

Improving the Enhanced Vold-Kalman Filter to Analyze Vibration in Rotating Machines

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Abstract

Vibration analysis is indispensable for different mechanical applications for early fault diagnosis, and many methods are used to analyze signals. Order tracking is one of these methods that is necessary for vibration analysis, especially for rotating machines. One critical and widely used method of order tracking is Vold-Kalman order tracking (VK-OT), which is used to diagnose faults in non-stationary machines. However, it has complicated and intensive calculations, requires special analysis tools, needs large memory, and takes a very long time to extract the results.

The proposed method aims to analyze signals by using Vold-Kalman filter order tracking with shorter time and less calculation memory with high accuracy by using partitioning method that separates the signal into many blocks with overlaps. The proposed method achieved less processing time and need much smaller memory than the original Vold-Kalman filter-based methods.

1. Introduction

Order tracking (OT) is an important technique in vibration analysis in rotating machinery to diagnose faults. The main feature of OT compared with other techniques of vibration analysis lies in an analysis of the non-stationary vibration and noise, which varies in frequency with the rotational reference shaft. Order domain analysis imputes a vibration signal to a shaft rotating speed, instead of the absolute frequency base. As a result, the vibration components which are proportional to the multiples of running speed can be identified easily. OT can perform in different ways each way has many advantages and drawbacks [1].

There are many methods of order tracking to analyze a non-stationary vibration signal. It can be represented by two major categories waveform non-reconstruction and reconstruction methods [2]. Each one includes a group of methods like (Vold-Kalman Filter Order Tracking (VKF-OT) and Gabor expansion for Order Tracking (GOT)) that are considered waveform reconstruction methods. Other methods such as Time Variant Discrete Fourier Transform Order Tracking (TVDFOT), Tacho-Less Order Tracking (Tacho-less-OT), Fast Fourier Transform order tracking (FFT-Based-OT), and Computed order tracking (COT)) are waveform non-reconstruction methods [3].

Order tracking analysis (OTA) is important, especially during the start-up or shutdown of the machine, where conventional analysis techniques, like Fourier transform, are unable to be used.

Vold and Leuridan proposed a scheme of Vold-Kalman order tracking (VKOT) in 1993, the objective of this method was to get a reconstructed version of an order that satisfies both

the data and structural equations. Then, Vold et al. introduced a method of second-generation VKOT, the purpose of this method was to obtain an envelope of the order, and by multiply the envelopes by a corresponding rotating, a signal could be reconstructed vectors [4].

Vold and Leuridan, [5] proposed an algorithm called Vold-Kalman filter order tracking (VKF-OT) based on the Kalman filter notions for high resolution. This technique defines the local constraints, which means the unknown orders have a slowly varying amplitude which means they are smooth. The smoothness condition is named (the structural equation), and a relationship from observed data is named (the data equation).

Vold et al. [6] suggested a developed technique of VKFT-OT led to the production of the second generation and allowed the appreciation of many orders at the same time. which makes VKFT-OT more efficient in separating the cross and close orders.

Tuma, [7] summarized the method of VKF-OT, explained the first and second generations of it, and showed the variation between them mathematically. Also, he provided a MATLAB-solvable system for Vold-Kalman filter equations.

Guoa and Kiong, [8] studied a new decoupling method for the (VKF-OT) that uses the Independent Component Analysis (ICA) to make it more effective in separating the mixed signals. This reduced successfully an order coupling distortion and, as a result, increases the accuracy of order-tracking results.

Feng and Liang, [9] proposed a method of a time-frequency analysis to extract the faults based on VKF and the separations of higher-order energy. VKF has the advantage of separating the complex signals into mono-components and the separation

of higher-order energy enables a high-accuracy estimation of the signals that have instantaneous frequency.

Zhao et al. [10] studied a developed algorithm of multi-fault diagnosis of gearbox and bearing under the conditions of variable speed, by extracting IDMM that is used in the PF calculations corresponding to FCF of bearing and the gear rotational frequency. Also, they used the Vold-Kalman filter to find the readjusted FCF and the rotating frequency.

Feng et al. [11] presented a technique for using the Vold-Kalman filter to diagnose the faults in nonstationary speeds wind turbine planetary gearboxes. The Vold-Kalman filter is effectively used to separate the harmonic components from the vibration signals of the rotating machine. This ability allows for precise estimation of instantaneous frequency.

Alsalaet [4] proposed an enhanced Split Vold-Kalman (SVK) filter in order tracking analysis. It used the terms of the split sine and cosine, which provided real-valued, results which minimized the errors, leading to smoother envelope tracking and the best optimization making the signal analysis more accurate.

2. Theoretical background

2.1. Second generation Vold-Kalman order tracking

The structural equation is given by [12]:

$$d(0)x(n) + d(1)x(n+1) + d(2)x(n+2) + \dots + d(p)x(n+p) = \varepsilon(n) \quad (1)$$

Where $d(\cdot)$ are filter coefficients that depend on the filter order. The work of the structural equation is like a low-pass filter. A second-generation data equation is given by [7], [13]:

$$y(n) = x(n) e^{i\varphi n} + \eta(n) \quad (2)$$

Where $y(n)$ is the observed signal, $x(n)$ is the filtered order, $\eta(n)$ the error term due to measurement noise and the other sinusoids contribution, and $\varphi(n)$ is an angular displacement of a required order.

For the multiple orders the structural and data equations are given by [14]:

$$\nabla^p x(n) = \varepsilon(n) \quad (3)$$

$$y(n) = \sum_{k=1}^M x(n) e^{i\varphi_k n} + \eta(n) \quad (4)$$

Where ∇ is the backward difference of the order p , p is the poles number, $\varepsilon(n)$ is the term of non-homogeneity, and M is number of the orders to be extracted.

The above equations can be written in matrix form as [15]:

$$y - \sum_{k=1}^M D_k x_k = \eta \quad (5)$$

$$A x_k = \varepsilon_k \quad (6)$$

Where D_k is the diagonal matrix with the elements of phase-samples of order k , that is mean $D_k = e^{i\varphi_k}$, 0 , $e^{i\varphi_k}$, 1 , \dots , $e^{i\varphi_k}$, $n-1$, and A is a filter coefficients matrix.

