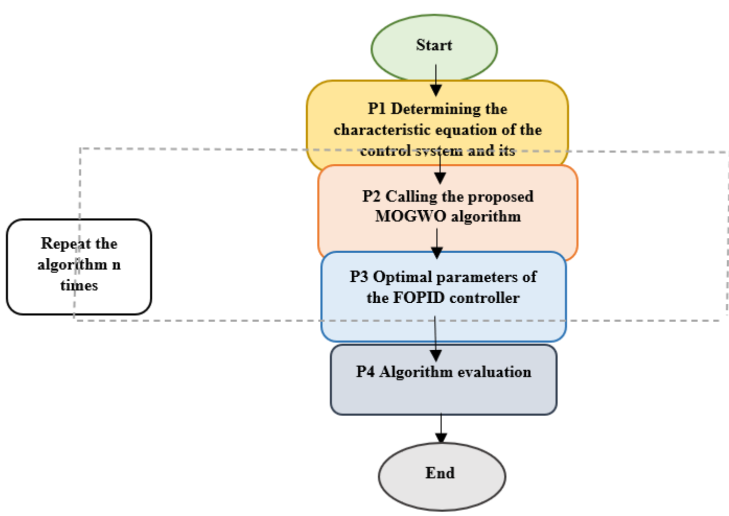


Fig. 3. Flowchart of the proposed method

2.2.1. Flowchart explanations

- P1 — Problem Formulation and Initialization

Determine the control system model and its characteristic equation, specify variable bounds for the FOPID parameters $K_p, K_i, K_d, \lambda, \mu$, and set algorithmic hyperparameters (population size, maximum iterations, and stopping criteria). Define the objective functions and any constraints used in the optimization.

- P2 — Calling the Proposed MOGWO Algorithm

Invoke the MOGWO routine to explore the decision space. For each iteration, generate or update a population of candidate controllers and evaluate them with respect to the defined objectives (e.g., steady-state errors, overshoot, and settling time). Non-dominated solutions are retained to approximate the Pareto front.

- P3 — Optimal Parameters of the FOPID Controller

From the current Pareto set, identify the best candidate(s) given the iteration state—either by diversity-preserving selection (within the archive) or by a decision criterion if a single controller is needed. The dashed loop in Fig. 3 indicates that P2–P3 are repeated for n iterations (or until a convergence/termination condition is satisfied).

- P4 — Algorithm Evaluation

After completing the iterations, finalize the controller parameters and evaluate the resulting FOPID design(s). Report quantitative metrics (e.g., time-domain indices and steady-state performance) and, if applicable, compare against baselines.

2.2.2. GWO algorithm

The GWO algorithm was presented in a 2014 study that focuses on the lives of gray wolves [18]. These animals form packs, with the alpha wolf serving as the decision-maker for various activities, including when to hunt. The hunting strategy that inspired this optimization algorithm consists of three key steps:

- Tracking, chasing and approaching the prey
- Pursuing, encircling, and harassing the prey till it stops moving.
- Attacking the prey

To simulate the social behavior of wolves, a random set of solutions is generated. The highest-ranking solution is designated as the best solution, while the second and third highest are identified as the second and third best solutions, respectively. β and δ, respectively. Also, other solutions are considered as wolves of the. The GWO algorithm hunting (optimization) is guided by. β , δ and the ω wolves follow them. In 3D modeling, the process begins by identifying the points surrounding the prey. Next, the model is moved closer to the prey, and finally, an attack is executed. To identify the points around the prey, Eq. (18) and Eq. (19) are used.

$$\vec{D} = \left|C.\overrightarrow{X_p}(t) - \vec{X}(t)\right| \tag{18}$$

$$\vec{x}(t+1) = \overrightarrow{X_p}(t) - \vec{A}.\vec{D} \tag{19}$$

In Eq. (18-19), t represents the latest version of the algorithm. $\vec{A}$ and $\vec{C}$ are the coefficient vectors, $\overrightarrow{X_p}$ is the prey position vector. $\vec{X}$ is the gray wolf position vector. The vectors $\vec{A}$ and $\vec{C}$ are obtained from the following Eq. (20) and Eq. (21).

