



DYNAMIC RESPONSE ANALYSIS OF PRE-TWISTED BEAMS UNDER HARMONIC LOADING

**Sadiq Emad Sadiq¹, Orhan Sabah Abdullah², Amina Hmoud Alikhan³,
Luay S. Al-Ansari⁴, Ameen Topa⁵, and Mujtaba A. Flayyih^{6,7}**

¹ Department of Aeronautical Technical Engineering, Technical Engineering College of Najaf, Al-Furat Al-Awsat Technical University, Najaf, Iraq,
Email:sadaiq.emad@atu.edu.iq.

² Mechanical Engineering Department, University of Technology, Baghdad, Iraq,
Email:20313@uotechnology.edu.iq.

³ Mechanical Engineering Department, Faculty of Engineering, Wasit University, Iraq,Email:adhaef@uowasit.edu.iq.

⁴ Mechanical Engineering Department, Faculty of Engineering, University of Kufa, Iraq,Email:luays.alansari@uokufa.edu.iq.

⁵ Department of Maritime Technology, Faculty of Ocean Engineering Technology, Universiti Malaysia Terengganu, Malaysia, Email:p4296@pps.umt.edu.my.

⁶ Prosthetics and Orthotics Engineering Department, College of Engineering and Technologies, Al-Mustaqbal University, Hillah, 51001, Iraq,
Email:mujtaba_abdulkadhim@uomus.edu.iq.

⁷ Department of Mechanical Engineering, University of Al-Qadisiyah, Al-Qadisiyah, 58001, Iraq, Email:mujtaba_abdulkadhim@uomus.edu.iq.

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ABSTRACT

The behavior of a non-prismatic beam under static and dynamic loads depends on the density distribution and geometry of the beam along its length. An essential type is a pre-twisting beam. In the current study, the dynamic response of a pre-twisted beam vibrated freely and forced under a harmonic force was investigated by establishing finite element modeling via ANSYS software. Beam length and twist angle were considered as pre-twist parameters. Beam length varying from 300 mm to 1000 mm and twist angle ranging from 0 to 360° with incremented 45°. Modal analysis and then harmonic response were implemented to calculate the natural frequency, total deflection and equivalent stress, and the damping ratio variation with pre-



twisting beam parameters. The model was validated against previous studies, showing perfect agreement. The obtained results show that the first natural frequency increases when the twisting angle rises till (180°) and then remains constant. Notably, the influence of the beam length on the harmonic response reduces when the twisting angle is greater than 180° . Similarly, the deflection of harmonic response increases when the frequency ratio is approximately unity. Finally, the influence of the length of the beam and twisting angle on the stress appears when the twisting angle is smaller than 180° .

KEYWORDS

dynamic response, force vibration, damping ratio, pre-twisting beam, finite elements model.

1. INTRODUCTION

In industrial applications, beams hold real importance when the geometry and density of the beam are changed along the length of the beam. This non-uniformity can enhance the ratio (strength/weight) or raise structural stability (Diwan et al., 2022). Beams with varying material distribution but constant dimensions are commonly found in applications involving axially functionally-graded materials (A-FGM) (Hashim et al., 2022; Shukur et al., 2023; Wadi et al., 2022). The beam cross-section may vary along its length, either linearly, such as in stepped (Alansari et al., 2019; Al-Saffar et al., 2020; Khalsan Al-Raheem and Alansari, 2019) and tapered beams (Al-Ansari Luay S. et al., 2018), or non-linearly (Alansari et al., 2022). The behavior of non-prismatic (or non-uniform) beams under static and dynamic loads depends on the geometry and density along the beam length, so several papers have been dedicated to it. Non-uniform (or non-prismatic) beams are used in different ways because of their economic, aesthetic, and other benefits.

For a stepped beam, (Farghaly and El-Sayed, 2016) Analyzed the dynamics of a multi-step Timoshenko beam with variable cross-sections and attachments, deriving global matrices for natural frequencies and mode shapes. Validated by parametric analyses, their work enhances understanding of complex beam dynamics in engineering. (Hu et al., 2022) Conducted simulations on the stepped beam within space manipulator systems to mitigate vibrations and improve system efficiency. The outcomes demonstrated consistent vibration amplitudes at the free end of the flexible beam across different configurations of the stepped cross-section. (Chen et al., 2020) present a novel transfer matrix method (TMM) for analyzing the free and forced vibrations of multi-step Timoshenko beams coupled with rigid bodies on springs, commonly utilized in robotic arms. The method simulates the system as a chain of elements, allowing for efficient and accurate computation without the need for high-order equations or spatial discretization. The study checked the validity of the method through four numerical examples, including applications in flexible robot arms and launch vehicles. The results demonstrate the method's precision and computational efficiency, making it suitable for various engineering structures. (Tang et al., 2022) used variable cross-section Euler-Bernoulli beams to model the aircraft body in their dynamics study using the Modified Transfer Matrix Method (MSTMM). They represented the rudder system as a rigid body and the connection as a space elastic hinge. Their research calculates the vibration characteristics of aircraft with tail axial thrust, validated through finite element software. The results demonstrate that thrust significantly impacts vibration characteristics, providing an effective method for analyzing and improving aircraft stability.

