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# Estimation of the Reliability Function of the Maxwell - Boltzmann Distribution Using the M-Robust Estimation Methods and Comparing them With Conventional Methods

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RESEARCH ARTICLE

# Estimation of the Reliability Function of the Maxwell - Boltzmann Distribution Using the M-Robust Estimation Methods and Comparing them With Conventional Methods

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## ABSTRACT

To achieve the best possible outcomes. The reliability function estimation was also established for the In this study, the scal parameter Maxwell distribution was estimated using both traditional and robust methods, and the results were compared typical estimation methods, which included the Maximum Likelihood method (MLE), the moments method (MOM), and the M- robust method The results were compared using the mean error squared (MSE). The M-robust technique was found to have the lowest mean square error across all sample sizes, as demonstrated by the findings hence, it is regarded as being more effective than the conventional estimate methods. In addition, it was determined that the statistical criterion, also known as MSE, is the most effective way for estimating, and thus, the results of the simulation are based on this. m-strong technique is used to determine the model values (M-strong). According to statistical theory the number of small and medium-sized enterprises (SMEs) decreases as the sample size increases. Because parameter estimates are significantly hindered when the data set has few outliers, m is the method of estimation that is the most reliable. Because of this robust estimation is an essential method for evaluating databases that contain problematic outliers Numerous robust regression estimators as well as M method estimators, are available in a wide variety. The least squares approach is one of the safest methods because of its remarkable efficiency in obtaining the possibilities

**Keywords:** Estimation methods, Maxwell-Boltzmann, M-robust estimation, Maximum likelihood, Reliability function

## Introduction

The parameters of this distribution play a significant role in understanding the nature of the data, and so there are a plethora of techniques for analyzing and interpreting it. Several situations in statistical mechanics use the Maxwell-Boltzmann Distribution,[1] which is one of the ways to estimate the parameters to get excellent estimators that have the features needed to be available in the estimator. The kinetic energy of gases, which in turn explains many of the fundamental features of gases including pressure and diffusion is based on the Maxwell-Boltzmann distribution.[2] It's also possible to refer to this pattern as the distribution of molecular speeds energies and kinetic energies.

Since it was first described by the Scottish physicist James Clark Maxwell and then again by Boltzmann with the addition of Some assumptions, the Maxwell distribution has been considered one of the continuous probability distributions with a measurement parameter ($\theta > 0$) that determines the probability density function of the distribution.[3] It has applications in physics and chemistry and serves as the foundation for the theory of gas motion, which is used to model the properties of gases such as their pressure, diffusion, particle velocity, and thermal equilibrium. A life model is the Maxwell distribution.[4] The Maxwell

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distribution can be thought of as a generalization of the Whipple distribution and the Rayleigh distribution. For reliability studies and life testing, when a constant failure rate is assumed, it possesses all the necessary qualities.

Box was the first to use the term "robustness" to describe a statistical approach whose statistical inference was unaffected by the violation of any of its basic requirements. Since then, several authors have provided similar definitions. Researchers are motivated to employ robust estimators because they are immune to the effects of outliers and do not assume a fixed shape for any given function. This is because data on some phenomena may contain abnormal values, leading to inaccurate results, which in turn can lead to conclusions that are at odds with the reality of the phenomenon under study.[5,6]

A robust estimate is insensitive to outliers and whose efficiency is comparable to that of least-squares estimates when outliers or outliers are present in data.[7]

## Maxwell distribution

The Maxwell distribution possesses all the characteristics necessary to make it particularly effective in life tests, especially in circumstances when the failure rate assumption is constant. It can be thought of as a variant of the generalized Whipple and Rayleigh distributions.[8] The probability density function of the Maxwell distribution is given by.[9]

$$f(x,\theta) = \frac{4}{\sqrt{\pi}}\frac{1}{\theta^{2}}x^{2}\exp^{-\frac{x^{2}}{\theta}}x,\ (\theta > 0) \quad (1)$$

