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Achieving Cost Balance in Wheat Production: A Strategy for Cost Reduction

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Abstract: The study focused on maximizing the amalgamation of production cost elements to achieve production equilibrium as a cost-reduction technique for wheat farming. A mathematical model utilizing the mixed-number equation was employed to reduce total costs, including production quantity and expenses, through the application of the Lagrange multiplier equation pertaining to production equilibrium. This technique can address the challenge of determining the appropriate production quantity for agricultural regions and the inability to identify the best production parameters that create equilibrium and reduce costs. The outcomes attained production equilibrium via the optimal amalgamation of production elements, considered a cost-reduction strategy for wheat farming in Iraq.

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تحقيق التوازن في التكاليف لإنتاج القمح: استراتيجية لخفض التكاليف

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المستخلص

ركزت الدراسة على تعظيم دمج عناصر تكلفة الإنتاج لتحقيق توازن الإنتاج كأسلوب لخفض تكاليف زراعة القمح. واستُخدم نموذج رياضي يستخدم معادلة الأعداد المختلطة لخفض التكاليف الإجمالية، بما في ذلك كمية الإنتاج والنفقات، من خلال تطبيق معادلة مضاعف لاغرانج المتعلقة بتوازن الإنتاج. ويمكن لهذه التقنية أن تعالج تحدي تحديد كمية الإنتاج المناسبة للمناطق الزراعية، والقدرة على تحديد أفضل معايير الإنتاج التي تحقق التوازن وتخفض التكاليف. وقد تم تحقيق توازن الإنتاج من خلال الدمج الأمثل لعناصر الإنتاج، والذي يُعتبر استراتيجية لخفض تكاليف زراعة القمح في العراق.

الكلمات المفتاحية: تكلفة الإنتاج، توازن الإنتاج، خفض التكاليف، محاسبة التكاليف، الاستراتيجية.

1 Introduction

Ensuring food availability is an essential goal for the economic progress of every nation. The cultivation of prime farmland is essential for stabilizing and improving grain production capacity. Food production and the accessibility of arable land are crucial for survival, advancement, regional security, and stability of humanity. The growing worldwide population is accompanied by an increasing demand for diverse food requirements. However, the limit of arable land on Earth has been mainly stable during the 21st century. This situation has increased the pressure on food production due to the inequality between the site of food production and the accession of fertile land resources (Hao et al., 2024). Land fragmentation and low conspiracy size, by spoiling the efficiency of farming, including labor, capital and land, adversely affect agricultural yields and costs. Increasing the size of the plot can lead to a decrease in average prices, which reflects the presence of economies of the scale level at the plot level. This suggests that the structure and integration of production factors including land, labor and capital can improve, increase efficiency and reduce costs (Lu et al., 2018). Production balance is an important concept in economics, defined as optimal amalgamation of production factors. Various strategies have been employed to achieve balance, including the application of the lagrange equation (Xiao et al., 2023). Production balance, or factor combination, refers to the ideal distribution of production elements, including labor, capital, and land, so that production can be maximized or reduced costs. The production balance refers to optimal allocation of various production inputs to maximize output. This idea can be used for expenses and production forecasting (Cui et al., 2022). Developing functioning that assess cost factors through both long -term and short -term approaches to prioritize the schedule of the project and analyze the results of assessing adaptive costs, assisting to determine the amount of production and control expenditure (Im et al., 2022).

2 Literature review

2.1 Production balance: Production is the process of transforming inputs into outputs. Inputs, or factors of production, include land, labor, and capital (Al-Jazaerli, 2020). To create an effective system, agricultural production must be thoroughly investigated. This evaluation will assist in analyzing agricultural practices, comparing various systems and approaches, and ultimately improving the agrarian framework. The goal is to protect the environment by combining multiple methods that maintain agricultural balance while also offering economic and environmental benefits (Shi & Umair, 2024). To develop a cost-effective manufacturing process, it is essential to maintain a consistent level of technological innovation that aligns with the organization's needs. The results will contribute to preserving productivity equilibrium and developing a more efficient production framework (Du et al., 2021).

