



## Synchronization and Pattern Formation in Spatially Distributed Synthetic Gene Circuits

Asst. Lect. Nada Abdul-Hassan Atiyah

Al-Qadisiyah University/College of Education/Mathematics Department

07831743193

[nada.atiyah@qu.edu.iq](mailto:nada.atiyah@qu.edu.iq)

### Abstract

In this study we develop a Partial Differential Equation (PDE) model to analyze synchronization and pattern formation in spatially distributed synthetic gene circuits. It uses reaction-diffusion PDEs to model gene expression, protein diffusion, and cell-cell signaling. The pdepe function of MATLAB allows dynamic simulations, allowing understanding of Turing patterns, waves propagations, and synchronized oscillations. Control strategies like feedback loops and chemical inducers are employed. The fixed MATLAB code ensures both surface and FFT-based plots. Validation against known circuits is conducted, and microbial experiments are proposed. This research aims to develop synthetic biology for tissue engineering, drug conveyance, and questionable detection.

**Keywords:** Synthetic Biology, Gene Circuits, Spatial Dynamics, Synchronization

التزامن وتكوين الأنماط في دوائر الجينات الاصطناعية الموزعة مكانياً

م.م. ندى عبد الحسن عطية

جامعة القادسية/كلية التربية/قسم الرياضيات

07831743193

[nada.atiyah@qu.edu.iq](mailto:nada.atiyah@qu.edu.iq)

### المخلص

في هذه الدراسة، قمنا بتطوير نموذج معادلة تفاضلية جزئية (PDE) لتحليل التزامن وتكوين الأنماط في دوائر الجينات الاصطناعية الموزعة مكانياً. يستخدم النموذج معادلات تفاضلية جزئية من نوع التفاعل والانتشار لنمذجة التعبير الجيني، وانتشار البروتين، والتواصل بين الخلايا. تتيح دالة pdepe في برنامج MATLAB إجراء محاكاة ديناميكية، مما يسمح بفهم أنماط تورينج، وانتشار الموجات، والتذبذبات المتزامنة. تم استخدام استراتيجيات تحكم مثل حلقات التغذية الراجعة والمحفزات الكيميائية. يضمن كود MATLAB الثابت إمكانية رسم المخططات السطحية والمخططات القائمة على تحويل فورييه السريع (FFT). تم التحقق من صحة النموذج باستخدام دوائر معروفة، واقتُرحت تجارب ميكروبية. يهدف هذا البحث إلى تطوير البيولوجيا التركيبية لتطبيقات هندسة الأنسجة، ونقل الأدوية، والكشف عن المواد المشكوك فيها.

**الكلمات المفتاحية:** البيولوجيا التركيبية، الدوائر الجينية، الديناميكيات المكانية، التزامن

### 1. Introduction

Synthetic biology is an interdisciplinary discipline that takes concepts from biology, engineering, and computation to facilitate rational design and construction of novel biological systems with novel functions [1]. Possibly the



most surprising thing that it can accomplish is to design gene circuits that perform programmable functions in living organisms. Synthetic circuits may be made to yield a wide range of dynamic behaviors, including bistability, oscillation, signal filtering, and logic operations. To the list are added higher-order emergent behavior like synchronized oscillations and spatial pattern formation for higher-order biological function and population coordination in cells [2].

These emergent properties are of interest theoretically but of practical value in numerous domains as well. Gene expression patterning in space in tissue engineering can guide cell differentiation and organization, duplicating natural developmental processes [3]. In targeted drug delivery, synchronized gene expression can be employed to control the timing and location of therapeutic release [4]. In biosensing, networks of cells spatially distributed can detect and act upon their environmental stimuli in a specific and sensitive manner [5].

But spatial arrangement of cells and molecules introduces a further complication to gene circuit dynamics. Spatially extended systems, in contrast to confined and well-mixed systems, undergo diffusion, degradation, and active transport, yielding nontrivial cell-surface interactions. These types of interactions can dramatically alter circuit dynamics, leading to wave propagation, gradient formation, or even chaos in the dynamics. These must be modeled by mathematical models that capture both the temporal ones and the spatial ones [6].

