

Effect of Flow Splitters on Energy Loss in C-Type Piano Key Weirs

Amirhossein Fathi¹ , Wadi Mohammed Wadi² , Ali Khoshfetrat^{2*} , Seyed Hamid Alavi³ 

¹Department of Civil Engineering, Isfahan University of Technology, Isfahan, Iran

²Department of Civil Engineering, Isf.C., Islamic Azad University, Isfahan, Iran

³Civil and Environmental Engineering Department, Tarbiat Modares University, Tehran, Iran

*Email: ali.khoshfetrat@iau.ac.ir

Article Info	Abstract
Received 16/12/2024	<p>Piano key weirs (PKWs) are a new type of labyrinth weirs. They have a longer crest length within a limited width and, unlike linear weirs, ultimately have a higher discharge coefficient. The higher energy loss in PKWs can reduce scour. Therefore, this study used flow splitters on the weir crest and two C-type PKWs with heights of 0.18 m and 0.20 m. The splitters had rectangular and circular cross-sections. They were arranged in three configurations: two-by-two, four-by-four, and six-by-six in each cycle. The results showed that the presence of a flow splitter increases energy loss. Additionally, as the number of splitters increases, energy loss increases, and the discharge coefficient decreases. Adding six rectangular flow splitters to the weir per cycle resulted in an average increase in energy loss of approximately 10.40% and a decrease in the discharge coefficient of 18.60%. On average, rectangular flow splitters had 1.24% higher energy loss than circular ones. The energy loss in weirs with rectangular and circular flow splitters increased by about 6.89% and 5.72%, respectively, compared to those without flow splitters. A 10% increase in weir height led to a decrease in both the discharge coefficient (by about 9.20%) and the energy loss (by about 1.68%).</p>
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1. Introduction

Since ancient times, humans have built weirs—barriers across flows—to increase upstream water levels for drinking and agriculture. Among these structures, labyrinth weirs are particularly effective for regulating water levels in dams and irrigation canals, making them among the most widely used hydraulic structures in water-transmission systems. Factors such as the upstream water head, wall angles, crest thickness, and crest shape influence the discharge coefficient of these weirs [1]. The Piano key weir (PKW) is another type of nonlinear long-crested weir, representing an evolution of the labyrinth weir characterized by sloped inlet/outlet keys and overhanging edges. A PKW can pass a discharge up to three to four times greater than a standard linear weir [2]. In addition to dams, PKWs are placed in drainage canals and rivers [3], [4]. The Loombah and Andhra Pradesh weirs in India are located on drainage canals and rivers [5], [6]. The PKWs in the plan are triangular, trapezoidal, or rectangular. They are further classified into four types: A, B, C, and D. These weirs have inlet and outlet keys, with the inlet keys having a negative slope and the outlet keys having a positive slope [7]. Numerous

researchers have investigated the discharge coefficients of PKWs. For instance, Dabling and Tullis [8] demonstrated that the coefficient is highly dependent on the weir's geometric parameters and is significantly higher under free-flow conditions than submerged ones. However, very little research has been conducted on energy loss and the factors affecting it. Researchers such as Torabi et al. [9], Bansal et al. [10], Shen and Oertel [11], and Mirkhorli et al. [12] have studied the energy loss of crests and PKWs. Al-Shukur and Al-Khafaji [13] found that increasing the outlet key slope of rectangular Type-B PKWs leads to a corresponding increase in energy loss. They also measured the distance from the hydraulic jump to the toe of the weir, which was zero for low slopes. Eslinger and Crookston [14] investigated energy loss in four rectangular A-type PKWs with different heights and inlet-to-outlet width ratios. They observed that energy loss is higher at lower flow depths and decreases logarithmically with increasing depth. In addition, they found that the energy loss was nearly independent of the inlet-to-outlet width ratio. In a weir with a greater height, the energy loss is lower. Naghibzadeh et al. [15] examined energy loss in rectangular Type-A PKWs with steps and baffles, finding that these elements induced maximum energy losses.

They also found that the energy loss decreased with increasing Weber number. Singh and Kumar [16] investigated the effects of different rectangular-type B PKW geometries on energy loss. Their investigation also showed that the presence of three steps in the outlet keys led to a significant increase in energy loss. In addition, the energy loss increases with the number of keys. Fathi et al. [17] experimentally investigated a stepped trapezoidal type-A PKW. They found that the presence of steps in the weir's outlet keys increased energy loss. Shen and Oertel [18] found that the PKW crest shape affects energy loss, with a flat crest being the most efficient configuration. Singh and Kumar [19] found that the energy loss of a type-A PKW decreases as the inlet-to-outlet width ratio increases. Rdhaiwi et al. [20] found that the energy loss in Type C PKWs was higher than in Type B PKWs.

According to the above studies, investigations of energy loss in type C PKW are limited. To our knowledge, this study is the first to investigate a trapezoidal C-type PKW utilizing flow splitters of varying shapes and numbers. The study tested configurations of 2, 4, and 6 splitters per cycle. Four flow rates of 0.025, 0.030, 0.035, and 0.040 m³/s and two weirs with heights of 0.18 and 0.20 m were also used. This study investigated the effects of flow rate, weir height, flow-splitter geometry, and arrangement on energy loss and discharge coefficient.