Where $\theta$ the scale parameter of the distribution, and cumulative distribution function (c. d. f) are given by:

$$\mathcal{F}(\mathcal{X},\theta) = \frac{1}{\Gamma\left(\frac{3}{2}\right)}\Gamma\left(\frac{\mathcal{X}^{2}}{\theta},\ \frac{3}{2}\right) 0 < x,\theta \quad (2)$$

Where $\Gamma(\frac{\mathcal{X}^2}{\theta}, \frac{3}{2}) = \int_x^\infty e^{-u}u^{\alpha-1}du$, $(x, \alpha > 0)$ incomplete Gama function, So that $\alpha = \frac{3}{2}$, $x = \frac{x^2}{\theta}$. As shown in the Fig. 1

## First: Reliability function

Stands for the likelihood that the device or system will continue functioning normally for a given time interval t. It is represented by the symbol $R(t)$. Mathematically, the reliability function is defined by the following equation:[9]

$$R(t) = Pr(T > t) \quad (3)$$

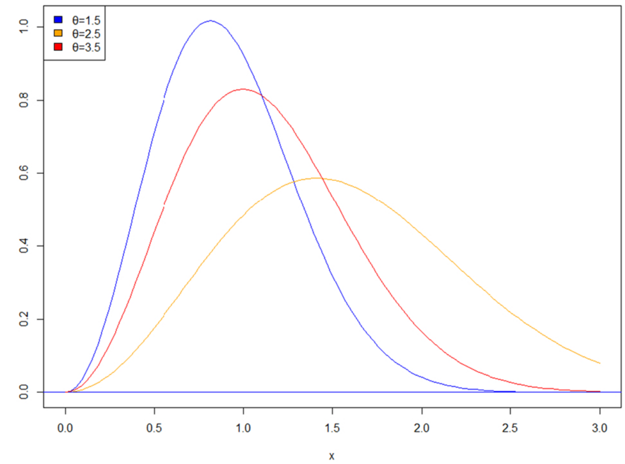


Fig. 1. Maxwell distribution's probability density function (P.D.F.).

The form of the survival function for continuous distributions is:

$$R(t) = P(T > t) = \int_t^\infty f(x)\ dx,\ 0 < t < \infty \quad (4)$$

Where f(t) is the failure probability density function, t is the length of time that starts from zero. That is the reliability function is a probability function confined between zero and the correct one.[10] In other words:

$$0 \le R(t) \le 1$$

The survival function is sometimes referred to as the complementary cumulative distribution function.

$$R(t) = \int_t^\infty f(x)\ dt = 1 - F(t) \quad (5)$$

$$R(t) = 1 - F(t) \quad (6)$$

So that

$$F(x,\theta) = \int_0^t \frac{4}{\sqrt{\pi}}\frac{1}{\theta^{\frac{3}{2}}}x^{2}e^{-\frac{x^{-2}}{\theta}}\,dx \quad (7)$$

$$R(t) = 1 - F(x,\theta) \quad (8)$$

$$R(t) = 1 - \int_0^t \frac{4}{\sqrt{\pi}}\frac{1}{\theta^{\frac{3}{2}}}x^{2}e\frac{x^{-2}}{\theta}dx \quad (9)$$

For this type of integral we use Taylor series for: - $e^{-\frac{x^2}{\theta}}$

$$e^{-\frac{x^{2}}{\theta}} = \sum_{j=0}^{\infty}\frac{(-1)^{j}}{j!}\frac{x^{2j}}{\theta^{j}} \quad (10)$$

$$R(t) = 1 - \frac{4}{\sqrt{\pi}} \frac{1}{\theta^{\frac{3}{2}}} \int_0^t x^2 \sum_{j=0}^{\infty} \frac{(-1)^j}{j!} \frac{x^{2j}}{\theta^j} dx \tag{11}$$

$$= 1 - \frac{4}{\sqrt{\pi}} \sum_{j=0}^{\infty} \frac{(-1)^j}{j!\theta^{\frac{3}{2}+j}} \int_0^t x^{2j+2} dx$$

$$R(t) = 1 - \frac{4}{\sqrt{\pi}} \sum_{j=0}^{\infty} \frac{(-1)^j}{j!\theta^{\frac{3}{2}+j}} \frac{t^{2j+3}}{2j+3} \tag{12}$$

