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HYBRID GEOMETRIC-BASED QUATERNION CONTROL FOR ALTITUDE AND AGGRESSIVE ORIENTATION TRACKING OF QUADROTOR UAV

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ABSTRACT

This paper proposes a hybrid control algorithm that combines geometric tracking control (GTC) with special orthogonal group SO(3) for position tracking and quaternion orientation-based attitude tracking control (QOATC) in order to achieve trajectory tracking with high attitude maneuverability for quadcopter systems suffering from aggressive orientation. Firstly, to solve the problem of the limitation of executing large-angle maneuvers in a quadrotor system caused by the lack of globality or uniqueness in Euler angles models, an improved geometric controller is constructed on SO(3). Secondly, an advanced attitude quaternion-based controller guarantees the tracking performances in finite time and addresses the persisting issue associated with Euler angle-based solutions. The proposed controller guarantees stability and precision by leveraging nonlinear control methods, with thrust and torque computed directly in the body frame. Furthermore, the theoretical research assures the asymptotic tracking and boundedness of all signals in the closed loop system by utilizing Lyapunov's stabilization theory on SO(3). Several numerical simulations are presented to demonstrate the superiority of the proposed approach, showing accurate trajectory tracking, reduced position and velocity errors, and robust performance under aggressive conditions. The hybrid controller effectively avoids singularities and ensures global stability, making it suitable for real-world UAV applications requiring high performance.

KEYWORDS

Quadrotor, Robust Control, Aggressive Maneuvers, Geometric Tracking Control, Quaternion-based Orientation Control.

1. INTRODUCTION

Quadrotor Unmanned Aerial Vehicles (UAVs) have attracted substantial attention in recent years due to their simplicity, versatility and ability to perform complex aerial maneuvers. The quadrotor UAV includes two sets of counterrotating rotors and propellers, which are situated at the corners of a square configuration. In contrast to conventional helicopters, quadrotors do away with intricate mechanical components like swash plates and teeter hinges, rendering them a more mechanically simple alternative. Quadrotors exhibit a structural simplicity that, along with their proficiency in Vertical Take-Off and Landing (VTOL), renders them highly suitable for various applications such as surveillance, mobile sensor networks, and educational activities. The effectiveness in terms of cost and the inherent agility have contributed to a growing acceptance of these solutions in both academic research (Madeiras et al., 2024; Kherkhar et al., 2023; Hassan et al., 2021; Invernizzi et al., 2020; Martins et al., 2024) and in the commercial sector (Benevides et al., 2019; Simplício et al., 2024; Sharma et al., 2023).

Even with the widespread deployment of quadrotor UAVs, challenges still exist in the design of effective and high-performance control systems. This problem has been the subject of many studies summarized in the literature review made by (Al-husnawy et al., 2024); particularly for executing complex and aggressive maneuvers such as single flips, double flips, or circular-wave trajectories. It is common to use traditional linear controllers, like proportional derivative (PD) controllers and linear quadratic regulators (LQR), to ensure the stability of quadrotors (Cabecinhas et al., 2021; Choutri et al., 2017; Jeaeb et al., 2025). However, these strategies are intrinsically constrained because they rely on linearized dynamics near equilibrium points, which restricts their effectiveness to slight perturbations. As a result, linear controllers find it challenging to operate effectively with highly nonlinear systems, including quadrotors, particularly during severe maneuvers or in the face of external disturbances.

The fundamental limitation of linear controllers arises from their inability to effectively represent the global dynamics of systems that operate within nonlinear manifolds. In the case of quadrotor UAVs, the configuration space is delineated within the *Special Euclidean Group* SE(3), which combines the aspects of rotational and translational dynamics. Approaches based on Euler angles, including Newton-Euler formulations (Gonzalez, 2019), are conceptually straightforward but are affected by singularities like "gimbal lock," where the alignment of axes leads to a loss of one degree of freedom. These formulations also entail significant computational costs due to the extensive trigonometric calculations required.

In order to mitigate these limitations, nonlinear control approaches, including Geometric Control and Quaternion-Based Control, have been developed as effective solutions. The purpose of Geometric Control techniques is to operate directly on nonlinear manifolds, offering formulations that do not rely on coordinates, which helps to avoid singularities and accomplish nearly global asymptotic stability (Zhao et al., 2022; Ramp et al., 2018; Wang et al., 2022). These approaches are particularly advantageous for dynamic systems, including quadrotors, as they require simultaneous control of both rotational and translational dynamics to ensure precise trajectory tracking. Geometric controllers are designed to maintain robust performance during aggressive maneuvers, such as recovering from upside-down orientations or tracking complex trajectories.

Likewise, Quaternions can effectively reduce computational overhead and avoid the singularities that are typically present in Euler angle approaches by representing rotations in a compact format and orientations using a four-dimensional unit vector. Furthermore, Compared to Direction Cosine Matrices (DCMs), quaternion dynamics can be depicted using only four coupled differential equations, which simplifies the overall system. While quaternions have been used in earlier works to stabilize attitude, many approaches still rely on converting quaternion errors back to Euler angles for regulation, which reintroduces the associated nonlinearities and singularities (Fresk et al., 2013; Zha et al., 2017).

Recent works in quadrotor UAV dynamics demonstrate that the dynamic model can be articulated globally (i.e., the special Euclidean group, SE(3)). Moreover, the ability to utilize nonlinear control systems through computational resources broadens the capability to track complex paths with different modes. Additionally, the use of a hybrid control architecture allows a quadrotor to achieve several aggressive maneuvers. The development of hybrid control systems based directly on SE(3) or SO(3) ensures that singularities, complexities, and ambiguities associated with minimal attitude representations are fully avoided (Zhao et al., 2022; Martins et al., 2024). A recent study proposed by (Zhao et al., 2022) exploits the robustness properties of model-free control on the SE(3) to quadrotor tracking control under windy conditions. In (Martins et al., 2024), a trajectory tracking control for input-saturated quadrotors was developed using unit quaternion-based SE(3). The hybrid formulation of the controller benefits from the unique properties of the modified Rodrigues parameters attitude description.

The motivation for this work stems from the need for advanced nonlinear controllers that can address the limitations of linear strategies and provide superior performance during complex maneuvers. This paper focuses on two controllers: GTC on SO(3) and Full Quaternion-based Attitude Control. These controllers will be implemented and simulated to test their performance under challenging trajectories, including single flips and helical-wave paths. Additionally, this work explores the integration of geometric and quaternion-based controllers to evaluate their combined effectiveness. The main contributions of this paper are summarized as follows:

- The proposed hybrid geometric-based quaternion control approach for quadrotor UAV aggressive angles tracking is the major contribution of this paper. The pose of the quadrotor is represented directly on the Lie group of rigid body transformations, the special Orthogonal group SO(3). Unlike the control designs reported by (Madeiras et al, 2024), (Martins et al, 2024) and (Zhao et al, 2022), who used Euler angles or quaternions for attitude representation or did not include attitude kinematics, like the control approach proposed by (Zhao et al, 2022) in reference trajectory track under windy conditions, the pose of the quadrotor in this paper is represented in SO(3) to avoid kinematic singularities. To the best of the authors knowledge, there is no existing paper on quadrotor agressive maneuvering using hybrid geometric-based quaternion approach with pose representation on SO(3).
- A novel control approach is developed that leverages the concept of SO(3)-based GTC and quaternions for executing aggressive maneuvers requiring precise knowledge of the mass and inertia matrix of the quadrotor, where the configuration of the quadrotor is represented globally and uniquely. Hence, compared to control schemes based on Euler angles, the proposed control scheme ensures the effectiveness of the quadrotor during large-angle maneuvers.
- The advanced attitude quaternion-based controller guarantees the tracking performances in finite time and addresses the persisting issue associated with Euler angle-based solutions, which include kinematic singularity and the inability of model representation in particular configurations.
- Using Lyapunov analysis, we proved that tracking is almost globally asymptotically stable and that all signals in the closed-loop system are bounded.

The remainder of this work is organized as follows: Section II introduces the mathematical formulations of the system behavior, Section III presents the controller algorithm design with simulation setup and test trajectories, Section IV proves the stability analysis of the proposed control algorithm by using Lyapunov approach, Section V illustrates the simulation results and performance comparison and Section VI concludes the paper with key findings and future directions.

Some of the commonly used abbreviations are provided in Table 1.

Table 1. Abbreviations.