Straight beams are simple, linear-geometries with parallel fibers such that the stress distribution is uniform and mechanical properties predictable and so are used in conventional applications such as bridges and buildings. Pre-twisted beams are beams with their cross-sectional areas twisted about the longitudinal axis. The twist pre-twist beam is helical in nature, which makes the geometry of the pre-twist beam non-uniformly distributed, necessitating technical analytical models and simulations. They are applied in more sophisticated applications such as aircraft blades and wind turbines, where exact vibration control is very important (Zeng et al., 2020). A study of pre-twist beam with the most impact was (Ramaswamy et al., 2023), which reanalyzed the buckling behavior of pre-twisted bars via the two-field Hellinger-Reissner mixed method. This approach passes the axial and bending stiffness computations and gives a more precise and secure estimate of buckling loads resolving the disparities between conventional theories and experimental findings. Moreover, (Khakalo and Niiranen, 2023) examined the buckling behavior of pre-twisted birch plywood stripes both experimentally and computationally. They study up to 400 degree twisting including linear and nonlinear buckling analysis after twisting to take residual stresses into consideration. They determined four important twisting angles, which strongly affect buckling behavior, such as mode jumps, losses of stability. This study advances the concept of the effect of twisting induced stresses on the structural integrity of columns. (Baziyar Hamzehkhani et al., 2024) developed a dynamic stiffness matrix for pre-twisted sandwich beams, revealing that pre-twisted beams exhibit significant mode shape transformations and coupled flexural displacements. Their findings were validated against ABAQUS simulations and other studies. (Hu et al., 2020) developed a refined beam theory for analyzing the dynamics of rotating pre-twisted beams. They utilized a three-dimensional beam element formulation based on the Carrera unified formulation (CUF), enhancing prediction accuracy for structures like fan blades. This advanced model effectively captures in-plane, out-of-plane, and torsional deformations, proving particularly valuable in applications involving complex rotational dynamics.

In recent years, researchers have noted the significant impact of material distribution along their length on the behavior of beams. (Al-Raheem et al., 2024) focused on the static deflection of axially functionally graded (FG) cantilever beams using Rayleigh and Finite Element methods based on Euler-Bernoulli beam theory. Their approach highlights how varying material properties according to a power-law distribution influence beam behavior under load, crucial for applications in aerospace, automotive, and defense industries, where optimized material properties enhance performance and safety. In practical engineering applications, many types

of beams are combined to optimize performance and meet specific structural requirements. For instance, research conducted by (Migliaccio and Ruta, 2021) the behavior of pre-twisted, tapered beams used in turbine and helicopter blades was analyzed. A mathematical model was developed to predict how pre-twist affects these beams, with equations established through a variational approach. Verified by 3D-FEM simulations, the findings highlight the complex interactions of bending, twisting, and traction forces, emphasizing the importance of advanced modeling in practical applications. (Hoskoti et al., 2021) Performed modal analysis on a rotating, twisted and tapered Rayleigh beam to explore the vibration characteristics of cantilever blades. They derived energy expressions, transformed equations into non-dimensional forms, and solved the eigenvalue problem. The study examined how natural frequencies change with rotation speed, taper ratio, and pre-twist angle, and analyzed resonance conditions using a Campbell diagram, providing insights into the dynamics of rotating blades.

Basing on the previous discussions about the impact of varying mechanical properties through changes in cross-sectional geometry or material distribution along the beam's length, no previous research has specifically addressed the response of beams when vibrated under harmonic force. This gap in the literature highlights a critical area for further exploration. The current study aims to bridge this gap by initially investigating the free vibration of pre-twisted beams and subsequently analyzing their forced vibration response under harmonic forces. Utilizing the Finite Element Analysis (ANSYS software), this research focuses on understanding how the pre-twist variable beam length and twisting angle affect the first three natural frequencies and damping ratio. Subsequently, the study examines how these variables affect the response of pre-twisted beams under harmonic forces, especially given the total deflection and equivalent stress. This comprehensive approach aims to deepen our understanding of pre-twisted beam dynamics and contribute to the development of more robust predictive models for their behavior under dynamic loading conditions, enhancing both theoretical and practical applications in structural engineering.

2. NUMERICAL MODELING

2.1. Problem statement

Pre-twist beams are directly useful in the engineering sector such as in marine structures, aircrafts, and wind turbines. Their distinctive form enhances the aerodynamic efficiency of aircraft wings, their load bearing capacity in naval constructions and performance of wing turbine blades. However, when these beams are exposed to harmonic loads, they tend to vibrate particularly the forced vibration. Forced vibration resonance may result in the development of vibrations on dangerous levels and structural failure can result through crack initiation in the

material, joint wear and damage to critical components. Such failures may lead to huge financial losses and disastrous accidents. Consequently, it is important to study the behavior of pre-twisted beams when they are subjected to harmonic loads in order to improve their design and provide safety. To achieve increased resistance and stability of pre-twist beams, the behavior of pre-twist beams under harmonic load to prevent the occurrence of resonance and the optimization of designs needs to be examined.

2.2. specimen geometry and harmonic loading conditions

ANSYS harmonic response analysis is an effective software utilized to find the steady-state reaction of a building to sinusoidal loads (harmonic loads). The analysis is essential in the study of the behavior of a structure to dynamic loading conditions which change sinusoidal with time. To simulate the harmonic vibration of the pre-twisting beam under harmonic load, the ANSYS Software version 2023 harmonic response analysis is performed in this study. The beam is a rectangular cross section that is 15 mm wide and 10 mm high. Table 1 revealed that the beam length varied. Fig.1 shows the beam was in a cantilever with one end of the beam attached and the other one not attached. It was exposed to a focused harmonic load on the free end of (0-30) Hz frequency range and a solution interval of 300.

$$F_{(x,t)} = F_0 \sin(wt) \quad (1)$$

Were

F_0 is 1 Newton

W is the force frequency

Pre-twist beam parameters such as twist angle (θ) and beam length (L) were carefully chosen as listed in Table 1.

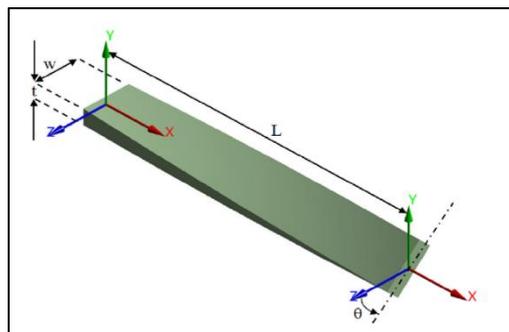


Fig. 1. Schematic diagrams of the pre-twisting beam.

