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IMPROVED NEW TWO-SPECTRAL CONJUGATE GRADIENT ITERATIVE TECHNIQUE FOR LARGE SCALE OPTIMIZATION

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ABSTRACT

Numerous strategies have been proposed in the field of unconstrained optimization to address various optimization challenges, particularly those associated with large-scale systems. Among the classical methods, Newton and Quasi-Newton approaches are well-known for their rapid convergence properties, especially when initiated with accurate initial estimates. However, these methods rely on second-order derivative information and require the computation or approximation of the Jacobian matrix, which is computationally expensive and thus impractical for large-scale problems. As a result, their direct applicability becomes limited when dealing with high-dimensional optimization tasks.Although originally developed for unconstrained optimization, these methods have been extended to solve systems of nonlinear equations and large-scale optimization problems. In contrast, spectral approaches utilize eigenvalue-based techniques to improve computational efficiency, while conjugate gradient (CG) methods minimize quadratic forms without requiring second-order derivatives, making them more suitable for complex, large-scale systems.The Dai–Yuan (DY) method improves upon the classical CG algorithm by providing a more efficient search direction, which can lead to faster convergence and improved numerical stability in certain scenarios. Similarly, the Hestenes–Stiefel (HS) method offers an alternative formulation that often enhances the convergence rate by more accurately approximating the ideal solution path.More recently, a two-spectral conjugate gradient approach has been explored as a promising technique for solving large-scale unconstrained optimization problems. This method enhances both robustness and efficiency by combining the current gradient direction with a previous search direction, resulting in a more

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effective search path. Furthermore, many recent CG algorithms incorporate a Wolfe–Powell-type inexact line search strategy, which enables efficient step length determination without the computational burden of exact line search methods. This inexact line search provides a favorable trade-off between computational efficiency and solution accuracy, enhancing the practicality of these methods for large-scale unconstrained optimization.

KEYWORDS

Conjugate Gradient, Spectral, Unconstrained Optimization.

1. INTRODUCTION

Optimization arose alongside the development of calculus and is regarded as a basic subject within applied mathematics because to its widespread applications in domains such as engineering, natural sciences, and economics. It is defined as an iterative process that starts with an initial estimate and progresses through a series of stages aimed at gradually improving the solution until a stopping requirement is satisfied, marking the algorithm's end (Zhou and Zhang, 2006).

Iterative methods are effective instruments for solving large systems of equations, and choosing the right method depends on the specific characteristics of the problem at view. Numerical optimization approaches are intended to find the global minimum of a function specified on a real-valued domain. These difficulties are commonly referred to as minimization problems.

According to Zhang (2006), optimization is one of the oldest and most influential fields in the evolution of scientific knowledge. It is based on the notion of determining the best potential solution to a problem whose outcome is quantifiable in terms of actual numbers. The basic question in this discipline is, "What is the optimal solution?". Optimization challenges are often classified into two major kinds, Constrained and unconstrained optimization.

Unconstrained optimization issues are those in which no conditions or constraints are imposed on the variables. Their purpose is to determine the maximum and lowest values of a function over its whole definition space.

It is frequently assumed that the function under consideration is differentiable across the entire domain and that its first-order partial derivatives are known. However, certain algorithms require the second derivative to be positively defined in order to improve convergence conditions and numerical efficiency.

1.1. Conjugate Gradient Methods

Conjugate gradient methods are among the most extensively used and successful approaches for addressing optimization problems, particularly in large-dimensions. These methods have seen significant development in recent decades, particularly with the expanding use of artificial intelligence techniques and evolutionary algorithms, which has contributed to the development of powerful and effective tools for obtaining high-quality approximate solutions in less computational time (Caraba, 2008). Hillier et al. (1973).

The spectacular expansion of the optimization field can be attributed to rapid technological advancements in computers and software, the rise of parallel processors, and the development of intelligent algorithms. This has resulted in the development of improved algorithms capable of discovering approximation solutions that meet optimality conditions before reviewing the solution inside the same process to confirm compliance with the initial issue.

Most optimization issues are derived from engineering applications, automated control systems, and the regulation of electronic devices, as well as complex issues in various sciences that were considered unsolvable until recently. Evolutionary algorithms are among the most prominent algorithms used in optimization, including:

• Genetic Algorithm (GA),

• Particle Swarm Optimization (PSO),

in addition to other algorithms classified under the heading of "meta-heuristic algorithms."(Ozule et al., 2025)

These algorithms are characterized by their reliance on two basic principles:

• Exploration: to search across different parts of space,

• Exploitation: to improve the discovered solutions.