## *The estimation methods*

### 1. Method of Moments

Due to its simplicity, it has become a standard technique for estimating parameters. The rationale behind this technique relies on establishing a near formula for the parameters, which is based on equating the community torque with the sample moment. The estimates obtained using the method of moments are straightforward.[11] The Maxwell distribution's torque can be calculated as follows:

$$E(X^r) = \frac{2}{\sqrt{\pi}} \theta^{\frac{r}{2}} \Gamma\left(\frac{r+2}{2}\right) \qquad r > -3 \tag{13}$$

$$\grave{\mu} = E(x) = 2\frac{\sqrt{\theta}}{\sqrt{\pi}} \tag{14}$$

$$\mu = \frac{1}{n} \sum_{i=1}^{n} x_i = \bar{x} \Rightarrow \acute{\mu} = \mu \tag{15}$$

$$2\frac{\sqrt{\hat{\theta}}}{\sqrt{\pi}} = \bar{x} \Rightarrow 2\sqrt{\hat{\theta}} = \bar{x}\sqrt{\pi} \Rightarrow \sqrt{\hat{\theta}} = \frac{\bar{x}\sqrt{\pi}}{2} \tag{16}$$

$$\hat{\theta} = \frac{\pi \bar{x}^2}{4} \tag{17}$$

## *Maximum likelihood*

When it comes to making reliable estimations, this technique is among the most significant ones available because of the stability with which it is associated. Scientist R.A. Fisher put forth the idea in 1920. The following is a definition of it:

If $(x_1, x_2, \ldots\ldots, x_n)$ then is the largest possible function MLE if it is larger than the normal probability function.

The following equation describes the maximum likelihood estimate (MLE).

$$L(x_i|\theta) = \prod_{i=1}^{n} f(x_i, \theta) \tag{18}$$

$$= \boldsymbol{ln}\, L(x, \theta) = \frac{ln\, L(x, \theta)}{d\theta} = 0 \tag{19}$$

Where $L(x_i|\theta)$ is the probability function or the logarithm of the probability function.[3] The information matrix is computed in turn from the second derivatives of the log-likelihood function, with respect to the vector of parameters

To estimate the maximum possibility function of the Maxwell distribution:

$$L(x, \theta) = \prod_{i=1}^{n} f(xi, \theta) \tag{20}$$

$$L(x_1, x_2, \ldots\ldots x_n/\theta) = \left(\frac{4}{\sqrt{\pi}}\right)^n \frac{1}{\theta^{\frac{3n}{2}}} \prod_{i=1}^{n} x_i^2 exp^{-\frac{\sum_{i=1}^{n} x_i^2}{\theta}} \tag{21}$$

the natural logarithm of both sides of the equation, we get:

$$\ln L(x, \theta) = nln\left(\frac{4}{\sqrt{\pi}}\right) - \frac{3n}{2} ln(\theta) + ln\left(\prod_{i=1}^{n} x_i^2\right) - \frac{\sum_{i=1}^{n} x_i^2}{\theta} \tag{22}$$

To find the estimated value we must find the derivative of the function and equate it to (zero).

$$\frac{ln\, L(x, \theta)}{d\theta} = 0 - \frac{3n}{2\theta} + 0 + \frac{\sum_{i=1}^{n} x_i^2}{\theta^2} = 0 \tag{23}$$

$$3n\theta = 2\sum_{i=1}^{n} x_i^2 \tag{24}$$

$$\hat{\theta}_{MLE} = \frac{2\sum_{i=1}^{n} x_i^2}{3n} \tag{25}$$

### 2. M-Robust Estimation Methods

Parameter estimations are severely hampered when a dataset contains even a few outliers.

As a result, robust estimation is a crucial technique for investigating databases with questionable outliers. There is a wide variety of robust regression estimators, the M-estimation method Estimators.

