The Administration & Economic College Journal
For Economics & Administration & Financial Studies
Vol.18,N1, P P -
ISSN PRINT 2312-7813
ISSN ONLINE 2313-1012
رقم الايداع ١٥٥٧ لسنة ٢٠١١
مجلة كلية الإدارة والاقتصاد للدراسات الاقتصادية
والإدارية والمالية
المجلد ١٨ العدد١ كانون الثاني ٢٠٢٦ ص ١- ص٢٥

Bayesian composite quantile regression with new regularization approach

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Abstract

Our proposed method introduces a hierarchical Bayesian regression model with an effective Gibbs sampler for parameter estimation and variable selection. It combines a regularization technique with Bayesian inference by reformulating the Laplace distribution as a mixture of Uniform and Exponential distributions. This mixed distribution is incorporated into the composite quantile regression model to form a new hierarchical model with strong statistical properties.

Through a simulation study, our method outperformed both Bayesian and non-Bayesian approaches in variable selection and estimation. Additionally, it demonstrated excellent stability. When applied to real-world Thrombocytopenia data, our method proved highly effective in both estimating parameters and selecting relevant variables.

1. Introoduction

Modeling the relationship between the dependent and independent variables is crucial for making accurate predictions. Quantile regression (QR) is an important tool for this, as it provides a comprehensive view of the relationship by estimating multiple quantile levels. However, determining the optimal QR model can be challenging. Composite quantile regression (CQR) addresses this by blending variable selection with QR, leading to powerful, predictive models by excluding non-significant variables.

This search uses three estimation methods: a non-Bayesian approach combining Lasso with QR, and two Bayesian methods, including a new hierarchical model that combines composite QR with a modified Laplace distribution as the prior. This modification blends uniform and exponential distributions, offering a robust and efficient approach for parameter estimation and variable selection.

Through simulation studies and real data analysis, the proposed method outperformed existing methods. In the real data analysis on Thrombocytopenia deficiency, the model successfully identified 14 significant variables and excluded six non-significant ones, leading to a strong predictive model.

2. Research problem :

Given the complexity of modeling relationships between a response variable and a large number of independent variables, to what extent does the use of a composite quantile regression model enhance the accuracy of estimating these relationships? Why is variable selection considered a crucial step in constructing a

quantitative regression model with strong predictive power? Considering the infinite number of possible regression lines, how can one accurately determine the most appropriate quantitative regression function? In this context, how does the integration of a new regularization method within the composite quantile regression framework contribute to improving the model's precision? Furthermore, what is the significance of employing a Bayesian approach in estimating the parameters of such a complex and refined regression model?

3. The objective of the research :

The objective of this search is to propose a new hierarchical prior distribution model based on a novel scale mixture introduced by Remah (2021). This model will be applied within the framework of Bayesian composite quantile regression (CQR). Additionally, a new Gibbs sampler algorithm will be developed based on the aforementioned model. A comparative study will also be conducted, comparing the proposed model with other regularization methods through simulation studies and real data analysis. Lastly, the proposed method will be utilized to identify the significant variables that contribute to building a model aimed at describing the relationship between blood platelet deficiency and specific other variables.

4. Variable selection(VS)

Variable selection (**VS**) is one of the basic principles in many applications, and it represents a subset selection of the important explanatory variables to be used in building the model, as it provides a good predictive model in addition to identifying the important variables to fit the model. (**Girffin and Brown , 2010**) .

The main assumption when using the **VS** is that the data contains many variables. Therefore, there are two main aims that must be achieved when building a regression model. The first is to select the important explanatory variables affecting the response variable and obtain insight into the relationship of the explanatory variables to the response variable through the structure of the model. The second is to create an accurate model that has the ability to predict well and has high explanatory power.

significant Variable selection**(VS)** is considered one of the requirements when building a regression model, so excluding non-significant variables is necessary, and this improves the accuracy of prediction. One of the techniques used in estimating and selecting variables is the Lasso technique proposed by **Tibshirani in 1996**, which works to make the non-significant variables equal to zero and preserve the significant variables(**Tibshirani ,1996**)

5. Bayesian Lasso composite quantile Regression Formulations

The quantile regression (**QR**) model is one of the most important regression models**.** QR does not require any statistical assumptions and is also very robust against outlier data (**Cade, B. S., & Noon, B. R. (2003**)). All these features make the **QR** an important regression model. Therefore, it is used in many scientific applications, such as econometrics, finance, biological sciences, agricultural sciences, and medicine. Quantile regression is adequate for distributions that are severely skewed and non-central. Using different quantile levels, the **QR** model can be used to estimate the relationship between independent variables and a dependent variable in any position of the dependent distribution

(**Levin, J. (2002**)).The quantile regression levels refer to an interval (0, 1). The **QR** model is evaluated through the relationship between the independent variables and a dependent variable via a conditional function at many quantile levels, where$Q_\tau(y_i|x_i)$ is the quantile function at the many quantile level τ_{th}.

The quantile function is equivalent to the inverse distribution function as follows$Q_\tau(y_i|x_i) = F^{-1}(\tau) \quad 0 \leq \tau \leq 1$. There are infinite quantile regression lines to describe the relationship between one dependent variable and many independent variables. But choosing the optimal quantile regression line is a hard matter. To overcome this problem , composite quantile regression(**CQR**) has been used. Consider Q different quantile levels ,$0 < \tau_1 < \tau_2 < \cdots < \tau_Q < 1$ Let $y_i = x_i^T \beta_\tau + \varepsilon_i$ for $i = 1, \ldots, n$. Then, the **CQR** (**Zou and Yuan, 2008**) is given by :

$$(\hat{\alpha}_{\tau_1}, \ldots, \hat{\alpha}_{\tau_q}, \hat{\beta}) = \underset{\alpha_{\tau_1}, \ldots, \alpha_{\tau_q}, \beta}{argmin} \sum_{i=0}^{n} \left[\sum_{q=1}^{Q} \rho_{\tau_q} \left(y_i - \alpha_{\tau_q} - x_i^T \beta_\tau \right) \right] \quad (1)$$

We can rewrite the optimization problem (1) by the following weighted optimization problem :

$$(\hat{\alpha}_{\tau_1}, \ldots, \hat{\alpha}_{\tau_q}, \hat{\beta}) = \underset{\alpha_{\tau_1}, \ldots, \alpha_{\tau_q}, \beta}{argmin} \sum_{i=0}^{n} \left[\sum_{q=1}^{Q} w_q \rho_{\tau_q} \left(y_i - \alpha_{\tau_q} - x_i^T \beta_\tau \right) \right] \quad (2)$$

Where $0 \leq w_q \leq 1, \ and \ \sum_{q=1}^{Q} w_q = 1$, $is \ q^{th}$ component.