2. Dimensional analysis

Fig. 1 and Equation (1) show the parameters affecting the amount of energy loss (E_L) in a trapezoidal type C PKW with different numbers and geometries of the flow splitters. The amount of energy loss is calculated from $(E_L = (E_1 - E_2)/E_1)$, where E_1 is the specific energy of the flow upstream of the weir, and E_2 is the specific energy of the flow downstream of the weir [21]. The specific energy of the flow upstream of the weir was also calculated from; $(E_1 = P + h + V_1^2/(2g))$, and the specific energy of the flow downstream of the weir was calculated from; $(E_2 = y + V_2^2/(2g))$, where P is the height of the weir, h and y are the upstream flow and tailwater depths, V_1 and V_2 are the upstream flow and tailwater velocities, respectively, and g is the acceleration due to gravity.

$$E_L = f(H, V_1, \rho, \sigma, \mu, P, W_i, W_o, W, L, B_o, B, N, S) \quad (1)$$

H is the flow depth plus the equivalent height of kinetic energy upstream of the weir, ρ is the density of water, σ is the surface tension coefficient, μ is dynamic viscosity, W_i is the width of the inlet keys of the weir, W_o is the width of the outlet keys of the weir, W is the width of the weir, L is the length of the weir crest, B_o is the length of the hanging edge downstream of the weir, B is the length of the weir wall, N is the number of flow splitters per cycle (2, 4, and 6), and S is the shape of the splitters (rectangular (R) and circular (C)).

Because the geometry of the weirs used in this study was constant, the parameters W_i , W_o , W , L , B_o , and B will be ignored. If L_a represents the effective length of the weir crest and is calculated as $(L_a = L - 2dN)$ Then the direct influence of N and S can be disregarded, where d represents the diameter for a circular flow splitter, or the length in the flow direction for a

rectangular one. The number 2 is due to the presence of flow splitters in both weir cycles. Therefore, energy loss can be considered as a function of the parameters in (2).

$$E_L = f(H, V_1, \rho, \sigma, \mu, P, L_a) \quad (2)$$

Using Buckingham's π theorem, and considering the three repeating parameters ρ , V_1 and H , the amount of energy loss is expressed as a function of (3).

$$E_L = f\left(\frac{H}{P}, R_e, W_e, \frac{L_a}{P}\right) \quad (3)$$

In the (3), $R_e = \rho V_1 H / \mu$ is the Reynolds number of the flow and $W_e = V_1 / \sqrt{\sigma / \rho H}$ is the Weber number. Owing to the turbulence of the flow and a Reynolds number greater than 4000, the Reynolds number was ignored. In addition, because of the sufficient depth of more than 0.03 m of flow on the crest of the weir, the Weber number was ignored [22].

$$E_L = f\left(\frac{H}{P}, \frac{L_a}{P}\right) \quad (4)$$

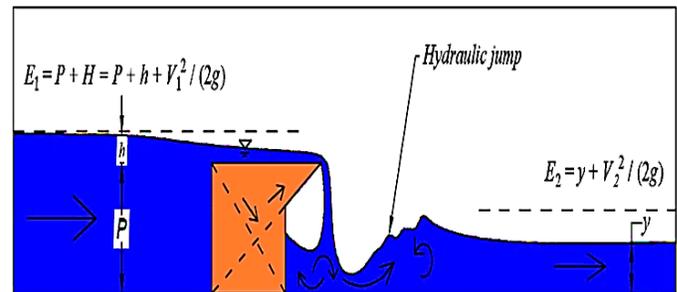


Figure 1. View of the flow over a trapezoidal type C PKW and parameters affecting energy loss.

3. Materials and Methods

The experiments were conducted in a flume with a length, width, and height of 10, 0.60, and 1.20 m, respectively. The flume floor was galvanized, and the walls were made of thick glass. After the pump was turned on, the flow passed through flow straighteners consisting of large gravel grains and metal screens, and after traveling 5.50 m, it entered the weir. The pump discharged the flow from the tank through a 0.1 m-diameter pipe at a maximum rate of 0.055 m³/s into the flume. Fig. 2a shows the Programmable Logic Controller (PLC) used to record the flow rate and depth at three points. The PLC unit was designed and manufactured by Radman Industry Company. Flow depth was measured using three sensors installed at the top of the flume. To ensure that surface-tension effects were negligible (Weber number > 1000), a sensor above the weir crest was used to confirm that the flow depth exceeded 0.03 m, in accordance with the criterion in [23]. The other two sensors were positioned at $2P$ upstream and $8P$ downstream of the weir's centerline, respectively [16]. Fig. 2b shows the laboratory flume. The pump and sensors provided flow rate and flow depth with an error of 0.1%. The 10 m flume had an elevation change of <4 mm, with water temperatures of 7 and 13 °C.



(a)



(b)

Figure 2. Specifications of the laboratory flume and pump: a) PLC and b) Laboratory flume.

Two trapezoidal-type C PKWs with constant geometric specifications were used. The width of the weirs (W) was 0.6 m, the length of the crest of the weirs (L) was 2.56 m, and the width of the inlet keys (W_i) and outlet keys (W_o) of the weirs were 0.215 and 0.075 m, respectively. The length of the side walls of the weirs (B) was 0.495 m, the length of the inlet side walls (B_i) was zero m, the length of the outlet side walls (B_o) was 0.13 m, and the thickness of the weirs (T_s) was 0.01 m. The only nonconstant geometric component of the weirs was their height. The heights of the first weir were 0.20 m, and the height of the second weir was 0.18 m. The weirs were then subjected to two cycles. The weirs were composed of iron sheets. Fig. 3 shows the geometric specifications of the trapezoidal-type C PKW.