This method is considered one of the safest methods due to the great efficiency with which it obtains capabilities. hence it is regarded to be one of the safest methods. Researchers have shown a lot of interest in this methodology for a number of reasons, including the fact that it is more adaptable and offers the prospect of direct generalization on multiple regression. It was proposed by the researcher Huber (1973), and the concept behind this method is to locate the value of a and b that is the least significant, and the immune estimator is obtained by testing the weight function $\psi$.[12,13]

$$Min \sum_{i=1}^{n} \rho\,(e_i) \tag{26}$$

$$Min \sum_{i=1}^{n} [Y - a - b\,(X_i)]^2$$

Estimating the value of the Maxwell distribution's undetermined parameter can be accomplished through a straightforward comparison of two distinct goal functions.

Bisquare and Huber's weight are the objective functions that were chosen.[14,15]

The M estimator can be obtained by reducing or decreasing the following amount:

$$Min \sum_{i=1}^{n} \rho\left(\frac{e_i}{s}\right) = Min \sum_{i=1}^{n} \psi\left(\frac{x_i - \mu}{s}\right) = 0 \tag{27}$$

Since s is an estimation of the scale, it may be calculated using the following equation:

$$s = \frac{median\left|e_i - median\,(e_i)\right|}{1.339} = \frac{MAD}{1.339} \tag{28}$$

MAD represents the average absolute deviation. When attempting to calculate the hippocampus estimator, it is necessary to test the weight function to solve the Eq. (5) assume that,

$$\text{So that}: u_i = \left(\frac{X_i - \mu}{\sigma}\right) \tag{29}$$

$$\mu_1 = \mu_0 + \sigma_0 \frac{\sum_{i=1}^{n} \psi\,(u_i)}{\sum_{i=1}^{n} \grave{\psi}\,(u_i)} \tag{30}$$

$\psi\,(u_i)$, $\grave{\psi}\,(u_i)$, are The first and second derivative $\rho\,(u_i)\grave{}$, respectively

Since the robustness of the estimator is dependent on the function of the weight functions, it is permissible to use the robust weight function (Huber) in the following calculation,[15,16] which can be viewed as a collection of functions on which the weights corresponding to the observations are calculated.[17]

$$\psi = \begin{cases} 1 & if\ d_i \le c \\ 0 & if\ d_i > c \end{cases} \tag{31}$$

❖ **Huber (1964) Objective Function $\rho(e_i)$:**

$$\rho\,(e_i) = \begin{cases} \frac{1}{2}e_i & if\ |e_i| < c \\ c\,|e_i| - \frac{1}{2}c^2 & if\ |e_i| \ge c \end{cases}, c = 1.345 \tag{32}$$

Score Function $\psi\,(e_i)$

$$\psi\,(e_i) = \begin{cases} e_i \\ c\ sign\ e_i \end{cases}, c = 1.345 \tag{33}$$

❖ **Tukey Function.**

Objective

Function $\rho\,(e_i) =$

$$\begin{cases} \frac{c^2}{6}(1-(1-(\frac{e_i}{c})^2)^3) & if\ |e_i| < c \\ \frac{1}{6}c^2 & if\ |e_i| \ge c \end{cases}, \ c = 1.345 \tag{34}$$

Score Function $\psi\,(e_i)$

$$\psi = \begin{cases} e_i(1-(\frac{e_i}{c})^2)^2 & if\ |e_i| < c \\ 0 & otherwise \end{cases}, \ c = 1.345 \tag{35}$$

## Results and discussion

In this research article, M-robust estimation methods, one of the modern estimation methods, and some of the conventional estimation methods have been compared in the estimation of the reliability function of the Maxwell-Boltzmann Distribution.