Table 1 pre-twisting beam parameters

Parameters	Values
Twist angle	0 – 360 with step of 45 degrees
Beam length	300, 500,700, and 1000 mm

2.3. Material Properties for Simulation

The material that has been selected in this study as the pre-twisting beam to use in the

simulations is PLA or Polylactic Acid. (Hassib et al., 2024; Hossain et al., 2024). PLA is a renewable thermoplastic biodegradable polymer which is produced with the use of cornstarch or sugar and, therefore, is environmentally friendly. that is why it is widely used in 3D printing because of its mechanical characteristics: elastic modulus of approximately 1.2 Gpa, 1360 kg/m³ density, Poisson ratio of 0.36 and these properties define the rigidity, lightness and flexibility of PLA, making it ideal in applications where low strength/weight ratio is to be considered (E. S. Sadiq et al., 2025).

2.4. Mashing

In the present research, ANSYS has developed a detailed mesh of the pre-twisted beam; materials of SOLID186 type were used due to the possibility of modeling the 3D structural behavior (Majid Jasim and Jawad Abdulsamad, 2025). The mesh has been optimized to be accurate particularly in the high-stress areas and convergence check was to provide the efficiency in the computation. The mesh of the pre-twist beam is depicted in Fig.2 with the length of the beam being 300 mm and twists with a 45- and 90-degree twist having 30498 nodes and 6040 elements depending on the length of the beam. Mesh quality checks were done very meticulously as acceptable aspect ratios, skewness and verticality were also checked. This led to simulation of the deflection and the same stress in harmonic loads and this gave understanding of the critical frequency and the stress points.

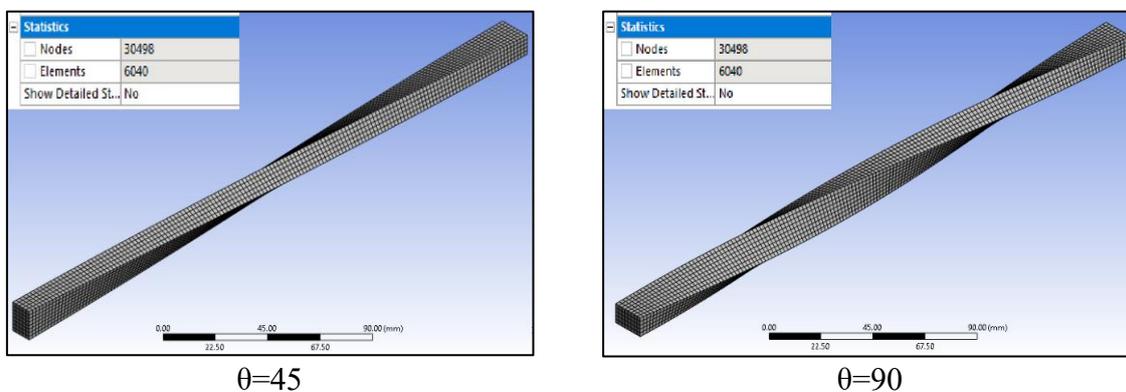


Fig.2. Meshing of pre-twisting beam with different twisting angles where L=300mm.

2.5. Postprocessing

The deflection and equivalent stress at each frequency were extracted through the analysis. This comprehensive analysis facilitates an understanding of the beam's behavior, including the identification of critical frequencies that lead to resonance, thereby aiding in the design of more resilient and stable structures.

2.6. model validation

In this research, the forced vibration under harmonic loading was studied to investigate the effect of pre-twisting beam parameters on the dynamic response using ANSYS version 2023

software with harmonic analysis. In previous work, (Shukur et al., 2024) The free vibration behavior of pre-twisted beams was experimentally and numerically investigated by using ANSYS software with Modal analysis. This research validates the finite elements model of the current study. They assumed the pre-twisted cantilever beam has a 350 mm length, 15 mm width, and 10 mm height, also the twist angles were 0° , 90° , 180° , 270° , and 360° .

One of the key outcomes of studying forced vibration under harmonic loading is the beam's deflection response at each frequency. In this response, a significant peak appears, characterized by a sharp rise, indicating the resonance condition. (Sadiq et al., 2021, 2020) The frequency at this peak represents the natural frequency. Fig.2 shows the deflection response at each frequency for the five twist angles (as per the previous study of (Shukur et al., 2024)). These frequencies were extracted and compared with the frequencies obtained experimentally and numerically in the previous study, as illustrated in Fig.3. The results showed good agreement, indicating the accuracy and validity of the simulation. Table 2 shows the comparison between the natural frequency obtained in the present work by harmonic analysis and both experimental and numerical methods obtained by (Shukur et al., 2024) Also, the error percentage with respect to the experimental results of (Shukur et al., 2024) are listed in Table 2. It can be observed that the natural frequency obtained by Harmonic analysis is smaller than that of the experimental results.

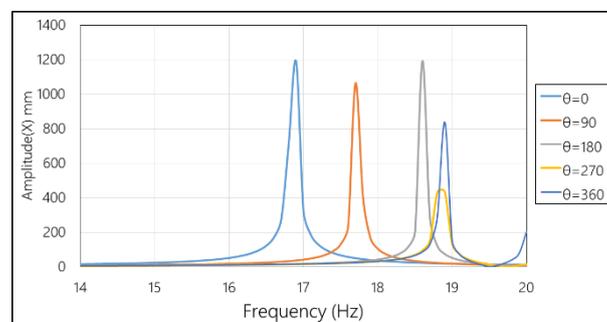


Fig. 2: Variation of Deflection Response of Cantilever pre-twisted beams with Five Pre-Twisting Angles Due to Frequency of Applied Harmonic Load

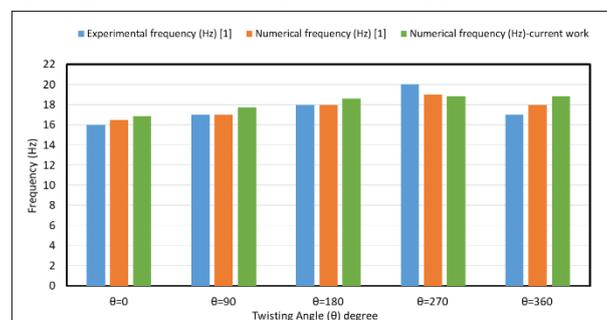


Fig. 3. Comparison Between Natural Frequency obtained by Shukur et al. (2024) (Experimental and Numerical Results) and Natural Frequency using Harmonic Analysis Obtained in the Present Work