Each algorithm also has unique behavioral and social characteristics that contribute to the development of search strategies, as indicated by numerous studies (Hachim and Swadi, 2024;Thanoon, Ban and Mitras, 2020).

Conjugate gradient methods are effective for two main reasons:

1. Their high ability to solve systems of large-dimensional nonlinear equations.

2. Their adaptability to efficiently solve nonlinear optimization issues (Nocedal and Wright, 1996).

These methods are intermediate in computational performance between the Steepest Descent method, which suffers from slow convergence, and Newton's method, which requires calculating and storing second-order derivatives (the Hessian matrix), which is numerically expensive, especially in cases with large dimensions(‘OPTIMIZATION THEORY AND METHODS N onlinear P rogramming Springer Optimization and Its Applications’, no date).

Conjugate gradient methods are divided into two main types:

• The first type: Linear Conjugate Gradient Methods

Also known as quadratic or "pure" conjugate gradient methods, they are used to minimize convex quadratic functions.

• The second type: Nonlinear Conjugate Gradient Methods

They are used to minimize general functions, whether convex or non-convex.The linear conjugate gradient method was first proposed by Hestenes and Stiefel in 1950 as an alternative to Gaussian elimination for solving systems of linear equations with positively defined matrices (Hestenes and Stiefel, 1952). Later, Fletcher and Reeves developed this method further, introducing its first nonlinear formulation in 1960. Since then, numerous derivatives of this method have been proposed, finding widespread application in various fields (Nocedal and Wright, 1996)

Numerous issues that are studied in scientific fields can be transformed into unconstrained optimization issues (UOI), as is well known. Because of its straightforward iterations, quick convergence, and minimal memory needs, the conjugate gradient method (CGM) is one of the most popular approaches for resolving large-scale optimization issues in nonlinear optimization (Zhou and Ni, 2015; Sheekoo, 2021). In this study, we evaluate the UOIs using the following form:

$$\min f(x), x \in \Re^n \tag{1}$$

The gradient of the continuously differentiable nonlinear function $f: \Re^n \rightarrow \Re$ is represented by $g(x) = \nabla f(x)$, and it is bounded below. CG uses an iterative process to provide a series of roughly ideal solutions, which is

$$x_n = x_{n-1} + \omega_{n-1} d_{n-1}, \qquad n = 1,2,3, \ldots, \tag{2}$$

where $\omega_{n-1} > 0$ is a step-size discovered by line search a long d_n , d_n is a search direction to effectively explore for an optimal solution to issues (1), x_0 is an arbitrarily selected initial solution and x_n denotes the current iterative point (Sulaiman *et al.*, 2024). The search directions for the traditional CGMs are often provided by:

$$d_n = \begin{cases} -g_n, & n = 1 \\ -g_n + \beta_n d_{n-1}, & n \geq 2 \end{cases} \tag{3}$$

where $g_n = g(x_n)$ and $\beta_n \in \Re$ are parameters whose variations result in different nonlinear CGMs. We have two categories of parameters β_n, the first one, include the Fletcher-Reeves (FR) (Fletcher, 1964) method, the conjugate descent (CD) (Fletcher, 1987)method and the Dai-Yuan (DY) (Yuan and Dai, 1999)method. Their form is as follows

$$\beta_n^{FR} = \frac{\|g_n\|^2}{\|g_{n-1}\|^2}, \beta_n^{CD} = -\frac{\|g_n\|^2}{g_{n-1}^T d_{n-1}}, \beta_n^{DY} = \frac{\|g_n\|^2}{y_{n-1}^T d_{n-1}} \quad .$$

Where $y_{n-1} = g_n - g_{n-1}$, the Euclidean norm vectors id denoted by $\|.\|$.These parameters contain the same numerator, $\|g_n\|^2$, resulting in significant convergence in theory. Despite its strong convergent characteristics, jamming may result in subpar performance in real-world scenarios. Numerical performance remains inefficient.

The second categories include the Hestenes-Stiefel (HS) (Hestenes and Stiefel, 1952) method, Polak-Ribiere and Polyak (PRP) (Opérationnelle *et al.*, 1969; Polyak, 1969) method and Liu-Storey (LS) (Liu and Storey, 1991) method. Their form is as follows:

$$\beta_n^{HS} = \frac{g_n^T y_{n-1}}{y_{n-1}^T d_{n-1}}, \beta_n^{PRP} = \frac{g_n^T y_{n-1}}{\|g_{n-1}\|^2}, \beta_n^{LS} = -\frac{g_n^T y_{n-1}}{g_{n-1}^T d_{n-1}}$$

These parameters contain the same numerator $g_n^T y_{n-1}$. However, they do not satisfy the convergence property, even when employing exact line searches for non-convex functions (Andrei, no date; Control and 2008, no date).