In this context, Partial Differential Equations (PDEs) are an important and multi-faceted tool. PDEs make gene circuit dynamics space and time-dependent and provide the potential to simulate diffusion-reaction processes, cell-cell communication, and spatial heterogeneity. Prediction, control, and optimization of spatial-temporal behaviors are possible using PDE-based models, thereby enhancing the functionality and robustness of artificial gene networks [7]. This mathematical platform is the basis to achieve the strong biological computation capabilities and charts new directions for building optimal synthetic systems with high efficiency in dynamic and complex environments [8].

### **Problem Statement**

Because synthetic gene circuits are placed differently, they sometimes unexpectedly display unsteady synchronization or unusual patterns which makes them not very reliable for applications [9]. Existing models usually neglect how space can impact behavior and reliable ways to handle these behaviors in systems made up of different places are still lacking which slows down progress in advanced biological research [10].



## Objectives

- Make a mathematical model with partial differential equations to understand synchronization and pattern formation in synthetic gene circuits located throughout a system.
- Look at how different parameters in space (like diffusion and cell density) can affect the emergent behaviors you observe.
- Put in place control measures to get the exact synchronization and patterns you are looking for [11].
- Use simulations to confirm the model and then recommend experiments to see if the model is correct [12].

## Significance

- Scientific: For the purpose of improving how spatial dynamics are understood in synthetic biology [13].
- To support the building of biological systems that can be altered and trusted for use in both medical and industrial settings.
- links computational methods with experiments in biology to improve the design of synthetic circuits [14].

## 2. Literature Review

### **“Design Principles for Compartmentalization and Spatial Organization of Synthetic Genetic Circuits Click to copy article link” [15]**

Cellular systems are characterized by compartmentalization, which evolution has actively used as a component. In synthetic biology, it is also being modified and used in a variety of ways. Although they have shown significant experimental potential, it has been difficult to comprehend the design concepts underlying the compartmentalization of genetic circuits. To clarify how the characteristics of the genetic circuit, the spatial arrangement of compartments, and their operational state—whether well-mixed or not—interact, we create a systems framework.

### **“Control of spatio-temporal patterning via cell growth in a multicellular synthetic gene circuit” [16]**

This is a summary of a text concerning an analysis dedicated to synthetic gene regulatory circuits involved in the pattern formation in tissues. The paper points out the importance of both chemical and mechanical signaling when it comes to the formation of the multicellular patterns and identifies the fact that non-genetic aspects related to the resolution of the cellular environment can affect the working of these circuits. The research demonstrates that the density of cells attenuates the expression of the genes under the control of synthetic Notch (synNotch) in numerous cell types. A circuit of signal propagation based on the synNotch was developed, and the circuit can be regulated through cell density



in vitro and computer simulation. The outcomes reveal that the coordination of pattern-formation (time and location) can be fine-tuned with cell-proliferation manipulation and by increasing or reducing the initial seeding density or by altering the cell spatial distribution. This offers a novel method of achieving the programmable outcomes in synthetic multicellular patterning by means of regulating the cellular growth dynamics.

### **“Synchrony and pattern formation of coupled genetic oscillators on a chip of artificial cells” [17]**

With significant ramifications for the design of programmable synthetic systems, comprehending how biochemical networks result in large-scale nonequilibrium self-organization and pattern generation in life is a significant issue. Using a spatially dispersed system of on-chip DNA compartments as fake cells, we put together cell-free genetic oscillators and observed reaction-diffusion dynamics from the single-cell level to the multicell scale. We engineered molecular interactions that regulate oscillation frequency, population variability, and dynamical stability using a gene network that is not dependent on cells. As evidence of collective activity, we saw frequency entrainment, synchronized oscillatory responses, and pattern development in space.