Table 2. Comparison Between Natural Frequency obtained by Shukur et al. (2024) (Experimental and Numerical Results) and natural frequency using harmonic analysis obtained in present work

θ	(Shukur et al., 2024)			Current Work	
	Natural Frequency (Hz). Experimental	Natural Frequency (Hz). Numerical	Numerical Error %	Frequency (Hz)	Error %
0	16.00	16.50	3.13	16.86	5.39
90	17.00	17.00	0.00	17.73	4.28
180	18.00	18.00	0.00	18.62	3.44
270	20.00	19.00	5.00	18.85	5.75
360	17.00	18.00	5.88	18.88	11.04

2.7. Damping Ratio

In this work, the forced vibration under harmonic loading is investigated to calculate the damping ratio of pre-twisted cantilever beam. The damping ratio is calculated using the following points:

- 1- Find the maximum peak of deflection (Q).
- 2- Calculate the value $(\frac{Q}{\sqrt{2}})$.
- 3- Calculate ω_2 and ω_1 as shown in Fig.4 Calculate the damping ratio

$$(\xi = \frac{\omega_2 - \omega_1}{2\omega_n}). \text{ (S. E. Sadiq et al., 2025)} \tag{2}$$

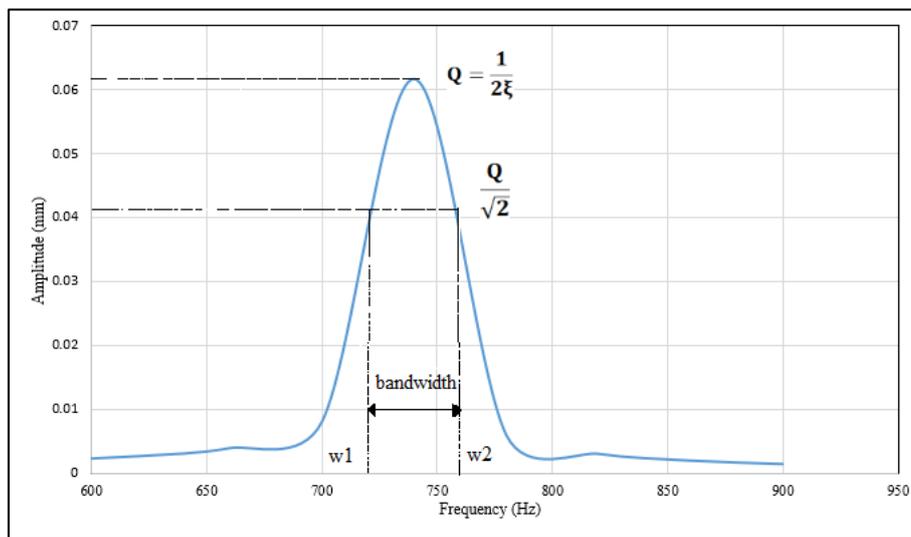


Fig.4. Frequency response under harmonic load excitation (Sadiq et al., 2020)

3. RESULTS AND DISCUSSION:

In this work, the influence of harmonic load on the dynamic response of a cantilever pre-twisted beam is examined. The twisting angle is changed from zero to 360 degrees with step of 45 degrees, and the length of the cantilever pre-twisted beam is (300, 500, 700, and 1000) mm. These two parameters were considered to investigate the dynamic response of a pre-twisted beam under a harmonic load. The following results were found:

3.1. Comparison Between Natural Frequencies Obtained by Modal and Harmonic Analyses:

In the beginning, the first natural frequency values of the cantilever pre-twisted beam calculated by harmonic analysis are compared with those calculated by modal analysis to check the accuracy of harmonic analysis. Fig.5 shows that there is a very good agreement between the modal and harmonic natural frequencies of pre-twisted beam with different length. The maximum error percentage between them is (2.275 %) when the length of pre-twisted beam is (1000 mm) and twisting angle is (360°) and the minimum error percentage between them is (-10.296 %) when the length of pre-twisted beam is (1000 mm) and twisting angle is (225°) as listed in Table 3. The maximum error percentage between the modal and harmonic natural frequencies of pre-twisted beam with different lengths depends on the frequency increment used in harmonic analysis. This increment affects on convergence between the frequency of modal and harmonic analysis. When the frequency increment decreases, the maximum error percentage decreases too, but the computation time increases.