When analyzing convergence and implementing nonlinear CG algorithms, the strong Wolfe (SW) inexact line search is frequently taken into consideration because exact line search for finding ω_{n-1} is typically costly and impracticable. The following two SW requirements must be met in order to find a step size n.

$$f(x_n + \omega_{n-1} d_{n-1}) \leq f(x_{n-1}) + \delta_1 \omega_{n-1} g_{n-1}^T d_{n-1} \quad (4)$$

$$|g(x_n + \omega_{n-1} d_{n-1})^T d_{k-1}| \leq \delta_2 |g_{n-1}^T d_{n-1}| \quad (5)$$

where $0 < \delta_1 < \delta_2 < 1$, are the parameters (Hassan and Al-Naemi, 2020).

Birgin and Martinez (Birgin and Martínez, 2001) combined the CG and spectral gradient approaches to create the SCG- technique (Raydan, 1997). Zhang et al. (Zhang *et al.*, 2006)adapted the FR method from the SCG approach.

Spectral gradient has become one of the prominent techniques for solving unconstrained optimization issues (Lai *et al.*, 2023). Raydan worked on developing the spectral conjugate gradient method. After this idea was introduced by Bariziliai & Borwein. It is based on the principle of small storage and calculations for convex quadratic functions as well as for non-quadratic functions (Raydan, 1997). The idea of working is to combine the spectral gradient with the conjugate gradient and obtain a convex linear combination of two different gradient coefficients (Qasim and Salih, 2022). One of its advantages is to ensure the appropriate descent direction as well as global convergence (Tang *et al.*, 2023).

If $\theta_n = \frac{y_{n-1}^T d_{n-1}}{\|g_{n-1}\|^2}$, then for all $k \geq 0$, $g_n^T d_n = -\|g_n\|^2$. This results in a CG method with global convergence given certain assumptions and the Armijo condition. The direction d_n can be calculated using the SCG method as follows:

$$d_n = \begin{cases} -g_n, & n = 1 \\ -\theta_n g_n + \beta_n d_{n-1}, & n > 1 \end{cases} \quad (6)$$

θ_n represents a spectral parameter. many authors proposed various methods (Enterprise and 2022, no date; Malik et al., no date; Deng et al., 2015; Zhou, Engineering and 20152015 ,; Zhu et al., 2017; Meansri et al., 2024; Yahaya et al., 2024).

2. DERIVATION OF THE NEW SCG-TECHNIQUE AND ITS SUFFICIENT DESCENT PROPERTY

In this section, we explore how to obtain the spectral parameter θ_n. We intend for the drop direction coming from Eq.6 to be close to the quasi-Newton direction. This technique tries to improve the algorithm's convergence while retaining the desired numerical performance.

The direction d_n is Newton direction determined by:

$$d_n = -B_n^{-1}.g_n \tag{7}$$

Specifically, from (6) and (7) we can write

$$-B_n^{-1}.g_n = -\theta_n g_n + \beta_n d_{n-1} \tag{8}$$

Multiply both sides (8) by $B_{\boldsymbol{n}} s_{n-1}^T$, we get

$$-s_{n-1}^T g_n = -\theta_n B_{\boldsymbol{n}} s_{n-1}^T g_n + \beta_n B_{\boldsymbol{n}} s_{n-1}^T d_{n-1} \tag{9}$$

where $s_{n-1} = x_n - x_{n-1} = \omega_{n-1} d_{n-1}$. In the quasi-Newton (QN) method, an approximation matrix B_{n-1} of the Hessian $\nabla^2 f(x_{n-1})$ is update such that the new matrix B_n satisfies the following QN equation:

$$B_{\boldsymbol{n}} s_{n-1}^T = y_{n-1}^T \tag{10}$$

We obtain by substituting (10), with some algebraic tricks, into (9).

$$\theta_n = \frac{g_n^T s_{n-1}}{g_n^T y_{n-1}} + \beta_n \frac{y_{n-1}^T d_{n-1}}{g_n^T y_{n-1}} \tag{11}$$