One of the important parameters that must be produced is weighting factor that is used to equalize an error of the data

equation and the structural equation, it is referred to a range from 0 up to a cutoff frequency of the low-pass filter. It is given by [16]:

$$r(n) = \frac{S_\eta(n)}{S_\varepsilon(n)} \quad (7)$$

Where $S_\eta(n)$ is a standard deviation of an error in data equation and $S_\varepsilon(n)$ is a standard deviation of an error in structural equation. The relationship between the normalized filter bandwidth Δf and the weighting factor r for the one-pole filter can be given by [7]:

$$r = \frac{\sqrt{2} - 1}{\sqrt{2(1 - \cos(\pi\Delta f))}} \quad (8)$$

2.2. Split Vold-Kalman order tracking

For the single-order signal, the data equation is given by [4]:

$$y(n) = a(n) \cos \theta(n) + b(n) \sin \theta(n) + \eta(n) \quad (9)$$

Represented using a matrix form:

$$y(Ca + Sb) = \eta \quad (10)$$

Where $C = \text{diag}\{\cos \theta(1), \cos \theta(2), \cos \theta(3), \dots, \cos \theta(n)\}$, and $S = \text{diag}\{\sin \theta(1), \sin \theta(2), \sin \theta(2), \dots, \sin \theta(n)\}$.

The sine and cosine envelopes follow a structural equation used in the VKF. For instance, for the structural equations of two-pole filter, have the following [4]:

$$a(n) - 2a(n-1) + a(n-2) = \varepsilon(n) \quad (11)$$

$$b(n) - 2b(n-1) + b(n-2) = \varepsilon(n) \quad (12)$$

The data equation of multi-component is written as [4]:

$$y(n) = \sum_{k=1}^M a_k(n) \cos \theta_k(n) b_k(n) \sin \theta_k(n) + \eta \quad (13)$$

Represented using a matrix form:

$$y - \sum_{k=1}^M C_k a_k + S_k b_k = \eta \quad (14)$$

Where $C = \text{diag}\{\cos \theta(1), \cos \theta(2), \cos \theta(3), \dots, \cos \theta(n)\}$, and $S = \text{diag}\{\sin \theta(1), \sin \theta(2), \sin \theta(2), \dots, \sin \theta(n)\}$.

2.3. Partitioning Vold-Kalman order tracking

This method will be known as partitioning Vold-Kalman order tracking. It is used to decrease the time calculation for signal analysis while maintaining the percentage of error at the same time. In this method, three parameters will be added. block size, overlap, and overlap start. This method partitions the long signals into many blocks in certain sizes, the result will be easier to calculate with a shorter time. Generally, it will be represented as:

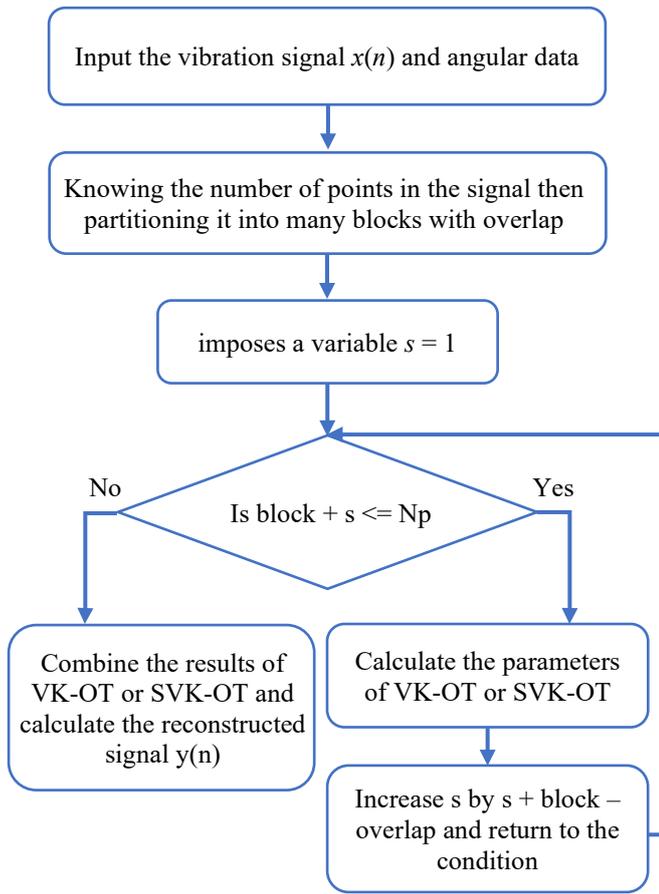


Fig. 1 The proposed method flowchart.

3. Results and discussion

3.1. Simulated run-up signal

This test produces a signal with three orders, which are 1X, 1.2X (where X is a shaft speed), and the constant frequency of 40 Hz. The shaft speed increases linearly from 5 Hz to 60 Hz over 50 sec, which test them in two cases with partition and without partition.

The calculation time in this case is equal to 0.4293 sec without partition, however it is equal to 0.2097 sec with partition in the method of VKOT, and equal to 1.3818 sec without partition, however it is equal to 0.7514 sec with partition in the method of SVKOT That is mean the time of process will decrease to half. To know the effectiveness of this method the mean absolute error (MAE) must be calculated.

The mean absolute error (MAE) is represented in the following equations:

$$MAEa = \frac{1}{N} \sum_{i=1}^N |a_i - \hat{a}_i| \quad (14)$$

$$MAEb = \frac{1}{N} \sum_{i=1}^N |b_i - \hat{b}_i| \quad (15)$$

Where N is the total length that record.

The maximum percentage amplitude error (MPAE) is another formula used to evaluate the tracked amplitude accuracy, it is given by:

$$MPAE = \frac{1}{N} \sum_{i=1}^N \left| \frac{\max(A_k(n) - \hat{A}_k(n))}{\max(A_k(n))} \right| \times 100 \% \quad (16)$$

Where:

$$A_k(n) = \sqrt{a_k(n)^2 + b_k(n)^2}$$

The maximum percentage amplitude error (MAPE) of this case for VK-OF and SVK-OT are Listed in the following tables:

Table 1. Simulated run-up signal analysis results with VK-OT.

weighting factor	Without partitioning			With partitioning		
	MPAE1	MPAE2	MPAE3	MPAE1	MPAE2	MPAE3
200	0.7661	1.2146	0.539	0.8193	1.2569	0.6286
400	0.5265	0.4683	0.3814	0.2165	0.4978	0.4025
450	0.5881	0.501	0.3531	0.2473	0.5185	0.3851
500	0.6515	0.5372	0.3283	0.2773	0.5558	0.3728
700	0.918	0.616	0.2556	0.4376	0.6646	0.3452
900	1.1969	0.72	0.2089	0.6358	0.7612	0.3288
1000	1.3389	0.7858	0.1914	0.7355	0.8062	0.3238

Table 2. Simulated run-up signal analysis results with SVK-OT.

weighting factor	Without partitioning			With partitioning		
	MPAE1	MPAE2	MPAE3	MPAE1	MPAE2	MPAE3
2	0.0064	0.0103	0.0062	0.0122	0.1095	0.0652
5	0.0064	0.0103	0.0062	0.0121	0.1095	0.0652
10	0.0064	0.0102	0.0062	0.0121	0.1096	0.0653
12	0.0064	0.0102	0.0062	0.0121	0.1097	0.0654
15	0.0064	0.0101	0.0062	0.012	0.1098	0.0654
20	0.0064	0.0101	0.0062	0.0119	0.11	0.0656

Tables 1 and 2 show the maximum percentage amplitude error (MAPE) for VK-OF and SVK-OT with and without partitioning. It still has relatively high MPAE and MAE for the orders. While it achieves a shorter time than the conventional method. Figure 2 and 3 show that the accuracy is still high with this method.