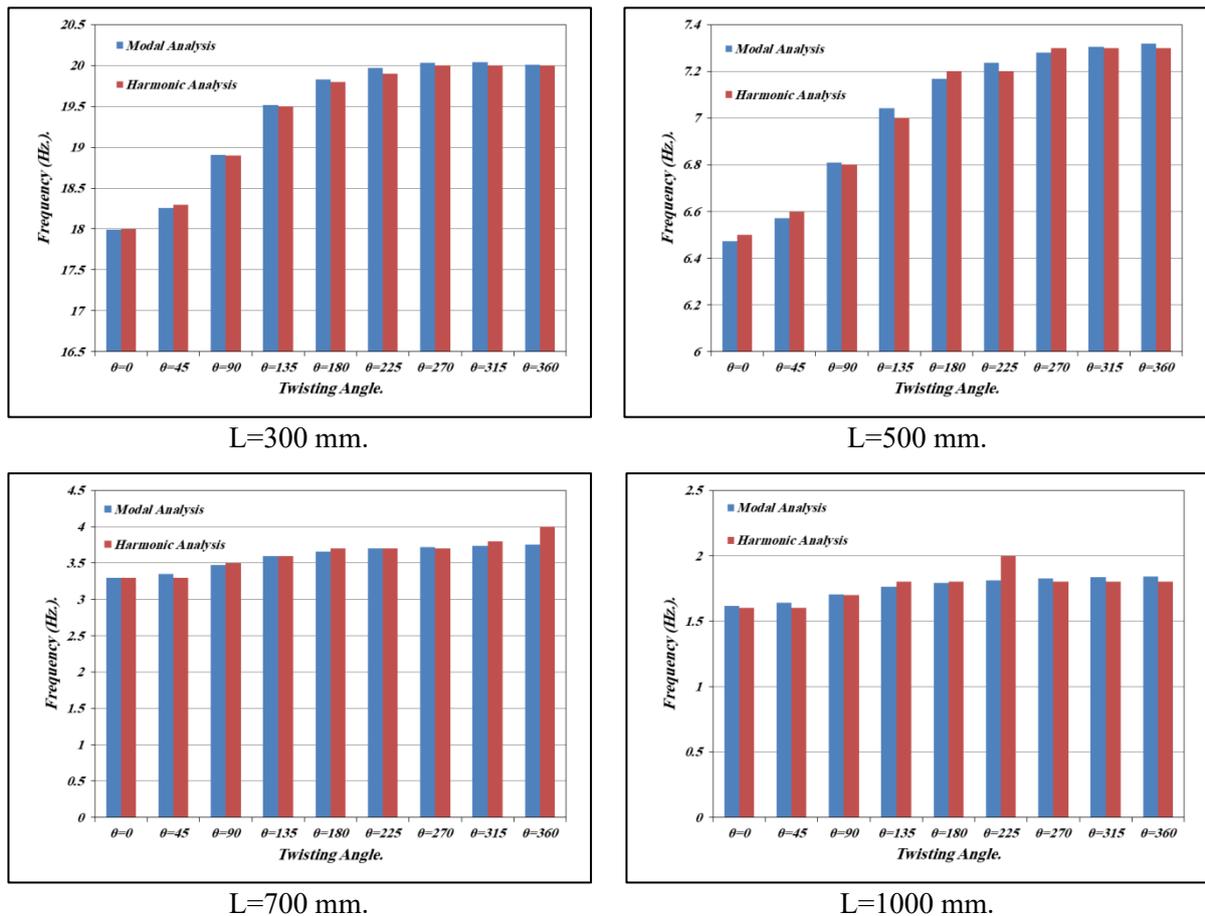


Fig.5. Comparison between the First Natural Frequency Values obtained by Modal and Harmonic Analyses for the Twisted Beam with Different lengths and Different Twisting angles.

Table 3. The Natural Frequency Values Obtained by Modal and Harmonic Analysis and the Error Percentage Between them for Different Beam Length and Different Twisting Angles.

(1) L=300mm									
Twisting Angle	$\theta=0$	$\theta=45$	$\theta=90$	$\theta=135$	$\theta=180$	$\theta=225$	$\theta=270$	$\theta=315$	$\theta=360$
Free Vibration	17.993	18.260	18.905	19.517	19.829	19.973	20.035	20.041	20.009
Harmonic Vibration	18.000	18.300	18.900	19.500	19.800	19.900	20.000	20.000	20.000
Error %	-0.039	-0.219	0.026	0.087	0.146	0.365	0.175	0.205	0.045
(2) L=500mm									
Twisting Angle	$\theta=0$	$\theta=45$	$\theta=90$	$\theta=135$	$\theta=180$	$\theta=225$	$\theta=270$	$\theta=315$	$\theta=360$
Free Vibration	6.473	6.571	6.810	7.041	7.167	7.236	7.279	7.304	7.317
Harmonic Vibration	6.500	6.600	6.800	7.000	7.200	7.200	7.300	7.300	7.300
Error %	-0.423	-0.435	0.153	0.579	-0.458	0.498	-0.291	0.051	0.235
(3) L=700mm									
Twisting Angle	$\theta=0$	$\theta=45$	$\theta=90$	$\theta=135$	$\theta=180$	$\theta=225$	$\theta=270$	$\theta=315$	$\theta=360$
Free Vibration	3.301	3.352	3.475	3.594	3.660	3.698	3.723	3.739	3.750
Harmonic Vibration	3.300	3.300	3.500	3.600	3.700	3.700	3.700	3.800	4.000
Error %	0.030	1.543	-0.728	-0.175	-1.087	-0.057	0.612	-1.629	-6.664
(4) L=1000mm									
Twisting Angle	$\theta=0$	$\theta=45$	$\theta=90$	$\theta=135$	$\theta=180$	$\theta=225$	$\theta=270$	$\theta=315$	$\theta=360$
Free Vibration	1.617	1.642	1.702	1.761	1.794	1.813	1.826	1.835	1.842
Harmonic Vibration	1.600	1.600	1.700	1.800	1.800	2.000	1.800	1.800	1.800
Error %	1.045	2.552	0.141	-2.209	-0.323	-10.296	1.445	1.929	2.275

3.2. Parameters Effect on Natural Frequency:

Several parameters affect on the natural frequencies of the cantilever pre-twisted beam. In this work, the effect of twisting angle on the first three natural frequencies of the cantilever pre-twisted beam is studied as shown in Fig.6. In the first mode (transverse vibration in x-y plane as illustrated in Fig.1, the frequency increases when the twisting angle rises till 180 degrees. Then, the frequency is constant when the twisting angle is greater than 180 degrees. To explain that, the following points must be considered:

1- The natural frequency of the beam is defined as:

$$\omega_n = (B_n L)^2 \sqrt{\frac{E \cdot I_{eq}}{\rho \cdot A \cdot L^4}} \quad (3)$$

2- The parameters affecting the natural frequency are (1) mass distribution (i.e., density when the cross-sectional area is constant) and stiffness of the beam (i.e., modulus of elasticity (E) *

equivalent second moment of area (I_{eq}). In the present case, the modulus of elasticity (E) is constant, but the second moment of area (I_{eq}) varies with respect to twisting angle, therefore, the stiffness of the beam varies with respect to twisting angle too.