Rectangular and circular flow splitters were also used. Ehsanifar et al. [24],[25] investigated the discharge coefficients of triangular, rectangular, and trapezoidal type-A PKWs with two flow splitters in each cycle. They also considered rectangular and square cross-section splitters similar, with circular cross-section splitters the most suitable. According to Ehsanifar et al. [24], because of the similar effects of rectangular and square splitters, only rectangular and circular cross-section splitters were used in this study. In this study, these splitters were glued to the crest of the weir in pairs, fours, and sixes in each cycle. The diameter of the circular splitters and the width of the rectangular splitters were equal to the weir thickness. The length of the rectangular splitters was twice their width. The height of the splitters was 0.08 m, so that they would not be submerged. The spacing was 0.119 m for circular splitters and 0.11 m for rectangular splitters. The two initial and end splitters were also 0.12 m and 0.11 m away from the edge of the weir. The splitters were made of metal. The presence of flow splitters promotes air entrainment below the inlet keys and creates flow gaps. These factors collectively increase the downstream flow distance, thereby increasing energy loss. It is

worth mentioning that the concept of artificial aeration using flow splitters was inspired by similar weirs, such as labyrinth weirs [26].



Figure 3. Geometric specifications of the trapezoidal type C PKW.

3.1. Procedure

The experiments were conducted in the following steps:

1. The weir was installed at a distance of 5.50 m from the beginning of the flume and was waterproofed using glue. This distance is due to the development of the flow. The velocity profiles were measured at 3.5, 4.0, and 4.5 m from the beginning of the upstream platform; they coincide, indicating that the flow is developed [8].
2. The pump was turned on, and the desired flow rate was set. The flow entered the tank and then the weir.
3. The depth of the flow was measured upstream and tailwater of the weir (at a distance of $2P$ upstream and at a distance of $8P$ downstream, relative to the center of the weir) by sensors [16]. The flow depth was measured upstream and downstream of the weir using a point gauge to ensure the two measurements were equal.
4. The downstream valve of the flume did not artificially control the depth of flow at the tailwater.
5. The average flow velocity upstream of the weir was calculated using a discharge-stage curve. The average flow velocity downstream of the weir was calculated using the continuity equation.
6. The amount of energy loss was calculated using the specific energy relationship upstream and in the tailwater of the weir.
7. The discharge coefficient was calculated using the general weir $C_d = 1.5Q/(W\sqrt{2g}H^{1.5})$ [27]. In this equation, C_d is the discharge coefficient, and Q is the flow rate.

Table 1 lists the hydraulic characteristics of the flow. As you can see, 28 tests were performed for the weir with a height of 0.20 m with and without flow splitters, and four tests were performed for the weir with a height of 0.18 m, without flow splitters. The study employed two different weir heights to isolate their effect on the discharge coefficient and energy loss; several configurations were deemed sufficient for this purpose.

Table 1. Hydrodynamic characteristics of the flow.

Q (m ³ /s)	P (m)	H (m)	N	S	E_L	C_d	Re
0.025	0.2	0.036	0	-	0.545	2.099	6054
0.03	0.2	0.048	0	-	0.540	1.605	9349
0.035	0.2	0.056	0	-	0.518	1.502	12220
0.04	0.2	0.063	0	-	0.507	1.416	15384
0.025	0.2	0.042	6	Rectangular	0.633	1.667	6944
0.03	0.2	0.054	6	Rectangular	0.602	1.349	10317
0.035	0.2	0.064	6	Rectangular	0.574	1.233	13633
0.04	0.2	0.073	6	Rectangular	0.536	1.142	17284
0.025	0.2	0.037	6	Circular	0.621	2.015	6206
0.03	0.2	0.049	6	Circular	0.599	1.557	9514
0.035	0.2	0.059	6	Circular	0.573	1.391	12760
0.04	0.2	0.065	6	Circular	0.544	1.353	15776
0.025	0.2	0.040	4	Rectangular	0.603	1.760	6726
0.03	0.2	0.052	4	Rectangular	0.585	1.426	10000
0.035	0.2	0.061	4	Rectangular	0.548	1.324	13114
0.04	0.2	0.069	4	Rectangular	0.529	1.240	16541
0.025	0.2	0.037	4	Circular	0.597	2.015	6206
0.03	0.2	0.048	4	Circular	0.572	1.605	9349
0.035	0.2	0.058	4	Circular	0.543	1.426	12582
0.04	0.2	0.065	4	Circular	0.516	1.368	15679
0.025	0.2	0.038	2	Rectangular	0.591	1.936	6356
0.03	0.2	0.049	2	Rectangular	0.559	1.557	9514
0.035	0.2	0.058	2	Rectangular	0.529	1.426	12582
0.04	0.2	0.066	2	Rectangular	0.505	1.323	15970
0.025	0.2	0.037	2	Circular	0.580	2.015	6206
0.03	0.2	0.048	2	Circular	0.549	1.605	9350
0.035	0.2	0.057	2	Circular	0.520	1.467	12402
0.04	0.2	0.064	2	Circular	0.502	1.384	15581
0.025	0.18	0.034	0	-	0.562	2.254	6289
0.03	0.18	0.042	0	-	0.544	1.992	8904
0.035	0.18	0.050	0	-	0.521	1.748	12078
0.04	0.18	0.058	0	-	0.513	1.611	15385