Figure 3(a) shows a signal of raw vibration of VKOT without partitioning and with partitioning, while Fig. 3(b) displays the orders frequency map of VKOT without partitioning and with partitioning. Figure 3(c), (d) show the analysis results of using the VKOT method with the weighting factor $r = 500$ without partitioning and with partitioning. Figure 3(e), (f) show the analysis results of using the SVKOT method with the weighting factor $r = 2$ without partitioning and with partitioning.

3.2. Response of the system with two-degree of freedom

Three parameters are considered as a matrix in the response of a system with two-degree of freedom (2-DOF), the mass, damping, and stiffness matrices given below:

$$M = \begin{bmatrix} 1.2 & 0 \\ 0 & 1 \end{bmatrix}, K = \begin{bmatrix} 90000 & -48000 \\ -48000 & 78000 \end{bmatrix}, C = \begin{bmatrix} 18 & 0 \\ 0 & 15 \end{bmatrix}$$

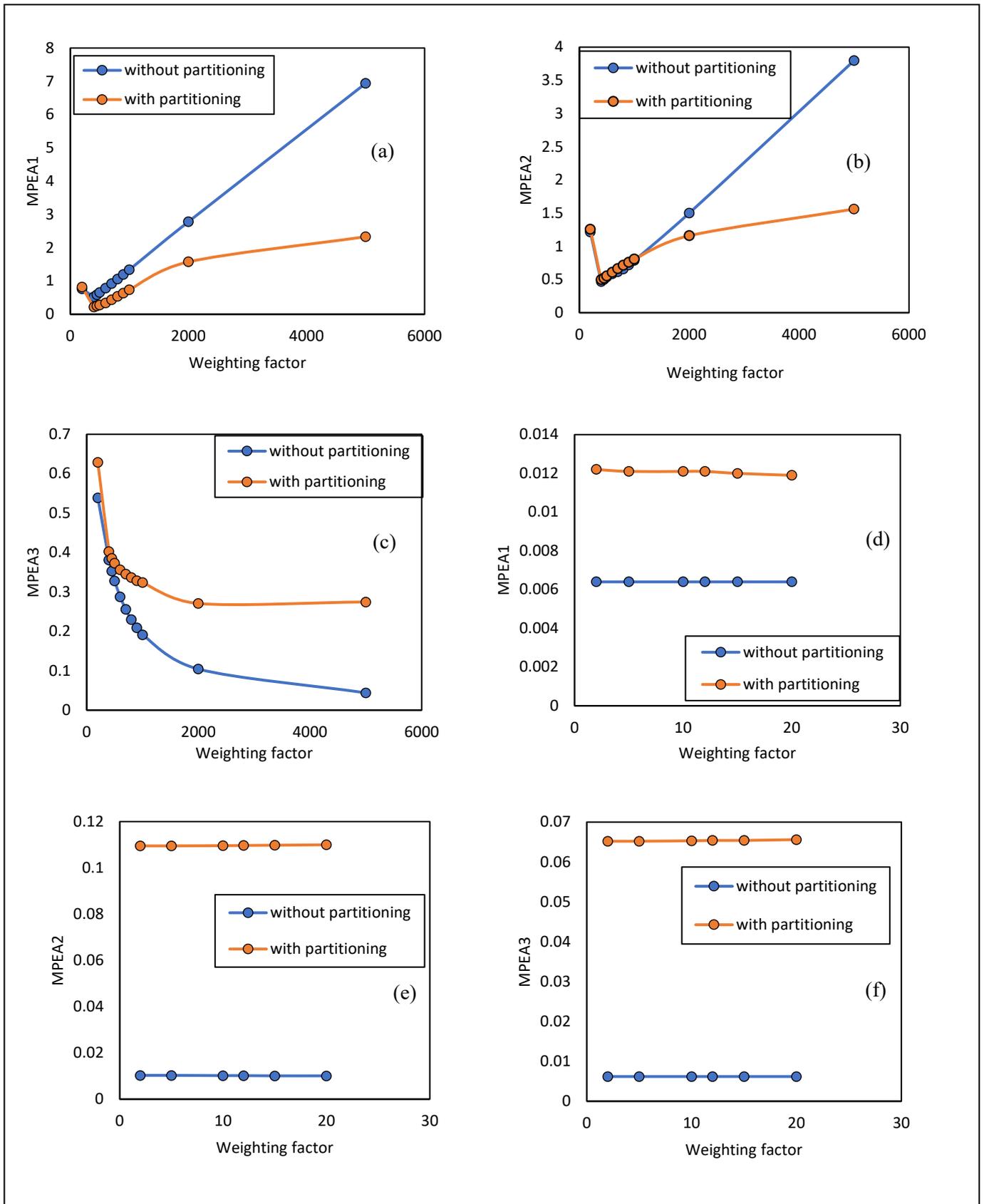


Fig. 2 Relationship between the MPAE and weighting factor for the response of the first case simulated signal; **(a)** MPAE1 for VKF-OT, **(b)** MPAE1 for VKF-OT, **(c)** MPEA for VKF-OT, **(d)** MPAE1 for SVKF-OT, **(e)** MPAE2 for SVKF-OT, **(f)** MPEA for SVKF-OT.