To generate two new spectral parameters by picking two classical parameters, we enter β_n^{DY} in (11). We get new SCG represented by θ_n^{RGSDY}, defined as:

$$\theta_n^{RGSDY} = \frac{g_n^T s_{n-1}}{g_n^T y_{n-1}} + \frac{\|g_n\|^2}{g_n^T y_{n-1}}. \tag{12}$$

Entering the value of $\beta_n^{DY} = \frac{\|g_n\|^2}{y_{n-1}^T d_{n-1}}$ and the value of θ_n^{RGSDY} from equation (12) results in the following spectral direction:

$$d_n^{RGSDY} = -\left[\frac{g_n^T s_{n-1}}{g_n^T y_{n-1}} + \frac{\|g_n\|^2}{g_n^T y_{n-1}}\right] g_n + \frac{\|g_n\|^2}{y_{n-1}^T d_{n-1}} d_{n-1} \tag{13}$$

When we insert β_n^{HS} into (11), we get a new SCG represented by θ_n^{RGSHS}, defined by:

$$\theta_n^{RGSHS} = 1 + \frac{g_n^T s_{n-1}}{g_n^T y_{n-1}}. \tag{14}$$

Entering the value of $\beta_n^{HS} = \frac{g_n^T y_{n-1}}{y_{n-1}^T d_{n-1}}$, and enter the value of θ_n^{RGSHS} from equation (14) results in the following spectral direction:

$$d_n^{RGSHS} = -\left[1 + \frac{g_n^T s_{n-1}}{g_n^T y_{n-1}}\right] g_n + \frac{g_n^T y_{n-1}}{y_{n-1}^T d_{n-1}} d_{n-1} \tag{15}$$

2.1. Algorithm:

The following algorithms are among the general algorithms used to compute the spectral gradient center. The difference between them lies in the value of θ_n, which is obtained from Eq.12 and applied in Eq. 13 during step seven of each algorithm, along with the assumed value of β_n. The same applies to the second algorithm as well.

2.2. Algorithm 1 RGSDY

1. initialize the starting point $x_0 \in \Re^n$ and parameters $0 < \delta_1 < \delta_2 < 1$.,Compute $f(x_0)$ and $g_0 = \nabla f(x_0)$. Put $d_0 = -g_0$ and n=0.
2. While $\|g_n\| > \varepsilon$, do
3. Evaluated ω_{n-1} by using (4) & (5).
4. Evaluated x_n by (2), $f(x_n)$ & $g_n = \nabla f(x_n)$.
5. If $|g_n^T g_{n-1}| \geq 0.2\|g_n\|^2$, then restart, put $d_n = -g_n$.
6. Else
7. Evaluated the direction by (13).
8. End if
9. Set $n = n + 1$
10. End if.

2.3. Algorithm 2 RGSHS

1. initialize the starting point$x_0 \in \Re^n$ and parameters$0 < \delta_1 < \delta_2 < 1$.,Compute $f(x_0)$ and $g_0 = \nabla f(x_0)$. Put $d_0 = -g_0$ and n=0.
2. While $\|g_n\| > \varepsilon$, do
3. Evaluated ω_{n-1} by using (4) & (5).
4. Evaluated x_n by (2), $f(x_n)$ & $g_n = \nabla f(x_n)$.
5. If $|g_n^T g_{n-1}| \geq 0.2\|g_n\|^2$, then restart, put $d_n = -g_n$.
6. Else
7. Evaluated the direction by (15).
8. End if
9. Set n=n+1
10. End if.

2.4. Theorem 1

If the sequences generate (2) and (13) computed by line search (4) and (5), then the sufficient descent criterion applies.

$$g_n^T d_n \leq -\zeta_1 \|g_n\|^2 \quad (16)$$

Proof: The spectral direction (13), is defined by

$$d_n^{RGSDY} = -\left[\frac{g_n^T s_{n-1}}{g_n^T y_{n-1}} + \frac{\|g_n\|^2}{g_n^T y_{n-1}}\right] g_n + \frac{\|g_n\|^2}{y_{n-1}^T d_{n-1}} d_{n-1} \quad (17)$$

Multiply equation (17) by $\frac{g_n}{\|g_n\|^2}$ .We obtain

$$\frac{g_n^T d_n}{\|g_n\|^2} = -\left[\frac{g_n^T s_{n-1}}{g_n^T y_{n-1}} + \frac{\|g_n\|^2}{g_n^T y_{n-1}}\right] + \frac{g_n^T d_{n-1}}{y_{n-1}^T d_{n-1}} \quad (18)$$

We have, $g_n^T d_{n-1} \leq y_{n-1}^T d_{n-1}$ (19)