4. Results and Discussion

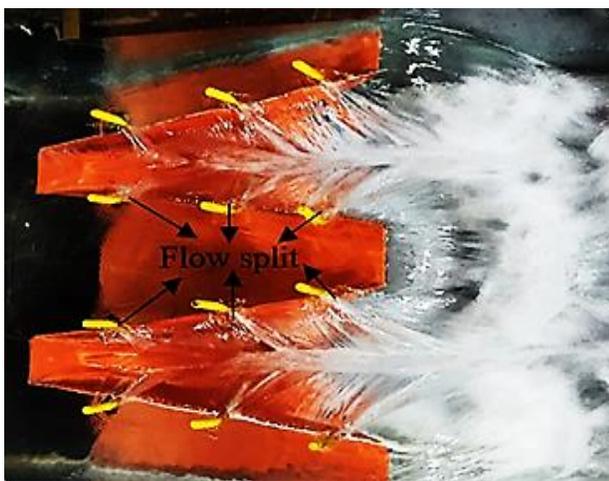
4.1. Laboratory Observations

Fig. 4 shows the flow over a trapezoidal-type C PKW with flow splitters. As shown in the figure, the flow is discharged in the form of an inclined jet from the outlet keys of the weir and a free-falling jet from the inlet keys to the tailwater and inside the outlet keys. Furthermore, the splitters divide the falling jet into several smaller jets. This jet division is more pronounced in the weir with rectangular splitters due to their greater length, and less so with the circular splitters. With increasing flow rate, these divisions became larger, and the flow was discharged at a greater distance from the toe of the weir. Fig. 4a shows the flow over a trapezoidal-type C PKW with six rectangular splitters per cycle, and Fig. 4b shows the flow over a trapezoidal-type C PKW with six circular splitters per cycle. As mentioned, the presence of the splitters causes the flow to be discharged at a greater distance from the weir downstream. For instance, under a flow rate of 0.04 m³/s and a weir height of 0.20 m, the measured flow distances from the inlet keys were 0.29 m (circular splitters), 0.35 m (rectangular splitters), and 0.27 m (no splitters).

In a weir with circular and rectangular splitters with four splitters at the mentioned flow rate, the distance is equal to 0.28 and 0.34 m, respectively. In a weir with circular and rectangular splitters with two splitters at the mentioned flow rate, the distance is equal to 0.27 and 0.32 m, respectively. With an increase in the number of splitters, this distance increased. The primary benefit of this increased distance was reduced energy loss, resulting in a significant reduction in scour potential and the longitudinal extent of the scour hole. In addition, with increasing height and flow rate, the distance from the toe of the weir increases. The maximum flow distance from the inlet keys was 0.31 m, measured in a weir with six rectangular splitters per cycle and averaged across the four tested flow rates. Increasing the length of the falling jet further enhances flow mixing, ultimately leading to increased energy loss. Additionally, the power of this jet of flow is considerable, and numerous eddies are generated upon impacting the downstream flow at the weir. These eddies induce rotational flow and, at a distance from the point of impact, dissipate and transfer to the bottom. This phenomenon also contributes to increased energy loss in the flow.



a) Flow passage over a weir with rectangular splitters



b) Flow passage over a weir with circular splitters

Figure 4. Flow passage and separation over a weir with rectangular and circular splitters.

Flow separation caused by the splitters allows air to enter the inlet keys, thereby aerating the flow. The flow depth increased after hitting the splitters and bulging. Furthermore, vortices are generated by the collision of the flow separated by the splitters with both the outflow from the outlet keys and the converging flows from adjacent splitters. These vortices are shallower in rectangular splitters and are transferred downstream. Rectangular splitters induce greater flow separation, thereby producing this effect. Conversely, the smaller gaps in circular splitters produce more powerful, non-superficial vortices that travel downstream. Fig. 5a shows the increase in the flow depth behind the splitters, and Fig. 5b shows the vortices formed in front of the circular splitters. Vortices reduce flow velocity and cause energy loss, but circular splitters mitigate this effect. In addition, the bulge of the flow behind the splitters reduced the flow velocity upstream of the weir.



a) Vortex formation in front splitters



b) Vortex formation behind the splitters

Figure 5. Vortex formation and flow bulge in front of and behind the splitters.

The flow converged in the inlet keys and at the beginning of the first splitter. This convergence resulted from backflow and bulging of the flow caused by the splitters. This flow convergence was clearly visible in the weirs with six and four splitters. In contrast, in the two-splitter weirs, convergence occurred near the end of the inlet key and was far less distinct. Flow convergence is more pronounced in weirs with rectangular splitters than in weirs with circular splitters. This convergence forces the flow to curve as it enters the downstream section, causing it to collide with the flow from the outlet keys and thereby reducing its velocity.

4.2. Discharge Coefficient

Fig. 6 shows the discharge-stage curve for a trapezoidal-type C PKW. As shown in the figure, the head increased with increasing flow rate. The minimum head is for a weir without flow splitters, with a height of 0.18 m, and the maximum head is for a weir with rectangular flow splitters with six splitters per cycle. The lower the weir height, the lower the head, and the easier the flow passes over the weir. Also, on average, the head on a weir without splitters, and with a height of 0.18 m, is about 9.20% less than the head on a weir without splitters with a

height of 0.20 m. Also, on average, the head on a weir with a height of 0.20 m and six splitters per cycle is approximately 12.70% greater than the head on a weir without splitters.