Where $\rho_{\tau_q}(.)$ is the loss function , $\tau_q = \frac{q}{Q+1} for \ q = 1, \ldots, Q.$

$\boldsymbol{\alpha_\tau}$ is the intercept parameter under the $\boldsymbol{\tau^{th}}$ quantile of error and the intercept of linear model .

It is impossible to differentiable equation (1) at the zero point. Therefore, minimization of the above equation can be achieved through an algorithm proposed by **Koenker and D'Orey (1987)**. However, this algorithm might be inefficient at some quantile level. In order to estimate the **CQR** parameters, a Bayesian approach has been used . Now, the random error term $\boldsymbol{\varepsilon_i}$ is being to asymmetric Laplace distribution (**ALD**). Where the probability density function of the asymmetric Laplace distribution with one assigned to the scale parameter is :

$$\boldsymbol{f(\varepsilon|\tau_q) = \tau_q(1-\tau_q)\ \eta\ exp\{-\eta\ \rho_{\tau_q}(\varepsilon)\}.} \qquad (3)$$

Where $\eta > 0$ is the scale parameter .

Then the joint distribution of $y = (y_1, \dots, y_n)\ given\ X = (x_1^T, \dots, x_n^T)^T, \beta = (\beta_1, \dots . \beta_k)^T$
for composite quantile regression is :

$$\boldsymbol{f(y|x,\alpha,\beta,\eta) \sim \prod_{i=1}^{n}(\sum_{q=1}^{Q} w_q\, \rho_{\tau_q}\left(y_i \middle| x_i, \alpha_{\tau_q}, \beta, \eta\right)}, \qquad (4)$$

Where the conditional probability of y_i given x_i ,α_{τ_q} ;β, η is defined as follows :

$$\boldsymbol{f_{\tau_q}\left(y_i \middle| x_i,\ \alpha_{\tau_q}, \beta, \eta\right) = \tau_q(1-\tau_q)\eta\ exp\left[-\eta\ \rho_{\tau_q}\left(y_i - \alpha_{\tau_q} - x_i^T\beta_\tau\right)\right]}$$
, for q=1 ,…, Q

The minimizing equation (1) is equivalent to maximizing the likelihood function of the dependent variable (3). **Yu et al. (2013)**); and **Benoit et al. (2013)** mentioned estimating a possible

parametric link between the minimization in Equation (1) and the maximum likelihood in Equation (1). But there are difficulty to solve Equation (3) directly because of the mixture of Q elements. Following (**Huang and Chen (2015)**) we use a cluster assignment matrix c whose element i,qth (c_{iq}) is equal to 1 if the subject belongs to the q cluster, otherwise is belong to $c_{iq} = 0$. The element is treated as a missing value problem. Therefore, likelihood takes the following form :

$$f(y|x,\alpha,\beta,\eta) \sim \prod_{i=1}^{n}\prod_{q=1}^{Q}\left(w_q \rho_{\tau_q}\left(y_i \middle| x_i, \alpha_{\tau_q}, \beta, \eta\right)\right)^{c_{iq}}$$

$$\sim \prod_{i=1}^{n}\prod_{q=1}^{Q}\left[w_q \tau_q(1-\tau_q)\eta\, exp(-\eta\, \rho_{\tau_q}(y_i - \alpha_{\tau_q} - x_i^T \beta_\tau)\right]^{c_{iq}} \quad (5)$$

By following **Li et al. (2010)**, **Kozumi and Kobayashi (2011),** and **Benoit et al. (13)** The dependent variable ε_i can be view as**:**

$$\varepsilon_i = \varphi_{1q}\ \eta^{-1} v_i + \varphi_{2q}\ \eta^{-1}\sqrt{v_i}\ z_i \quad (6)$$

Where $v_i \sim$ standard normal**.**

Where :

$$\varphi_{1q} = \frac{1-2\tau_q}{\tau_q(1-\tau_q)} \text{ and } \varphi_{2q} = \sqrt{\frac{2}{\tau_q(1-\tau_q)}} \quad .$$

With $\tau\epsilon(0,1), then, \varepsilon\ can$ be distributed to ALD with mean and variance as follows :

$E(\varepsilon) = \frac{(1-2\tau)}{\tau(1-\tau)\eta}$ **,** $\mathbf{Var}(\varepsilon) = \frac{1-2\tau+2\tau^2}{\tau^2(1-\tau)^2\eta^2}$

See **lixin (2010)** and **yu and zhang(2005)** for more details.

By letting $\tilde{v}_i = \eta^{-1} v_i$ then we can say that $\tilde{v}_i$ distributed as exponential distribution with parameter η, because if we let $p(\tilde{v} \leq v)$, then $p(\tilde{v} \leq v) = p(\eta^{-1} v \leq \tilde{v}) = p(v \leq \eta \tilde{v})$

$$= \int_0^{\eta \tilde{v}} e^{-v} \, d = 1 - e^{-\eta \tilde{v}}$$

$= 1 - e^{-\eta \tilde{v}}$, with the cdf of exponential distribution

Then the formula **(6)** can be rewritten as follow :

$$\varepsilon_i = \varphi_{1q} \eta^{-1} v_i + \eta^{-1} \varphi_{2q} \sqrt{v_i} \, z_i$$

$$= \varphi_{1q} \tilde{v}_i + \eta^{-1} \varphi_{2q} \sqrt{\eta \tilde{v}_i} \, z_i$$

$$= \varphi_{1q} \tilde{v}_i + \eta^{-1} \eta^{1/2} \varphi_{2q} \sqrt{\tilde{v}_i} \, z_i$$

$$= \varphi_{1q} \tilde{v}_i + \eta^{-1/2} \varphi_{2q} \sqrt{\tilde{v}_i} \, z_i$$