We have, $\cos \vartheta_n = \frac{g_n^T g_{n-1}}{\|g_n\| \cdot \|g_{n-1}\|}$

$g_n^T y_{n-1} = g_n^T (g_n - g_{n-1}) = \|g_n\|^2 - g_n^T g_{n-1}$

$$= \|g_n\|^2 \left(1 - \frac{\|g_{n-1}\|}{\|g_n\|} . \cos \vartheta_n\right), |\cos \vartheta_n| \leq 1 \quad (20)$$

Substitute (20) in (18), after some algebraic manipulations, we obtain

$$\frac{g_n^T d_n}{\|g_n\|^2} \leq -\left[\frac{\|s_{n-1}\|}{\|g_n\|\left(1 - \frac{\|g_{n-1}\|}{\|g_n\|}\right)} + \frac{1}{\left(1 - \frac{\|g_{n-1}\|}{\|g_n\|}\right)} - 1\right]$$

$$= \left[\frac{\|s_{n-1}\|\|g_n\| + \|g_{n-1}\|}{(\|g_n\| - \|g_{n-1}\|)}\right]$$

$$\|y_{n-1}\| = \|g_n - g_{n-1}\| \geq \|g_n\| - \|g_{n-1}\| \quad (21)$$

$$\frac{g_n^T d_n}{\|g_n\|^2} \leq -\left[\frac{\|s_{n-1}\| \cdot \|g_n\| + \|g_{n-1}\|}{\|y_{n-1}\|}\right]$$

Our presumption is that

$$\breve{a} \leq \|s_{n-1}\| \leq a, \breve{b} \leq \|g_n\| \leq b, \breve{d} \leq \|g_{n-1}\| \leq d, \breve{c} \leq \|y_{n-1}\| \leq c \quad (22)$$

Let $\zeta_1 = \frac{ab+d}{\breve{c}} > 0$.

Then (16) holds.

2.5. Theorem 2

If the sequences generate (2) and (15) computed by line search (4) and (5), then the sufficient descent criterion applies.

$$g_n^T d_n \leq -\zeta_2 \|g_n\|^2 \quad (23)$$

Proof: The spectral direction (13), is defined by

$$d_n^{RGSHS} = -\left[1 + \frac{g_n^T s_{n-1}}{g_n^T y_{n-1}}\right] g_n + \frac{g_n^T y_{n-1}}{y_{n-1}^T d_{n-1}} d_{n-1} \quad (24)$$

Multiply equation (24) by $\frac{g_n}{\|g_n\|^2}$ .We obtain

$$\frac{g_n^T d_n}{\|g_n\|^2} = -\left[1 + \frac{g_n^T s_{n-1}}{g_n^T y_{n-1}}\right] + \frac{g_n^T y_{n-1}}{\|g_n\|^2 (y_{n-1}^T d_{n-1})} g_n^T d_{n-1}$$

Use (19) and (20), after some algebraic manipulations, we obtain

$$\frac{g_n^T d_n}{\|g_n\|^2} \leq -\left[\frac{\|s_{n-1}\| \cdot \|g_n\|^2 - \|g_{n-1} \cdot\| \|y_{n-1}\|}{\|y_{n-1}\| \cdot \|g_n\|}\right]$$

using (22) in the above inequality, we get

$$\frac{g_n^T d_n}{\|g_n\|^2} \leq -\left[\frac{a.b^2 - d.c}{\breve{c}.\breve{b}}\right] \text{ with } a.b^2 > d.c$$

Let $\zeta_2 = \frac{a.b^2 - d.c}{\breve{c}.\breve{b}} > 0$.

So, (23) is holds.

3. THE CONVERGE ANALYSIS OF THE TWO SCG ALGORITHMS

3.1. Presumptions (I)

A1-The level set $\Upsilon = \{x \in \Re^n, f(x) \leq f(x_1)\}$, is bounded.

A2-The function f is smooth, and its gradient is Lipschitz continuous in a specific neighborhood $\mathbb{N}$ of Υ; specifically, there is a constant $\check{\mathcal{L}}$ greater than zero, so that:

$$\|\nabla f(x_1) - \nabla f(x_2)\| \leq \check{\mathcal{L}}\|x_1 - x_2\|, \forall x_1, x_2 \in \mathbb{N} . \quad (25)$$

The relationship will be satisfied when the function is uniformly convex (Gilbert and Nocedal, 1992).

$$g_n^T y_{n-1} \geq \mu \|g_n\| . \|y_{n-1}\| \quad (26)$$

3.2. Lemma (1)

Suppose that Presumptions (I) holds. Then for the SCG-algorithm 1 the direction d_n^{RGSHS} is a descent, and the step size ω_{n-1} is achieved by (4) & (5) if

$$\sum_{n\geq 0} \frac{1}{\|d_k\|^2} = \infty , \quad (27)$$

then

$$\lim_{n\to\infty} \inf \|g_n\| = 0 . \quad (28)$$

3.3. Theorem (3)

Assume that Presumptions (I) are true, where d_n^{RGSDY} is defined by (13), then the method satisfies (28).