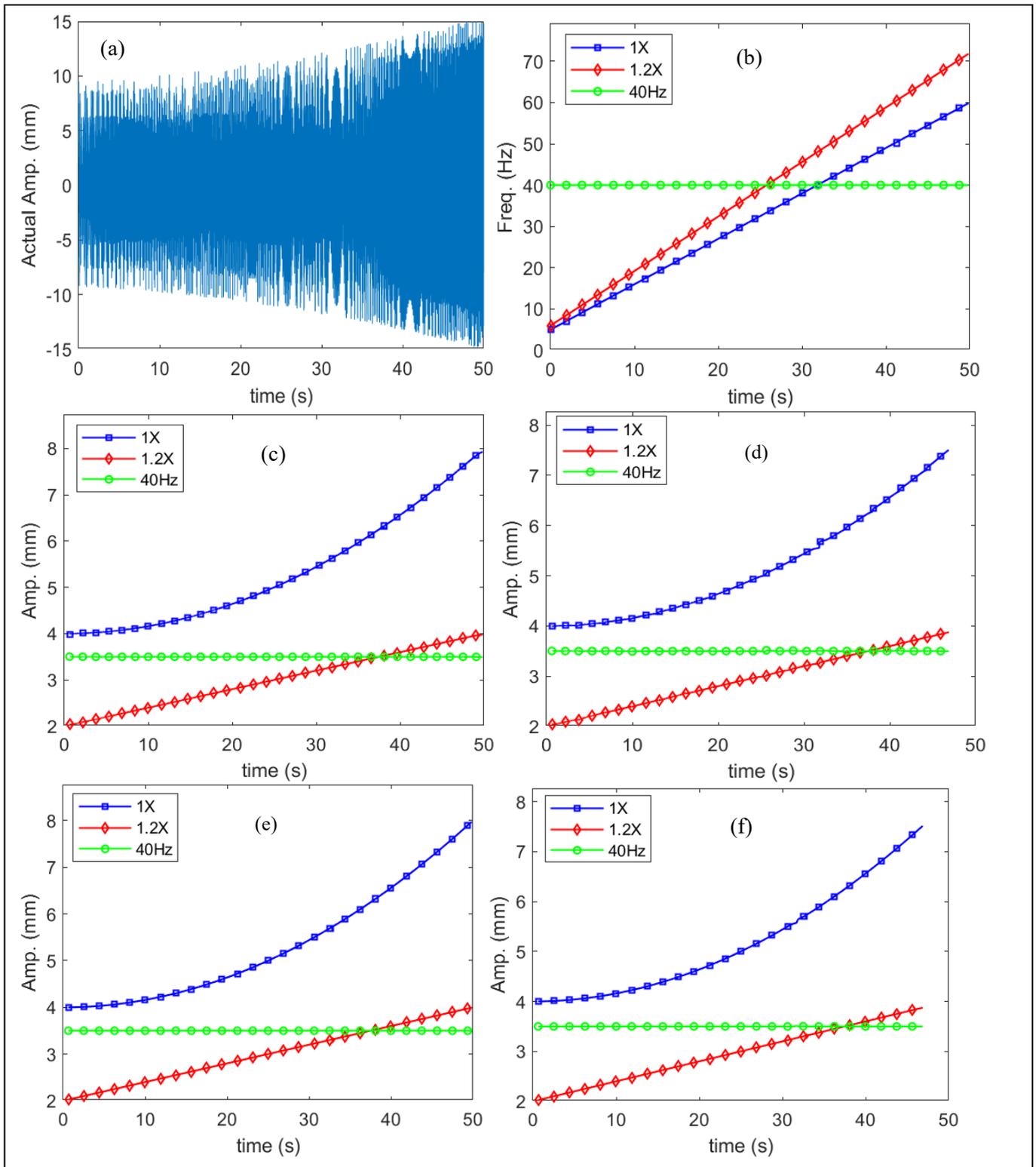


Fig. 3 The response of the Simulated run-up signal and related analysis result, (a) raw signal, (b) frequency map, (c) VK-OT $r = 500$, (d) VK-OT $r = 500$ with partitioning, (e) SVK-OT $r = 2$, (f) SVK-OT $r = 2$ with partitioning.

The calculation time in this case is equal to 0.2281 sec with partition, however it is equal to 0.2663 sec without partition in the method of VKOT, and equal to 0.742 sec with partition, however it is equal to 1.1713 sec without partition in the method of SVKOT that is mean the time of process will decrease.

The maximum percentage amplitude error (MAPE) of this case for VK-OF and SVK-OT are listed in tables 3 and 4.

Table 3. the case of 2-DOF simulated signal with VK-OT.

weighting factor	Without partitioning			With partitioning		
	MPAE1	MPAE2	MPAE3	MPAE1	MPAE2	MPAE3
35	3.5809	2.3612	2.93	3.5809	2.3612	2.93
50	1.9484	2.2266	1.6874	1.9484	2.2266	1.6874
60	1.3997	2.5603	1.3149	1.3997	2.5603	1.3149
70	1.0728	3.0309	1.0815	1.0728	3.0309	1.0815
80	1.1909	3.5947	0.923	1.1909	3.5947	1.7304
90	1.3552	4.2273	0.8087	1.3552	4.2273	1.5635
100	1.5549	4.9126	0.7222	1.5549	4.9126	1.4296

Table 4. the case of 2-DOF simulated signal with SVK-OT.

weighting factor	Without partitioning			With partitioning		
	MPAE1	MPAE2	MPAE3	MPAE1	MPAE2	MPAE3
35	0.2396	1.0089	0.5551	0.2396	1.0089	0.5551
43	0.3266	1.3252	0.5444	0.3266	1.3252	0.5444
50	0.4165	1.6449	0.5341	0.4165	1.6449	0.5341
60	0.5663	2.1638	0.5183	0.5663	2.1638	0.5183
70	0.7403	2.7465	0.5015	0.7403	2.7465	0.5015
71	0.759	2.8079	0.4998	0.759	2.8079	0.4998
80	0.9372	3.3832	0.4844	0.9372	3.3832	0.4844
90	1.1555	4.065	0.4671	1.1555	4.065	0.4671
100	1.3939	4.784	0.45	1.3939	4.784	0.45