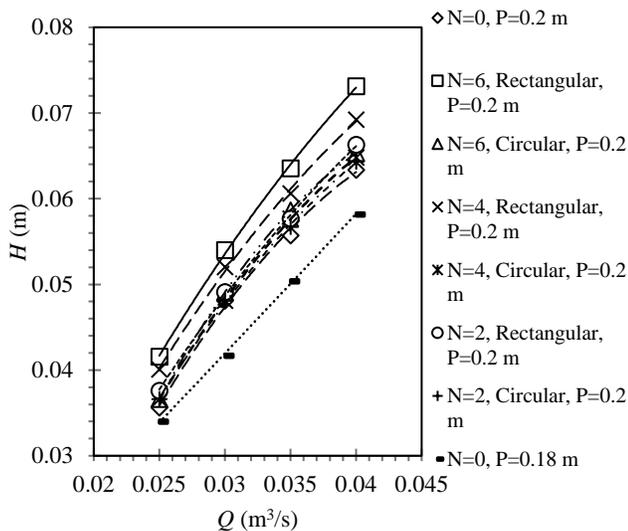


Figure 6. discharge-stage curve.

Fig. 7 shows the weir discharge coefficient as a function of the dimensionless H/P parameter. With increasing H/P parameter, the discharge coefficient decreased. The discharge coefficient was higher in a weir with a lower height. In addition, the splitters cause backflow and reduce the flow passage; ultimately, the discharge coefficient decreases in the presence of splitters. In addition, the higher the number of splitters, the lower the discharge coefficient. The average discharge coefficient in a weir with a height of 0.20 m and 0.18 m is 1.66 and 1.90, respectively. In contrast, the discharge coefficient in a weir without splitters with a height of 0.18 m is about 12.90% higher than the discharge coefficient in a weir without splitters with a height of 0.20 m. In weirs with rectangular and circular splitters with several two splitters and in weirs with circular splitters with four splitters, they are almost similar to each other and have a negligible difference. However, owing to the larger dimensions of rectangular splitters, the discharge coefficients in this type of weir are much lower. This can be attributed to the rotation of the flow around the splitters, backflow, and a higher rise in the flow behind the splitters. This backflow collided with the flow passing through the inlet keys, causing the flow to converge. This convergence reduced the flow passage over the weir crest. In the worst case, in a weir with rectangular splitters and with 6 in each cycle, the discharge coefficient is about 18.60% lower than the weir without splitters, with a height of 0.20 m. Also, in Table 1, the discharge coefficients for the different cases are presented. On average, rectangular splitters reduce the discharge coefficient by approximately 12.50%, and circular splitters reduce it by approximately 3.40%.

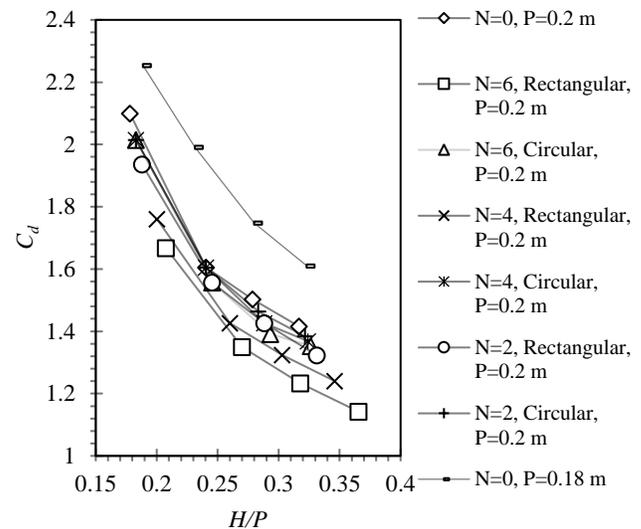


Figure 7. Weir discharge coefficient versus H/P parameter.

Fig. 8 illustrates the discharge coefficient of the PKW as a function of the dimensionless parameter H/L_a . As mentioned above, increasing the number of flow splitters decreased the discharge coefficient. Additionally, reducing the value of L_a (effective length of the weir crest) resulted in less flow discharge over the weir, thereby decreasing the discharge coefficient.

Equation 5 was used to calculate the discharge coefficient in the trapezoidal PKW, both with and without the flow splitters. The correlation coefficient in this equation was 99.12%. Fig. 9 shows the observed and calculated discharge coefficient values. According to Fig. 9, this equation is suitable for estimating the discharge coefficient in trapezoidal PKW with and without flow splitters, with very little error ($\pm 10\%$). Additionally, the data from Rdhaiwi et al. [20] are shown in Figs. 5 and 9. It is evident that this equation is not applicable for the trapezoidal PKW type A with a height of 0.20 m due to significant error; however, it is suitable for the trapezoidal PKW type B with a height of 0.20 m, with less error.

$$C_d = 7.6 \left(\frac{H}{P} \right)^{0.2} \left(\frac{H}{L_a} \right)^{1.3H/P} \quad (5)$$

4.3. Energy Loss

Fig. 10 shows the effect of the discharge per unit width ($q = hV_1$) on relative energy ($E_r = E_2/E_1$). As shown, the relative energy increases with increasing discharge per unit width and depth of flow upstream of the weir. With increasing discharge per unit width, the energy upstream and downstream of the weir increases; the downstream energy is higher, reducing the energy loss. The relative energy also increased with increasing weir height. With a 10% increase in the weir height, the relative energy increased by approximately 1.68%.