Overall instability has intensified, notably in agricultural operations and environmental security, resulting in increasingly significant changes. Due to variations in factor levels, long-term production rearrangements result in significantly different outputs. Understanding sustainable agricultural production is crucial for studying the relationship between humans and the land, charting a sustainable path within this complex balance, and achieving the optimal composition of production materials while maintaining productivity (Qu et al., 2024). Addressing production time and length issues is crucial because they help achieve balance and increase production efficiency (Prata et al., 2024). Technological reserves, temporal elements, attributes, market dynamics, community requirements, and production-related variables all influence production forecasting model selection (Wang et al., 2024). According to work in (Lia et al., 2021), capital, labor costs, and additional inputs such as land and equipment are crucial for cost-effective and profitable production. Production equilibrium, also known as factor combination, is a crucial concept in economics. Production equilibrium is the condition in which the optimal mix of production factors (including labor, capital, and raw materials) can be achieved to maximize project output. The equilibrium is determined by the marginal productivity of each component and its corresponding relative prices. Companies endeavor to achieve production equilibrium by adjusting their use of production factors until the marginal product of each factor equals its price. This process ensures the organization minimizes expenses and optimizes profits (Maeda & Watts, 2019). The producer's purpose is to enhance production efficiency or reduce costs. The producer adopts the first technique when constrained by budgetary limitations and the second strategy when restricted by production limitations. To achieve maximum production at minimal cost, the producer seeks the most efficient combination of production factors, specifically labor (L) and capital (K). This can be achieved by utilizing the subsequent approaches(Nassira, 2023). Engineering method: Through the drawing, the product equilibrium point is the point of contact of the equal cost line with the highest output curve, and accordingly, the combination between K and L is the optimal combination that achieves product equilibrium. Mathematical method: The Lagrange multiplier method can be used to obtain the optimal combination of production factors to reach product equilibrium in both cases (maximization and minimization)

2.2 Reduces production costs: The impact of diverse production systems on the balance between agricultural output and economic sustainability. The statement emphasizes the importance of integrating several components to achieve optimal equilibrium in the production system, considering it a strategy for cost reduction (Shi & Umair, 2024). The concept of optimal is essential in contemporary times and is viewed as a standard for the farmer's judicious economic decision-making. The optimal production amount is the level at which costs are minimized. The optimal production volume is characterized as the quantity that attains the lowest price and the highest return per unit of output (Al-Jashamy & Al-Azzy, 2012).

The Cobb-Douglas production function is used to determine the optimal combination of inputs (land, labor, and capital) that minimizes production costs. The Lagrange multiplier method is utilized to achieve this objective, with the Lagrange multiplier acting as the shadow price or marginal cost of production (Mohajan, 2021). Analytical approaches provide a thorough analysis of the production process and the composition of various production components, yielding a more accurate cost estimate (Aram et al., 2014). Infrastructure is a key factor influencing agricultural development, particularly in resilience to natural disasters and reducing agricultural inputs. Infrastructure generally includes public utilities (such as electricity, telecommunications, piped water supply, sewerage, solid waste collection and disposal, and gas pipelines), public works, and other transportation sectors. It can reduce input costs and help achieve significantly higher output prices, thereby reducing costs. In particular, irrigation facilities and roads have a substitution or complementary effect on labor, capital, and other factors of agricultural production(Wu et al., 2019) In the context of managing and enhancing agricultural production systems, accurately identifying the components of an optimization model helps save costs by minimizing production waste. This methodology may be unfamiliar to farmers and other stakeholders, such as manufacturing firms, seeking to manage agricultural operations, as it offers a structured framework for operational planning processes. They can obtain substantial advantages by minimizing environmental impact, labor, machinery, and waste, enhancing quality, timing, movement, and other resources utilized in sowing, planting, and harvesting, as well as ascertaining the optimal area and quantity of planting to maximize yield(Solano et al., 2022).

3 **Materials and Methods:** This section describes the data collection, production function specification, and optimization technique used to identify the most cost-effective input mix for wheat cultivation in Iraq for the 2024 season.