$$\vec{A} = 2\vec{a}.\overrightarrow{r_1} - \vec{a} \tag{20}$$

$$\vec{C} = 2.\overrightarrow{r_2} \tag{21}$$

In Eq. (20-21), $\overrightarrow{r_1}$ and $\overrightarrow{r_2}$ are random vectors within the interval $[0,1]$. $\vec{a}$ The parameter decreases from 2 to 0 as the algorithm executes. Since there is no information about the hunting position in the initial search space, it is assumed that.α, β and δ wolves are respectively the three best answers that have been obtained so far. Other solutions should change their position in order to converge towards the optimal solution, according to Eq. (22)- Eq. (28).

$$\overrightarrow{D_\alpha} = \left|\overrightarrow{C_1}.\overrightarrow{X_\alpha} - \vec{X}\right| \tag{22}$$

$$\overrightarrow{D_\beta} = \left|\overrightarrow{C_2}.\overrightarrow{X_\beta} - \vec{X}\right| \tag{23}$$

$$\overrightarrow{D_\delta} = \left|\overrightarrow{C_3}.\overrightarrow{X_\delta} - \vec{X}\right| \tag{24}$$

$$\overrightarrow{X_1} = \overrightarrow{X_\alpha} - \overrightarrow{A_1}.\left(\overrightarrow{D_\alpha}\right) \tag{25}$$

$$\overrightarrow{X_2} = \overrightarrow{X_\beta} - \overrightarrow{A_2}.\left(\overrightarrow{D_\beta}\right) \tag{26}$$

$$\overrightarrow{X_3} = \overrightarrow{X_\delta} - \overrightarrow{A_3}.\left(\overrightarrow{D_\delta}\right) \tag{27}$$

$$\vec{X}(t+1) = \frac{\overrightarrow{X_1}+\overrightarrow{X_2}+\overrightarrow{X_3}}{3} \tag{28}$$

As mentioned, wolves attack when they stop hunting. To create a mathematical model as the prey is consumed, the value of "A" decreases. It's important to mention that the range of changes will also be limited to a smaller interval. In other words, it will be a random value within that specified range.$[-2a, 2a]$ where A gradually decreases from 2 to 0 during the algorithm's iterations, utilizing random values within the defined range. $[-1,1]$, The search agent's next position can be anywhere between its current location and that of the prey. In the gray algorithm, the positions of the wolves are modified according to the locations of the alpha, beta, and delta wolves. While searching for prey, wolves may spread apart, and there is no guarantee of convergence. To model these phenomena, a random value is utilized that is either greater than 1 or less than -1. A. Fig. 4 shows that if $|A| > 1$, wolves will avoid hunting. While for $|A| < 1$, wolves are forced to attack towards prey. The flowchart of the GWO algorithm is shown in Fig. 5.

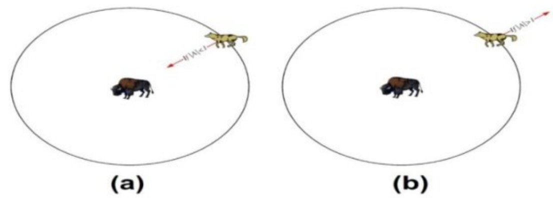


Fig. 4. (a) Attacking prey, (b) searching for prey[18]