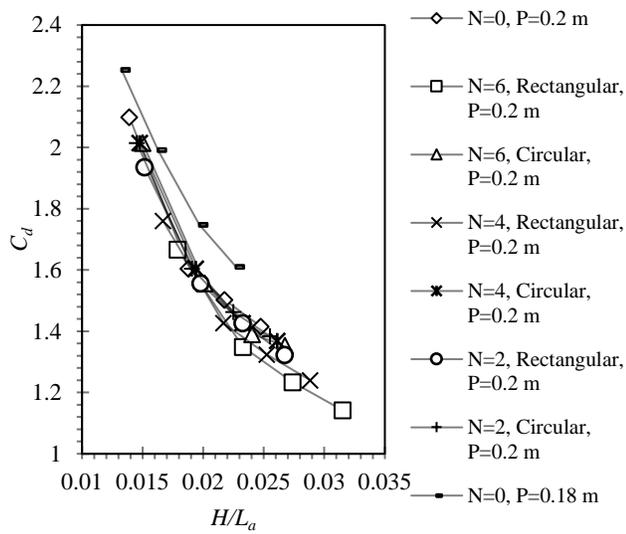


Figure 8. Weir discharge coefficient versus H/L_a parameter.

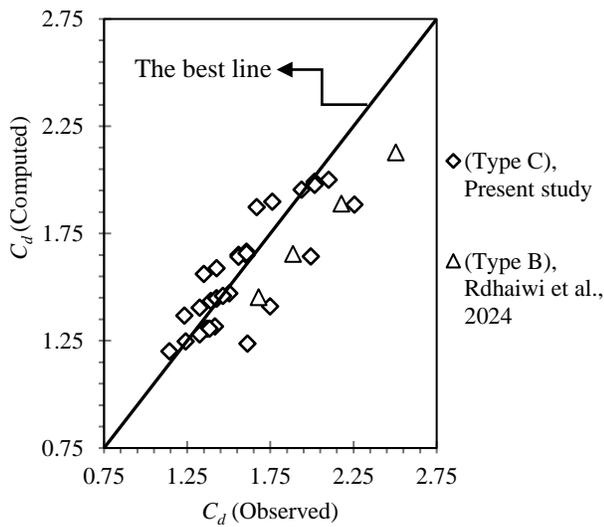


Figure 9. Observed and calculated values of the discharge coefficient.

Fig. 11 shows the energy loss as a function of the dimensionless H/P parameter. As shown in the figure, increasing the ratio of the flow depth to the equivalent height of the kinetic energy to the height of the weir reduces the energy loss. This is because the depth of the flow upstream of the weir increases, or the energy downstream of the weir increases owing to the increase in the flow rate. In addition, by increasing the height of the weir, energy loss decreased. That is, with a 10% increase in weir height, the energy loss decreased by approximately 1.5%. With an increase in the number of splitters per cycle, energy loss also increased. The highest energy loss was observed in weirs with six splitters in each cycle in the rectangular and circular splitters.

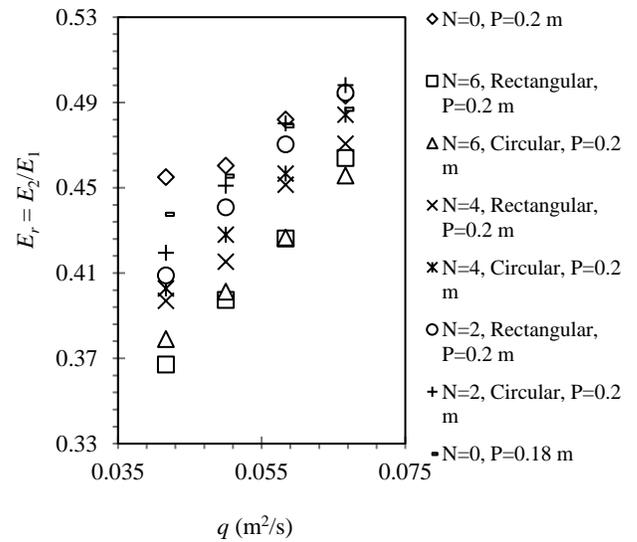


Figure 10. Effect of discharge per unit width on relative energy.

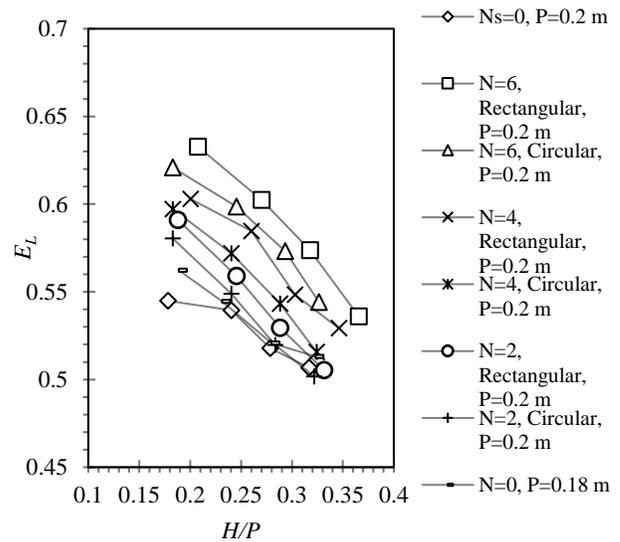


Figure 11. Energy loss versus dimensionless H/P parameter.