In this part of the research, a Monte Carlo simulation study is carried out, Methods such as the maximal likelihood estimate (MLE) and the method known as (mom) are examples of non-robust methodologies

M- method is the Robust method. The maxwell distribution with the supplied parameter values is used to produce complete data with outliers at random. The Monte Carlo simulation program is written in the Matlab programming language,[18–20]

We use the MATLAB script editor to begin the process of writing the simulation algorithm, as shown in the steps below: Considering several possible (actual) values for the Maxwell-Boltzmann distribution parameter: $\theta$: 1.5, 2.5, 3.5 (Selecting many different samples sizes.[21–23] (n 10, 20, 40, 50, 100)

Assuming many values of $a$, $b$, and $k$ as the following:

Considering the number of sample replicate $L = 1000$,Generating random number by the following algorithm suggested by Krishna and Malik estimated the *reliability function in Maxwell distribution* by *using* type-two censored sample.

**a.** Using the uniform distribution U(0,1), generate two random numbers X1 and X2 and write them down.

**Table 1.** The reliability estimates for each estimate method.

| | $\theta = 1.5$ | | | | $\theta = 2.5$ | | | | $\theta = 3.5$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| n | R(t) | R(MLE) | R(MOM) | R_M | R(t) | R(MLE) | R(MOM) | R_M | R(t) | R(MLE) | R(MOM) | R_M |
| | 0.9938 | 0.993 | 0.9923 | 0.9936 | 0.9971 | 0.9966 | 0.9968 | 0.9969 | 0.9982 | 0.9987 | 0.9985 | 0.9979 |
| 10 | 0.9536 | 0.9526 | 0.9574 | 0.9534 | 0.9776 | 0.9768 | 0.9763 | 0.9772 | 0.9982 | 0.9861 | 0.9862 | 0.9977 |
| | 0.7212 | 0.7164 | 0.7397 | 0.721 | 0.8495 | 0.8446 | 0.8419 | 0.8491 | 0.9982 | 0.9019 | 0.9026 | 0.9976 |
| | 0.9938 | 0.5488 | 0.5804 | 0.9936 | 0.741 | 0.7335 | 0.7292 | 0.737 | 0.8272 | 0.8254 | 0.8266 | 0.8269 |
| | 0.3909 | 0.3838 | 0.419 | 0.3906 | 0.6149 | 0.6051 | 0.5995 | 0.6146 | 0.7325 | 0.73 | 0.7318 | 0.7322 |
| 20 | 0.9938 | 0.9936 | 0.9943 | 0.9935 | 0.9971 | 0.997 | 0.9969 | 0.9965 | 0.9982 | 0.9982 | 0.9985 | 0.9981 |
| | 0.9536 | 0.9326 | 0.9394 | 0.9533 | 0.9776 | 0.9679 | 0.9704 | 0.9774 | 0.9982 | 0.9879 | 0.9886 | 0.998 |
| | 0.7212 | 0.6267 | 0.6557 | 0.7209 | 0.8495 | 0.7946 | 0.808 | 0.8492 | 0.9982 | 0.9139 | 0.9186 | 0.9978 |
| | 0.5553 | 0.435 | 0.4705 | 0.5549 | 0.741 | 0.6581 | 0.6779 | 0.738 | 0.8272 | 0.8456 | 0.8535 | 0.82668 |
| | 0.3909 | 0.3838 | 0.419 | 0.3905 | 0.6149 | 0.6051 | 0.5995 | 0.6147 | 0.7325 | 0.73 | 0.7318 | 0.7323 |
| 40 | 0.9938 | 0.9947 | 0.9945 | 0.9934 | 0.9971 | 0.9964 | 0.9964 | 0.9966 | 0.9982 | 0.9978 | 0.9979 | 0.9979 |
| | 0.9536 | 0.9605 | 0.959 | 0.9532 | 0.9776 | 0.9724 | 0.9723 | 0.9773 | 0.9982 | 0.9828 | 0.9837 | 0.9978 |
| | 0.7212 | 0.7553 | 0.7478 | 0.7207 | 0.8495 | 0.8193 | 0.8191 | 0.849 | 0.9982 | 0.8812 | 0.8867 | 0.9976 |
| | 0.5553 | 0.602 | 0.5915 | 0.555 | 0.741 | 0.6948 | 0.6945 | 0.734 | 0.8272 | 0.7915 | 0.8004 | 0.8268 |
| | 0.3909 | 0.4437 | 0.4317 | 0.3906 | 0.6149 | 0.5555 | 0.5552 | 0.6146 | 0.7325 | 0.6827 | 0.695 | 0.7324 |
| 50 | 0.9938 | 0.9941 | 0.9855 | 0.9937 | 0.9971 | 0.9974 | 0.9932 | 0.9968 | 0.9982 | 0.998 | 0.9979 | 0.9977 |
| | 0.9536 | 0.9567 | 0.9559 | 0.9534 | 0.9776 | 0.9792 | 0.9798 | 0.9774 | 0.9982 | 0.9845 | 0.984 | 0.9979 |
| | 0.7212 | 0.7363 | 0.7324 | 0.7206 | 0.8495 | 0.8594 | 0.8629 | 0.8491 | 0.9982 | 0.8918 | 0.8886 | 0.9978 |
| | 0.5553 | 0.5757 | 0.5704 | 0.5549 | 0.741 | 0.7566 | 0.7621 | 0.737 | 0.8272 | 0.8088 | 0.8036 | 0.8266 |
| | 0.3909 | 0.4137 | 0.4077 | 0.3906 | 0.6149 | 0.6355 | 0.6429 | 0.6145 | 0.7325 | 0.7067 | 0.6994 | 0.7324 |
| 100 | 0.9938 | 0.9945 | 0.9946 | 0.9937 | 0.9971 | 0.9972 | 0.9972 | 0.9969 | 0.9982 | 0.998 | 0.9979 | 0.9978 |
| | 0.9536 | 0.959 | 0.9596 | 0.9532 | 0.9776 | 0.9782 | 0.9783 | 0.9773 | 0.9982 | 0.9844 | 0.9835 | 0.9977 |
| | 0.7212 | 0.7476 | 0.7509 | 0.7211 | 0.8495 | 0.853 | 0.8537 | 0.8492 | 0.9982 | 0.8913 | 0.8858 | 0.9976 |
| | 0.5553 | 0.5913 | 0.5959 | 0.5551 | 0.741 | 0.7466 | 0.7476 | 0.739 | 0.8272 | 0.8079 | 0.7989 | 0.8266 |
| | 0.3909 | 0.4314 | 0.4366 | 0.3907 | 0.6149 | 0.6222 | 0.6236 | 0.6145 | 0.7325 | 0.7055 | 0.6929 | 0.7323 |