3- The cross-sectional area of the pre-twisted beam at $x=0$ is (15 mm width and 10 mm height), and the second moment of area is ($I_{\theta=0} = \frac{width*(height)^3}{12} = \frac{15*(10)^3}{12}$) For any twisting angle.

While the cross-sectional area of the pre-twisted beam at $x=L$ is (10 mm width and 15 mm height) when the twisting angle is 90 degree, and the second moment of area is ($I_{\theta=90} = \frac{width*(height)^3}{12} = \frac{10*(15)^3}{12}$). It can be noted that ($I_{\theta=90} > I_{\theta=0}$) and this leads to a rise the equivalent second moment of area (I_{eq}).

4- From the above points, the rising the value of (I_{eq}) leads to increase the natural frequency as appears in [Fig.2](#).

In the second mode , the transverse vibration happens in ($x-z$) plane and that means:

The cross section area of the pre-twisted beam at $x=0$ is (10 mm width and 15 mm height) for any twisting angle, and the second moment of area is ($I_{\theta=00} = \frac{width*(height)^3}{12} = \frac{10*(15)^3}{12}$) .

And , cross section area of the pre-twisted beam at $x=L$ is (15 mm width and 10 mm height) when the twisting angle is 90 degree and the second moment of area is ($I_{\theta=90} = \frac{width*(height)^3}{12} = \frac{15*(10)^3}{12}$). Also, it can be noted that ($I_{\theta=0} > I_{\theta=90}$) and this causes a decrease in (I_{eq}) , therefore, the natural frequency decreases with respect to the natural frequency of a uniform beam with zero twisting angle.

According to the same procedure, the equivalent second moment of area increases due to rise in twisting angle and the maximum. the equivalent second moment of area is found when the twisting angle is 90 degree and then the equivalent second moment of area decreases in the third mode.

From [Figs.6](#) and [7](#), the natural frequency of the first three modes decreases with increase in the length of pre-twisted beam for any value of twisting angle.

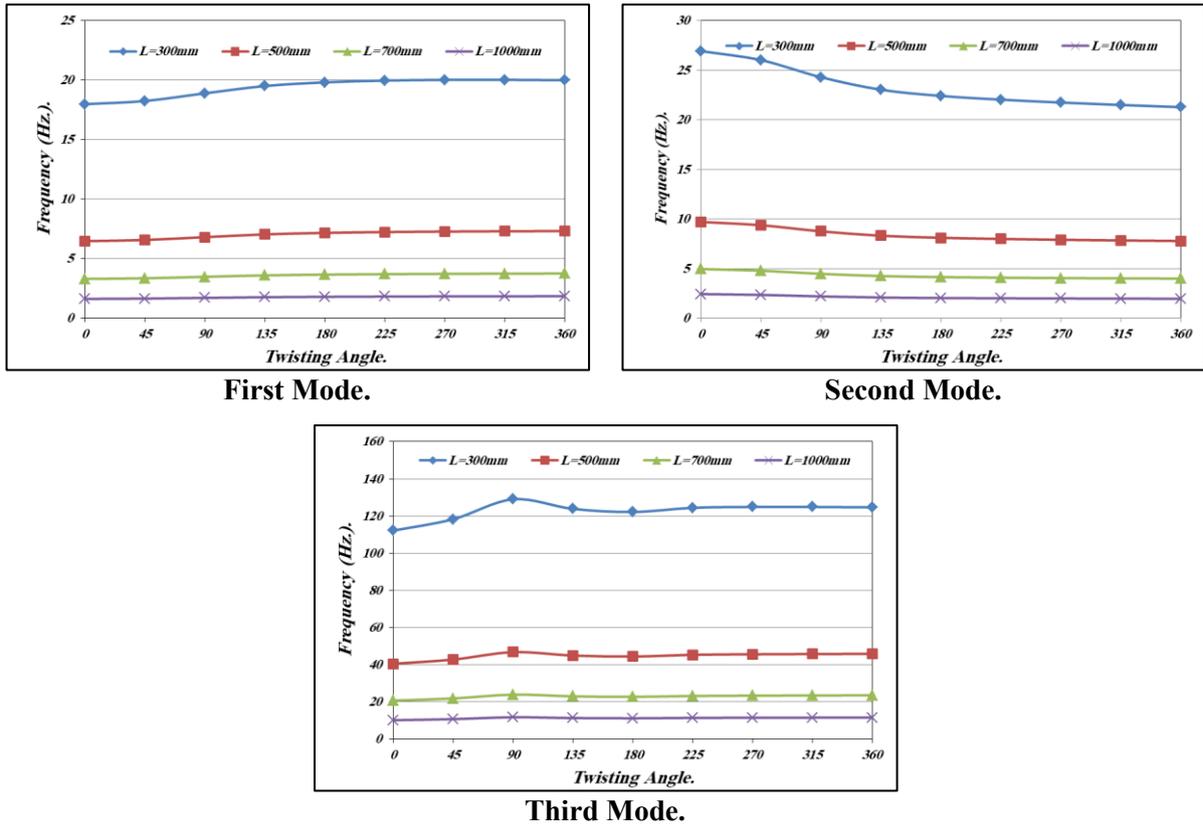


Fig.6. Variation of First , Second and Third Natural Frequencies Due to Increase of Twisting Angle with Different Length of Pre-Twisted Beam.