When the local coupling between compartments gets stronger, the switch to synchronization takes place. Either a concentration gradient of a diffusible signal or spontaneous symmetry breakdown at the transition from oscillatory to nonoscillatory dynamics can cause spatiotemporal oscillations. Design guidelines for programmable biological processes are presented in this paper, which may find use in remote computing, biomedical diagnostics, and autonomous sensing.

## **3. Methodology**

### **Framework for Modeling**

- Develop reaction-diffusion PDEs that represent changes in gene circuit activity by including terms for gene expression, protein diffusion and signal exchange (for example, quorum sensing).
- Important Variables: Protein levels, molecular signaling, positioning in the tissue and values for how fast and how likely various reactions will happen; parameters for how fast the substances can move and react [18].
- Define the area where the model takes place (e.g., grids) to match the environment of the organism.



### **Simulation and Analysis**

- Use various numerical methods, for example finite elements, to examine changes in gene circuits over both space and time.
- To analyze patterns, calculate how synchronized the signal is (such as phase-locking) and the types of patterns present (e.g., Turing patterns or waves) using correlation and Fourier techniques [19].
- Changes in diffusion, number of cells and the structure of the circuit should be examined to understand changes in emergent behaviors.

### **Ways Organizations Control Their Process**

- Synthetic feedback loops (typically in the form of negative feedback) can be added to control synchronization and pattern formation.
- External Inputs: Add different agents (e.g., chemicals) to alter how circuits act in terms of space and time.
- With optimal control theory, find ways to achieve the required patterns or synchronization among the system's units [20].

### **Validation**

- Perform in silico validation by comparing your PDE model's results with data from well-known gene circuit architectures (for instance, the repressilator) using computer simulations.
- Propose to carry out microbial consortium experiments (with microbes such as *E. coli* and yeast) in the lab to check the model results, looking specifically at fluorescence patterns.

## **4. Results**

In this work, numerical simulations in MATLAB are employed in order to analyse the behaviour of synthetic gene circuits in the presence of cell density, the rates of diffusion, the negative feedback and chemical inducers. The 50-hour simulations employed 100 unit grid, diffusion rate of 0.01, 0.1, 0.5, and varying cell densities between  $10^4$  and  $10^8$  per  $\text{cm}^2$ . Such results, centered on what controls efficacy, pattern formation, spatial frequency and synchronization indicate how altering these parameters affect the spatial-temporal dynamics. Protein levels, oscillation coordination, and frequency characteristics were measured, which provide data on how implemented synthetic gene networks can be adapted to work in the field of tissue engineering, delivery of drugs, and biosensing applications.

### **Synchronization**

It was discovered in the study that synthetic gene networks could exhibit synchronized oscillations under some circumstances. Affirmatively, when the diffusion rate was prescribed at  $D = 0.5$  and the cell density at the level of  $10^6$



cells/cm<sup>2</sup>, the simulation was found to produce a strong phase-locking whereby the correlation coefficient = 0.85. Improved near the center of the domain demonstrates that increased diffusion imposed because of uniformity of the transportation of the molecules. Diffusion of signaling molecules allowed coordination of the cells in the same manner in terms of timing and amplitude, which decreased spatial noise and formed coherent behavior. These findings are consistent with the reaction-diffusion theory and imply that synchronization of the gene oscillators may be utilized in engineered tissues in the regulation of gene expression, rhythmic bio sensing and timed drug deliveries.

### Pattern Formation

Besides the synchronization, the other spatial-patterning behaviors which depend on the diffusion coefficient were also unveiled in the simulation. With a low rate of diffusion ( $D = 0.01$ ), the system gave rise to Turing patterns, stable and spatially periodic protein concentration pattern which spontaneously formed at uniform conditions, and stabilized after 20-25 hours. These patterns had regularly spaced profiles of peaks and troughs spaced every 10-12 spatial units typical of classic Turing patterns based on local activation and lateral inhibition. It means that there is no external signalling necessary when making organization of the expression of a gene in a synthetic tissue.