3.1 Study Area and Data Collection: Structured, in-person interviews were administered to 50 wheat producers from March to July 2024 in significant agricultural areas of central and northern Iraq. Each participant provided detailed information on their per-acre input use and associated expenses. Using the farm-level data, we calculated the following average cost parameters:

Table (1): Per-acre input parameters and associated costs.

parameter	Cost in IQD per unit
Capital input (K)	5 000
Labor input (L)	100 000
Total production budget (T)	500 000

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All cost estimations represent current market prices and were validated by cross-checking with local cooperative societies.

Data for the 2024 wheat crop were collected from the website of the Central Statistical Agency of the Ministry of Planning. The researchers summarised the data results for wheat quantity, cultivated area, and average yield for 2024, as shown in the following tables and figures.

The Central statistics Organization, along with the Iraqi Ministry of Planning, publishes its annual statistics report on the wheat harvest. Wheat is a crucial crop in Iraq and globally, integral to the national economy and essential for supplying food to the populace (Ezzat & Sayed, 2023).

Wheat production is estimated at 5,234,000 tons for the winter season of 2024, an increase of 23.2% compared to last year's production of 4,248,000 tons. Nineveh Governorate ranked first in terms of production, which was estimated at 1,394,000 tons, representing 26.6% of the total production, followed by Salah al-Din Governorate, which was estimated at 854,000 tons, representing 16.3% of the total production, and Wasit Governorate, which was estimated at 581,000 tons, representing 11.1% of the total production. The rest of the governorates accounted for 46% of the total production, as shown in Table 2. Wheat production on irrigated lands was estimated at 4,195,000 tons, representing 80.1%, as shown in Table 4، while wheat production on rain-fed lands was estimated at 1,039,000 tons, representing 11.1% of the total production. 19.9%, as in Table 3.

The cultivated area for wheat was estimated at 8,177,000 dunums for the 2024 winter season, a decrease of 2.9% compared to the previous season, which was estimated at 8,420,000 dunums, the harvested area for this season was estimated at 8,119,000 dunums, representing 99.3% of the total cultivated area, as shown in Table 2. The cultivated area in irrigated lands was estimated at 5,036,000 dunums, representing 61.6%, as shown in Table 4، while the cultivated area in rain-fed lands was estimated at 3,141,000 dunums, representing 38.4% of the total cultivated area, as shown in Table 3.

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Table (2): Cultivated area, average yield per donum, and production of Wheat by governorate for 2024

Governorates	Cultivated area (Donum)			Production (Ton)	percentage	Average yield KG/Donum	
	Total area	Harvested area	Damaged area			Total area	Harvested area
Ninevah	3,532,179	3,505,685	26,494	1,393,738	26.6	394.6	397.6
Kirkuk	532,251	532,251	0	439,535	8.4	825.8	825.8
Diala	336,798	336,798	0	248,190	4.7	736.9	736.9
Anbar	433,252	433,252	0	373,067	7.1	861.1	861.1
Baghdad	35,029	34,779	250	30,112	0.6	859.6	865.8
Babylon	143,912	135,118	8,794	138,498	2.6	962.4	1025.0
Karbala	182,737	182,672	65	165,458	3.2	905.4	905.8
Wasit	685,724	685,724	0	580,739	11.1	846.9	846.9
Salah-Aldeen	957,154	954,293	2,861	854,272	16.3	892.5	895.2
Al-Najaf	219,256	219,256	0	174,932	3.3	797.8	797.8
Al-Qadisiya	441,728	441,728	0	355,265	6.8	804.3	804.3
Al-Muthnna	344,128	326,109	18,019	231,549	4.4	672.9	710.0
Thi-Qar	128,458	126,945	1,513	90,044	1.7	701.0	709.3
Maysan	164,899	164,899	0	127,566	2.4	773.6	773.6
Al-Basra	39,672	39,672	0	31,206	0.6	786.6	786.6
Total	8,177,177	8,119,181	57,996	5,234,171	100.0	640.1	644.7