UAV	Unmanned aerial vehicle
PID	Proportional-integral-derivative
LQR	Linear Quadratic regulator
GTC	Geometric Tracking Control

SO(3)	Special orthogonal group
SE(3)	Special Euclidean Group
QOATC	Quaternion Orientation-Based Attitude Tracking Control
VTOL	Vertical Take-Off and Landing
PD	Proportional-derivative
DCM	Direction Cosine Matrices

2. SYSTEM DESCRIPTION AND PRELIMINARIES

2.1. Assumptions

To create our QUADROTOR UAV control algorithm, it is necessary to take into account the following hypotheses:

- Assumption 1: The structure and propellers are assumed rigid and symmetrical; the frame center is assumed to coincide with the center of gravity, the axes are set according to Fig. 1, and only differential forces from propeller rotation are considered to affect the drone rotation.
- Assumption 2: The roll and pitch equations assume decoupling between translational and rotational dynamics. For aggressive maneuvers, this assumption can lead to inaccuracies.
- Assumption 3: Euler angles can lead to singularity issues (gimbal lock). If the desired trajectory or dynamics approach such configurations, the quaternion controller may not perform as expected.
- Assumption 4: There's no explicit saturation handling for control inputs (maximum thrust or torque limits). This could lead to unrealistic simulations.

2.2. UAV system description

The world and robot frames of the quadrotor UAV system are assigned as shown in Fig. 1 (Castillo et al., 2019; Sanwale et al., 2020; Shi et al., 2018).

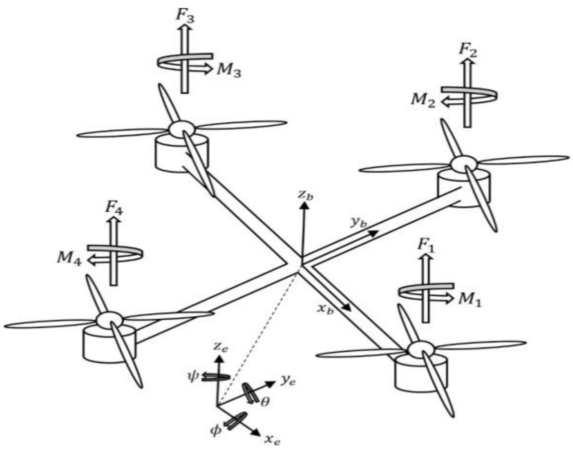

Fig. 1. Quadrotor model and coordinate frames.

Where {A} is the global or world frame and {B} is the robot frame whose origin is placed at the center of gravity of the drone. In the global frame, the linear position is $\xi = [x, y, z]^T$ and the angular position is $\eta = [\phi, \theta, \psi]^T$ in which the authors in (Sanwale et al., 2020) use X-Y-Z

Euler angles, whereas the authors in (Shi et al., 2018) have used Z-X-Y Euler angles and a vector q which contains the linear and angular position of the quadrotor.

$$\xi = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad \eta = \begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix}, \quad q = \begin{bmatrix} \xi \\ \eta \end{bmatrix} \tag{1}$$

In the robot frame, the linear and angular velocities are given by:

$$V_B = \begin{bmatrix} \upsilon_{xB} \\ \upsilon_{yB} \\ \upsilon_{zB} \end{bmatrix}, \quad \upsilon = \begin{bmatrix} p \\ q \\ r \end{bmatrix} \tag{2}$$

If we use X-Y-Z Euler angles; the rotation matrix from frame {A} to {B} will change as follows (Madeiras et al., 2024):

$$\Re_{XYZ} = \begin{bmatrix} c_\psi c_\theta & c_\psi s_\theta s_\phi - s_\psi c_\phi & c_\psi s_\theta c_\phi + s_\psi s_\phi \\ s_\psi c_\theta & s_\psi s_\theta s + c_\psi c_\phi & s_\psi s_\theta c_\phi - c_\psi s_\phi \\ -s_\theta & c_\theta s_\phi & c_\theta c_\phi \end{bmatrix} \tag{3}$$

where $s_{(.)}$ and $c_{(.)}$ stand for $\sin(.)$ and $\cos(.)$ respectively.

The linear velocity in global frame is given as:

$$\upsilon = \dot{\xi} \tag{4}$$

We also have a transformation matrix for angular velocities from global frame to robot frame, which is given by W_η.

$$\upsilon = W_\eta \dot{\eta} \tag{5}$$

This matrix is only invertable if $\theta \neq (2k-1)\phi/2,\ k \in \mathbb{Z}$ (Sanwale et al., 2020).

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} 1 & 0 & -s_\theta \\ 0 & c_\phi & c_\theta s_\phi \\ 0 & s_\phi & c_\theta c_\phi \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} \tag{6}$$

Assuming that the quadrotor UAV is symmetric, the inertia matrix is given as:

$$I = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix} \tag{7}$$

The off-diagonal entries are zero and $I_{xx} = I_{yy}$.

2.2.1. Newton-Euler Model

Using the Newtonian Mechanics, we derive the dynamical equations of the drone. The force on the drone is given as:

$$m\dot{\upsilon}_{(3\times1)} = mg\vec{a}_{3(3\times1)} + \Re_{(3\times3)} F_{(3\times3)} \tag{8}$$

Where $F = \sum_{i=1}^{4} f_i \in \mathbb{R}$, $i = 1,\dots,4$ is the force or thrust vector, $\Re$ is the rotation matrix from world frame to robot frame, m is the mass of the drone, g is the gravity and $\vec{a}_3$ is the unit vector in the z direction. The following equation is developed:

$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = -g \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + \frac{1}{m} \Re_{(3\times3)} \begin{bmatrix} 0 \\ 0 \\ F \end{bmatrix} \quad (9)$$

The angular equations are given as:

$$I_{[3\times3]}\dot{\upsilon}_{[3\times1]} = -\upsilon_{[3\times1]} \times I_{[3\times3]}\upsilon_{[3\times1]} + \tau_{[3\times1]} \quad (10)$$

This equation is derived in (Spong et al., 2020), where I is the inertia tensor of the drone, τ is the torque applied on the drone and $(I\dot{\upsilon})$ is the inertia of the drone. Eq. 10 can be written as:

$$I_{[3\times3]} \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} \tau_\phi \\ \tau_\theta \\ \tau_\psi \end{bmatrix} - \begin{bmatrix} p \\ q \\ r \end{bmatrix} \times I_{[3\times3]} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \quad (11)$$

2.3. Geometric Tracking Control on SE(3)

The approach to controlling aggressive maneuvers is based on closed-loop control. The use of this approach offers the advantage of generality, as it can be applied to a wide variety of aggressive maneuvers and provides robustness and adaptability to external disturbances. The first step in this method is to create a reference trajectory for the maneuver, which is then given to a nonlinear controller to achieve precise reference tracking.

The main purpose for the application of GTC is based on the seminal work by (Escamilla et al., 2023). This approach leverages the mathematical structure of the Special Euclidean Group SE(3) and the Special Orthogonal Group SO(3) to provide a globally defined, singularity-free formulation for attitude and position control. Geometric control ensures precise tracking of desired trajectories by directly formulating the control laws on these manifolds.

2.3.1. Quadrotor motion equations on SO(3)

The dynamics of the UAV system can be described using the Newton-Euler equations, assuming a rigid body model with symmetric structure. The state of the quadrotor is defined by its position $x \in \mathbb{R}^3$, orientation $\Re \in SO(3)$, linear velocity $\upsilon \in \mathbb{R}^3$ and angular velocity $\Omega \in \mathbb{R}^3$ in the body-fixed frame. The translational and rotational dynamics are given as (Martins et al., 2024, Ramp et al., 2018, and Wang et al., 2022):

$$m\ddot{x} = (mg - F.\Re).e_3 \quad (12)$$

$$J\dot{\Omega} = M - \Omega(J\Omega) \quad (13)$$

Whith m is the mass of the quadrotor, F is the total thrust produced by the rotors, $\Re \in SO(3)$ is the rotation matrix representing the attitude, $e_3 = [0\ 0\ 1]^T$ is the vertical unit vector, g is the

gravitational acceleration, J is the inertia matrix and $M \in \mathbb{R}^3$ is the moment vector.
The force F acts along the third body axis $\Re e_3$ and the torques M determine the rotational motion of the UAV.
The Eq. 12 represents linear dynamics and the Eq. 13 represents angular dynamics. In these equations F is the input for the linear dynamics so the altitude controller outputs F and M is the input of the angular dynamics so the attitude controller outputs M.
The GTC approach introduces errors for position, velocity, attitude and angular velocity that are directly defined on the manifold SE(3). The errors are represented as follows:

$$e_x = x - x_d \tag{14}$$

With x_d and e_x are the desired position and position error respectively.

$$e_\upsilon = \dot{x} - \dot{x}_d \tag{15}$$

With $\dot{x}_d$ and e_υ are the desired velocity and velocity error respectively.