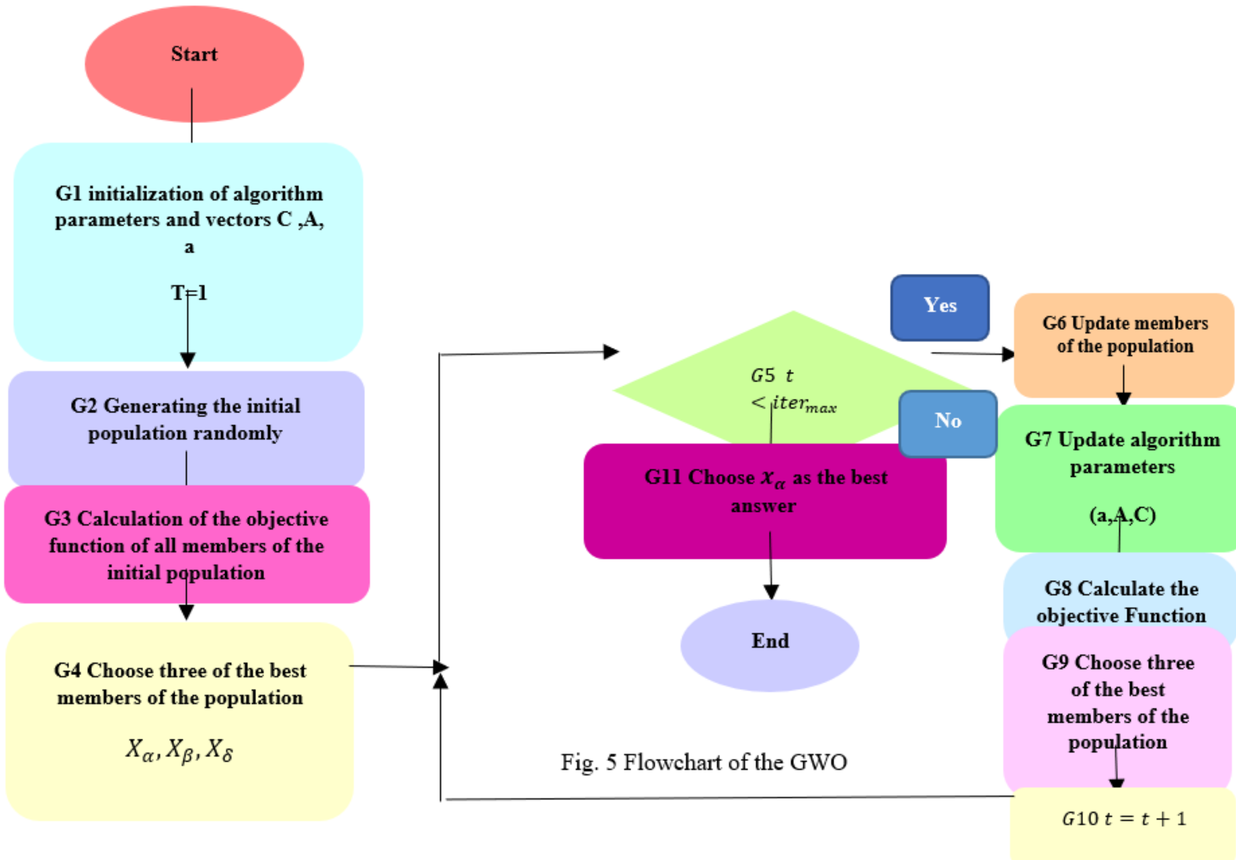


Fig. 5. Flowchart of the gray wolf optimization (GWO) algorithm; G1: Initialization, Initialize the algorithm parameters (a, A, C) and set the initial iteration $t = 1$, G2: Generate Initial Population, Randomly generate the initial population of candidate solutions, G3: Objective Function Calculation, Compute the objective function for each member of the population, G4: Selection of Leaders, Choose the top three best members of the population: X_a, X_B and X_δ, G5: Iteration Check, Check if the stopping criterion (maximum iterations t_{max}) is met: If yes, proceed to G11 (choose the best solution and terminate), If no, proceed to G6, G6: Update Population Members, Update the positions of the members in the population based on the positions of the leaders, G7: Update Algorithm Parameters, Update the algorithm parameters (a, A, C) for the next iteration, G8: Recalculate Objective Function, Recalculate the objective function for the updated population, G9: Re-select Leaders, Re-select the top three best members of the population, G10: Increment Iteration, Increment the iteration counter $t \leftarrow t + 1$, G11: Final Solution Selection, If the maximum number of iterations is reached, select the best solution X_a and terminate the algorithm.

2.2.3. Proposed multi-objective gray wolf optimization algorithm

This article focuses on addressing a multi-objective optimization problem, which requires the use of multi-objective algorithms for its solution. The GWO algorithm is primarily designed for single-objective optimization tasks. Consequently, one of the aims of this article is to adapt and extend the GWO algorithm for use in multi-objective optimization scenarios.

In the first step, we consider a list as an archive whose members are exclusively non-dominant answers. Non-superior solutions have two characteristics: these solutions are the optimal solutions of the problem, and secondly, they do not have any solution that is preferable to any other. At the beginning of the algorithm, this list is empty, and there is no answer in it. After the evaluation of the population by the objective functions, all the answers are compared, and the answers that are not defeated by any of the members of the population are transferred to this list. During the iteration process of the algorithm, the optimal solutions obtained so far are compared with the current members of the archive. One of the following three situations may occur:

- If the new member is defeated by at least one of the current members of the archive, the new member will not be allowed to enter the archive.