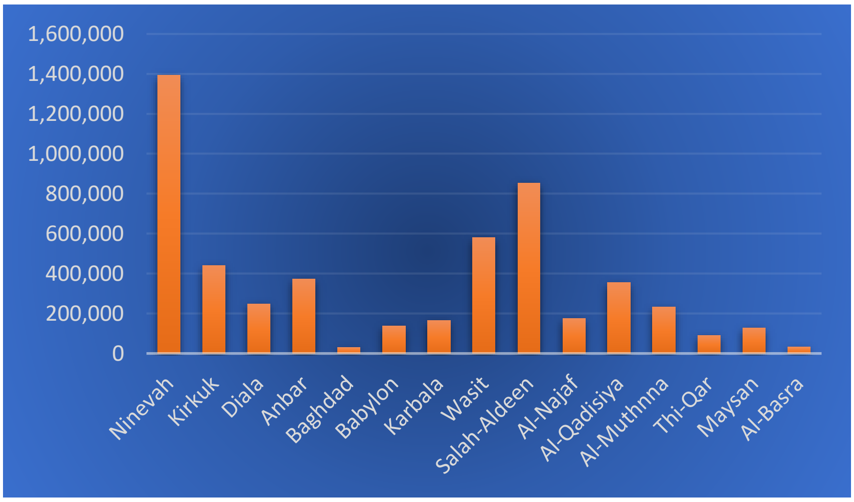

Fig. 1. Quantity of wheat production by governorate for 2024

Figure 1 elucidates to researchers that the disparity in wheat output among Iraqi governorates is attributable to the imbalance between production volume and the agricultural areas capable of achieving optimal yields. This variation in production and instability necessitates a focus on

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identifying effective strategies for achieving compositional balance in production. This can affect the value of Y, which represents the equilibrium production and is a linear multiplier function of the capital and labour inputs.

Table (3): Cultivated area, average yield per donum and quantity of wheat production in rain fed area by governorate for 2024

Governorates	Cultivated area (Donum)			Production (Ton)	average yield KG/Donum	
	Total area	Harvested area	Total area		Total area	Harvested area
Ninevah	2,956,575	2,930,081	26,494	955,698	323.2	326.2
Kirkuk	131,516	131,516	0	57,602	438.0	438.0
Diala	28,163	28,163	0	15,576	553.1	553.1
Salah-Aldeen	21,964	19,103	2,861	9,459	430.7	495.2
Maysan	2,870	2,870	0	1,148	400.0	400.0
Total	3,141,088	3,111,733	29,355	1,039,483	330.9	334.1

Table (4): Cultivated area, average yield per donum and production quantity of Wheat in irrigated area for private sector by governorate for 2024

Governerates	Cultivated area (Donum)			Production (Ton)	average yield KG/Donum	
	Total area	Harvested area	Damaged area		Total area	Harvested area
Ninevah	575,604	575,604	0	438,040	761.0	761.0
Kirkuk	400,735	400,735	0	381,933	953.1	953.1
Diala	308,635	308,635	0	232,614	753.7	753.7
Anbar	433,252	433,252	0	373,067	861.1	861.1
Baghdad	35,029	34,779	250	30,112	859.6	865.8
Babylon	143,912	135,118	8,794	138,498	962.4	1025.0
Karbala	182,737	182,672	65	165,458	905.4	905.8
Wasit	685,724	685,724	0	580,739	846.9	846.9
Salah-Aldeen	935,190	935,190	0	844,813	903.4	903.4
Al-Najaf	219,256	219,256	0	174,932	797.8	797.8
Al-Qadisiya	441,728	441,728	0	355,265	804.3	804.3
Al-Muthnna	344,128	326,109	18,019	231,549	672.9	710.0
Thi-Qar	128,458	126,945	1,513	90,044	701.0	709.3
Maysan	162,029	162,029	0	126,418	780.2	780.2
Al-Basra	39,672	39,672	0	31,206	786.6	786.6
Total	5,036,089	5,007,448	28,641	4,194,688	832.9	837.7