The error $e_\Re$ between the current orientation $\Re$ and the desired orientation $\Re_d$ is defined using the Lie-algebra of SO(3) as:

$$e_\Re = \frac{1}{2} vee(\Re_d^T \Re - \Re^T \Re_d) \tag{16}$$

Where *vee* extracts the vector from a skew-symmetric matrix. The angular velocity error is expressed as:

$$e_\Omega = \Omega - \Re^T \Re_d \Omega_d \tag{17}$$

Where Ω_d is the desired angular velocity.

2.3.2. Geometric Control Law

The geometric control law generates the thrust F and moment M required to stabilize the UAV and track the desired trajectory. The control inputs are designed as follows:

$$F = m(\ddot{x}_d + g e_3^T \Re e_3) - k_x e_x - k_\upsilon e_\upsilon \tag{18}$$

$$M = \Omega \times (J\Omega) - k_\Re e_\Re - k_\Omega e_\Omega \tag{19}$$

With k_x, k_υ present the control gains for position and velocity errors.

$k_\Re$, k_Ω denote control gains for attitude and angular velocity errors and $\ddot{x}_d$ the desired acceleration.
The thrust F acts along the body-fixed z-axis, and the control moment M stabilizes the attitude using the error metrics defined on SO(3).

2.4. Quaternion-based Angular Dynamic Model of a Quadrotor

One of the most used techniques to model the dynamics is the Newton-Euler model, which provides linear and angular dynamics. Euler angles are commonly used in dynamic implementations due to their ease of interpretation and understanding, making them one of the most popular techniques for working with rotations. While Newton-Euler is considered fundamental, it has three drawbacks due to the Euler Angles (nonlinearity, computational cost and singularity phenomenon) (Andersen et al., 2017). A quaternion is a hyper-complex number of ranks 4, which can be represented in many ways. The most common representation is (Fresk et al., 2013):

$$q = q_0 + q_1 i + q_2 j + q_3 k \tag{20}$$

$$q = \begin{bmatrix} q_0 & q_1 & q_2 & q_3 \end{bmatrix} \tag{21}$$

The quaternion units from q_1 to q_3 are called the vector part of the quaternion, while q_0 is the scalar part. The multiplication is given as (Gonzalez, 2019):

$$q \otimes q = \begin{pmatrix} p_0 q_0 - p_1 q_1 - p_2 q_2 - p_3 q_3 \\ p_0 q_1 + p_1 q_0 + p_2 q_3 - p_3 q_2 \\ p_0 q_2 - p_1 q_3 + p_2 q_0 + p_3 q_1 \\ p_0 q_3 + p_1 q_2 - p_2 q_1 + p_3 q_0 \end{pmatrix} \tag{22}$$

The norm of the quaternion is similar to the norm of a vector.

$$Norm(q) = \|q\| = \sqrt{q_0^2 + q_1^2 + q_2^2 + q_3^2} \tag{23}$$

The inverse and the conjugate of the quaternions are given as:

$$inv(q) = q^{-1} = \frac{q^*}{\|q\|^2} \tag{24}$$

$$conj(q) = q^* = [q_0 - q_1 - q_2 - q_3]^T \tag{25}$$

The derivative of the quaternion $\dot{q}$ given by the quaternion q and the angular velocity vector Ω, is as follows as (Castillo et al., 2019):

$$\dot{q}_{\Omega}(q,\Omega) = \frac{1}{2} q \otimes \begin{bmatrix} 0 \\ \Omega \end{bmatrix} \tag{26}$$

One of the significant transformations when using quaternions are the Euler to quaternion and quaternion to Euler transformations. The quaternion q, obtained from the Euler angles ϕ, θ and ψ (roll, pitch and yaw), is given by:

$$q = \begin{pmatrix} c_{(\phi/2)}c_{(\theta/2)}c_{(\psi/2)} + s_{(\phi/2)}s_{(\theta/2)}s_{(\psi/2)} \\ s_{(\phi/2)}c_{(\theta/2)}c_{(\psi/2)} - c_{(\phi/2)}s_{(\theta/2)}s_{(\psi/2)} \\ c_{(\phi/2)}s_{(\theta/2)}c_{(\psi/2)} + s_{(\phi/2)}c_{(\theta/2)}s_{(\psi/2)} \\ c_{(\phi/2)}c_{(\theta/2)}s_{(\psi/2)} - s_{(\phi/2)}s_{(\theta/2)}c_{(\psi/2)} \end{pmatrix} \quad (27)$$

The Euler angles obtained from $q = [q_0\ q_1\ q_2\ q_3]^T$, are given by (Andersen et al., 2017):

$$\begin{pmatrix} \phi \\ \theta \\ \psi \end{pmatrix} = \begin{pmatrix} \tan^{-1}\left(2(q_0q_1 + q_2q_3)/1 - 2q_1q_1 + q_2q_2\right) \\ \sin^{-1}(2(q_0q_2 - q_3q_1)) \\ \tan^{-1}\left(2(q_0q_3 + q_1q_2)/1 - 2(q_2q_2 + q_3q_3)\right) \end{pmatrix} \quad (28)$$

In order to model the drone dynamics using quaternions, we start by considering the above assumptions. If we use the Newton-Euler model for rotational and translational dynamics, then the dynamics of the system can be represented using forces F, torques τ and angular velocities Ω. The equations are given as follows (Gonzalez., 2019):

$$\begin{pmatrix} F \\ \tau \end{pmatrix} = \begin{pmatrix} m & 0 \\ 0 & I_{cm} \end{pmatrix} \begin{pmatrix} a_{cm} \\ \dot{\Omega} \end{pmatrix} + \begin{pmatrix} 0 \\ \Omega(I_{cm}\Omega) \end{pmatrix} \quad (29)$$

Here, F can be represents the external force acting on the system. τ is the torque acting on the system. m is the mass of the system. I_{cm} is the inertia matrix about the center of mass. a_{cm} is the linear acceleration of the center of mass. $\dot{\Omega}$ is the angular acceleration. $\Omega(I_{cm}\Omega)$ represents the gyroscopic torque caused by rotational motion.

$$\Omega = \begin{pmatrix} \Omega_x \\ \Omega_y \\ \Omega_z \end{pmatrix} \quad (30)$$

Where Ω_x, Ω_y and Ω_z are the components of the angular velocity vector along the *x*, *y* and *z*-axes, respectively.

Then we combine the quaternion derivative from Eq. 26 with the rotation dynamics from Eq. 33, that results in an equation system describing the entire rotation dynamics of the aerial vehicle in quaternion form (Andersen et al., 2017):

$$\dot{q} = -\frac{1}{2} q \otimes \Omega_{quat} \quad (31)$$

$$\Omega_{quat} = [0; \Omega] \quad (32)$$

$$\dot{\Omega} = J^{-1}(\tau - \Omega(J\Omega)) \quad (33)$$

3. UAV TRACKING CONTROLLERS DESIGN

3.1. GTC Tracking Controller

The design of GTC is demonstrated through two maneuvers:

A. Flip Trajectory: The Flip Trajectory involves performing a 360-degree backflip while maintaining stability and accurate tracking of desired states. The main steps of the GTC controller are detailled as follows:

Algorithm 1: UAV Flip Trajectory Generation and Control Law

Input: Initial conditions: $x_0, \upsilon_0, \Re_0, \Omega_0$, Quadrotor parameters: m, J, g, $k_x, k_\upsilon, k_\Re, k_\Omega$

Initialize: Flip time parameters: T_{flip}, T_{start}, T_{end}

for $t = t_0$ to t_{end} with step Δt **do**

  if $t \le T_{start}$ **then**

    Hover phase: $\phi(t) = 0$, $\theta(t) = 0$, $\psi(t) = 0$

  else if $T_{start} \le t \le T_{end}$ **then**

    Compute roll angle for flip: $\phi(t) = 2\pi \dfrac{t - T_{start}}{T_{flip}}$, $\theta(t) = 0$, $\psi(t) = 0$

  Else

    Stabilization phase: $\phi(t) = 2\pi$, $\theta(t) = 0$, $\psi(t) = 0$

  end if

  Compute desired thrust direction: $b_3 = \dfrac{-k_x e_x - k_\upsilon e_\upsilon + m a_d + m g e_3}{\left\| -k_x e_x - k_\upsilon e_\upsilon + m a_d + m g e_3 \right\|}$, (34)

  Compute control moments: $M = -k_\Re e_\Re - k_\Omega e_\Omega + \Omega \times (J\Omega)$ (19)

  Update state variables: $x, \upsilon, \Re, \Omega$

end for

Output: Trajectory $x(t), \upsilon(t), \Re(t), \Omega(t) = 0$

A.1. System Parameters: The system parameters define the physical properties of the UAV, including its mass, inertia matrix J and gravity g, as well as control gains k_x, k_υ, $k_\Re$, k_Ω for position, velocity and attitude control.