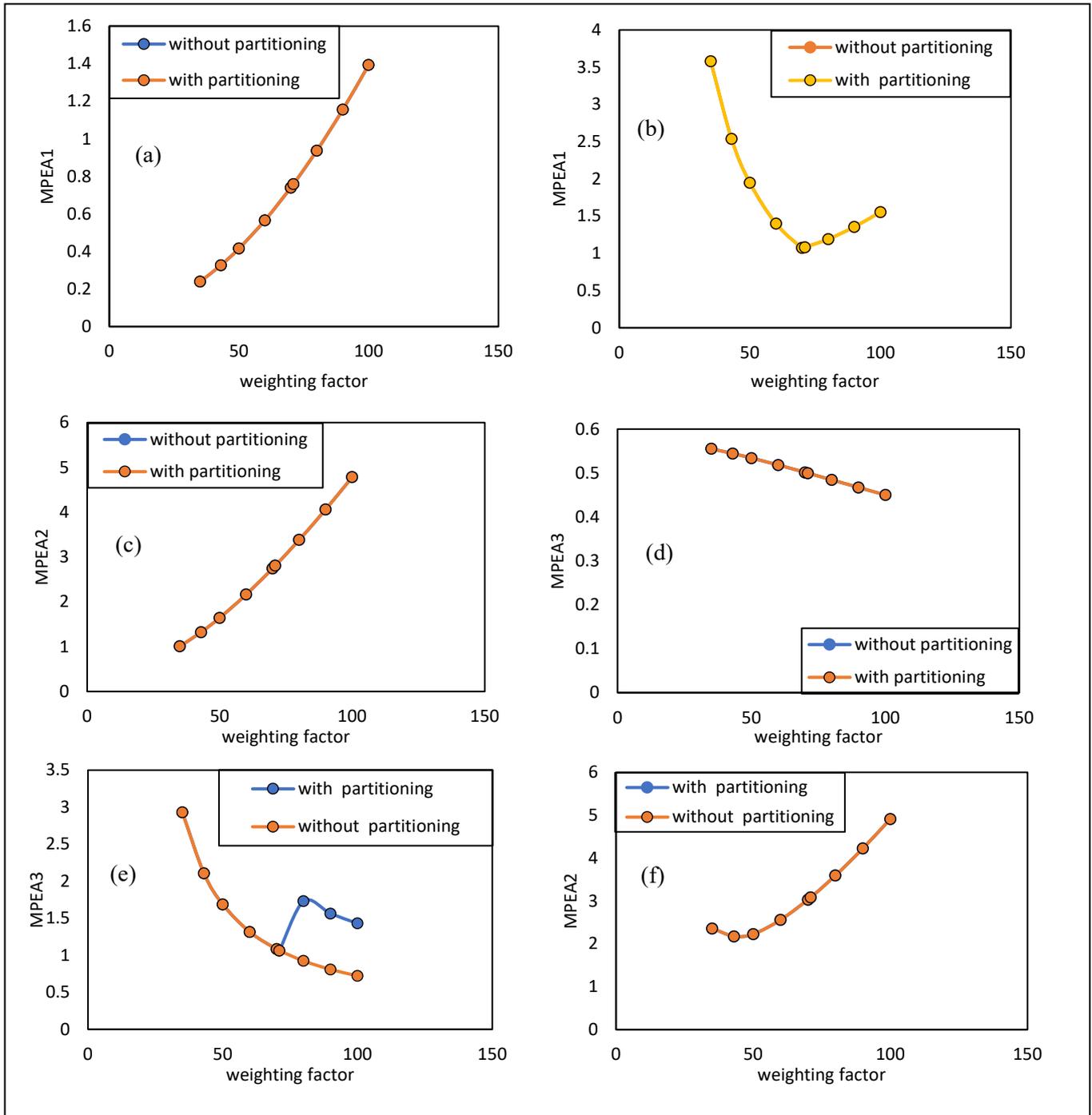


Fig. 4 Relationship between the weighting factor and MPAE for the response of 2-DOF; (a) MPEA1 for VKF-OT, (b) MPEA1 for VKF-OT, (c) MPEA3 for VKF-OT, (d) MPEA1 for SVKF-OT, (e) MPEA2 for SVKF-OT, (f) MPEA3 for SVKF-OT.

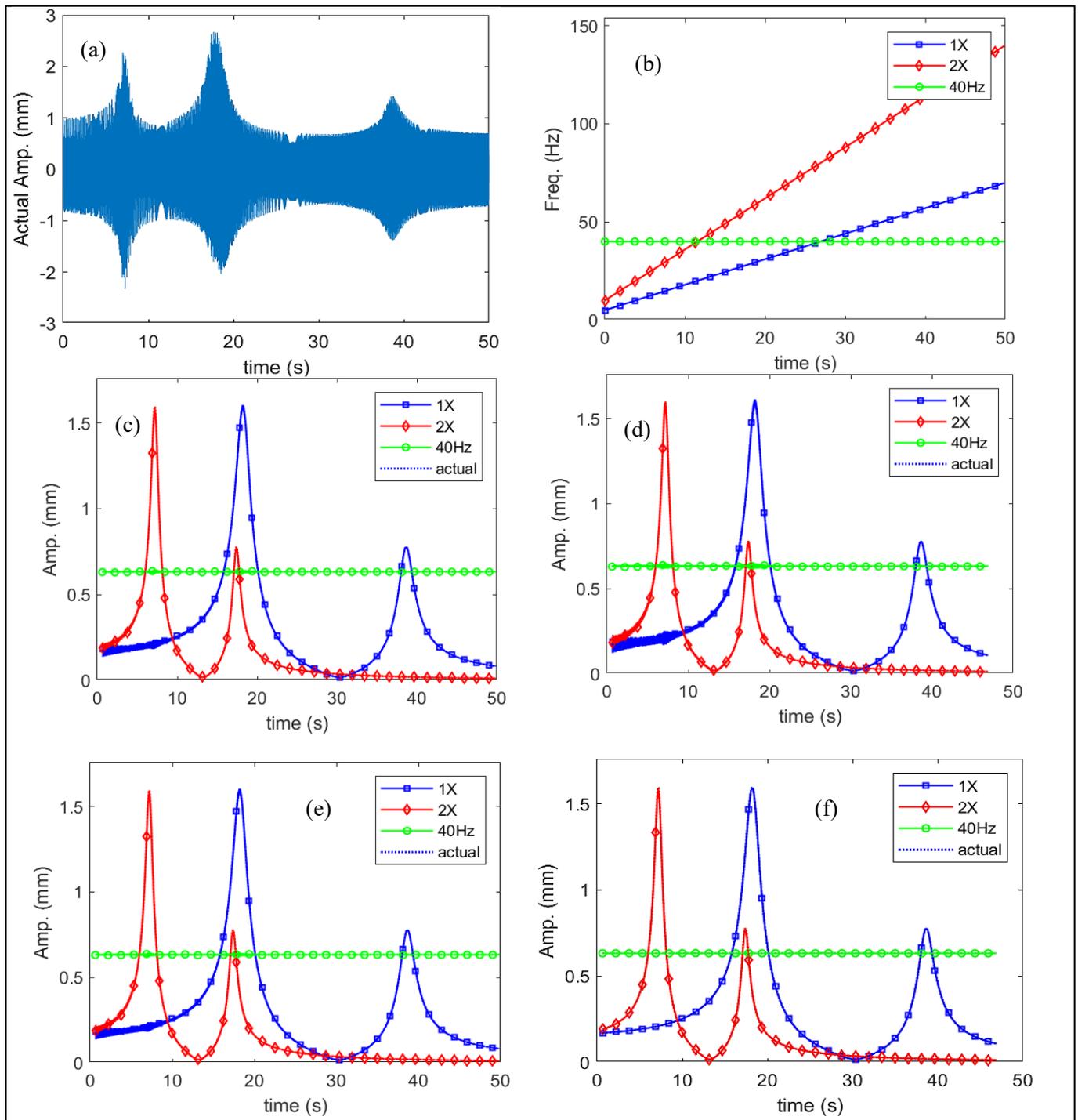


Fig. 5 The response of 2-DOF and related analysis result, (a) raw signal, (b) frequency map, (c) VK-OT $r = 35$, (d) VK-OT $r = 35$ with partitioning, (e) SVK-OT $r = 35$, (f) SVK-OT $r = 35$ with partitioning.