Table (5). Comparison of cultivated area, production quantity and average yield of wheat for 2019-2024

Details	yaer	Crop	
		Wheat	Annual rate of change
Cultivated Area (1000) Donum	2019	6,331	100.7
	2020	8,574	35.4
	2021	9,464	10.4
	2022	7,487	-20.9
	2023	8,420	12.5
	2024	8,177	-2.9
Production (1000) Ton	2019	4,343	99.4
	2020	6,238	43.6
	2021	4,234	-32.1
	2022	2,765	-34.7
	2023	4,248	53.6
	2024	5,234	23.2
Average Yield (Kg/Donum)	2019	686.1	-0.6
	2020	727.6	6.0
	2021	447.3	-38.5
	2022	369.3	-17.4
	2023	504.5	36.6
	2024	640.1	26.9

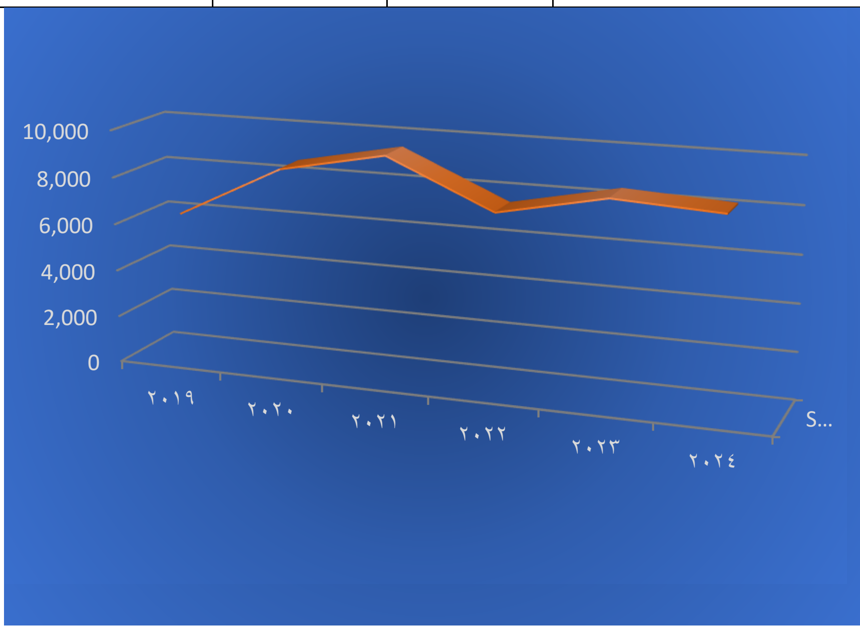

Fig. (2). Area cultivated for Wheat for (2019 - 2024)

Figure 2 shows the limit of land cultivated from 2019 to 2024. Researchers have discovered that arable land in the governor has demonstrated instability

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during these years, requiring an examination of the underlying causes of this variance and ups and downs. The cause may be inability to employ procedures or techniques that ensure production and balance in the region. This can affect the yield and production results of wheat and the use of the Lagrange multiplier method adopted in the study model.

Fig. 3. Wheat production for (2019 - 2024)