A.2. Trajectory Function: The desired trajectory consists of a hover phase followed by a full backflip. The roll angle $\phi(t)$ progresses linearly during the flip, completing a full 360-degree rotation. Pitch and yaw remain zero.

A.3. Control Law: The control law for thrust and moments is implemented as follows:

- The thrust direction b_3 is computed using the position and velocity errors:

$$b_3 = \frac{-k_x e_x - k_\upsilon e_\upsilon + m a_d + m g e_3}{\left\| -k_x e_x - k_\upsilon e_\upsilon + m a_d + m g e_3 \right\|}, \quad (34)$$

where we assume that: $\left\| -k_x e_x - k_\upsilon e_\upsilon + m a_d + m g e_3 \right\| \neq 0$

with $\|.\|$, is the standard euclidian norm.

- The moments M are calculated using the attitude and angular velocity errors:

$$M = -k_{\Re} e_{\Re} - k_{\Omega} e_{\Omega} + \Omega \times (J\Omega) \quad (35)$$

B. Helical Trajectory: The Helical Trajectory consists of a smooth circular path in the y-z plane with a progressive ascent, forming a helical shape. The desired trajectory expression is given as:

$$\xi_d = [x_d = 0.4t; y_d = 0.4\sin(\pi t); z_d = 0.6\cos(\pi t)] \quad (36)$$

B.1. Desired Trajectory: The position, velocity, and acceleration of the helical trajectory are derived analytically. The trajectory ensures smooth motion with tangential alignment.

B.2. Control Law: The control law aligns the UAV's nose tangentially to the trajectory by computing the desired rotation matrix $\Re_d$ as follows:

$$\Re_d = [\mathrm{b}_2 \times \mathrm{b}_3, \mathrm{b}_2, \mathrm{b}_3] \quad (37)$$

Algorithm 2: Quadrotor Helical Trajectory Generation and Control Law

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Input: Initial conditions: $x_0, \upsilon_0, \Re_0, \Omega_0$, Quadrotor parameters: $m, J, g, k_1, k_2, k_{\Re}, k_{\Omega}$

Define deisred Trajectory: $x_d = 0.4t,\ y_d = 0.4\sin(\pi t),\ z_d = 0.6\cos(\pi t)$ (36)

Compute velocity and acceleration: $\upsilon(t) = \dot{x}(t),\quad a(t) = \ddot{x}(t)$

for $t = t_0$ to t_{end} with step Δt **do**

  Compute desired thrust direction: $\mathrm{b}_3 = \dfrac{-k_1 e_x - k_2 e_\upsilon + m a_d + m g e_3}{\|-k_1 e_x - k_2 e_\upsilon + m a_d + m g e_3\|}$, (34)

  Compute desired rotation matrix: $\Re_d = [\mathrm{b}_2 \times \mathrm{b}_3, \mathrm{b}_2, \mathrm{b}_3]$, (37)

  Compute control moments: $M = -k_{\Re} e_{\Re} - k_{\Omega} e_{\Omega} + \Omega \times (J\Omega)$ (19)

  Update state variables: $x, \upsilon, \Re, \Omega$

end for

Output: Trajectory $x(t), \upsilon(t), \Re(t), \Omega(t) = 0$

B.3. Numerical Results and discussion: In this part, the simulations are carried out to evaluate the performance of Algorithm 2 by applying a helical trajectory for a quadrotor UAV. The effectiveness of the proposed controller as outlined in Algorithm 2 is illustrated in Figs. 2-8. Figs. 2, 3 and 4 show that the trajectories for both attitude and position successfully follow the desired values. Meanwhile, the total thrust is used to generate the attitude desired values to address the underactuated complexity associated with the UAV. In Figs. 5 and 6, the altitude trajectories follow desired values.

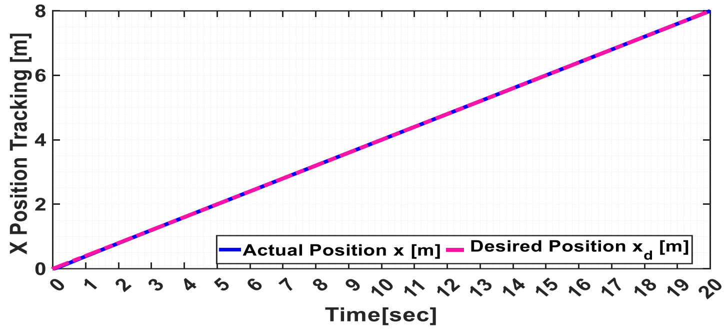

Fig. 2. Tracking position x [m].

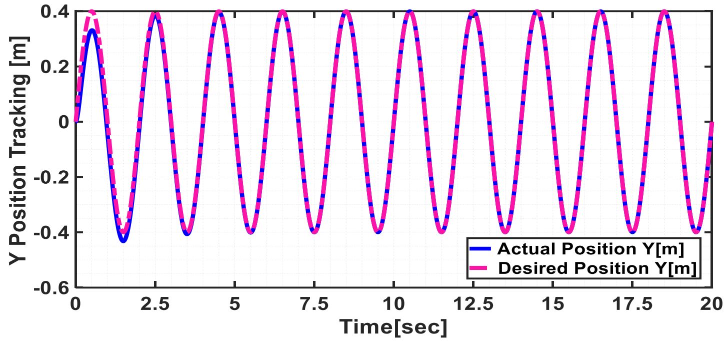

Fig. 3. Tracking position y [m].

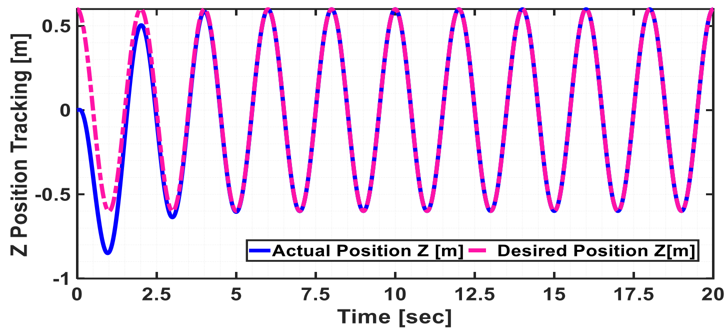

Fig. 4. Tracking position z [m].

The flight trajectory of UAV, position and linear velocity tracking errors are shown in Fig. 7 and 8, respectively. There is a successful and smooth convergence of the error components towards zero. The UAV's position and orientation, as shown in Fig. 7, exhibit stability and a smooth tracking of the desired trajectories with a high accuracy and even significant initialization applied.

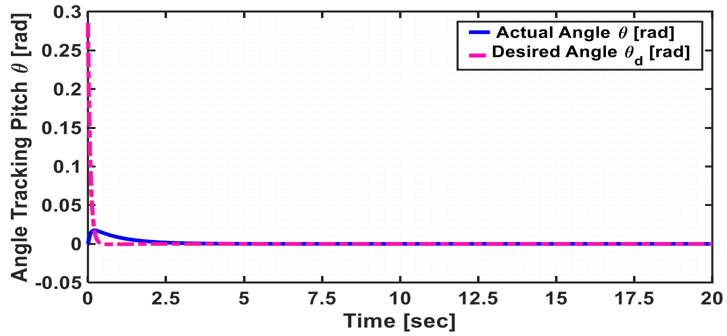

Fig. 5. Angle tracking Pitch [rad].

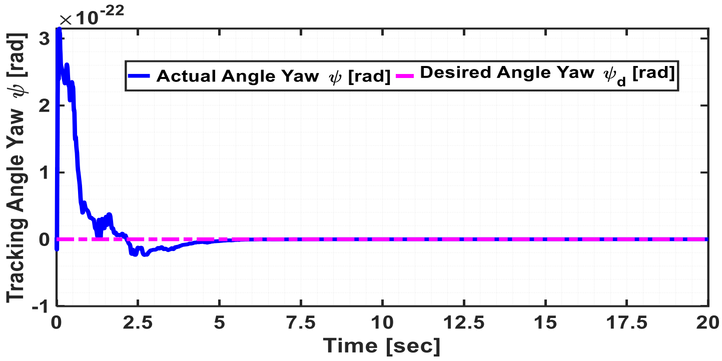

Fig. 6. Angle tracking Yaw [rad].