Then the hierarchical structure model with $\tilde{v}_i, z_i, i = 1, 2, \ldots . n$ is :

$$y_i = \alpha_q + x_i^T \beta_\tau + \varphi_{1q} \tilde{v}_i + \eta^{-1/2} \varphi_{2q} \sqrt{\tilde{v}_i} \, z_i$$

$$(\tilde{v}|\eta) \sim \prod_{i=1}^{n} \eta \, exp(-\eta \tilde{v}_i)$$

$$z \sim \left(\frac{1}{\sqrt{2\pi}}\right)^n exp\left(-\frac{1}{2}\sum_{i=1}^{n} z_i^2\right)$$

The Bayesian Hierarchical of composite quantile regression

By following **fadel alhusseini (2017)** the lasso composite quantile penalized regression solution is :

$$\widehat{\boldsymbol{\beta}} = \boldsymbol{argmin} \sum_{q=1}^{Q} \left\{ \sum_{i=1}^{n} \boldsymbol{\rho}_{\tau_q} \ (\boldsymbol{y}_i - (\boldsymbol{\alpha}_q + \boldsymbol{x}_i^T \boldsymbol{\beta}_\tau) \right\} + \boldsymbol{\lambda} \sum_{j=1}^{k} |\boldsymbol{\beta}_j| \qquad (7)$$

The Bayesian lasso composite quantile regression model based on **(6)** required to impose prior distribution for $\boldsymbol{\beta}$. Following **Tibshirani (1996)** the prior distribution of $\boldsymbol{\beta}$ is a Laplace density in loss quantile regression model is :

$$\boldsymbol{f} \ (\boldsymbol{\beta}|\boldsymbol{b}) = \frac{\mathbf{1}}{\mathbf{2}\boldsymbol{b}} \boldsymbol{exp}(-\frac{|x|}{\boldsymbol{b}}) \qquad (8)$$

Now we can rewrite the prior **(6)** as scale mixture of Uniform distribution mixing with standard exponential distribution as in the following proposition ,see **Mallick and Yi (2014)** for more details .

Proposition: Laplace distribution can be written as scale mixture of uniform distribution mixing with standard exponential distribution In mathematics facts its well known that :

$$\boldsymbol{e}^{-\boldsymbol{\eta\lambda}|x|} = \int_{\boldsymbol{z}>\boldsymbol{\sigma}|x|}^{\cdot} \boldsymbol{\lambda e}^{-\boldsymbol{\lambda z}} \, \boldsymbol{dz}$$

$$\frac{\boldsymbol{\eta\lambda}}{\mathbf{2}} \boldsymbol{e}^{-\sigma\boldsymbol{\lambda}|x|} = \int_{\boldsymbol{z}> \boldsymbol{\eta}|x|}^{'} \frac{\boldsymbol{\eta\lambda}}{\mathbf{2}} \, \boldsymbol{\lambda e}^{-\boldsymbol{\lambda z}} \boldsymbol{dz}$$

$$\boldsymbol{\lambda z} = \boldsymbol{m}$$

$$= \int_{\boldsymbol{m}>\boldsymbol{\lambda\eta}|x|}^{'} \frac{\boldsymbol{\eta}\,\boldsymbol{\lambda}}{\mathbf{2}} \, \boldsymbol{\lambda e}^{-\boldsymbol{m}} \frac{\mathbf{1}}{\boldsymbol{\lambda}} \, \boldsymbol{dm}$$

$$\frac{\boldsymbol{\eta\lambda}}{\mathbf{2}} \, \boldsymbol{e}^{-\boldsymbol{\eta\lambda}|x|} = \int_{\boldsymbol{m}> \boldsymbol{\eta\lambda}|x|}^{'} \frac{\boldsymbol{\eta}\,\boldsymbol{\lambda}}{\mathbf{2}} \, \boldsymbol{e}^{-\boldsymbol{m}} \quad \boldsymbol{dm} \qquad (9)$$

Now by letting $\mathbf{x} = \boldsymbol{\beta}$ in **(9), we get**

$$f(\beta|\eta,\lambda) = \frac{\eta\lambda}{2}\, e^{-\eta\lambda|\beta|}$$

$$= \int_{m>\eta\lambda|\beta|}^{'} \frac{\eta\lambda}{2}\, e^{-m} \quad dm \qquad (10)$$

More details see (Remah ,2021)

Hence , the Bayesian hierarchical model is :

$$y_i = \left(\alpha_\tau + x_i^T\, \beta_\tau + \ \varphi_{1q}\ \tilde{v}_i\ + 2\ \varphi_{2q}\, \eta^{-1/2} \sqrt{v_i}\ z_i\right)$$

$$f(y|x,\tilde{v}_i,\beta,\eta,z\,)$$

$$= \left\{\prod_{i=1}^{n}\prod_{q=1}^{Q}\left(\frac{1}{\sqrt{2\pi\eta^{-1}\varphi_{2q}^2\tilde{v}_i}}\right)^{c_{iq}}\right\} exp\left[-\frac{1}{2}\sum_{i=1}^{n}\sum_{q=1}^{Q}\frac{c_{iq}\left(y_i - \alpha_{\tau_q} - x_i^T\beta_\tau - \varphi_{1q}\right.}{\eta^{-1}\varphi_{2q}^2\tilde{v}_i}\right.$$

$$p(\alpha_q) \propto 1$$

$$(\tilde{v}|\eta) \sim \eta^n exp\left(-\eta\sum_{i=1}^{n}\tilde{v}_i\right)$$

$$z \sim \left(\frac{1}{\sqrt{2\pi}}\right)^n \quad exp\left(-\frac{1}{2}\sum_{i=1}^{n} z_i{}^2\right)$$

$$\beta|\,\eta,\lambda \sim\ uniform\left(-\frac{1}{\eta\lambda},\frac{1}{\eta\,\lambda}\right)$$

$$m \sim\ standard\ exponential$$

$$\eta \sim\ \ \eta^{a-1}\, exp^{(-b\eta)}$$

$$\lambda \sim\ \lambda^{c-1}e^{-d\lambda}$$

$w \sim$ ***Dirichlet*** $(d_1,\ldots,d_h)$

(a,b,c and d) are hyper parameter

Based on the hierarchical model that described above , we can write down the joint posterior distribution as follows :