- If the new member overcomes one or more current members of the archive, the defeated solutions are that the previous entry has been removed from the archive, and the new solution has been added.
- If neither the new solution nor the current members of the archive outperform the other, the new solution will be included in the archive.

2.2.4. Sensitivity analysis of algorithm parameters

This section analyzes the sensitivity of the MOGWO algorithm's performance to the fixed parameters of the optimization algorithms, including initial population size, number of repetitions, and mutation/crossover rates in the Genetic Algorithm (GA).

- Initial Population: A larger initial population increases solution diversity but requires higher computational resources, while a smaller population may lead to faster convergence at the cost of potentially missing the global optimum.
- Number of Repetitions: More repetitions enhance solution accuracy but increase computational time. The choice of this parameter involves balancing efficiency and precision.
- GA Parameters: The crossover (70%) and mutation (20%) rates were chosen to balance exploration and exploitation in the search space.

2.3. Objective function

Choosing appropriate objective (cost) functions is very important in system control. Objective functions are generally of several categories: objective functions of the time domain (maximum overshoot, rise time), error objective functions in the time domain (the integral of the squared error and the integral of the absolute value of the error) and objective functions used in the frequency domain. (The phase limit and the objective functions related to the output of the controller and the objective functions related to the transfer function of the system and the controller are divided.) In this Paper, two series of objective functions are used in this domain due to the greater acceptability of time-domain objective functions and error functions in the time domain.

2.3.1. The proposed first objective function

This cost function is defined as the total steady-state error of two systems. G_{12} and G_{21} according to Eq. (30).

$$\mathrm{function} = \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} \tag{29}$$

$$\mathrm{object\_function} = \sum_{i=1}^{n} e_i^2 \tag{30}$$

In Eq. (30), e_1 indicates the steady-state error of the G_{12} system, e_2 indicates the steady-state error of the G_{21} system, and the errors of each system have the same weight.

2.3.2. The proposed second objective function

This objective function pertains to the time domain and is calculated based on Eq. (31).

$$\mathrm{object\_function} = w_1 \times MP + w_2 \times T_S \tag{31}$$

In Eq. (31), MP is the maximum overshoot and T_S is the settling time. w_1 and w_2 are given weights. The weights w1 = 0.3 and w2 = 1 were selected based on the relative importance of the performance criteria. w1 represents the maximum overshoot (MP), which has a lower weight because it is less critical compared to settling time (T_S). w2 is assigned a higher value because minimizing settling time is crucial for achieving a fast and stable response in the system. The chosen weights reflect a balance that prioritizes faster stabilization while still considering the effect of overshoot.

3. Implementation and Test Results

In this paper, the QTP system has been transformed into two Single-Input and Single-Output (SISO) systems through the decoupling method. A FOPID controller was developed for the four-tank system using both classical techniques and the proposed MOGWO method. The parameters for the FOPID controller are detailed in Table 2. Next, will outline the evaluation criteria and present the test results.

Table 2. Range of FOPID controller parameters

Parameters	Values
k_p	$(0, 15]$
k_i	$(0, 15]$
k_d	$(0, 15]$
λ	$[0,1]$
μ	$[0,1]$

3.1. Evaluation criteria

This section outlines the criteria utilized to evaluate the effectiveness of the proposed method. Evaluation criteria are generally divided into two categories: evaluation criteria requiring reference and evaluation criteria without reference. Our problem has no reference, so evaluation criteria without reference have been used. In the following, the description of some evaluation criteria without reference used in this article will be discussed.

- Spacing Metric (SP)

The SP is used to measure the dispersion of the Euclidean distance in the different answers obtained. This criterion is computed according to Eq. (32).

$$SP = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(\bar{d} - d_i\right)^2} \quad (32)$$

In this Eq. (32), d_i indicates the Euclidean distance between the i^{th} answer of the algorithm and the closest possible answer. The parameter $\bar{d}$ is the average of the obtained values. In this regard, n represents the number of times the algorithm is executed.