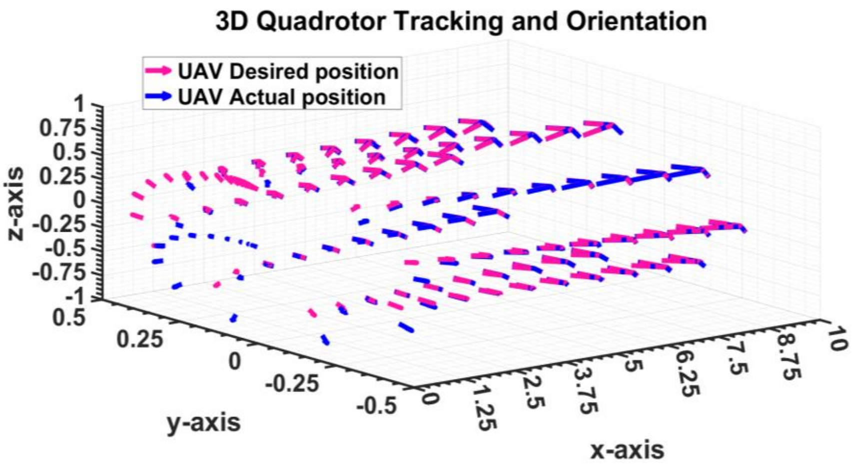

Fig. 7. UAV flight Orientation trajectories.

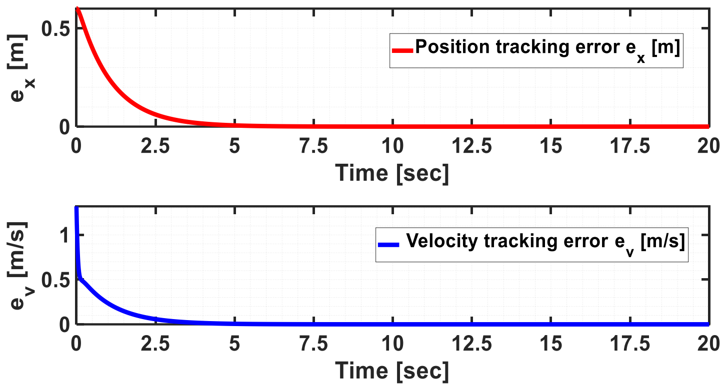

Fig. 8. Tracking error in position e_x and velocity e_v during the maneuver.

The results obtained from the simulation of this algorithm confirm the effectiveness of the proposed controller. The UAV achieved excellent tracking performance, and ensure that for large-angle rotational maneuvers can be exponentially stable and has better tracking performance with a large initial attitude error.

3.2. Quaternion-Based Attitude Control

The quaternion-based attitude controller employs the quaternion representation to track a desired trajectory while stabilizing angular velocity. This approach ensures a singularity free attitude control solution, suitable for high-precision applications such as quadrotor UAVs.

A. System Dynamics: The quaternion kinematics and angular velocity dynamics evolve as follows:

$$\dot{\mathrm{q}} = -\frac{1}{2}\mathrm{q} \otimes \Omega_{quat}, \quad \Omega_{quat} = [0; \Omega] \tag{38}$$

$$\dot{\Omega} = J^{-1}(\tau - \Omega \times (J\Omega)) \tag{39}$$

B. Control Law: The control law minimizes the orientation error using a proportional controller on the quaternion error and angular velocity. The torque τ is given as:

$$\tau = -k_{\mathrm{q}} e_{\mathrm{q}} - k_{\Omega}\Omega \tag{40}$$

Where e_{q} is the quaternion error vector, k_{q} and k_{Ω} are proportional gains for orientation and angular velocity control, respectively.

The quaternion is normalized at each time step to ensure numerical stability.

Algorithm 3: Quaternion-Based Attitude Control Law

Input: Initial conditions: q_0, Ω_0, Quadrotor parameters: $J, k_{\mathrm{q}}, k_{\Omega}$ and desired trajectory $\mathrm{q}_d(t), \Omega_d(t)$

Initialize: Time step Δt

for $t = t_0$ to t_{end} with step Δt **do**

Compute quaternion error: $e_{\mathrm{q}} = \mathrm{q}_d \otimes \mathrm{q}^*$

Extract vector part of quaternion error: $e_{\mathrm{q}} = \begin{bmatrix} e_{\mathrm{q}_1} & e_{\mathrm{q}_2} & e_{\mathrm{q}_3} \end{bmatrix}^T$

Compute control torque: $\tau = -k_{\mathrm{q}} e_{\mathrm{q}} - k_{\Omega}\Omega$ (40)

Update angular velocity: $\dot{\Omega} = J^{-1}(\tau - \Omega \times (J\Omega))$ (39)

Integrate angular velocity: $\Omega(t + \Delta t) = \Omega(t) + \Delta t \dot{\Omega}$

Update quaternion kinematics: $\dot{\mathrm{q}} = -\frac{1}{2}\mathrm{q} \otimes \Omega_{quat}, \quad \Omega_{quat} = [0; \Omega]$ (38)

Normalize quaternion: $\mathrm{q(t+}\Delta\mathrm{t)} = \frac{\mathrm{q(t)+}\Delta\mathrm{t}\dot{\mathrm{q}}}{\|\mathrm{q(t)+}\Delta\mathrm{t}\dot{\mathrm{q}}\|}$

end for

Output: Quaternion Trajectory $\mathrm{q}(t)$, angular velocity $\Omega(t)$, and torque $\tau(t) = 0$

C. Trajectory Tracking: The controller is tested on the following reference trajectories:

- **Step Trajectory:** Discrete steps in roll (ϕ), pitch (θ) and yaw (ψ).
- **Flip Trajectory:** Smooth 360° roll over a defined duration.
- **Circular Trajectory:** Constant angular velocity resulting in rotation in the xy-plane.

D. Numerical Results: The simulation evaluates the controller's performance through the following metrics:

- **Angular Velocity:** Evolution of $\Omega = [\Omega_{\phi}, \Omega_{\theta}, \Omega_{\psi}]$.

- **Control Torque:** The torque components τ_x, τ_y, τ_z, for the attitude stabilization of the UAV are shown in the Fig. 12, these were limited to ±1.5x10^{-3} N·m.
- **Orientation Tracking:** In Fig. 10, the actual Euler angles (ϕ, θ and ψ) tracks and compared with its desired signals (ϕ_d, θ_d and ψ_d).

Fig. 9 shows the dynamics of the quaternion error and their magnitude. The quaternion error expression is defined as follows: $e_q = q \otimes q_d^{-1}$, so in this context, the attitude (q) approaches to the desired path (q_d), making the absolute value of the error magnitude $|e_q|$ achieve convergence to the unit quaternion. As illustrated in Fig. 9, the quaternion controller developed in Algorithm 3 successfully achieves the defined goals, including trajectory tracking and minimizing the singularity issues. Notably, the Unit-quaternion orientations converge the desired chosen value of $q_d = [1,0,0,0]^T$.

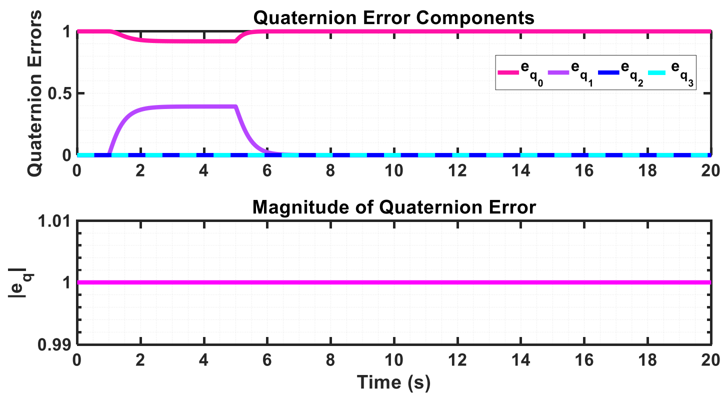

Fig. 9. Quaternion error components and their magnitude.

Note that for attitudes crossing the Euler angle singularities, there is a discontinuity in the roll tracking that is shown in Fig. 10. It means that the roll angle is fundamentally related to the aircraft's rotational dynamics in relation to its x-axis. This simulation demonstrates the main advantage of the quaternion attitude representation over that of Euler angles.

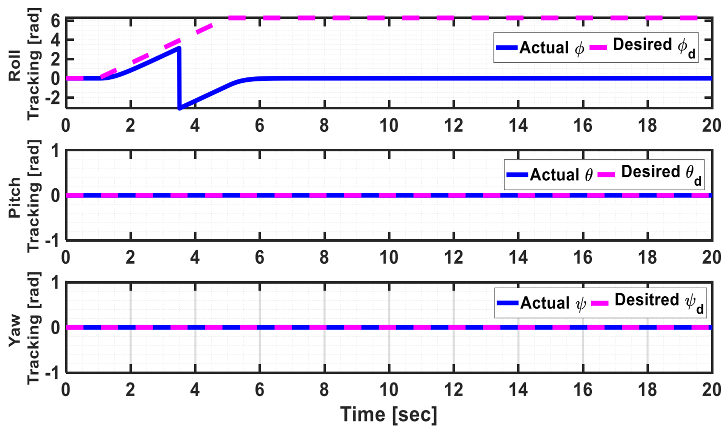

Fig. 10. Attitude Tracking by Quaternion over Time.