$$\pi\left(\boldsymbol{\beta}, \boldsymbol{\alpha_q}, \boldsymbol{\eta}, \tilde{\boldsymbol{v}}, \boldsymbol{w}, \boldsymbol{c}, \boldsymbol{z}, \boldsymbol{\lambda} \middle| \boldsymbol{y}, \boldsymbol{x}\right)$$

$$\propto \pi\left(\boldsymbol{y} \middle| \boldsymbol{x}, \boldsymbol{v}, \boldsymbol{\beta}, \boldsymbol{\alpha_q}, \boldsymbol{c}, \boldsymbol{\eta}, \boldsymbol{w}\right) \prod_{\boldsymbol{i=1}}^{\boldsymbol{n}} \pi(\tilde{\boldsymbol{v}}_{\boldsymbol{i}} | \boldsymbol{\eta}) \prod_{\boldsymbol{j=1}}^{\boldsymbol{k}} \pi\left(\boldsymbol{\beta_j} \middle| \boldsymbol{\eta}, \boldsymbol{\lambda}\right) \pi(\boldsymbol{\eta}) \pi(\boldsymbol{w})$$

6. Simulation stady

6.1.1 The First Simulation Scenario

In this section, we will used very sparse case where the data of first simulation scenario is generated by the following model

$$\boldsymbol{y_i} = \boldsymbol{x^t \beta} + \boldsymbol{ui}$$

where $y_i[i = 1,2, \dots, n]$ is response variable and x^t is matrix of independent variables are generated with multiple normal distribution , where $X \sim N_k(0, \Sigma_x)$ with $(\Sigma_x)_{lg} = (0.5)^{|l-j|}$. $\beta = (5,0,0,0,0,0,0,0)^{\mathrm{t}}$ are vector of true parameters in very sparse case. ui is random distribution as shown in above speech

In below table , we presented the values of **MAE** and **SD** for three methods under compressions as shown below:

Table -1- the mean absolute error and standard deviation for first simulation scenario

methods	$ui \sim N(2,4)$	$ui \sim N(0,1)$	$ui \sim t_{(5)}$	$ui \sim Lab(0,1)$	$ui \sim \chi^2_{(5)}$	$ui \sim 0.5N(2,2) + 0.5N(-2,2)$
$LqReg_{(\tau_1=0.14)}$	1.892 (0.497)	1.886 (0.534)	1.935 (0.493)	1.784 (0.672)	1.805 (0.672)	1.675 (0.686)
$LqReg_{(\tau_2=0.29)}$	1.563 (0.673)	1.673 (0.561)	1.707 (0.634)	1.694 (0.593)	1.581 (0.693)	1.496 (0.732)
$LqReg_{(\tau_3=0.43)}$	2.110 (0.951)	1.904 (0.763)	1.484 (0.507)	1.854 (0.564)	1.795 (0.845)	1.898 (0.674)

$LqReg_{(\tau_4=0.57)}$	1.896 (0.844)	1.574 (0.873)	1.793 (0.742)	1.509 (0.688)	1.829 (0.637)	1.444 (0.793)
$LqReg_{(\tau_5=0.71)}$	1.264 (0.573)	1.342 (0.434)	1.462 (0.523)	1.456 (0.466)	1.378 (0.387)	1.193 (0.181)
$LqReg_{(\tau_6=0.86)}$	1.193 (0.487)	1.165 (0.394)	1.186 (0.267)	1.141 (0.275)	1.128 (0.187)	1.067 (0.063)
BCQRegU	0.985 (0.363)	0.946 (0.163)	1.089 (0.572)	1.174 (0.692)	1.341 (0.677)	0.926 (0.315)
$BCQR_{new\ prior}$	**0.784** **(0.129)**	**0.851** **(0.193)**	**0.906** **(0.117)**	**0.824** **(0.156)**	**0.684** **(0.142)**	**0.572** **(0.116)**

Note: The results are averaged over 100 independent simulations.

From the results listed in table 1, the performance of **BCQR new prior** $\mathbf{BCQR_{new\ prior}}$ appears to be far better than **LqReg** and **BCQRegU**.

In general, the **MAE** and **SD** generated by our proposed method $\mathbf{BCQR_{new\ prior}}$ is appear quite much smaller than the **MAE** and **SD** generated by other methods under comparison through all various distributions of random error. Also ,we see the two methods **BCQRegU** and **BCQR new prior** appear quite better than **LqReg** method ,Perhaps the reason for this is in these two methods **BCQRegU** and **BCQR new prior** used composite quantile regression instead of single quantile regression . To verify the stability of the algorithms, it is necessary to draw the trace plot to observe the possibility of its stability through the iterations imposed by the researcher. As followin