Fig. 12 shows the energy loss of the flow as a function of the dimensionless parameter H/L_a . As mentioned above, increasing the number of flow splitters results in greater energy loss of the flow. In addition, reducing the value of L_a results in less flow over the weir. Consequently, the generation of eddies around the flow splitters makes the flow more dissipative, leading to greater energy loss.

Table 2 presents the average energy loss of the flow in weirs with and without flow splitters at different weir heights. The highest energy loss occurred in the weir with the rectangular six-flow splitters, followed by the weir with the circular six-flow splitters. On average across all experiments, the energy loss was approximately 1.24% higher with rectangular splitters than with circular splitters. In weirs with rectangular and circular splitters, the energy loss increased by approximately 6.89% and 5.72%, respectively, compared to the weirs without

a splitter. Fathi et al. (2023) found that the average hydraulic-jump contribution to energy loss was approximately 3% in laboratory studies of stepped PKWs [17]. Given the amount of energy loss observed in Table 2 in the weirs with splitters, the contribution of the hydraulic jump can be considered negligible.

Equation 6 shows the energy loss of PKWs with and without the flow splitters. This relationship was calculated using a correlation coefficient of 99.84%. Fig. 13 also illustrates the observed and calculated values of the flow energy loss with minimal error ($\pm 7\%$), thereby demonstrating its usefulness.

$$E_L = 2.67 \left(\left(\frac{H}{P} \right)^{0.49} \left(\frac{H}{L_a} \right)^{0.9H/P} \right) \quad (6)$$

Table 2. The average energy loss of the models studied in the present study.

<i>P</i> (m)	<i>N</i> =0	<i>N</i> =2	<i>N</i> =2	<i>N</i> =4	<i>N</i> =4	<i>N</i> =6	<i>N</i> =6
-	-	R	C	R	C	R	C
0.20	0.527	0.546	0.538	0.566	0.557	0.586	0.584
0.18	0.535	-	-	-	-	-	-

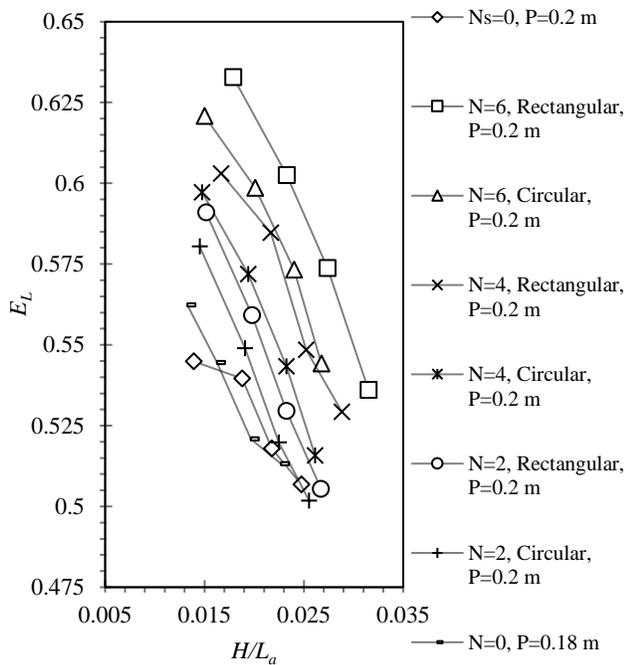


Figure 12. Energy loss versus dimensionless H/L_a parameter.

As noted, increasing the number of flow splitters decreases the discharge coefficient. However, increasing the number of flow splitters (i.e., reducing the effective length of the weir crest) increases the energy loss. The best energy-loss case was observed for the weir with six rectangular flow splitters per cycle, but its discharge coefficient is the lowest. The present study assumed a steady flow, but the discharge coefficient is essential in the case of flood flow. For this reason, the best-case scenario in the present study cannot be determined, and more comprehensive numerical studies are needed.

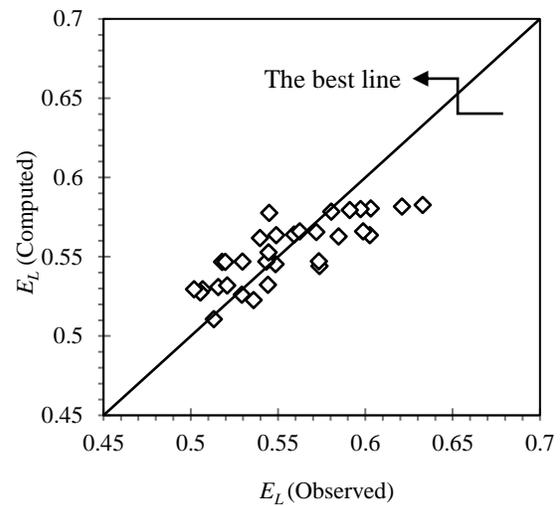


Figure 13. observed and calculated values of energy loss.