Proof: Take the norm of the both sides (13)

$$\|d_n^{RGSDY}\| = \|-\theta_n^{RGSDY} g_n + \beta_n^{DY} d_{n-1}\|$$

$$\leq |\theta_n^{RGSDY}| . \|g_n\| + |\beta_n^{DY}| . \|d_{n-1}\| \quad (29)$$

In [6], they proved that $\beta_n^{DY} > 0$, so

$$|\beta_n^{DY}| \leq \tau_1, \tau_1 > 0 . \quad (30)$$

$$|\theta_n^{RGSDY}| = \left| \frac{g_n^T s_{n-1}}{g_n^T y_{n-1}} + \frac{\|g_n\|^2}{g_n^T y_{n-1}} \right|$$

Using (22) & (26) in the above equation, we obtain

$$|\theta_n^{RGSDY}| \leq \frac{\|s_{n-1}\| + \|g_n\|}{\mu \|y_{n-1}\|} \leq \frac{a+b}{\mu \check{c}}$$

$$|\theta_n^{RGSDY}| \leq \tau_2, \tau_2 > 0. \quad (31)$$

Substitute (30) and (31) in (29), we obtain

$$\|d_n^{RGSDY}\| \leq \tau_2 \|g_n\| + \tau_1 \|d_{n-1}\| = \tau_3 .$$

$$\sum_{n\geq 1} \frac{1}{\|d_n\|^2} \geq \frac{1}{\tau_3^2} \sum_{n\geq 1} 1 = +\infty .$$

3.4. Theorem (4)

Assume that Presumptions (I) are true, where d_n^{RGSHS} is defined by (13), then the method satisfies (28).

Proof: Take the norm of the both sides (15)

$$\|d_n^{RGSHS}\| = \|-\theta_n^{RGSHS} g_n + \beta_n^{HS} d_{n-1}\|$$

$$\leq |\theta_n^{RGSHS}|.\|g_n\| + |\beta_n^{HS}|.\|d_{n-1}\| \quad (32)$$

In (Yousif *et al.*, no date), they proved that $\beta_n^{HS} > 0$, so

$$|\beta_n^{HS}| \leq \alpha \frac{\|g_n\|^2}{\|d_{n-1}\|^2} = \tau_4, \alpha > 1 \;\&\; \tau_4 > 0. \quad (33)$$

$$|\theta_n^{RGSHS}| = \left|1 + \frac{g_n^T s_{n-1}}{g_n^T y_{n-1}}\right|$$

Using (22) & (26) in the above equation, we obtain

$$|\theta_n^{RGSHS}| \leq 1 + \frac{\|g_n\|.\|s_{n-1}\|}{\mu\|g_n\|\|y_{n-1}\|} \leq \frac{\mu\check{c}+a}{\mu\check{c}}$$

$$|\theta_n^{RGSHS}| \leq \tau_5, \tau_5 > 0. \quad (34)$$

Substitute (33) and (34) in (32), we obtain

$$\|d_n^{RGSHS}\| \leq \tau_5\|g_n\| + \tau_4\|d_{n-1}\| = \tau_6$$

$$\sum_{n\geq 1} \frac{1}{\|d_n\|^2} \geq \frac{1}{\tau_6^2} \sum_{n\geq 1} 1 = +\infty .$$

4. NUMERICAL RESULTS AND DISSUSSION

The SCG techniques' numerical performance is shown in this section. A (46) test functions have been chosen for the new method's efficiency study, every test function is created as an experiment using N=5000 and N=10000 variables, accordingly. These roles are taken into from CUTEr (Andrei, 2010) and Andrei (Bongartz *et al.*, 1995). Using SW line search (4) and (5), the novel two SCG method RGSDY and t RGSHS CG methods are compared based on the number of iterations and functions. To test how quickly iteration techniques, approach the ideal, let $\delta_1 = 0.001$, $\delta_2 = 0.9$. be set to. The terminating criterion in this case is

$$\|g_n\| \leq 1 * 10^{-6}.$$

We analyze the two suggested SCG algorithms to determine how effective they are. RGSDY represents the first one, whilst (DY) represents the traditional DY CG [6] approach and (SDY) represents the SCG suggested by Birgin and Martınez [14]. RGSHS represents the second suggested SCG algorithm, HS represents the classical HS CG [7] technique, and SHS represents the SCG described by Ghanbari and et al.(Ghanbari et al., no date). Every code is written in FORTRAN (Microsoft Developer Studio Fortran Power Station V4.0) using double precision. MATLAB (2020a) was used for graphics (Dolan and Moré, 2002).