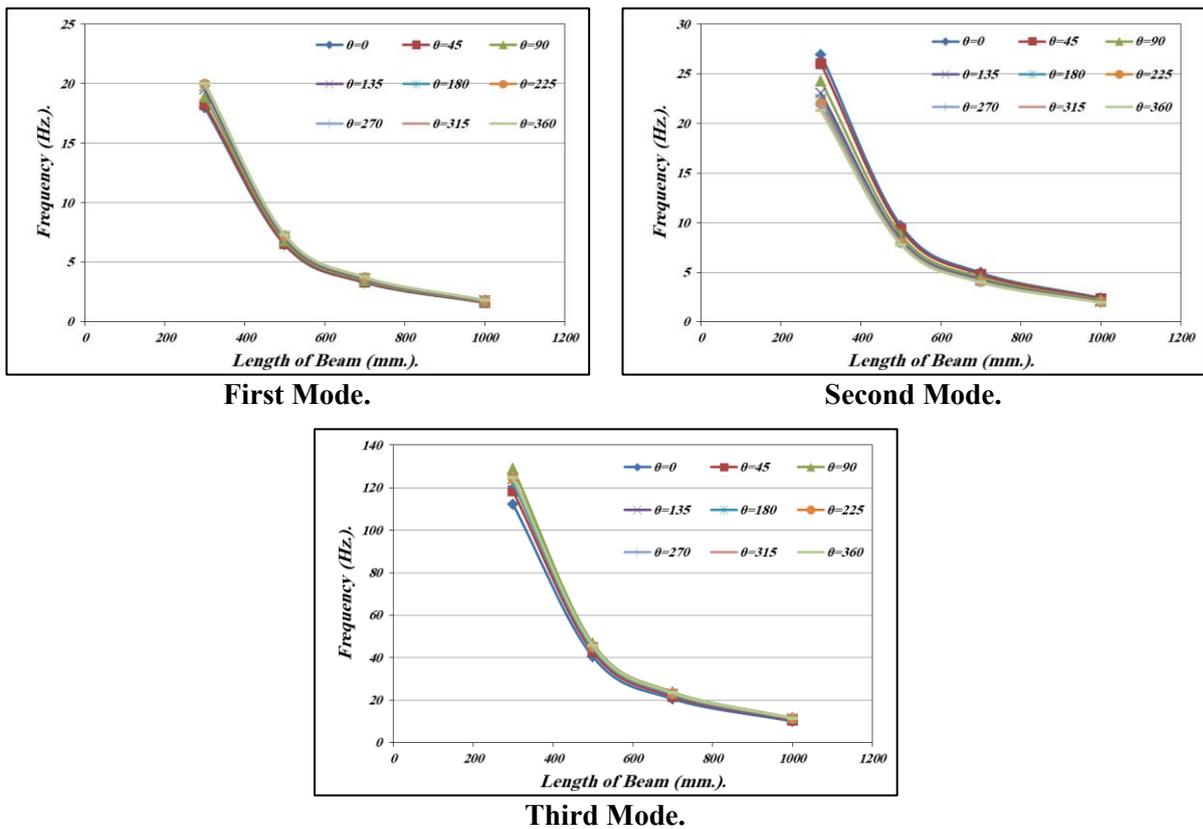
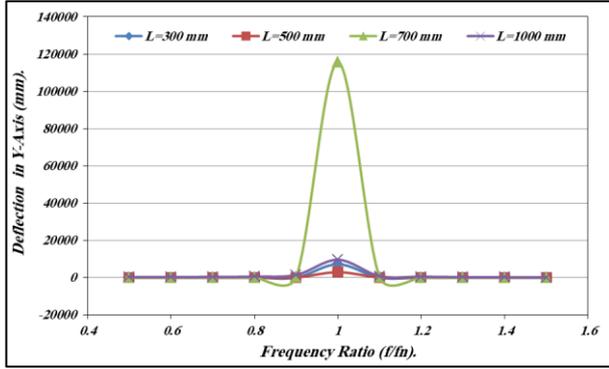
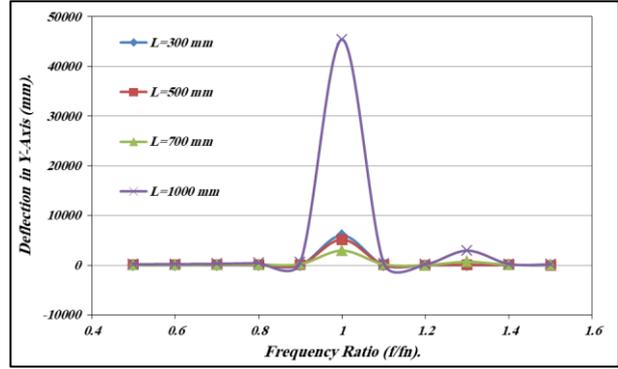


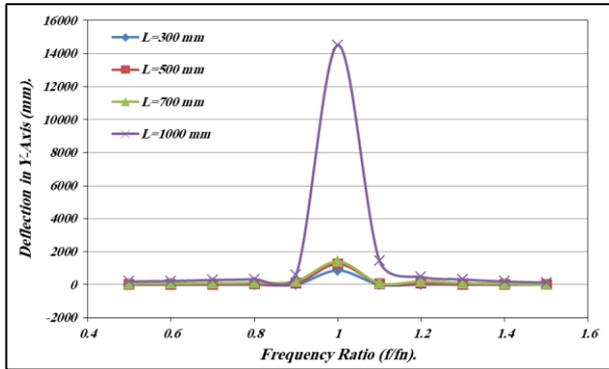
Fig.7. Variation of First, Second and Third Natural Frequencies Due to Increase Length of Pre-Twisted Beam with Different Twisting Angle.



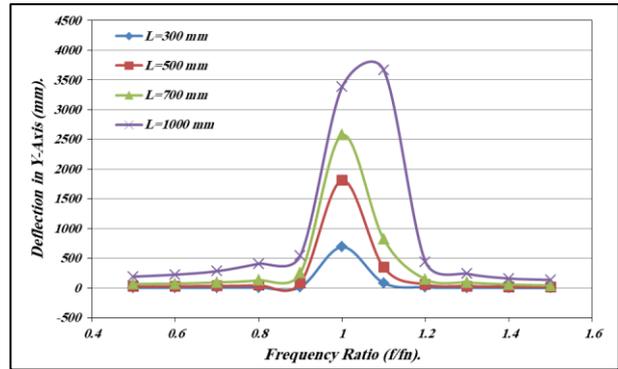
$\theta=0$ deg.