Fig. 14 compares the energy loss values from the present study with those of Fathi et al. [17]. They conducted their studies on a trapezoidal-type A PKW with a height of 0.20 m and three cycles. The average energy loss in this work was 0.50. They also observed an average energy loss at 5, 10, and 15 steps (N_s) in the output keys equal to 0.60, 0.67, and 0.62, respectively. They stated that the 10-step PKW was the most effective weir for increasing energy loss. In the best-case scenario of the present study, the energy loss in the trapezoidal-type C PKW with six rectangular splitters per cycle was 0.59. However, in the best case, the energy losses in the present study are approximately 1.50, 12.30, and 4.90% less than those of the 5-, 10-, and 15-step weirs, respectively. This may be due to the loss and damping of the flow caused by the steps in the output keys. As shown, for the splitter-free weir with the height of 0.20 m, the energy loss is approximately 4.74% higher than that of the type A weir. The higher energy loss in the trapezoidal type C PKW than in type A can be due to the lack of a hanging edge upstream of the weir, fewer cycles, and the shorter length of the weir.

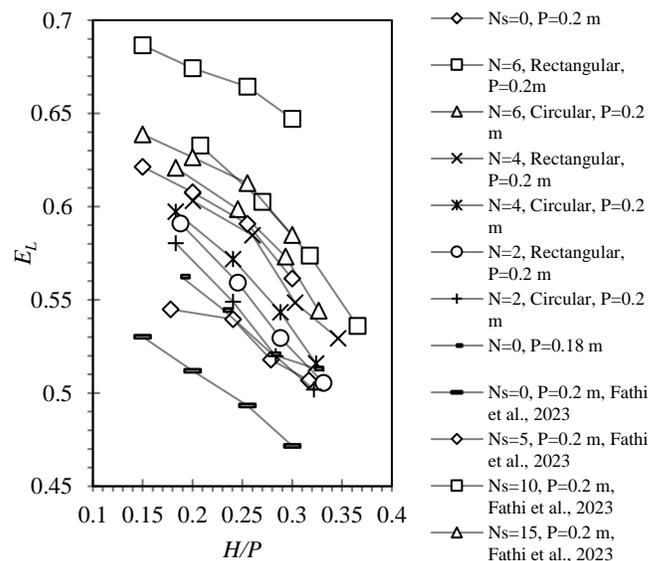


Figure 14. Comparison of energy losses.

5. Conclusions

The presence of flow splitters in PKWs primarily increases energy loss by creating a gap between the weir surface and the flow, thereby promoting flow separation and spreading. The larger flow gap created by rectangular splitters, compared to circular ones, is a primary reason for their superior energy loss. Furthermore, weir height influences dissipation capacity, with higher weirs dissipating more energy. Thus, the key determinants of the PKW discharge coefficient are the presence of the splitters, their shape, and the weir height.

1. The amount of energy loss decreases with increasing flow rate.
2. The average amount of energy loss in weirs with rectangular flow splitters is about 1.24% higher than in weirs with circular flow splitters.
3. Increasing the weir height by 10% reduced the discharge coefficient and, concurrently, decreased energy loss by 1.5%.
4. The configuration with six rectangular flow splitters per cycle yielded the highest overall energy loss, while the six-circular-splitter configuration produced the maximum loss among the circular splitter designs.
5. With the increase in the number of flow splitters, the amount of energy loss also increases.
6. The experimental results demonstrate that flow splitters significantly enhance energy loss, with the magnitude of loss increasing proportionally to the number of splitters. Increasing the H/P parameter reduced both the discharge coefficient and the energy loss. Conversely, shortening the effective crest length reduces the discharge coefficient but increases energy loss.
7. The bigger decrease in the discharge coefficient, 18.6%, occurred in the weir with six rectangular flow splitters.
8. Equations (5) and (6) calculate the discharge coefficient and energy loss for PKWs without and with flow splitters, respectively, and both exhibit a high correlation coefficient.

This study has several limitations, including the assumption of steady-flow conditions and a uniform velocity-correction factor. Furthermore, the model does not account for the presence of mud and debris during flood events, which can significantly impact excess flow capacity. Addressing these factors presents a valuable opportunity for future research.

List of symbols

B	the length of the weir wall
B_o	the length of the hanging edge downstream of the weir
C_d	the discharge coefficient
d	diameter of circular flow splitters and length of rectangular flow splitters in the flow direction
E_1	specific energy of the flow upstream of the weir
E_2	specific energy of the flow downstream of the weir
E_r	relative energy

E_L	energy loss
g	acceleration due to gravity
h and y	upstream flow and tailwater depths, respectively
H	flow depth plus equivalent height of kinetic energy upstream of the weir
L	the length of the weir crest
L_a	the effective length of the weir crest
N	the number of flow splitters per cycle
P	height of the weir
Q	the flow rate
q	the discharge per unit width
R_e	Reynolds number
S	the shape of the splitters
V_1 and V_2	upstream flow and tailwater velocities, respectively
W_i	the width of the inlet keys of the weir
W_o	the width of the outlet keys of the weir
W	the width of the weir
W_e	Weber number

Greek symbols

ρ	density of water
σ	surface tension coefficient
μ	dynamic viscosity

Conflict of interest

The authors confirm that the publication of this article causes no conflict of interest.

Contribution Statement of Author

Amirhossein Fathi and Ali Khoshfetrat developed the theory and performed the computations. All authors proposed the research problem, verified the analytical methods, and supervised the findings of this work. All authors discussed the results and contributed to the final manuscript.

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