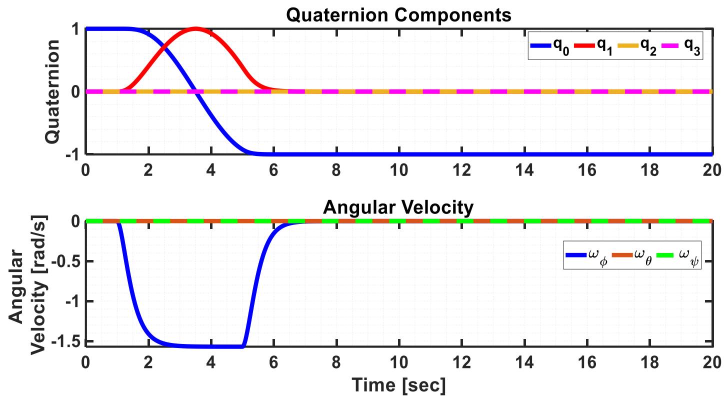

Fig. 11. Quaternion and Angular Velocity over Time.

Fig. 11 shows the stabilization of the quadrotor attitude; where it is clearly observable that the quaternion at the equilibrium point is stabilized. Likewise, we can see that the angular velocity errors attain zero. The outstanding outcomes confirm the efficacy of the Algorithm 3. In addition, control inputs are presented in Fig. 12.

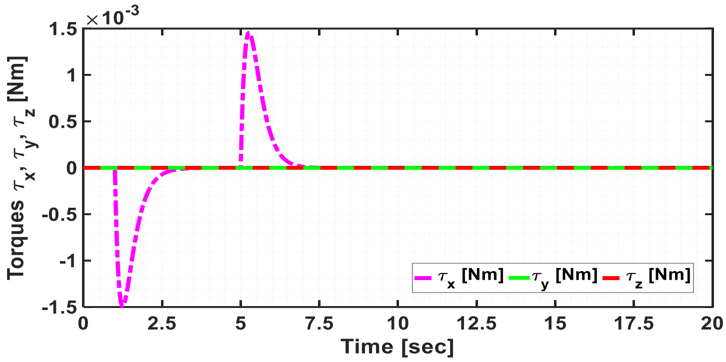

Fig. 12. Control Torque Components.

4. PROPOSED CONTROL ALGORITHM

In order to perform accurate and aggressive maneuver tracking of a predefined path, the control strategy proposed in this work is based on a hierarchical control law comprising two sub-controllers as shown in Fig. 13, an attitude controller using quaternion controller, and a position controller by using GTC controller. The key idea behind this subdivision is that the position controller will steer the vertical and horizontal displacements by generating the adequate thrust force F expression defined in Eq. 42. Then the attitude controller takes this information and manages the quadrotor orientation by generation the required rolling, pitching, and yawing torques (τ_x, τ_y and τ_z), these later will be transformed via the nonlinear decoupling of RPY-angles to quaternion model in Eqs. 17, 30 and 44 into a desired quaternion q_d, and disered angular velocity Ω_d. From the present analysis we can see that the vertical displacement, the roll, the pitch, and the yaw angles are controlled directly using the input signals (F, τ_x, τ_y and τ_z), whereas the quadrotor position is indirectly controlled using the roll and pitch rotations.

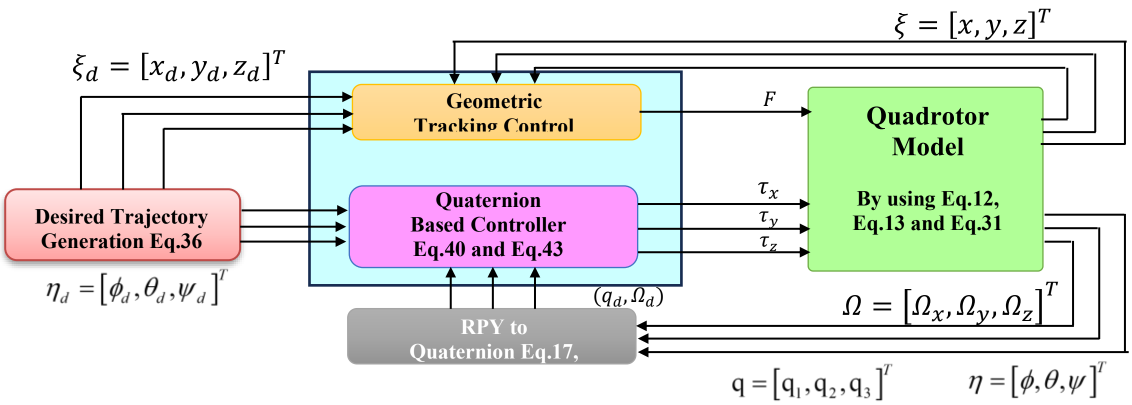

Fig. 13. Hierarchical scheme of the proposed control strategy.

Position and orientation are used to determine the desired trajectory, which is expressed as:

$$x_d = \begin{bmatrix} x_d & y_d & z_d & \dot{x}_d & \dot{y}_d & \dot{z}_d & \ddot{x}_d & \ddot{y}_d & \ddot{z}_d & \phi_d & \theta_d & \psi_d \end{bmatrix}^T \tag{41}$$

In the proposed algorithm, there are two independent controllers that output F and τ, which are used to control drone dynamics.

The altitude controller is based on GTC. The equation for altitude controller is given in Eq. 18 as:

$$F = m(\ddot{x}_d + g e_3^T \Re e_3) - k_x e_x - k_\upsilon e_\upsilon \tag{42}$$

The attitude controller uses a full quaternion-based approach to attitude control, effectively reducing the error to zero. The attitude controller formulation is given as:

$$\tau = -k_q \begin{bmatrix} e_{q_1} & e_{q_2} & e_{q_3} \end{bmatrix}^T - k_\Omega \begin{bmatrix} \Omega_x & \Omega_y & \Omega_z \end{bmatrix}^T \tag{43}$$

$$e_q = q \otimes q_d^{-1} \tag{44}$$

Where, e_q represents the quaternion error expression and q_d denotes the desired quaternion path vector.

The complete steps of the proposed algorithm are given as follows:

Algorithm 4: Quadrotor with Geometric Tracking Quaternion-based control

Input: $x_d(t), \dot{x}_d(t), \ddot{x}_d(t), q_d(t), \Omega(t), x_0, \dot{x}_0, \Omega_0$

Initialize: $\Delta t, m, g, I, k_x, k_\upsilon, k_q, k_\Omega$

for $t = t_0$ to t_{end} with step Δt **do**

$e_x = x(t) - x_d(t)$ (14)

$e_q = e_q(q_d, q)$ (44)

$\tau = -k_q \begin{bmatrix} e_{q_1} & e_{q_2} & e_{q_3} \end{bmatrix}^T - k_\Omega \begin{bmatrix} \Omega_x & \Omega_y & \Omega_z \end{bmatrix}^T$ (43)

if $4 < t < 8$ **then** $F = [0; 0; 0]$

else $F = m(\ddot{x}_d + g e_3^T \Re e_3) - k_x e_x - k_\upsilon e_\upsilon$ (42)

end if

$\Re = Rot\_matrix(quanternion\_to\_euler(\mathrm{q}))$ (3)

$\ddot{x}(t) = [0;0;-g] + \frac{1}{m}\Re.F$ (18)

$\dot{\Omega} = J^{-1}(\tau - \Omega \times J\Omega)$ (40)

$\Omega \leftarrow \Omega + \Delta t\dot{\Omega}$

$\dot{\mathrm{q}} = -0.5 \times \mathrm{q}\_mult([0;\Omega],\mathrm{q})$ (31)

$\mathrm{q} \leftarrow \mathrm{q} + \Delta t\dot{\mathrm{q}},\ \ \mathrm{q} \leftarrow \mathrm{q}/\|\mathrm{q}\|$