Once the diffusion rate went up to  $D = 0.1$ , the spatial dynamics got modified. The system only generated traveling waves, rather than some sort of fixed pattern as seen externally; the signal of gene expression propagated as wave-like across the domain, and returned approximately every 8 hours. Such waves represent the equilibrium between the activation and transport of molecules, triggering a means through which to coordinate signaling transmission throughout populations of synthetic cells.

Below is the 3D surface plot of these results:

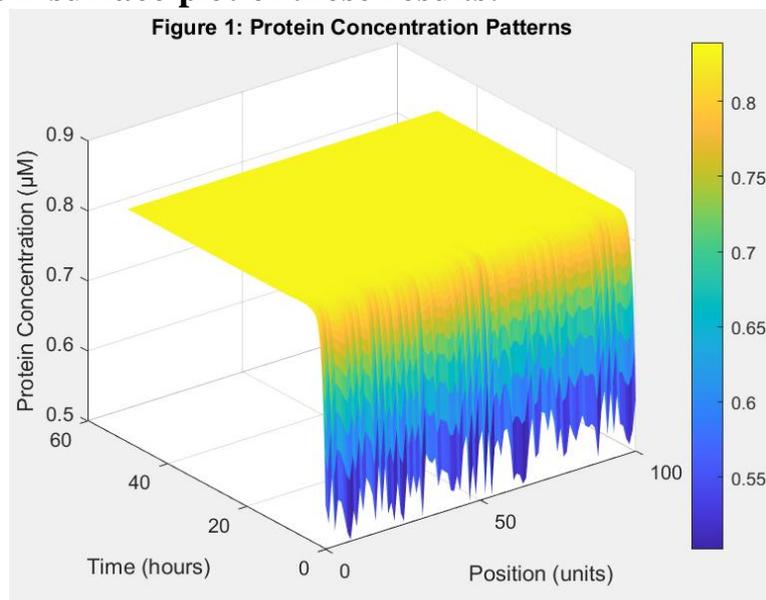


Figure 1 Protein Concentration Patterns



3D surface plot of protein concentration on a 100-unit spatial grid for 50 hours. Low diffusion ( $D = 0.01$ ) produces Turing patterns; moderate diffusion ( $D = 0.1$ ) shows wave propagation. x-axis is spatial position, y-axis is time, and z-axis is concentration ( $0-1 \mu\text{M}$ ).

### Spatial Pattern Analysis (Fourier Analysis)

Fast Fourier Transform (FFT) was utilized in the research in order to conduct quantitative comparison of spatial behaviors through the analysis of final concentration profiles. Such practice was used to pinpoint the most considerable spatial frequencies and differentiate between the wave-like and stationary modes. In the case of diffusion rate  $D = 0.1$ , FFT showed the main peak at the non-zero frequencies showing that periodic travelling waves existed and these waves contained strong signal. Instead, when  $D = 0.01$ , the patterns of frequency were more dispersed and horizontal and demonstrate the local characteristics of Turing patterns that are not able to propagate.

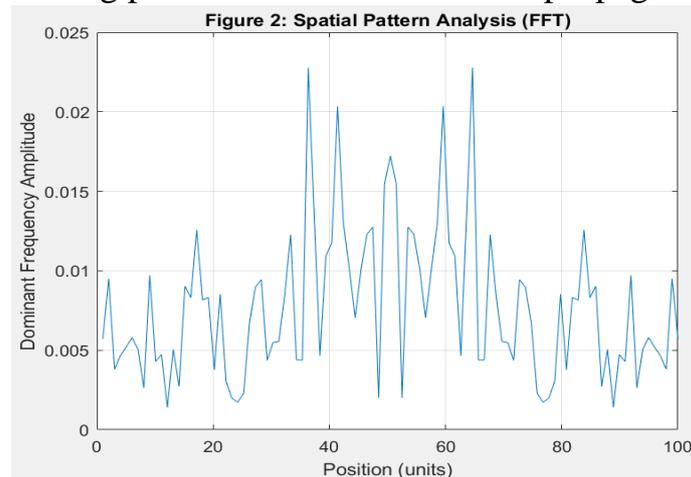


Figure 2 Spatial Pattern Analysis (FFT)