Tables 3 and 4 show that the proposed partitioning method outperforms the normal VKOT or SVKOT methods. It still has relatively high MPAE and MAE for the orders, particularly the MAE, which achieves the same accuracy as can be seen in Fig. 4 and 5. In addition, this method achieves a shorter time than the conventional method of VK-OT.

Figure 5 (a) shows a signal of raw vibration of VKOT without partitioning and with partitioning, while Fig. 5 (b) displays the orders frequency map of VKOT without partitioning and with partitioning. Figures 5 (c) and (d) show the analysis results of using the VKOT method with the weighting factor $r = 35$ without partitioning and with partitioning. Figures 5 (e) and (f) show the analysis results of

using the SVKOT method with the weighting factor $r = 35$ without partitioning and with partitioning.

3.3. Response of rectangular plate

The response signal to the sinusoidal excitation is obtained from the rectangular plate of steel response. A plate is 500 mm long, 400 mm wide, and 1 mm thick, with simply supported at the edges. These boundary conditions are selected because an analytical solution is easily available. The excitation force is applied in three orders; two of them have varying frequencies and one with constant frequency. The frequency of the first-order varies from 5 Hz to 60 Hz across 40 sec.

The calculation time in this case is 0.2217 sec without partition, however it is equal to 0.1806 sec with partition using the method of VKOT, furthermore the processing time is 0.582 sec without partition. While, it is equal to 0.8036 sec with partition using the method of SVKOT that means the time of process will be decreased.

To know the effectiveness of this method the reconstruction mean absolute error (RMAE) must be calculated. The reconstruction-mean absolute error (RMAE) is represented in the following equation:

$$RMAE = \sqrt{\frac{\sum_{n=1}^N |y(n) - \hat{y}(n)|}{N}} \quad (17)$$

Where $\hat{y}(n)$ is a reconstructed signal which can be obtained in the VKOT method from the following equation:

$$\hat{y}(n) = \sum_{k=1}^M x(n)e^{i\phi_k n} + \eta(n) \quad (18)$$

While can be obtained in the SVKOT method by adding the sine and cosine terms.

The error crest factor (ECF) is represented in the following equation:

$$ECF = \frac{\max |y(n) - \hat{y}(n)|}{RMS(y)} \quad (19)$$

Which is a ratio between a peak error and a value of signal root mean square RMS(y).

The Mean Absolute Error (MAE) and the error crest factor (ECF) of this case for VK-OF and SVK-OT are listed in the following tables:

Table 5. Analysis results of the case of rectangular plate simulated signal with VK-OT.

weighting factor	Without partitioning		With partitioning	
	ECF	MAE	ECF	MAE
35	0.2311	0.1056	0.2095	1.0089
43	0.1636	0.0894	0.1483	1.3252
50	0.1256	0.08	0.1138	1.6449
60	0.1018	0.0718	0.0923	2.1638
70	0.1254	0.0678	0.1136	2.7465
80	0.1538	0.0665	0.1394	2.8079
90	0.1855	0.0669	0.1681	3.3832
100	0.2172	0.0685	0.1969	4.065

Table 6. Analysis results of the case of rectangular plate simulated signal with SVK-OT.

weighting factor	Without partitioning		With partitioning	
	ECF	MAE	ECF	MAE
35	0.0395	0.0245	0.0358	0.024
43	0.0573	0.0295	0.0519	0.029
50	0.0745	0.0338	0.0676	0.0333
60	0.1012	0.0396	0.0917	0.0391
70	0.1294	0.0451	0.1173	0.0446
80	0.1591	0.0504	0.1442	0.0499
90	0.1896	0.0555	0.1719	0.055
100	0.2205	0.0603	0.1999	0.0599

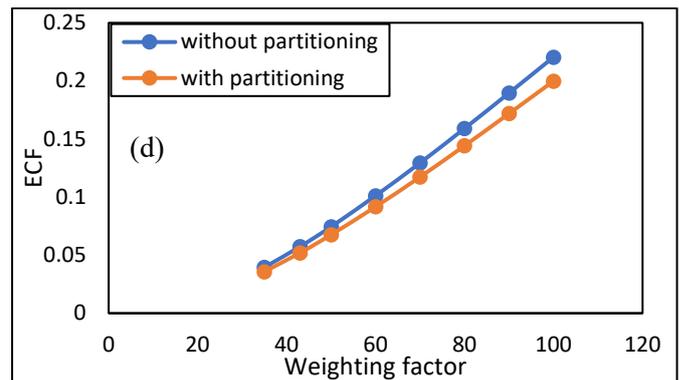
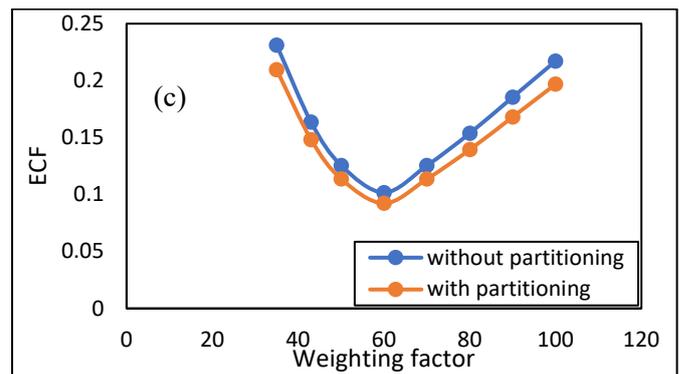
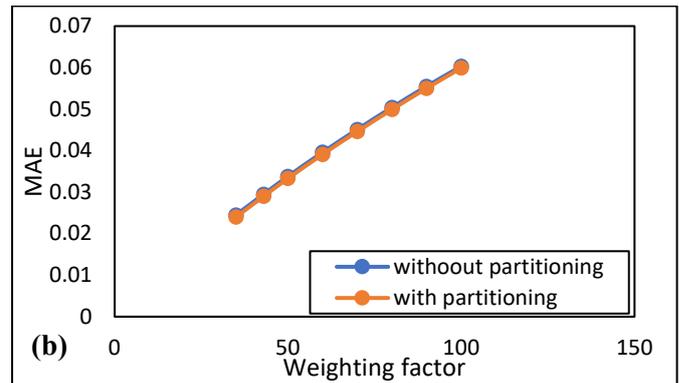
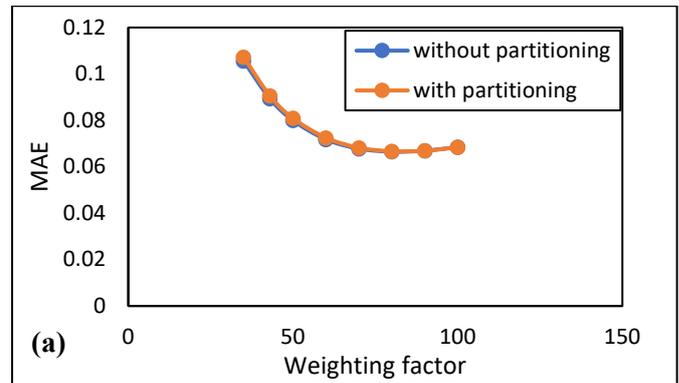