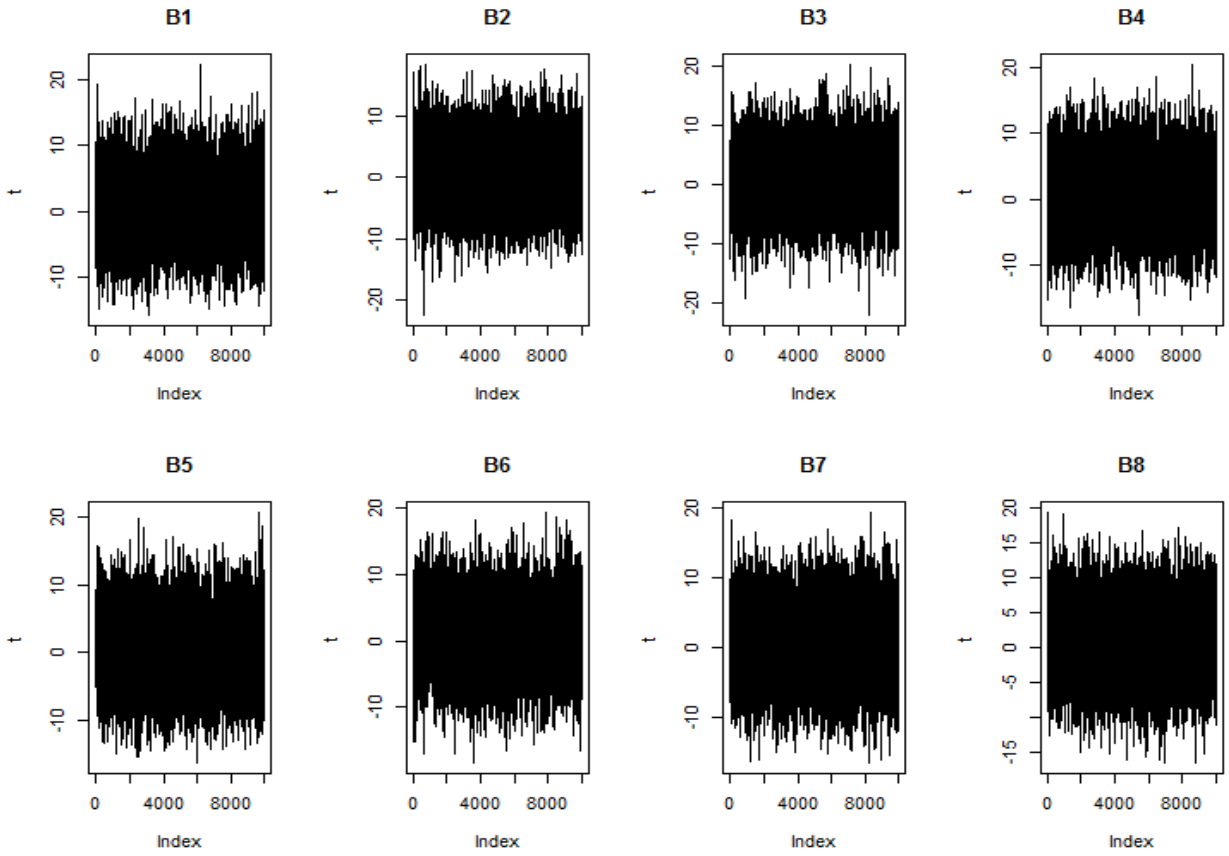

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Figure -1- Trace plots of our proposed method (BCQR $_{\text{new prior}}$) for simulation in very sparse case

from trace plot shown in figure1 ,we see our **MCMC** samples of the posterior distribution was convergence to very stationary

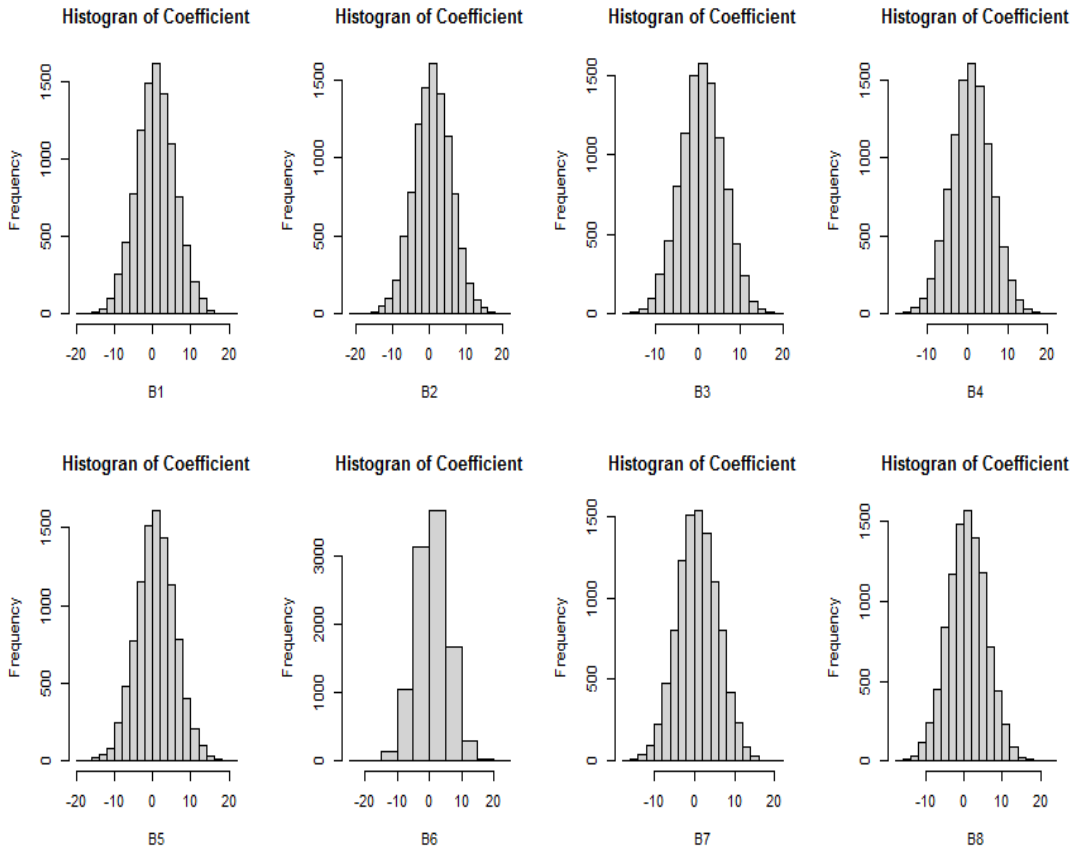


figure-2- Histogram of our proposed method (BCQR $_{new\ prior}$) parameter estimate for simulation in very sparse case

From the above histogram graphs to parameters estimates that belong to our method (**BCQRegN lasso**) is very closed from histogram of normal distribution.

6.1.2 The second Simulation Scenario

In this section, we will used sparse case where the data of second simulation scenario is generated by the following model

$$\boldsymbol{y_i = x^t \beta + ui}$$

where $y_i[i = 1,2, \dots, n]$ is response variable and x^t is matrix of independent variables are generated with multiple normal distribution , where $X \sim N_k(0, \Sigma_x)$ with $(\Sigma_x)_{lg} = (0.5)^{|l+g|}$. β =

$(0,1,0,5,0,0,0,1)^{t}$ are vector of true parameters in sparse case. ui is random distribution as shown in above speech

In below table , we presented the values of **MAE** and **SD** for three methods under compressions as shown below:

Table -2- the mean absolute error and standard deviation for first simulation scenario

methods	$ui \sim N(2,4)$	$ui \sim N(0,1)$	$ui \sim t_{(5)}$	$ui \sim Lab(0,1)$	$ui \sim \chi^2_{(5)}$	$ui \sim 0.5N(2,2) + 0.5N(-2,2)$
$LqReg_{(\tau_1=0.14)}$	1.319 (0.737)	1.892 (0.865)	1.979 (0.872)	1.003 (**0.568**)	1.562 (**0.873**)	1.041(**0.793**)
$LqReg_{(\tau_2=0.29)}$	1.182 (**0.815**)	1.813 (**0.968**)	1.635 (**0.883**)	1.314 (**0.652**)	1.983 (**0.978**)	1.963 (**0.986**)
$LqReg_{(\tau_3=0.43)}$	1.813 (**0.968**)	1.172 (**0.623**)	1.614 (**0.856**)	1.872 (**0.902**)	1.983 (**0.978**)	1.952 (**0.896**)
$LqReg_{(\tau_4=0.57)}$	1.739 (0.596)	1.599 (0.509)	1.949 (0.781)	1.572 (**0.751**)	1.452 (**0.768**)	1.963 (**0.986**)
$LqReg_{(\tau_5=0.71)}$	1.521 (0.382)	1.849 (0.680)	1.745 (0.569)	1.476 (0.294)	1.417 (0.295)	1.581 (0.407)
$LqReg_{(\tau_6=0.86)}$	1.302 (0.627)	1.589 (0.519)	1.394 (0.606)	1.3691 (0.762)	1.290 (0. 670)	1.152 (0.779)
BCQRegU	0.850 (0.323)	0.956 (0.215	0.808 (0.349)	0.772 (0.281)	0.789 (0.108)	0.949 (0.247)
$BCQR_{new\ prior}$	0.643 (0.283)	0.731 (0.153)	0.690 (0.144)	0.592 (0.221)	0.673 (0.231)	0.724 (0.174)

Note: The results are averaged over 100 independent simulations.

From the results listed in table 2, the performance of **BCQR new prior** $\mathbf{BCQR_{new\ prior}}$ appears to be far better than **LqRe**g and **BCQRegU**.

In general, the **MAE** and **SD** generated by our proposed method $\mathbf{BCQR_{new\ prior}}$ is appear quite much smaller than the **MAE** and **SD** generated by other methods under comparison through all various distributions of random error. Also ,we see the two methods **BCQRegU** and **BCQR new prior** appear quite better than **LqReg**

method ,Perhaps the reason for this is in these two methods **BCQRegU** and **BCQR** $_{\textbf{new prior}}$ used composite quantile regression instead of single quantile regression . To verify the stability of the algorithms, it is necessary to draw the trace plot to observe the possibility of its stability through the iterations imposed by the researcher. As following :

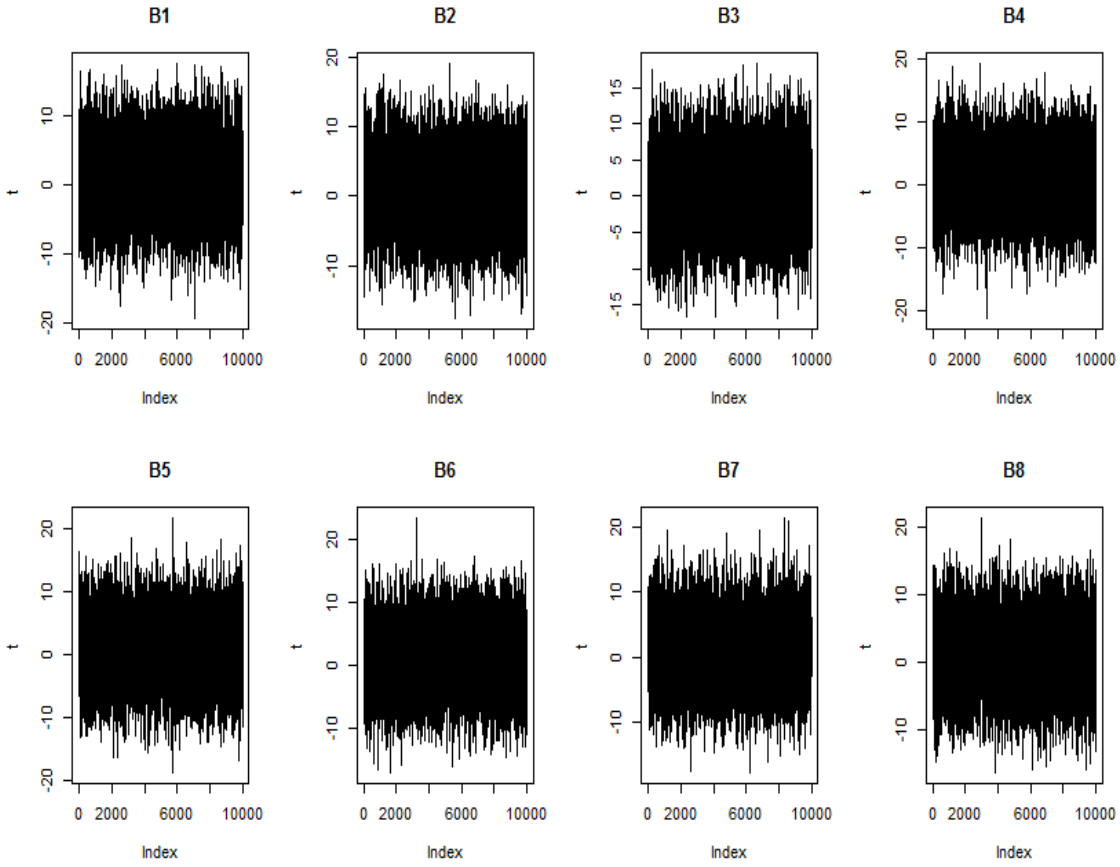


Figure -3- Trace plots of our proposed method (BCQR $_{\text{new prior}}$) for simulation in sparse case

from trace plot shown in figure1 ,we see our **MCMC** samples of the posterior distribution was convergence to very stationary.