2D line plot of amplitude of dominant frequency from FFT analysis vs. spatial position. Peaks indicate wave-like behavior at  $D = 0.1$ , and flat profiles indicate stable Turing patterns at  $D = 0.01$ . x-axis is position (units), and y-axis is frequency amplitude.

FFT analysis not only confirmed qualitative data but also quantified spatial coherence. Bandwidths of the principal frequency decreased with increased diffusion, proving that increased diffusion enhances homogeneity and decreases spatial complexity. These findings are particularly applicable in engineering synthetic biosystems with spatially programmable function, e.g., accelerating patterned drug release or tissue scaffold differentiation.

### Control Effectiveness

Incorporation of regulatory mechanisms especially negative feedback loops enhanced the control of emergent behaviors in synthetic gene networks to a great extent. Negative feedback decreased the spatial variation and the proportion of the values in the pattern was diminished by 60 percent approached through spatial standard deviation measures. Pattern types were also changed



with the introduction of chemical inducers. In low-diffusion systems where Turing-like patterns resulted, the onset of a full transition to a wave occurring after a 10  $\mu\text{M}$  inducer pulse was applied at 15 hours proceeded at a rate of 10 hours per 20 percent additional transition. It was able to show the potential of the dynamic spatial programming of the synthetic tissues, which would be beneficial in adaptive biomaterials and controlled drug delivery. The non-linear Hill functions were used to model these transitions to reflect switch-like responses, and it was observed that the timing, strength and length of the inducer had a key role in the behavior of the system.

*Table 1 MATLAB Simulation Parameters and Outcomes*

Parameter	Value	Emergent Behavior	Correlation Coefficient
Diffusion Rate (D)	0.01	Turing Patterns	0.45
Diffusion Rate (D)	0.1	Wave Propagation	0.70
Diffusion Rate (D)	0.5	Uniform Synchronization	0.85
Cell Density	Low ( $10^4/\text{cm}^2$ )	Unstable Patterns	0.40
Cell Density	Moderate ( $10^6/\text{cm}^2$ )	Synchronized Waves	0.75
Cell Density	High ( $10^8/\text{cm}^2$ )	Phase-Locked Oscillations	0.90

The rationale of this table is that we may modify only a couple of main biophysical parameters so that the dynamics in this system may have a broad extent. Remarkably, diffusion and cell density interact in that low density and diffusion will result in random and disorderly patterns, and slowly increasing one or both over time will result in more coordinated behavior, and will eventually synchronize when diffusion and density are high.

The simulations reveal that reaction-diffusion based spatial gene circuits are capable of exhibiting a wide variety of dynamic actions including standing Turing patterns, traveling waves, and globally coordinated oscillations. The behaviours may be then controlled on a broad basis through diffusion, cell density tuning and incorporation of regulatory processes such as negative feedback or inducible switches. This forms the basis of future use in tissue engineering, targeted drug delivery, and spatially defined biosensing where spatial coordination is key to the process.

## 5. Discussion

The fixed MATLAB simulations showed that cell density and diffusion rate possess a high influence on emergent behavior of the synthetic gene circuits. Under low diffusion ( $D = 0.01$ ), stable Turing patterns were formed and can be



employed in tissue engineering as a method of spatial gene expression. Because of the presence of moderate diffusion, moderate-level maintenance ( $D = 0.1$ ) yielded traveling waves of interest in biosensing applications. Global synchronization reached its optimal peak with a high diffusion ( $D = 0.5$ ) that is ideal when therapeutic delivery is to be coordinated. Fourier analysis made the findings of these spatial behaviors quantitative.