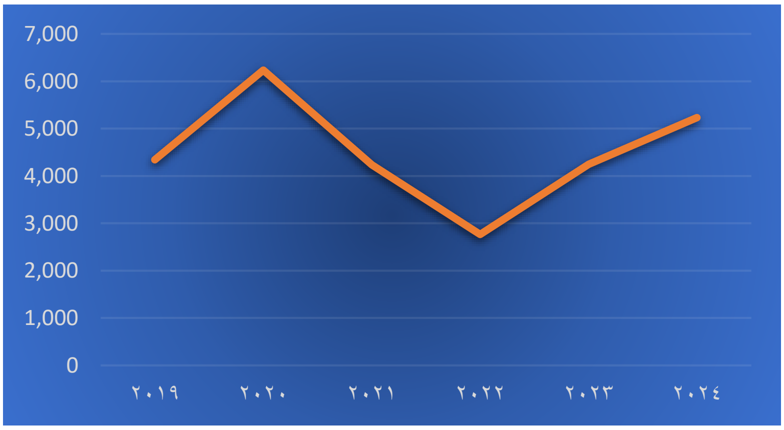

Figure 3 shows the production of wheat from 2019 to 2024. Researchers have determined that wheat crop cultivated in the governor over the years shows volatility, which is characterized by ups and downs in yield. As a result, it is necessary to examine factors contributing to this variation. The reason for this may be inability to employ methods or strategies that align production with the region. It can affect wheat yield and production results and the use of the Lagrange multiplier method adopted in the study model.

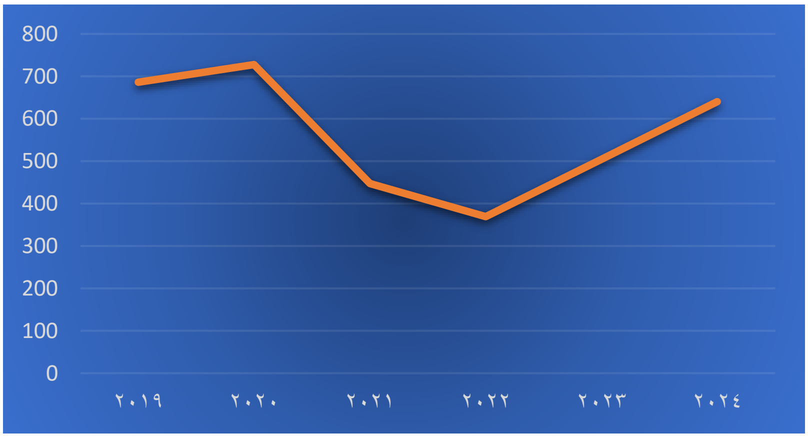

Fig. (4): Average yield of Wheat per donum for (2019 - 2024)

Figure 4 shows the average wheat production yield for a period from 2019 to 2024. Researchers have determined that the production of average wheat crop in the governor in these years is irregular, performing ups and rashes.

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As a result, it is necessary to examine the factors caused by this change. The cause may be insufficiency of the processes planned to achieve the appropriate production balance for the area. It can affect wheat yield and production results and the use of the Lagrange multiplier method adopted in the study model.

3.2 Production Function: The Cobb-Douglas framework, as used by (Fadheliya & Khudur, 2008), models wheat yield per acre ($\boldsymbol{Y}$) as a linear multiplicative function of capital and labor inputs**.**

$$Y = 2\,KL, \qquad 1$$

where Y is yield in quintals per acre, K is the number of capital units deployed, and L is the number of labor-unit equivalents.

3.3 Optimization Scenarios: To identify the most efficient allocation of inputs ($\boldsymbol{K, L}$), we formulated two mixed-integer programming problems:

Scenario I: Output Maximization

Formulate the production problem as a constrained maximization $\underset{K,L}{max}[Y]$, subject to

$$5000\,K + 100000L \leq 500000. \qquad 2$$

This scenario determines the (K, L) combination that yields the highest production given a fixed total expenditure, T.

Scenario II: Cost Minimization

Minimize total cost

$$C = 5000\,K + 100000L. \qquad 3$$

Subject to achieving a target yield (Y), this scenario yields the lowest-cost input mix necessary to attain a predetermined production level.

3.4 Solution Methodology: Both optimization problems were solved using the Lagrangian multiplier method: (Afra & Behnamian, 2021).

We created the Lagrangian functions for each situation, added multipliers for the budget or yield constraints, and determined the first-order conditions to define the optimal input ratios.

To ensure practicality, continuous solutions were rounded and validated using a mixed-integer programming solver, as K and L must be integer units.

To assess robustness, we adjusted input prices (±10%) and budget levels (±10%), resolving both situations and compared changes in optimal K and L.

4 **Results and Discussion:** There are two scenarios in which production equilibrium can be achieved:

The first case: finding the optimal combination of cost elements that yields the largest possible total output.