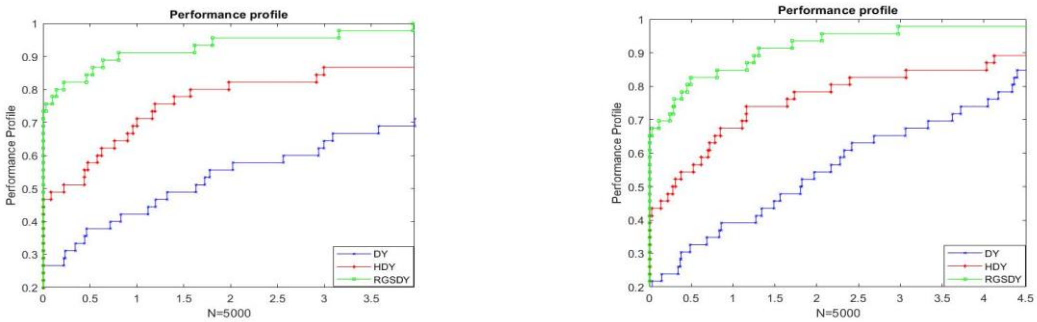
The following is included in the comparison:

NI: number of iteration.

NF: number of function.

If the process is repeated more than 2000 times, it is regarded to have failed.

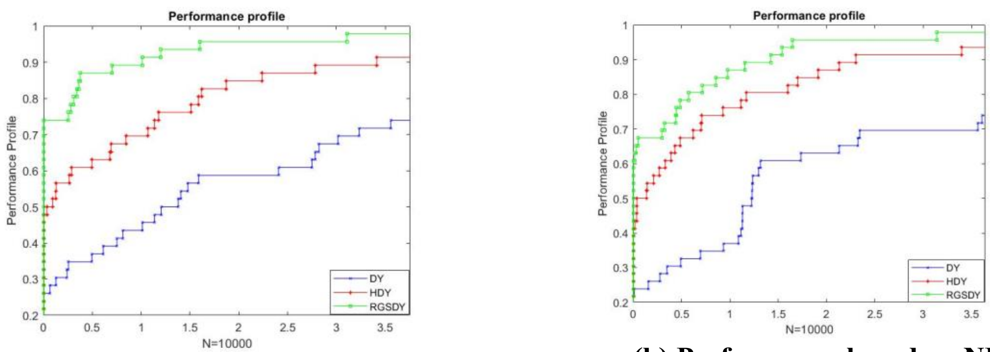
Fig.1 a shows that the new RGSDY method achieves the best result in terms of NI, compared with SDY and DY at dimension (N=5000). Fig.1 b shows that the new RGSDY method achieves the best result in terms of NF, compared with SDY and DY at dimension (N=5000). As shown by the top curve in the graph. (As a result, the new technique outperforms traditional CG approaches.



(a) Performance based on NI **(b) Performance based on NF**

Fig.1. Log_{10} scaled performance profiles of RGSDY methods

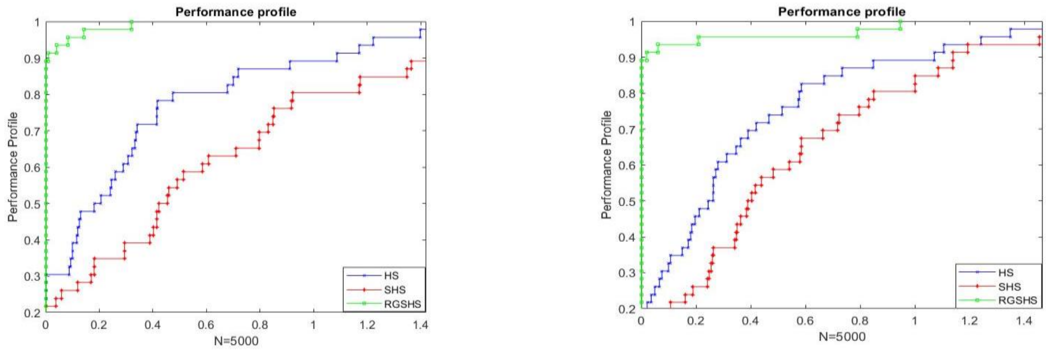
Fig.2 a shows that the new RGSDY method achieves the best result in terms of NI, compared with SDY and DY at dimension (N=10000). (b) shows that the new RGSDY method achieves the best result in terms of NF, compared with SDY and DY at dimension (N=10000). As shown by the top curve in the graph. (As a result, the new technique outperforms traditional CG approaches.