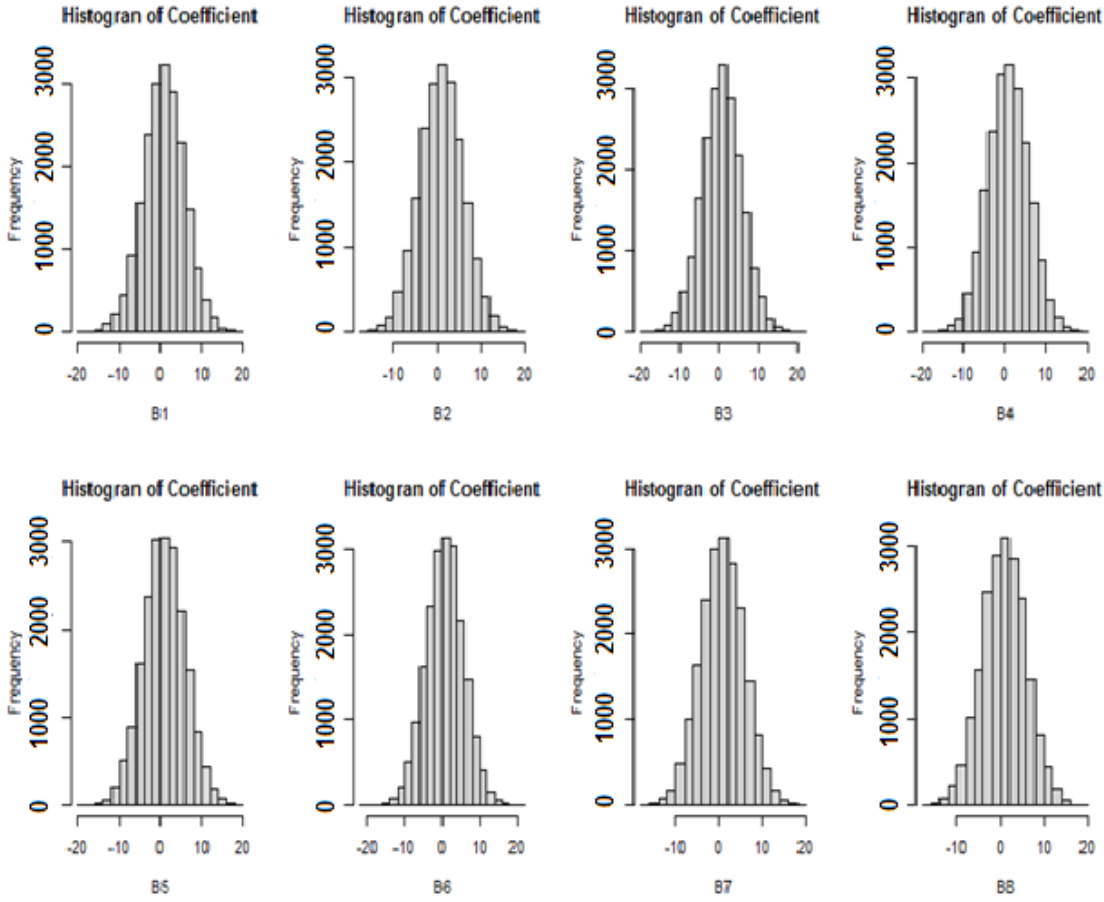


figure-4- Histogram of our proposed method (BCQR $_{new\ prior}$) parameter estimate for simulation in sparse case

From the above histogram graphs to parameters estimates that belong to our method (**BCQRegN lasso**) is very closed from histogram of normal distribution

Table (3) shows the study variables

Symbol of	Name of Variables	Term of Variables	Description of Variables

Variables			
y_1	Thrombocytopenia	$plt \downarrow$	Thrombocytopenia : less than 150000 per microliter of blood flowing
x_1	Serum Ferritin	S. Ferritin	S. Ferritin is a blood protein that contains iron. normal measurements of S. Ferritin is man 30-350ng/l and woman 20-250ng/l
x_2	Vitamin D	Vit D	Normal measurements of Vitamin D is 62.4-199.68L/nmol
x_3	Vitamin C	Vit C	Normal measurements of Vitamin C is 0.4 to 2.0 mg/dL
x_4	Immune thrombocytopenia	ITP	It is autoimmune disorder
x_5	human immunodeficiency virus	HIV	It is virus that attacks white blood cells and weakens the immune system
x_6	Chronic lymphocytic *leukemia*	CLL	It is a type of cancer in which the bone marrow .
x_7	Mean Platelet Volume	MPV	Normal measurements of MPV is 9.0-17.0/Fl
x_8	Platelet *distribution* width	PDW	Normal measurements of MPV is 9.2-16.7/Fl
x_9	Random blood sugar	R.B.Sugar	Normal measurements of R.B.Sugar F:80-120Mg/dl Normal measurements of

			R.B.Sugar F:80-120 R:Upto180Mg/dl
x_{10}	White blood cells	WBC	Normal measurements of WBC(3.50-9.50) $\times 10^3/UL$
x_{11}	Red blood cells	RBC	Normal measurements of RBC(3.80-5.80) $\times 10^6/UL$
x_{12}	Granulocyte test	Gran	Normal measurements of Gran(50. 0-70. 0) UL
x_{13}	Mean corpuscular volume	MCV	Normal measurements of MCV(82. 0-100. 0) FL
x_{14}	Mean corpuscular hemoglobin concentration	MCHC	Normal measurements of MCHC (31. 6-35. 4) g/dL
x_{15}	Blood urea nitrogen	B.urea	Normal measurements of B.urea (15-40.) mg/dL
x_{16}	Serum Creatinine	S. Creatinine	Normal measurements of S. Creatinine (1.8-1.9.) mg/dL
x_{17}	Low density lipoprotein	LDL	Normal measurements of LDL (2-30.) mg/dL
x_{18}	High density lipoprotein	HDL	
x_{19}	Packed cell volume	PCV	
x_{20}	Age	Age	

We use these real data to compare our proposed method with another method in the same filed. These data set collected from al-rafidin laboratory ,the sample size under our study was 120 observations . To coefficients estimation of independent variables our algorithm has been used . Similar to simulated scenarios,

when our algorithm runs 13,000 times and the initial 3,000 times are eliminated. To compare methods under current study