**b.** Using the transformation, obtain two normal N(0,1) variants designated as $Y_1$ and $Y_2$:

$$Y_1 = \sqrt{-2\log(X_1)}cos2\pi\ (X_2),$$

$$Y_2 = \sqrt{-2\log(X_1)}sin2\pi\ (X_2),$$

and find $Z = \frac{Y_1+Y_2}{\sqrt{2}} \sim N(0,1)$.

**c.** Performing steps a and b a total of three times in order to produce $\chi_3^2\ T = \sum_{i=1}^{3} z_i^2$ which is gamma $G(\frac{3}{2}, \frac{1}{2})$ variate.[24,25]

**d.** Using the transformation $V = \sqrt{\frac{T\theta}{2}}$,get a number generated from $MW(\theta)$ variate.

Finding the mean of estimators for all methods, given by the following equation:[26] $\hat{\theta}_{mean} = \frac{\sum_{i=1}^{L}\hat{\theta}_i}{L}$ where, $L$ represent the number of replications in each experiment and equal to (1000).

Final step of this algorithm is to employ measure of mean squares error[27] (which is defined as distance between the estimate value and actual value), and to compare between all methods of estimation, given by the following formula:

$$\text{MES}\left(\hat{\theta}\right) = \frac{1}{L}\sum_{i=1}^{L}\left(\hat{\theta}_i - \theta\right)^2$$

where, $\hat{\theta}_i$ is the estimate of ($\theta$ a) the $\underline{L}^{th}$ run, and $\theta$ is the true value.

In this research article, ten different estimation methods were compared that random or subjective gravities highly affected the estimation in the comparison of the M-robust and the conventional estimation methods.