$\dot{x}(t) \leftarrow \dot{x}(t) + \Delta ta,\ \ x(t) \leftarrow x(t) + \Delta t\dot{x}(t)$

end for

Output: $x(t)$, $\Omega(t)$, $\mathrm{q}(t) = 0$

4.1. Stability Analysis of the proposed controller

For the system model presented in Eqs. 12 and 13, the controller Eq. 42 can solve the problem of the UAV attitude tracking control presented in Algorithm 4. To track the desired trajectory, the controller must drive the position error $e_x = \xi - \xi_d$ to zero. We choose the first candidate Lyapunov function as follows:

$$V_1 = \frac{1}{2}e_x^T e_x \quad (45)$$

Computing the time-derivative of function V_1, yields:

$$\begin{aligned}\dot{V}_1 &= -W_1(e_x) + k_x e_x^T(e_x + \frac{1}{k_x}\dot{e}_x) \\ &= -W_1(e_x) + k_x e_x^T(e_x + \frac{1}{k_x}e_\upsilon)\end{aligned} \quad (46)$$

Where $trace(k_x) > 0$, and:

$$W_1(e_x) = -k_x e_x^T \quad (47)$$

Than, we have:

$$\begin{aligned}\dot{e}_x &= \dot{\xi} - \dot{\xi}_d \\ &= \Re\upsilon - \dot{\xi}_d = e_\upsilon\end{aligned} \quad (48)$$

By applying the standard Lyapunov stability procedure, we define a second error and second candidate Lyapunov function as below:

$$e_2 = e_x + \frac{1}{k_x}e_\upsilon \quad (49)$$

$$V_2 = \frac{1}{2}e_x^T e_x + \frac{1}{2}e_2^T e_2 \quad (50)$$

Similarly, we compute its time-derivative, yields

$$\dot{V}_1 = -W_2(e_x, e_2) + k_x k_\upsilon e_2^T (e_2 + \frac{1}{k_x^2 k_\upsilon} \ddot{e}_x) \tag{51}$$

Where, $W_2(e_x, e_2)$ must be a positive definite function as:

$$W_2(e_x, e_2) = k_x e_x^T e_x + k_x (k_\upsilon - 1) e_2^T e_2 \tag{52}$$

Therfore, $trace(k_\upsilon) > 1$. Then we obtain the error dynamic equation of the position system:

$$\ddot{e}_x = -\frac{1}{m} \Re F e_3 + g e_3 - \ddot{x}_d \tag{53}$$

In order to garantee the negative definite of $\dot{V}_2$, we can construct an appropriate control input for the quadrotor system so that:

$$\ddot{e}_x = k_x e_x + k_\upsilon e_v \tag{54}$$

Then, we have obtain the following relation:

$$\frac{1}{m} \Re F e_3 = g e_3 - k_x e_x - k_\upsilon e_v - \ddot{x}_d \tag{55}$$

In this case, we can only obtain the input force amplitude F as follows:

$$F = m(\ddot{x}_d + g e_3^T \Re e_3) - k_x e_x - k_\upsilon e_\upsilon \tag{56}$$

The Eq. 56 is the same with as obtained in Eq. 42. Where, $\Re$ is the actual attitude of the quadrotor system and the desired attitude $\Re_d$ is defined in Eq. 37. In Eq. 34, b3 is defined as the third column, and b2 is chosen to be orthogonal to b3 to ensure $\Re_d \in SO(3)$, as stated in Eq. 37. Then, the zero equilibrium of the tracking errors of the complete dynamics is attractive.

5. SIMULATION RESULTS AND DISCUSSION

In this sub-section, numerical results are presented to demonstrate how our proposed control approach in Algorithm 4 can be used to perform aggressive flight maneuvers of a quadrotor UAV. The nominal parameters of the quadrotor are: $m = 0.5\ kg$, $g = 9.81\ m/s^2$, and $J = 1e^{-2}.diag(0.558, 0.557, 1.05)\ kg.m^2$. According to the process of the control design above, the parameters of the controllers are given in Table 2.

Table 2. Controller Gains.

Gain	Value
k_q	0.025
k_Ω	0.007
k_x	$diag(7,5,100)$
k_υ	$diag(5,5,9)$

A helical trajectory is obtained by using the following desired trajectories:

$x_d = 0$, $y_d = 0$, $z_d = \begin{cases} 0, & \text{if } 0 \le t \le 1 \\ 1, & \text{if } t \ge 1 \end{cases}$ and $\psi_d = 0$ rad, $\theta_d = 0$ rad, and $\psi_d = 0$ rad.

The controller has been tested in the aggressive 360° flip maneuver, and the results are provided below:

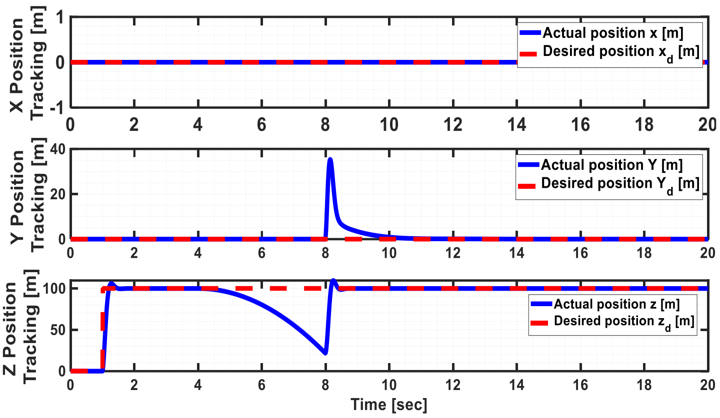

Fig. 14. Quadrotor actual position Vs desired trajectory.

Stabilization of both position and attitude of the quadrotor system was ensured by establishing the simulation results in Figs. 14-16. The quadrotor's position tracking during a 360° flip can be seen in Fig. 14. This is achieved by combining two control phases, as demonstrated in the figure. The quadrotor is commanded to take off vertically at time t=1 sec and to stop hovering at a height of 100 m.

Fig. 15 shows the stabilization of the vehicle position. In this figure the stabilization of the quaternion at the equilibrium point $q_d = [1, 0, 0, 0]^T$ is clearly observable. Likewise, we can see that the angular velocities converge to zero. This means that the quadrotor will exponentially converge to a given attitude and stay still in it. Notably, as shown in Fig. 16, during the flip around ϕ_d, the quadrotor deviates from its desired position in the x-axis due to the temporary deactivation of the controller and the roll angle relation with the rotational dynamics of the UAV in its x-axis (Assumption.3). The applied control inputs are presented in Fig. 17. Obviously, these signals can be achieved by a general quadrotor. In this Figure it can clearly see the influence of this deviation of the roll angle, as shown by the force applied on the rotational dynamics of the drone on its x-axis.

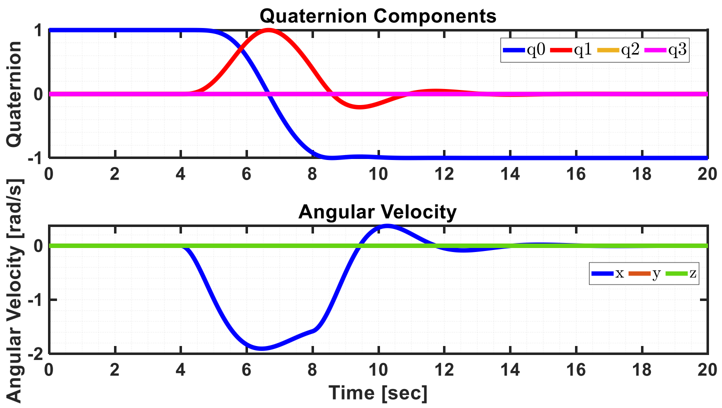

Fig. 15. Quadrotor quaternion and angular velocities.

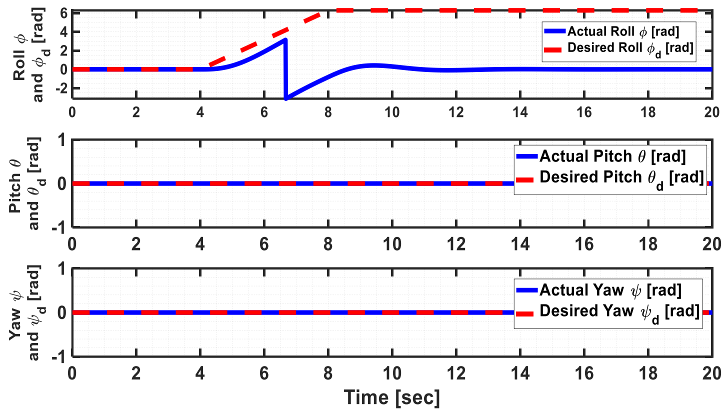

Fig. 16. Eulers angles tracking.