Fig. 6 Relationship between the weighting factor and MAE, the weighting factor and ECF for the response of rectangular plate; (a) MAE for VKF-OT, (b) MAE for SVKF-OT, (c) ECF for VKF-OT, (d) ECF for SVKF-OT.

Tables 5 and 6 show that the proposed partitioning method outperforms the normal VKOT or SVKOT methods. It still has relatively high ECF and MAE for the orders, even have less value of the ECF and MAE in some cases, which achieves the higher accuracy as can be seen in Fig. 6 and Fig. 7. in addition, this method achieves a shorter time than the conventional method.

Figure 7(a), show a signal of raw vibration of VK-OT without partitioning and with partitioning, while Fig. 7(b) displays the orders frequency map of VKOT without partitioning and with partitioning. Figures 7(c) and (d) show the analysis results of using the VK-OT method with the weighting factor $r = 35$ without partitioning and with partitioning. Figures 7(e) and (f) show the analysis results of using the SVK-OT method with the weighting factor $r = 35$ without partitioning and with partitioning.

3.4. Response of the system with two-degree of freedom with long process time

Three parameters are also considered as a matrix in the response of a system with two-degree of freedom (2-DOF), the mass, damping, and stiffness matrices given below:

$$M = \begin{bmatrix} 1.2 & 0 \\ 0 & 1 \end{bmatrix}, K = \begin{bmatrix} 90000 & -48000 \\ -48000 & 78000 \end{bmatrix}, C = \begin{bmatrix} 18 & 0 \\ 0 & 15 \end{bmatrix}$$

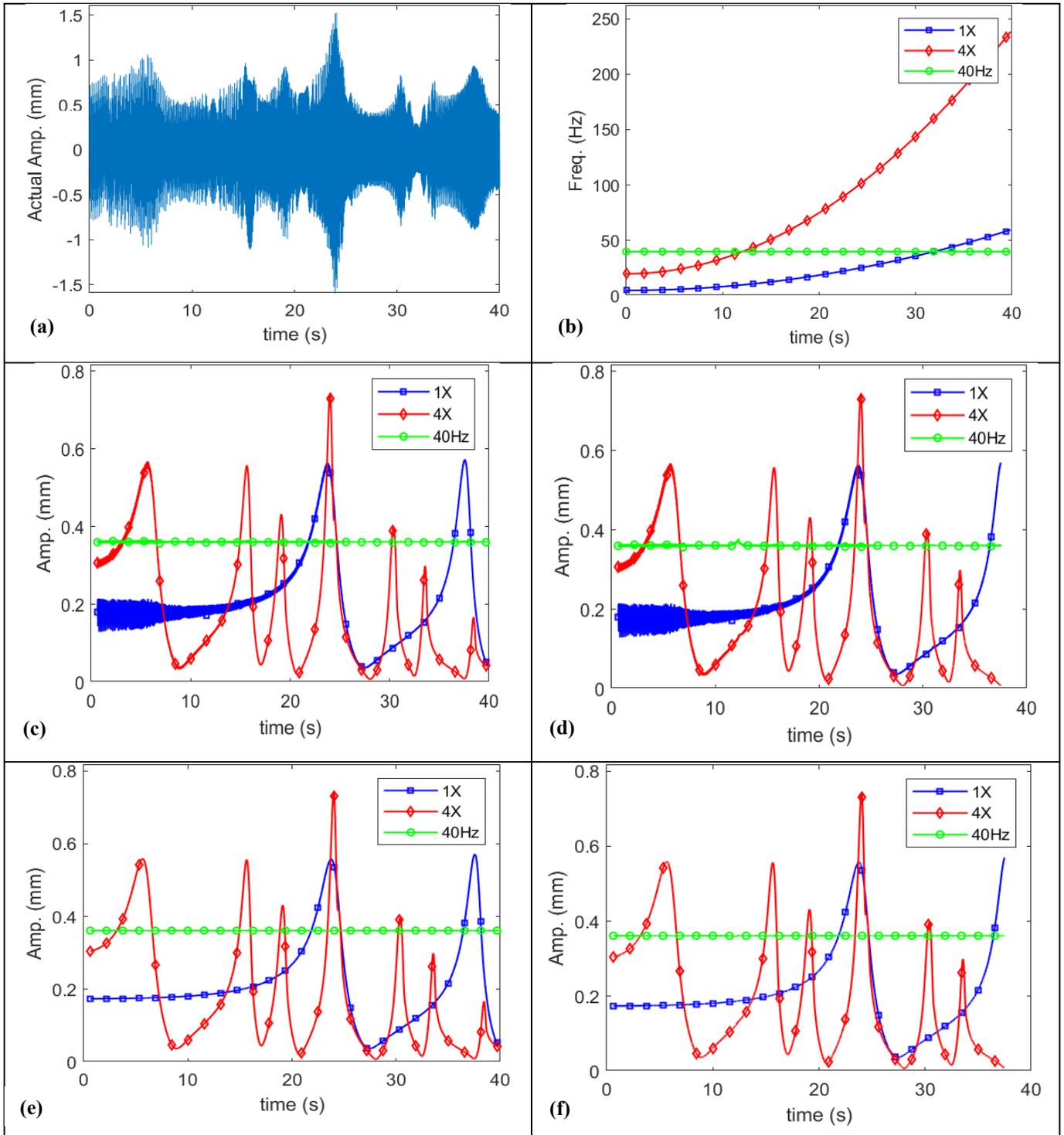


Fig. 7 The response of rectangular plate and related analysis result, (a) raw signal, (b) frequency map, (c) VK-OT $r = 43$, (d) VK-OT $r = 43$ with partitioning, (e) SVK-OT $r = 43$, (f) SVK-OT $r = 43$ with partitioning.