Table 1 shows the reliability function values for each estimate method. ($\theta$ = 1.5, 2.5, 3.5) according to sample sizes and the number of replications (L = 1000).

Table 2 shows the (MSE) of the estimation methods for the reliability function (MLE, MOM and the M_robust) for all sample sizes (10, 20, 40, 50, and 100) and for all experiments.

The results will show that, in general, the pseudo-robust estimate could be a good estimate when the normality assumption holds. In real simulation data, the pseudo-robust estimate still behaves poorly compared to the non-robust estimate. So, in general, based on coefficients of correlation and bias parameters, it can be said that the M-robust approaches, in particular the proposed one, perform better when the non-normality and contamination levels are greater. Moreover, we will show that the proposed M-robust estimation method is able to reach the highest power coefficient and the lowest type II errors in real and robust environments. By means of gamma MME value of $\beta$M, for some values of Ruse and N, the minimum value is attained by the proposed method. This set of

Table 2. Mean square error for the used estimation methods and for all sample sizes.

| $\theta$ | *N* | *MLE* | *MOM* | *M* | *Best* |
|---|---|---|---|---|---|
| **1.5** | 10 | 0.1198 | 0.0941 | 0.00030 | M |
| | 20 | 0.0076 | 0.0079 | 0.00010 | M |
| | 40 | 0.0038 | 0.0041 | 0.000013 | M |
| | 50 | 0.0028 | 0.0029 | 0.000001 | M |
| | 100 | 0.0014 | 0.0015 | 0.000003 | M |
| **2.5** | 10 | 0.0113 | 0.0116 | 0.0011 | M |
| | 20 | 0.0058 | 0.0059 | 0.00017 | M |
| | 40 | 0.0025 | 0.0027 | 0.000024 | M |
| | 50 | 0.0022 | 0.0023 | 0.000021 | M |
| | 100 | 0.001 | 0.001 | 0.00001 | M |
| **3.5** | 10 | 0.0078 | 0.0077 | 0.00062 | M |
| | 20 | 0.0036 | 0.0037 | 0.000022 | M |
| | 40 | 0.0017 | 0.0017 | 0.00001 | M |
| | 50 | 0.0013 | 0.0014 | 0.00002 | M |
| | 100 | 0.0007 | 0.0007 | 0.000001 | M |

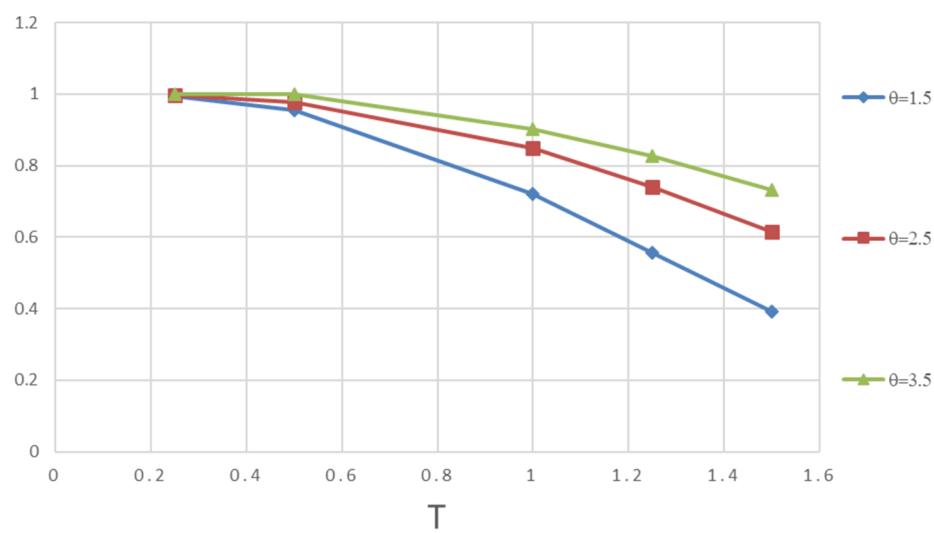


Fig. 2. Shows the reliability curve of the Maxwell distribution.

trials contains two Ruse values and three sample sizes N. From the environmental perspective, this range is rather rich. Thus, this procedure can be recommended to practitioners as a useful and efficient one.

where the stability function was estimated with the simplicity of the different method in application and ease of implementation to obtain an estimate of the stability function. In addition to Fig. 2.