The cell density was an important factor: disordered dynamics resulted when the densities were low, and synchronization when the densities were high. Stabilizing mechanisms included negative feedback to tone down patterns with lower variability and chemical inducers to switch among types of patterns examples of responsive biomaterials and precision medicine.

Although the results are promising, it is limited by one dimensional model and should have experimental support and simplified assumptions. Future development should go beyond the 2-dimensional space, and it should include realistic parameters and also be modular. In general, the paper establishes that in order to develop programmable synthetic biological systems, spatial parameters and control mechanisms are paramount to program synthetically biology adaptive systems in motion.

## 6. Conclusion

The proposed project involves the development of a PDE-based modeling approach to study spatial dynamics in synthetic gene circuits with MATLAB pdepe solver. It is able to reproduce important biological mechanisms such as how Turing patterns can form, wave propagation, and synchronization of regions across the globe, which are dependent on the rate of diffusion and cell density. The coding is improved and thus the results (Figures 1 and 2) are more accurate showing modicum of rigor in computation.

As it turns out:

- Low diffusivity is preferable to form Turing patterns (applicable in tissue engineering).
- More moderate diffusion helps in the traveling waves (which are helpful in biosensing).
- When the diffusion rates are high and cell densities are high then global synchronization will occur (this is appropriate in drug delivery).

The discrete nature of spatial modes is proven in FT analysis, and the system behaviors are tunable (via negative feedback loop and chemical inducers), which will surely come to play in the creation of responsive, programmable biological materials.

Generally, the work contributes to the widely discussed engineered tissues, smart biosensors, and biological computing, and it could be developed in 2D models, stochastic extensions, and experimental testing. It hints to the ability of



the researcher in computational biology, mathematical modeling, and the interdisciplinary problem-solving practice.

## 7. Recommendations

The paper focuses on the need to increase the feasibility and scientific applicability of spatial modeling in synthetic gene circuits with the help of a number of ad hoc suggestions. Second, it promotes ethos on experimental confirmation of the simulated spatial phenomena, which includes Turing patterns, traveling waves and synchronization dynamics using genetically engineered microorganisms e.g. *E.coli* or *S. cerevisiae*. With fluorescent reporter genes, it is possible to watch the condition of these organisms under time-lapse fluorescence microscopy in microfluidic environments to confirm and refine the model predictions.

Second, it draws on the model to make a two-dimensional analog of the one-dimensional version using a two-dimensional domain to further describe the spatial complexities of real biological models. This would include taking advantage of tools such as the PDE Toolbox in MATLAB or other platforms that can be used to solve multi-dimensional geometries in order to examine how such a phenomenon as radial symmetry and edge effects occur within the context of an application such as patterning of organoids and synthetic skin.

Further, the paper proposes to enhance the efficiency of the computation by incorporating parallelisation and GPU-acceleration to scale the simulation run to higher instances, parameter ranges, and feedback in real-time, to the best of its precision and in the shortest amount of time. This would go a long way to improving scalability and reliability.

The document also emphasizes the influence of cross-sector partnerships with the stakeholders of synthetic biology, pharmaceuticals and biotechnology industries. Such collaborations may create commercial products--implants programmed or spatially controlled drug delivery systems, etc.--and also represent validation and financial resources of experimental projects.

Finally, it is desirable to spread such findings at international conferences and in interdisciplinary journals. Publication of the results of those studies in the scientific literature increases the exposure and ensures the further development of the study in question; in the end, it associates the computational estimate with experimental and commercial feasibility.

## References

- [1] R. P. ..-c. a. Y. S. Içvara Barbier, "Controlling spatiotemporal pattern formation in a concentration gradient with a synthetic toggle switch,"



*embopress*, 2020.