(Amounts in Iraqi dinars, 1 US dollar = 1500 Iraqi dinars in 2024)

The first requirement is to calculate the optimal combination of the two production elements (K and L) that achieves the project's balance and allows for the largest number of products.

Therefore, the cost or spending budget equation is as follows:

$$500000 = 5000\,K + 100000\,L$$

We set this equation as an implicit function equal to zero and multiply it by the Lagrange multiplier:

$$\lambda\,(500000 - 5000\,K - 100000\,L)$$

We add the above result to the original production function, and we obtain the new function (V):

$$V = 2\,K.L + \lambda\,(500000 - 5000\,K - 100000\,L)$$

We derive this function for the three variables:

$$VK = 2\,L - 5000\,\lambda = 0 \qquad (I)$$

$$VL = 2\,K - 100000\,\lambda = 0 \qquad (II)$$

$$V\lambda = 500000 - 5000\,K - 100000\,L = 0 \quad (III)$$

We attribute equation (I) to equation (II) and move its second side to the other side:

$$(2\,L)/(2\,K) = (5000\,\lambda)/(100000\,\lambda)$$

We shorten and remove (λ) from the numerator and denominator, and then calculate the value of one variable in terms of the other:

$$10000\,K = 200000\,L \qquad K = 200000/10000\,L$$

$$L = 10000/200000\,K$$

$$K = 20\,L$$

We substitute in equation (III):

$$500000 - 5000\,K - 100000\,L = 0L$$

$$500000 - 5000\,(20\,L) - 100000\,L = 0$$

$$500000 - 100000\,L - 100000\,L = 0$$

$$500000 - 200000\,L = 0$$

$$200000\,L = 500000$$

$$L = \frac{500000}{200000} = 2.5$$

We substitute the value of L into the following equation to obtain the value of K:

$K = 20L, K = 20 * 2.5 = 50$

The optimal combination of production elements that achieves production balance for the project is shown to us as follows:

$\frac{L}{2.5}$، $\frac{K}{50}$ This confirms the required condition for optimality.

The second requirement is to ensure that the answer to the first requirement achieves the project’s balance:

The rule of project balance is: $UM = F\_K/P\_K = F\_L/P\_L$

knowing that:

(F_K) It is the marginal product of the capital element (K) and is calculated by deriving the original production function of (K).

(F_L) It is the marginal product of the labor cost element (L) and is calculated by deriving the original production function of (L).

(UM) It is the marginal utility of money, or the purchasing power of one unit of money.

The original production function is: $Y = 2K.L$

Derivative of the original production function with respect to (K):

$$Y\_K = F\_K = 2L = 2(2.5) = 5$$

Derivative of the original production function with respect to (K):

$$\mathrm{Y\_L = F\_L = 2K = 2(50) = 100}$$

We apply the consumer equilibrium rule to (UM):

$$\mathrm{UM = F\_K/P\_K = F\_L/P\_L}$$
$$= 5/5000 = 100/100000 = 0.001$$

The correctness of the answers to the optimal structure can also be verified by substituting them into the expenditure equation:

$500000 = 5000\,\mathrm{K} + 100000\,\mathrm{L}$

$500000 = 5000 * 50 + 100000 * 2.5 = 500000$

This confirms the required condition for optimality.

The third requirement is to calculate the total output (Y) from the answers obtained through the original production function:

$\mathrm{Y = 2K.L}$

$\mathrm{Y} = 2(50 * 2.5) = 250$ units

The cultivated area, average yield per acre, and production quantity of the wheat crop in Iraqi governorates for the agricultural season of 2024,

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From Tables No. 2, 3 and 4, we can reach the results shown below in Table No. 6.

Table (6). Optimal production quantity

average yield KG/Donum	Harvested area	Optimal production quantity	Production deviation
Irrigated	837.7	250	587.7
Rain fed	334.1	250	84.1
Total	644.7	250	394.7

(Unit = kilograms per donum)

We show in Table No. 5 that the average production yield for all Iraqi governorates is 644.7, as recorded in Table No. 1. It becomes clear to researchers that the average quantity that achieves production balance with the optimal combination of production elements is 250 units, and the difference appears to be 394.7 units from the average total output, and this indicates that using this method achieves the optimal combination of production elements to achieve balance, and this is what achieves the research objective.