- Non-uniformity of Pareto Front

The NPF criterion is calculated according to Eq. (33).

$$NPF = \sqrt{\frac{\sum_{i=1}^{n}\left(\frac{d}{\bar{d}}-1\right)^2}{|n|-1}} \quad (33)$$

This measure is the standard deviation of the distance normalized by $\bar{d}$. Therefore, as the value of the Pareto front increases, the space of the Pareto front becomes more non-uniform.

3.2. Test results

In these experiments, to assess how the selection of parameter values affects the outcomes of the FOPID controller in the four-tank system, the parameters of the fixed algorithm are selected in the following sequence. The weights of w_1 and w_2 of the proposed second objective functions are 0.3 and 1, respectively. The number of the initial population for genetic algorithm (GA)[19] , grasshopper[20] and Imperialist Competitive Algorithm (ICA) [21]is considered 80, and the number of population members in the proposed MOGWO method is 40. The number of archives is equal to the population's members. For a fair evaluation, the number of repetitions of all algorithms is 200. The percentage of combination and mutation rate in the GA algorithm is 70% and 20%, respectively. The number of initial empires was 12 and the revolution rate was 0.3. Four tests are performed. In the first test, the proposed objective functions are examined. In the second experiment, the dominant responses of FOPID controller parameters are examined. The step response of the proposed MOGWO method is evaluated in the third test. In the final experiment, its performance is compared to that of other methods.

3.2.1. Justification for population size selection in MOGWO

A population size of 40 was chosen for MOGWO to balance computational efficiency with search quality. While a larger population size enables a broader exploration of search space, it increases the computational cost. For GA and ICA, a population size of 80 was selected to ensure greater diversity in the search, leading to better solutions for multi-objective optimization problems.

3.2.2. Rationale for Weight Assignment (w_1 = 0.3 and w_2 = 1)

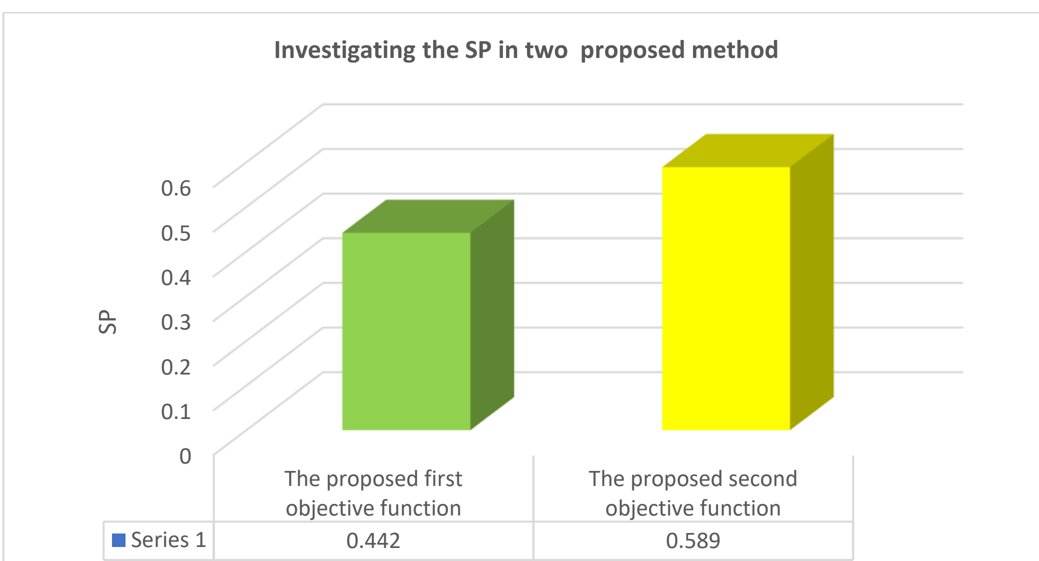
The weight w_1 = 0.3 was assigned to the maximum overshoot (MP), as it is considered less critical compared to settling time, for which w_2 = 1. Settling time is a key performance metric in control systems, as it indicates how quickly the system stabilizes. The goal is to minimize the time required for the system to reach a stable state.