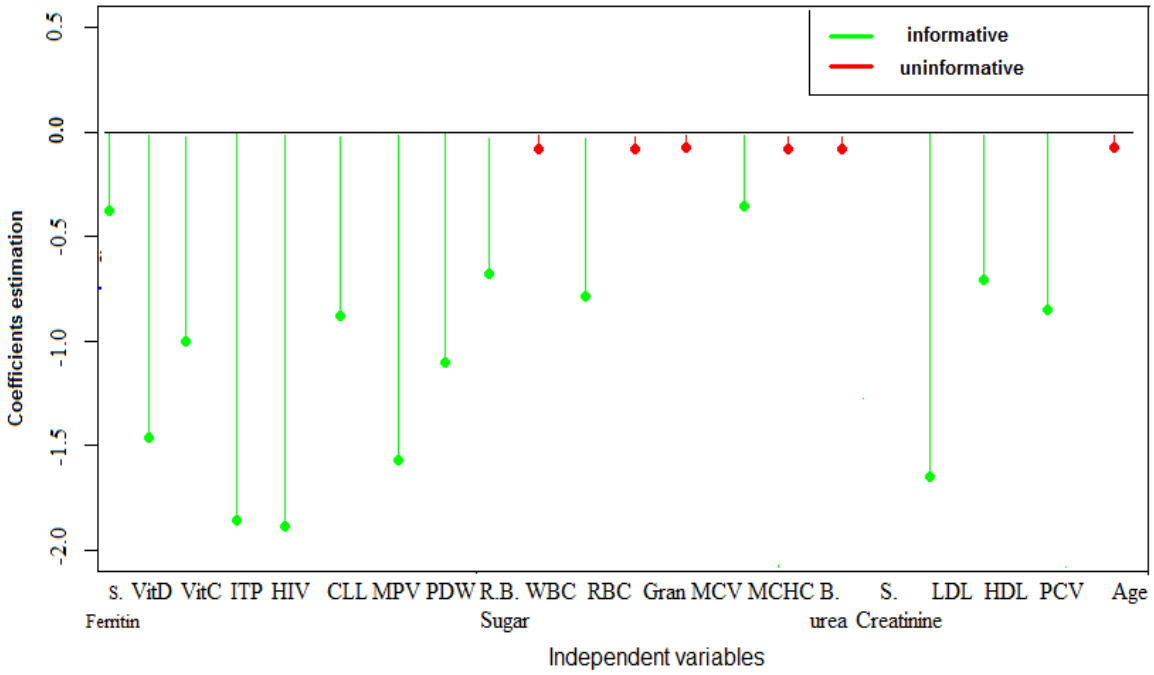
the mean square error **(MSE)** has been calculated for non-Bayesian method ($\mathbf{LqReg}\ at\ six\ quatile\ levels$) and Bayesian method **(BCQRegU)**($\mathbf{BCQR_{new\ prior}}$)as as the following $table$

Table - 4- Mean square errors for our proposed method ($\mathbf{BCQR_{new\ prior}}$),(**BCQRegU**)and$\mathbf{LqReg}\ at\ six\ quatile\ level$) methods

Methods	$LqReg$						BCQRegU	$BCQR_{new\ prior}$
Quantile levels	$\tau_1 = 0.14$	$\tau_2 = 0.29$	$\tau_3 = 0.43$	$\tau_4 = 0.57$	$\tau_5 = 0.71$	$\tau_6 = 0.86$	Q=6	Q=6
MSE	32.672	31.452	28.341	27.056	25.451	20.162	10.251	8.895

The mean square error (**MSE**) is generated by our proposed method

$\mathbf{BCQR_{new\ prior}}$ is equal 8.895. The mean square error**(MSE)** is generated by **(BCQRegU)** is equal 10.251 ,and the mean square error (MSE) of non-Bayesian method ($\mathbf{LqReg}\ at\ six\ quantile\ levels$) are (32.672, 31.452, 28.341, 27.056, 25.451 and 20.162) respectively . Clearly, we can see that the **MSE** is generated by our proposed method $\mathbf{BCQR_{new\ prior}}$ is much smaller than the **MSE** generated by Bayesian and non-Bayesian methods (**BCQRegU** and $\mathbf{LqReg}$) respectively. Therefore , the our proposed method ($\mathbf{BCQR_{new\ prior}}$) is outperformed on Bayesian and non- Bayesian methods under current study .Therefore, we will use our proposed method ($\mathbf{BCQR_{new\ prior}}$) to coefficients



estimation and variables selection for the our real data . The following figure shows the coefficients estimation and variables selection for independent variable

Figure –5 -shown the coefficients estimation of independent variables

From above figure , there are fourteen independent variables have non-zero coefficients and also they have negative effecting on Thrombocytopenia variable. But, there are six independent variables have zero coefficients, these six independent variables are uninformative on Thrombocytopenia variable. this mean ,we can exclude these six independent variables from construction our model.

7.Conclusion and Recommendation

7.1 Conclusion

In this thesis, a new hierarchical Bayesian new **lasso** composite quantile regression**(CQR)**method was introduced. When compared to other algorithms, our Gibbs sampler algorithm was easy to use and efficient. The simulation methodology and real dataset clearly show that our method, **BCQRegN lasso**, performs well when compared to other approaches in the same field. Therefore, we will conclude the following points:

1- In our current study, a hierarchical model has been proposed, characterized by ease and clarity in estimating the model parameters. Consequently, the algorithm employed in our proposed method exhibited remarkable stability during the initial iterations. Therefore, our algorithm demonstrates efficiency and speed in completing the model estimation process under study.

2- Our proposed hierarchical model enjoys ease and flexibility in to arriving the statistical distributions associated with the parameters of this model in a easy and straightforward method.

3- We find that all estimated model parameters of the studied model conform to a normal distribution, which aligns with the theoretical distribution of the parameters.

4- The our proposed method was highly effective in estimating the parameters of the model that models the relationship between the response variable, Thrombocytopenia and a set of explanatory variables.

5- Our proposed method successfully identified and excluded 6 non-informative explanatory variables in modeling the relationship

between Thrombocytopenia and a set of explanatory variables. Also, included only the important variables in constructing and modeling this studied model.

6- we see that all the important variables in constructing and modeling the relationship between Thrombocytopenia and some independent variables had an inverse effect

7.2 Recommendation

We recommend using mixture distributions to reframe the Laplace distribution in methods concerned with variable selection processes.

1- Developing and proposing new hierarchical models that incorporate both parameter estimation and variable selection processes in a more flexible method.

2- Constructing new hierarchical models by combining the new regularization methods with composite quantile regression**(CQR)** models, ensuring a seamless approach for parameter estimation and variable selection.

3- Utilizing new regularization techniques in estimating coefficients and selecting variables for a regression model with a response variable (Thrombocytopenia) and a set of explanatory variables.

4- Expanding the current study by modeling the relationship between Thrombocytopenia and a set of independent variables through the utilization of a Tobit regression model or other alternative models.

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