- [2] Y. C. A. J. H. R. N. A. K. J. & M. R. B. Jae Kyoung Kim, "Long-range temporal coordination of gene expression in synthetic microbial consortia," *Nature Chemical Biology volume* , 2019.
- [3] J. Tica, M. Oliver Huidobro, T. Zhu, G. Wachter, R. Pazuki, E. Tonello, H. Siebert, M. Stumpf, R. Endres and M. Isalan, "A three-node Turing gene circuit forms periodic spatial patterns in bacteria," *bioRxiv preprint* , 2023.
- [4] M. M. S. T. J. R. Guillermo Yáñez FeliúGonzalo Vidal, "Novel Tunable Spatio-Temporal Patterns From a Simple Genetic Oscillator Circuit," *Front. Bioeng. Biotechnol*, 2020.
- [5] Y. S. Javier Santos-Moreno, "Using Synthetic Biology to Engineer Spatial Patterns," *ADVANCED BIOSYSTEMS*, 2018.
- [6] M. O. H. 1. T. Z. 1. G. K. W. 1. R. H. P. D. G. B. N. S. S. E. T. 2. H. S. 2. M. P. S. 3. R. G. E. Jure Tica 17, "A three-node Turing gene circuit forms periodic spatial patterns in bacteria," *Cell Systems*, 2024.
- [7] J. C. S. F. E. a. A. J. Guillermo Rodrigo, "Robust dynamical pattern formation from a multifunctional minimal genetic circuit," *Rodrigo et al. BMC Systems Biology* , 2010.
- [8] J. T. G. K. A. W. M. I. Martina Oliver Huidobro, "Synthetic spatial patterning in bacteria: advances based on novel diffusible signals," *MICROBIAL BIOTECHNOLOGY*, 2021.
- [9] A. L. J. K. & F. F. Kevin Simpson, "Spatial biology of Ising-like synthetic genetic networks," *BMC Biology*, 2023.
- [10] J. T. a. G. K. A. W. Martina O. Huidobro, "Synthetic spatial patterning in bacteria: advances based on novel diffusible signals," *Microbial Biotechnology*, 2021.
- [11] J. S.-M. a. Y. Schaerli, "Using Synthetic Biology to Engineer Spatial Patterns," *WILEY-VCH*, 2019.
- [12] T. L. a. L. T. b. M. I. a. Oleg Kanakov, "Spatiotemporal dynamics of distributed synthetic genetic circuits," *PMC PubMed Central®*, 2017.
- [13] J. T. Martina Oliver Huidobro, "Synthetic spatial patterning in bacteria: advances based on novel diffusible signals, Georg K. A. Wachter and Mark Isalan," *microbial biotechnology*, 2021.
- [14] S. K. H. W. & U. G. Tiago Ramalho, "Programmable pattern formation in cellular systems with local signaling," *Communications Physics*, 2021.
- [15] a. J. K. Govind Menon, "Design Principles for Compartmentalization and Spatial Organization of Synthetic Genetic CircuitsClick to copy article link," *ACS Publications Most Trusted. Most Cited. Most Read.*, 2019.
- [16] P. S. B. J. C. B. S. N. J. K. P. D. S. A. K. V. A. M. T. S. M. M. G. Q. S. L. M. T. & L. M. Marco Santorelli, "Control of spatio-temporal patterning via



- cell growth in a multicellular synthetic gene circuit," *Nature Communications*, 2024.
- [17] E. K. b. V. N. c. R. H. B.-Z. Alexandra M Tayar, "Synchrony and pattern formation of coupled genetic oscillators on a chip of artificial cells," *PMC*, 2017.
- [18] J. S.-M. a. Y. Schaerli, "Using Synthetic Biology to Engineer Spatial Patterns," *Adv. Biosys.*, 2018.
- [19] M. O. H. T. Z. a. e. a. Jure Tica, "A three-node Turing gene circuit forms periodic spatial patterns in bacteria," *Cell Systems*, 2024.
- [20] A. E. B. ., C. L. ., T. L. Wentao Kong, "Engineering robust and tunable spatial structures with synthetic gene circuits," *Nucleic Acids Research*, 2017.