3.2.3. Justification for Setting the Number of Iterations to 200

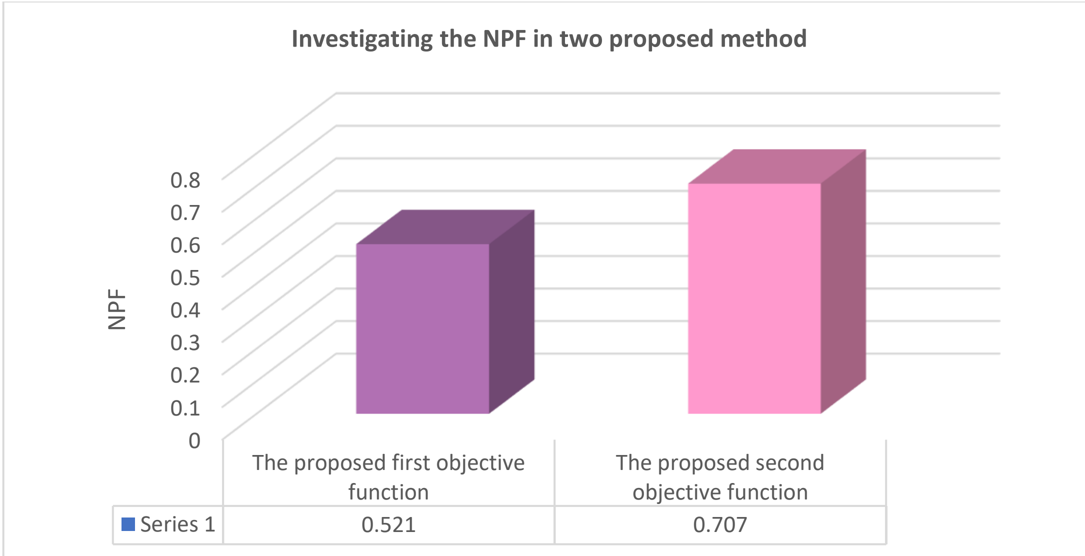
The number of iterations was set to 200 after preliminary testing showed that this value is sufficient for achieving convergence without excessively increasing computational time. A total of 200 iterations represents an effective stopping point for reaching an optimal solution while maintaining computational efficiency.

3.2.4. Check the Proposed Objective Functions

In this experiment, the two proposed objective functions were investigated with the proposed MOGWO algorithm in the four-tank system, and the results are shown in Fig. 6.



(a)



(b)

Fig. 6. Examining the proposed objective functions using the proposed MOGWO algorithm in the QTP; (a) Examination of the first objective functions proposed by the SP criterion, (b) Examination of the second objective functions proposed by the NPF criterion

As seen in Fig. 6, the two proposed objective functions perform well in the four-tank system. The proposed first objective function (the sum of steady-state error of two systems) has a better performance than the proposed second objective function (the rise time background and settling time) in terms of SP and NPF.

3.2.5. Investigation of dominant responses of FOPID controller parameters

In this section, a set of dominant responses in the QTP is reviewed (Table 3).

Table 3. Dominant values of FOPID controller parameters in QTP

First System					Second System				
μ	λ	k_D	k_I	k_P	μ	λ	k_D	k_I	k_P
0.975	0.932	0.943	0.583	0.654	0.955	0.775	13.794	6.389	2.430
0.821	0.894	0.510	0.285	0.572	0.940	0.799	12.001	10.693	5.417
0.512	0.618	0.491	2.148	1.851	0.713	0.542	8.01	12.951	8.013

Table 3 shows three dominant answers for each of the coefficients. This shows that in most articles, only one dominant response is calculated for each coefficient of the FOPID controller, but three dominant responses are calculated for each coefficient in the proposed method, which indicates the effective performance of the proposed method.

3.2.6. Checking the step response of the control system

In this experiment, will examine the step response of the proposed method, the root locus method, and Ziegler-Nichols. The method was examined, and the results are presented. In Figs. 7 and 8.