To further illustrate the effect of this method, the total time will be 300 instead of 50 sec as in the previous cases.

In this case, the calculation time is equal to 3.8716 sec with partition; while, it is equal to 1.36 hour without partition in the method of VKOT, the other hand the processing time is 14.2697 sec with partition; while, it is 1.55 hour without partition in the method of SVKOT. That means the time of the process will significantly decrease.

Figure 8(a), shows a signal of raw vibration of VK-OT without partitioning and with partitioning, while Fig. 8(b) displays the orders frequency map of VK-OT without partitioning and with partitioning. Figures 8(c) and (d) show analysis results of using the VK-OT method with the weighting factor $r = 35$ without partitioning and with partitioning. Figures 8(e) and (f) show analysis results of using the SVK-OT method with the weighting factor $r = 35$ without partitioning and with partitioning.

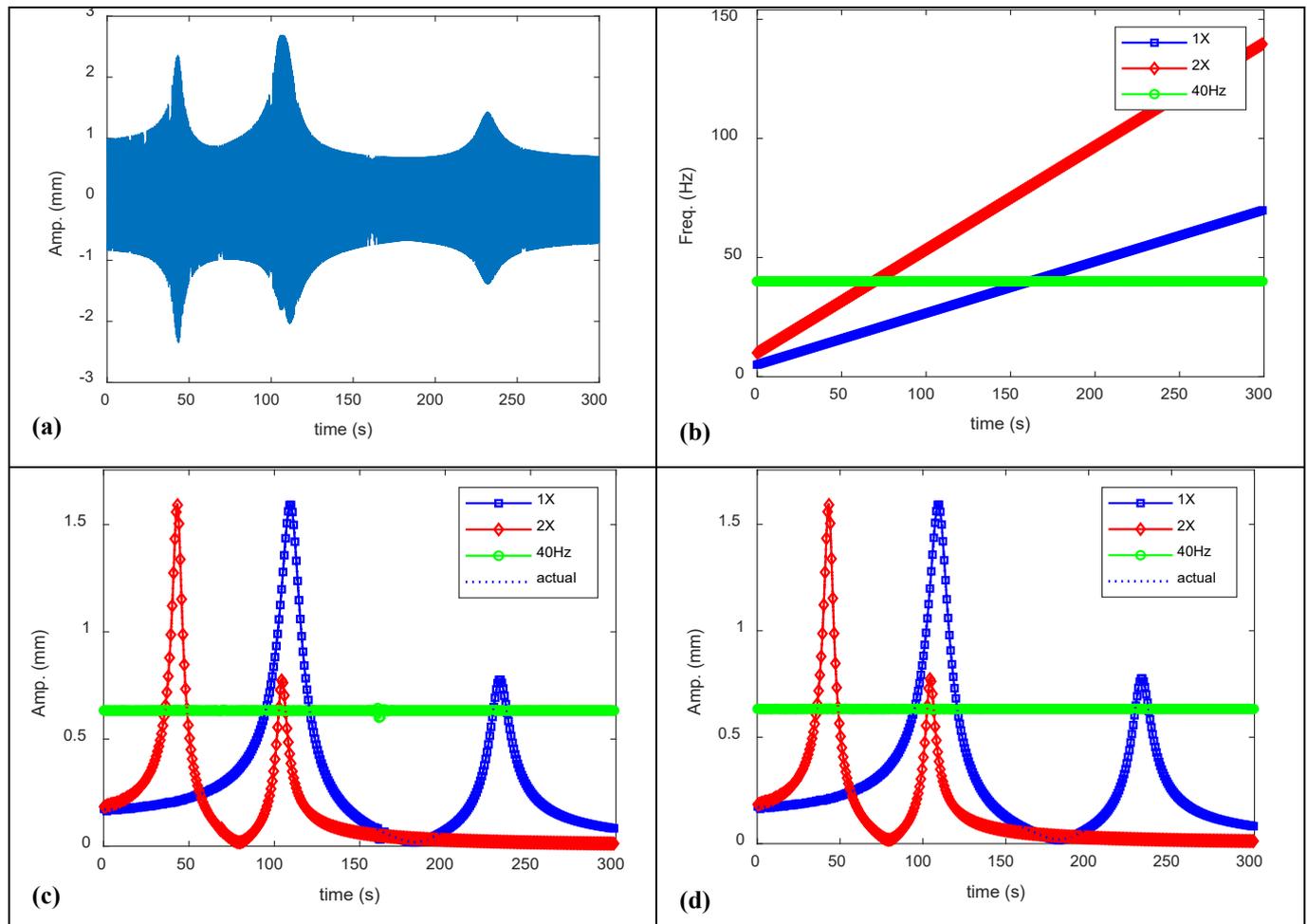


Fig. 8 The response of 2-DOF with long process time and related analysis result, (a) raw signal, (b) frequency map, (c) VK-OT $r = 200$, (d) VK-OT $r = 200$ with partitioning.

5. Conclusions

Some of the important limitations of the normal VKOT and the split VKOT methods are the complexity of the calculation and the time required. An enhanced VKOT is proposed for the analysis of order tracking. Instead of using a complete signal, the new filter is based on partitioning a signal into many blocks that overlap.

The signal results analysis shows that the proposed method can have maximum percent amplitude error (MPAE) and mean absolute error (MAE) close to conventional method the for many filter bandwidths, and even less than the conventional method in some cases.

The signal results analysis of the two DOF response systems exhibit that a proposed method has high performance over the VKOT and SVOT methods for many filter bandwidths, the proposed method can have the same maximum percent amplitude error (MPAE) and mean absolute error (MAE) for many filter bandwidths.

The signal results analysis of the rectangular plate response shows that the reconstruction mean absolute error (RMAE) and the error crest factor (ECF) values of the proposed method are close to the values of the conventional method.

The results analysis indicated that the three cases for the proposed method have a shorter time than the conventional method and have easier calculations.

The signal results analysis of the two DOF response systems with long process time shows approximately the same maximum percent amplitude error (MPAE) and mean absolute error (MAE) for many filter bandwidths. In addition, the calculation time was reduced from many hours to many seconds.

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