(a) Performance based on NI. **(b) Performance based on NF**

Fig.2. Log_{10} scaled performance profiles of RGSDY methods

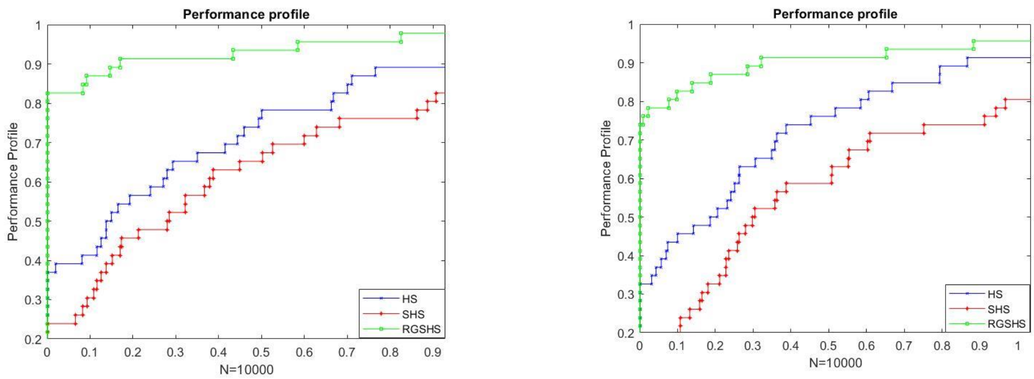
Fig.1a shows that the new RGSHS method achieves the best result in terms of NI, compared with SHS and HS at dimension (N=5000). Fig. b shows that the new RGSHS method achieves the best result in terms of NF, compared with SHS and HS at dimension (N=5000). As shown by the top curve in the graph. (As a result, the new technique outperforms traditional CG approaches.



(a) Performance based on NI **(b) Performance based on NF**

Fig.3. $\boldsymbol{Log_{10}}$ **scaled performance profiles of RGSDY methods**

Fig. 2 a shows that the new RGSHS method achieves the best result in terms of NI, compared with SHS and HS at dimension (N=10000). (b) shows that the new RGSHS method achieves the best result in terms of NF, compared with SHS and HS at dimension (N=10000). As shown by the top curve in the graph. (As a result, the new technique outperforms traditional CG approaches.



(a) Performance based on NI **(b) Performance based on NF**

Fig.4. $\boldsymbol{Log_{10}}$ **scaled performance profiles of RGSDY methods**

5. CONCLUSION

This paper proposes two unique spectral conjugate gradient (SCG) algorithms that are inspired by the classical Dai-Yuan (DY) and Hestenes-Stiefel (HS) conjugate gradient methods. The presented algorithms are specifically designed to solve large-scale unconstrained optimization problems, a category in which existing methods frequently fail due to computational inefficiency.

A fundamental aspect of the new approaches is that they satisfy the sufficient descent criterion independent of the line search methodology used, making them more adaptable and robust in actual situations. Furthermore, the algorithms' global convergence was theoretically demonstrated under simple assumptions, ensuring reliability across a wide range of optimization problems.

Preliminary numerical experiments were conducted to validate the proposed approaches' effectiveness. The results showed that the novel SCG algorithms outperformed classical conjugate gradient methods in terms of iteration count and function evaluations for large-scale issues.

These findings show that the suggested approaches are both efficient and practical, making them excellent candidates for real-world applications requiring high-dimensional optimization. Future research may focus on further improving the algorithms' performance, extending them to limited optimization situations, or incorporating them into machine learning frameworks.

6. RECOMMENDATIONS

Any parameter, whether traditional or modern, can be added to produce a new parameter that produces results different from previous ones. This represents the essence of optimization, as any change in parameters leads to a change in the results, which may be towards improvement or regression. Not all algorithms guarantee numerical improvement or achieving the desired goal.

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