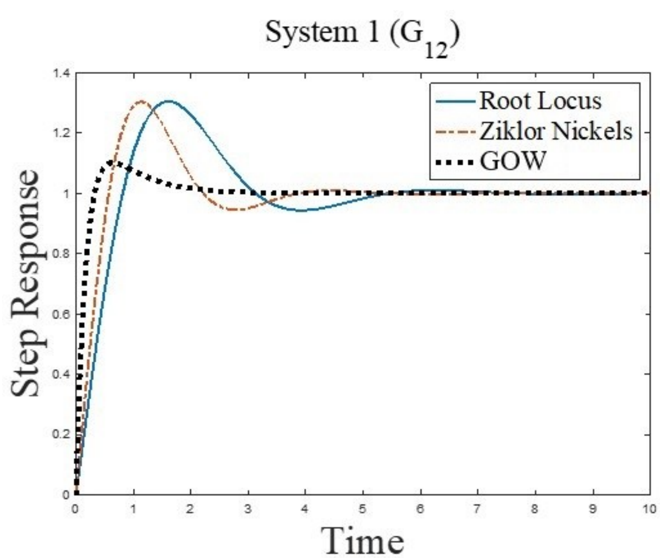


Fig. 7. System step response G_{12}

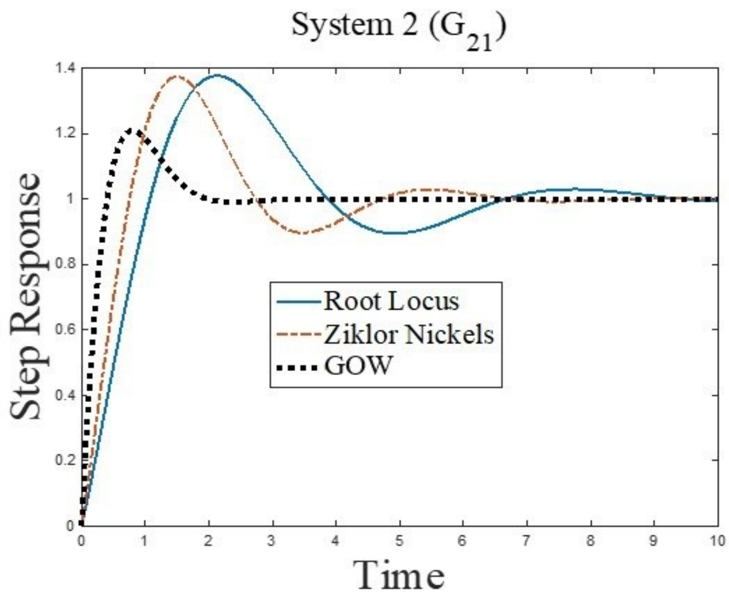


Fig. 8. System step response G_{21}

Figs. 7 and 8 illustrate that the proposed method outperforms the other two in terms of overshoot, rise time, and settling time, underscoring its effectiveness. The system managed by this approach exhibits superior performance compared to the alternatives.

3.2.7. Checking the performance of the proposed method with recent methods

In this section, evaluated the performance of the proposed method against recent approaches, with the results presented in Table 4.

Table 4. Checking the performance of the proposed method with recent methods

Methods	SP	NPF
multi-objective genetic	0.623	0.985
Grasshopper algorithm	1.494	1.058
Imperialist Competitive Algorithm (ICA)	1.772	1.073
Proposed MOGWO algorithm	0.442	0.521

As can be seen in Table 4, the multi-objective genetic algorithm outperforms both the grasshopper algorithm and the Imperialist Competitive Algorithm (ICA). The grasshopper algorithm and ICA show similar performance based on the SP and NPF criteria. Additionally, the proposed multi-objective gray wolf method demonstrates superior performance in relation to the other methods.

4. Conclusion

This study presents a novel approach for controlling the Quadruple Tank Process (QTP) using a Fractional Order Proportional-Integral-Derivative (FOPID) controller, with parameters optimized using a multi-objective Gray Wolf Optimization (GWO) algorithm. Two objective functions were proposed: one based on time-domain performance and another based on time-error. The time-error-based objective function demonstrated superior performance in comparison to the time-domain-based function, particularly in terms of system response. The proposed method outperforms conventional approaches in key performance indicators such as setpoint tracking (SP), normalized performance factors (NPF), and step response, offering enhanced efficiency and precision for the QTP .Future research will focus on applying this methodology to the QTP without decoupling, as well as exploring its potential for multi-input multi-output (MIMO) systems. Additionally, investigating the real-time implementation of the proposed approach and assessing its scalability for more complex systems could further enhance its practical applicability and robustness.

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