The second case: finding the optimal combination of production factors that produces a specific amount of output at the lowest cost.

We take the total production from Table No. 5 to substitute it in the production equation:

$$Y = 2\,K.L$$

$$644.7 = 2K.L$$

The spending budget equation is: $T = PK.K + PL.L$

$$T = 5000K + 100000L$$

We make the production function an implicit function equal to zero and multiply it by the Lagrange multiplier:

$$\lambda\,(644.7 - 2K.L)$$

$$V = 5000K + 100000L + \lambda\,(644.7 - 2K.L)$$

We derive the new function with respect to the three variables (λ, L, K):

$VK = 5000 - \lambda\,2L = 0$ (I)

$VL = 100000 - \lambda\,2K = 0$ (II)

$V\lambda = 644.7 - 2K.L = 0$ (III)

We relate equation (I) to equation (II), and transfer to the other side:

$(\lambda\,2\,L)/(\lambda\,2\,K) = (5000)/(100000)$

$10000K = 200000L$

$K = 20L$

$V\lambda = 644.7 - 2K.L = 0$ (III)

$644.7 - 2(20L) = 0$

$644.7 = 40L2$

$$L2 = \frac{644.7}{40} = 16$$

$L = 4, K = 20L, K = 20(4), K = 80$

So the optimal combination of factors of production that achieves equilibrium for each unit is:

$\frac{L}{4}, \frac{K}{80},$ This confirms the required condition for optimality.

This indicates that the average production quantity (644.7) is required for each unit of capital (80) and wages (4) for each unit in order to achieve production balance and reduce costs.

The researchers find that the above results represent the lowest achievable cost per unit of capital and labour cost because the production balance is not achieved according to comparing the results with the first case, where each unit of capital was 50, while in the second case, each unit of capital was 80. The labour cost in the first case was 2.5 per unit, while in the second case, it was 4 per unit. This is evidence that confirms the need to achieve a balance in production to find the optimal combination of production elements, which are capital and labour cost, and this achieves the research objective.

While the current model provides valuable insights for improving labour and capital costs, it approaches the wheat production system in Iraq in a simplified manner to focus on cost management. There are some important factors that have recently been documented in field studies but have not been addressed: water scarcity, which affects 78% of Iraqi farms, according to a recent FAO report(FAO, 2023). Soil fertility has deteriorated due to salinization, which has reached critical levels in 35% of agricultural land (Sissakian et al., 2025). The impact of climate variability has led to a 12% decrease in average annual rainfall over the past decade, as documented by the World Meteorological Organization(Jaff, 2023). These factors combined explain the gap between the model's theoretical results and the actual data recorded in Tables 4-6.

5 Conclusions:

The purpose of this paper is to identify the optimal combination of production factors to achieve production balance and reduce costs, aims to provide a decision -making equipment for cost reduction strategies in the agricultural business sector. The analysis was organized on the basis of data from the Iraqi Plan/Central Statistical Organization for 2024.

The results indicated that implementing the lagration equation for balance production can help detect the best amounts of production required for balance, with the best mix of production factors (capital, labor wages) at the lowest cost. Farmers can use this approach to increase the production of wheat and allocate equally suitable agricultural areas to prevent deviations from balance production. Farmers in Iraq can also use this approach as a strategy to reduce costs for wheat cultivation.

This concept can also be applied to other objects, including corn, rice, barley, and others. The subsequent investigation in this domain may test additional variables affecting the dynamics of the wheat market value, including agricultural technology, wheat inventions, anticipated demand for wheat, and others.

6 Declarations

- The authors declare that they have no known competing financial interests or personal relationships.
- No funding was received to assist with the preparation of this manuscript.
- The authors have no relevant financial or non-financial interests to disclose.
- The manuscript does not include humans or animals.

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