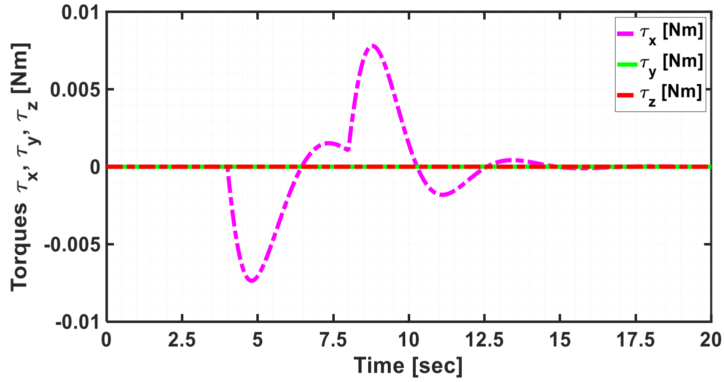

Fig. 17. Control torques.

The effectiveness of the proposed controller exceeds that of the two alternative controllers. With the increase in pitch (θ) and roll (ϕ) angles during the aircraft's maneuvers, the performance differences between the two controllers become markedly more significant. Quaternion attitude control proves effective at low pitch (θ) and roll (ϕ) angles; however, it becomes inadequate at higher angles, primarily due to the interactions between the elevator and rudder.

Overall, the hybrid control strategy offers enhanced responsiveness and accuracy, especially in challenging conditions characterized by high roll and pitch angles. Its ability to differentiate performance across various channels underscores its potential for improving control systems in dynamic environments.

6. CONCLUSION

This paper presented a hybrid control scheme for quadrotor UAVs, combining GTC on SE(3) for position and altitude tracking and Quaternion-Based Attitude Control for precise orientation tracking. The proposed method addresses the limitations of classical controllers, which struggle with aggressive maneuvers and large angular displacements. By leveraging the mathematical rigor of SE(3) and quaternions, the hybrid controller ensures global stability, singularity-free operation, and robust performance under dynamic conditions. The effectiveness of the proposed control strategy was performed through simulations on two aggressive maneuvers: a 360-degree flip and a helical trajectory. The controller showed precise trajectory tracking with minimal position and velocity errors, and it maintained stable attitude control during high-rotation maneuvers. Therefore, it can be concluded that the roll channel's setpoint tracking is effectively uniform across both controllers, reinforcing the understanding that roll, being the last of the Euler angles, directly relates to the aircraft's rotational dynamics around its x-axis. Simulation results confirmed that the hybrid approach effectively balances the complementary strengths of geometric and quaternion-based control, achieving smooth and robust control of UAV dynamics. The study highlights the potential for applying this control method to real-world scenarios, including high-precision UAV applications and tasks requiring aggressive maneuvering. By addressing these areas, the hybrid control framework can be further developed into a future comprehensive solution for a wide range of UAV applications, including high-precision tasks, cooperative operations, and autonomous navigation in dynamic and uncertain environments.

7. REFERENCES

Al-husnawy, T., & Al-Ghanimi, A. (2024) 'A review of control methods for quadrotor uav', Kufa Journal of Engineering, 15(4), 98-124. https://doi.org/10.30572/2018/KJE/150408

Andersen, T. S., & Kristiansen, R. (2017) 'Quaternion guidance and control of quadrotor', Int Conf Unmanned Aircraft Systems (ICUAS), USA, pp. 1567-601. https://doi.org/10.1109/ICUAS.2017.7991509

Benevides, J. R. S., Inoue, R. S., Paiva, M. A. D., & Terra, M. H. (2019) 'ROS-Based Robust and Recursive Optimal Control of Commercial Quadrotors', IEEE 15th Int Conf Aut Sci Eng (CASE), Canada, pp. 998-1003. https://doi.org/10.1109/COASE.2019.8843004

Castillo, A., Sanz-Diaz, R., García-Gil, PJ., Qiu, W., Wang, H., & Xu, C. (2019) 'Disturbance observer-based quadrotor attitude tracking control for aggressive maneuvers', Control Engineering Practice., 82:14-23. https://doi.org/10.1016/j.conengprac.2018.09.016

Choutri, K., Lagha, M., Dala, L., & Lipatov, M. (2017) 'Quadrotors trajectory tracking using a differential flatness-quaternion based approach', 7th Int Conf on Model, Sim, and App Opt (ICMSAO).

Escamilla, L., Carrillo, L. R. G., Sandoval, S., & Quesada, E. S. E. (2023) 'Stabilization and trajectory tracking of a subactuated aircraft based on a Geometric Algebra approach', Americ Cont Conf (ACC), pp. 3233-3238.

Fresk, E., & Nikolakopoulos, G. (2013) 'Full Quaternion Based Attitude Control for a Quadrotor'. Eur Contr Conf (ECC), Switzerland.

Gonzalez, H.A. (2019) 'Robust tracking of dynamic targets with aerial vehicles using quaternion-based techniques', Ph.D. dissertation, University of Technology of Compiègne.

Hassan, M., & Abulhail, S. (2021) 'FPGA BASED P/PI/PD/PID-like interval type -2 FLC design for control systems', Kufa Journal of Engineering, 9(3), 165-179. https://doi.org/10.30572/2018/KJE/090312

Invernizzi, D., Lovera, M., & Zaccarian, L. (2020) 'Integral ISS-Based Cascade Stabilization for Vectored-Thrust UAVs', IEEE Control Syst, 43–48.

Jeaeb, S.G.R., Saad Salih, R., & Makki Jiaad, S. (2025) 'Artificial Neural Network (ANN) based Proportional Integral Derivative (PID) for Arm Rehabilitation Device', Kufa Journal of Engineering, 16(1), 80-103. https://doi.org/10.30572/2018/KJE/160106

Kherkhar, A., Didi, F., Chiba, Y., & Tlemçani, A. (2023) 'Proportional Derivative (PD)-Based Interval Type-2 Fuzzy Control Design of a Quadrotor Unmanned Aerial Vehicle', Tobacco Regulatory Science (TRS), 3419-3440.

Madeiras, J., Cardeira, C., Oliveira, P., Batista, P., & Silvestre, C. (2024) 'Saturated Trajectory Tracking Controller in the Body-Frame for Quadrotors', Drones, 8(4), 163. https://doi.org/10.3390/drones8040163

Martins, L., Cardeira, C., & Oliveira, P. (2024) 'Global trajectory tracking for quadrotors: An MRP-based hybrid strategy with input saturation', Automatica, 162, 111521.

Ramp, M., & Papadopoulos, E. (2018) 'Geometric Surface-Based Tracking Control of a Quadrotor UAV for aggressive maneuvers', 26th Mediter Conf on Contr and Autm (MED), Croatia.

Sanwale, J., Trivedi, P., Kothari, M., & Malagaudanavar, A. (2020) 'Quaternion-based position control of a quadrotor unmanned aerial vehicle using robust nonlinear third-order sliding mode control with disturbance cancellation', Proc IMechE, Part G: J Aerospace Engineering, 234(4):997-1013.

Sharma, S., Khurana, P., Sharma, M., & Kumar, A. (2023) 'Optimization of Design Parameters for a Quadcopter Using Taguchi Design Methodology', Advances in Manuf Tech Manag, pp.598.

Shi, X., Sun, Y., & Shao, X. (2018) 'Robust output feedback trajectory tracking for quadrotors', Proc IMechE, Part G: J Aerospace Engineering, 233: 1596–1610.

Simplício, P. V. G., Benevides, J. R. S., Inoue, R. S., & Terra M. H. (2024) 'Robust and intelligent control of quadrotors subject to wind gusts', IET Control Theory & Applications. https://doi.org/10.1049/cth2.12615

Spong, M.W., Hutchinson, S., & Vidyasagar, M. (2020) 'Robot modeling and control'. John Wiley & Sons.

Wang, J., Yuan, X., & Zhu, B. (2022) 'Geometric control for trajectory-tracking of a quadrotor UAV with suspended load', IET Control Theory Appl. 16, 1271–1281. https://doi.org/10.1049/cth2.12301

Xie, W., Cabecinhas, D., Cunha, R., & Silvestre, C. (2021) 'Adaptive Backstepping Control of a Quadcopter with Uncertain Vehicle Mass, Moment of Inertia, and Disturbances', IEEE Trans. Ind. Electron., 69, 549–559. https://doi.org/10.1109/TIE.2021.3055181

Zha, C., Ding, X., Yu, Y., & Wang, X. (2017) 'Quaternion-based Nonlinear Trajectory Tracking Control of a Quadrotor Unmanned Aerial Vehicle', Chinese journal of mechanical engineering, Volume 30, 1.

Zhao, C., & Burlion, L. (2022) 'Geometric Model Free Trajectory Tracking Control on SE(3)', IFAC-Papers On Line, Volume 55:387-393. http://dx.doi.org/10.1016/j.ifacol.2023.03.065