## Conclusion

The statistical standard (MSE) is the best way to estimate, so that is what the simulation results are based on. The values of the model are the robust m method (M - robust). Statistical theory says that the MSE numbers get smaller as the sample size gets bigger. It is possible to adopt robust estimation methods to estimate the reliability function for other distributions and figure the reliability curve of the Maxwell distribution in this work, the stability function was estimated using a method different from the traditional methods followed by some researchers,

The M-robust technique was found to have the lowest mean square error across all sample sizes, as demonstrated by the findings hence, it is regarded as being more effective than the conventional estimate methods. In addition, it was determined that the statistical criterion, also known as MSE, is the most effective way for estimating, and thus, the results of the simulation are based on this. m-strong technique is used to determine the model values (M-strong).


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## Authors' declaration

- Conflicts of Interest: None.
- I hereby confirm that all the Figures and Tables in the manuscript are mine. Furthermore, any Figures and images that are not mine have been included with the necessary permission for re-publication, which is attached to the manuscript.
- No animal studies are present in the manuscript.
- No human studies are present in the manuscript.
- Ethical Clearance: The project was approved by the local ethical committee at Mosul University.

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# تقدير دالة المعولية لتوزيع ماكسويل- بولتزمان باستخدام طرق تقدير M - الحصينة ومقارنتها بالطرق التقليدية

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**الملخص**

في هذه الدراسة تم تقدير معلمات توزيع ماكسويل باستخدام الطرق التقليدية و,الحصينة وتمت مقارنة النتائج لتحقيق أفضل النتائج الممكنة. تم أيضًا إنشاء تقدير دالة الموثوقية لطرق التقدير النموذجية والتي تضمنت طريقة الامكان الاعظم (MLE)، وطريقة (MOM)، وطريقة M- strong. وتمت مقارنة النتائج باستخدام متوسط مربع الخطأ (MSE). وُجد أن تقنية M-robust تتمتع بأدنى مربع متوسط خطأ في جميع أحجام العينات، كما يتضح من النتائج؛ وبالتالي تعتبر أكثر فعالية من طرق التقدير التقليدية. بالإضافة إلى ذلك، تم تحديد المعيار الإحصائي المعروف أيضًا باسم MSE وهو الطريقة الأكثر فعالية للتقدير وبالتالي فإن نتائج المحاكاة تعتمد على ذلك. يتم استخدام تقنية m-strong لتحديد قيم النموذج (M-strong). ووفقا للنظرية الإحصائية فإن عدد المعلمات الصغيرة والمتوسطة تتناقص مع زيادة حجم العينة. نظرًا لأن تقديرات المعلمات يتم إعاقتها بشكل كبير عندما تحتوي مجموعة البيانات على عدد قليل من القيم المتطرفة فإن m هي طريقة التقدير الأكثر موثوقية ولهذا السبب يعد التقدير الحصين طريقة أساسية لتقييم قواعد البيانات التي تحتوي على القيم المتطرفة الإشكالية. تتوفر العديد من مقدرات الانحدار القوية بالإضافة إلى مقدرات الطريقة M في مجموعة واسعة. يعتبر أسلوب المربعات الصغرى من أكثر الطرق أمانا لما له من كفاءة ملحوظة في الحصول على الإمكانيات. هذه الطريقة هي واحدة من الاستراتيجيات الأكثر أمانا ونتيجة لحقيقة أن الهدف من هذا النهج هو تقليل بعض وظائف الخطأ بدلاً من مجموع المربعات فإنه يعتبر أحد أكثر الطرق أمانًا.

**الكلمات المفتاحية:** طرق التقدير، تقدير M القوي، الاحتمالية القصوى، ماكسويل-بولتزمان، دالة الموثوقية.