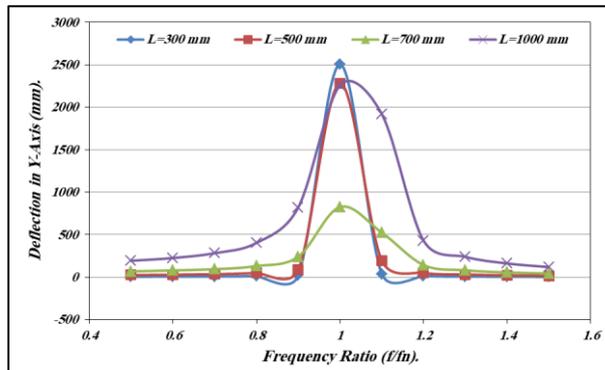
$\theta=90$ deg.



$\theta=180$ deg.

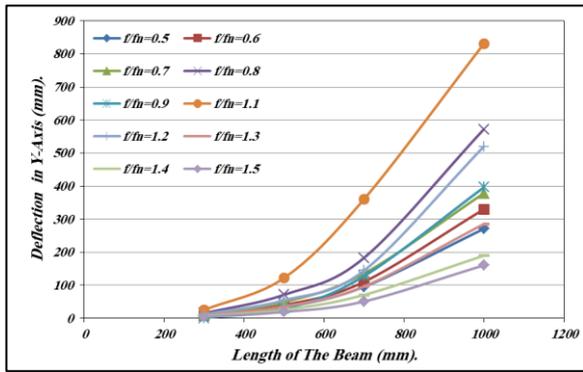


$\theta=270$ deg.

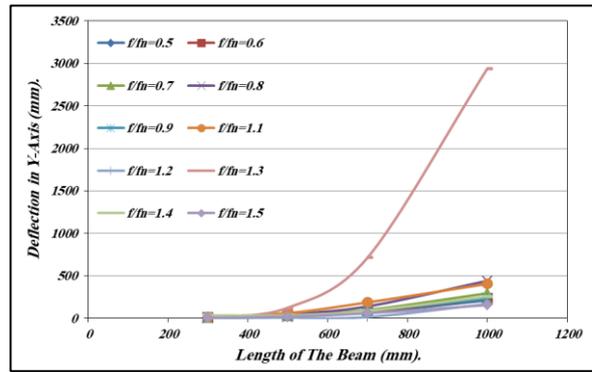


$\theta=360$ deg.

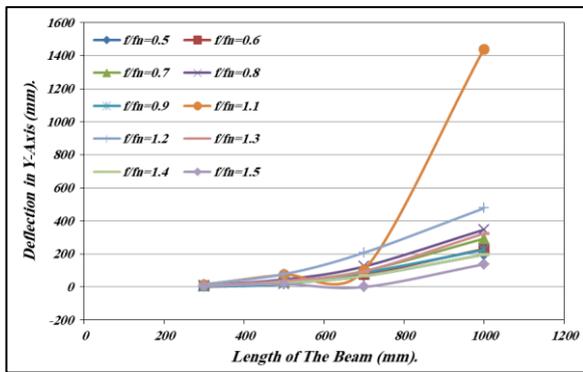
Fig. .9. Variation of Harmonic Deflection Due to Increase Frequency Ratio Near the First Natural Frequency of Beam with Different Length and Different Twisting Angle .



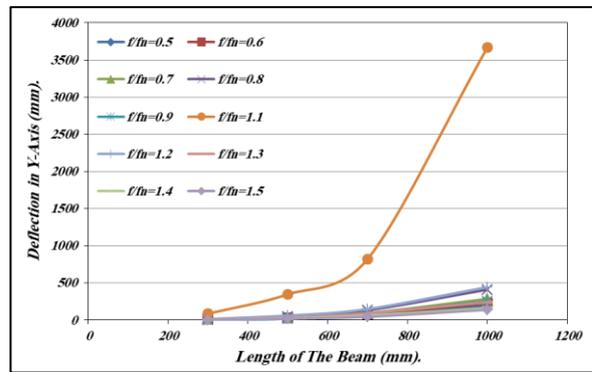
$\theta=0$ deg.



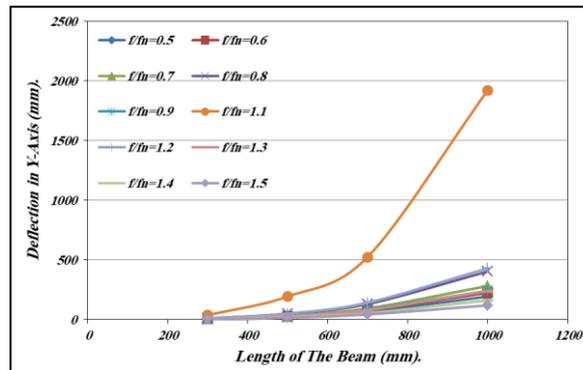
$\theta=90$ deg.



$\theta=180$ deg.



$\theta=270$ deg.



$\theta=360$ deg.

Fig.10.Variation of Harmonic Deflection Due to Increase Length of Beam with Different Frequency Ratio and Different Twisting Angle .

3.4. Stress of Pre-Twisted Beam Under Harmonic Vibration:

Fig.11 shows the stress of pre-twisted beam under harmonic load with different twisting angle and different length. As shown in Fig.11 the position and value of stress peak varies depending

on length of beam and twisting angle. In other side in order to compare between the pre-twisted beam different length and different twisting angle, the stress against the frequency ratio (f/f_n) is plotted as shown in Fig.12. It can be noted that the influence of the length of beam and twisting angle on the stress appears if the twisting angle is less than 180 degrees. Also, the stress rises linearly when the frequency ratio is smaller than 0.9 or greater than 1.2 as shown in Fig.13. Finally, the influence of twisting angle on the stress is illustrated in Fig.14. When the frequency ratio is less than 0.9 and greater than 1.3, the stress decreases with increasing twisting angle because the stiffness of the pre-twisted beam rises. While the stress fluctuates with increasing the twisting angle when the interval of frequency ratio is (0.9,1.3). This happens due to convergence or divergence of the frequency of harmonic load to the natural frequency of the pre-twisted beam.

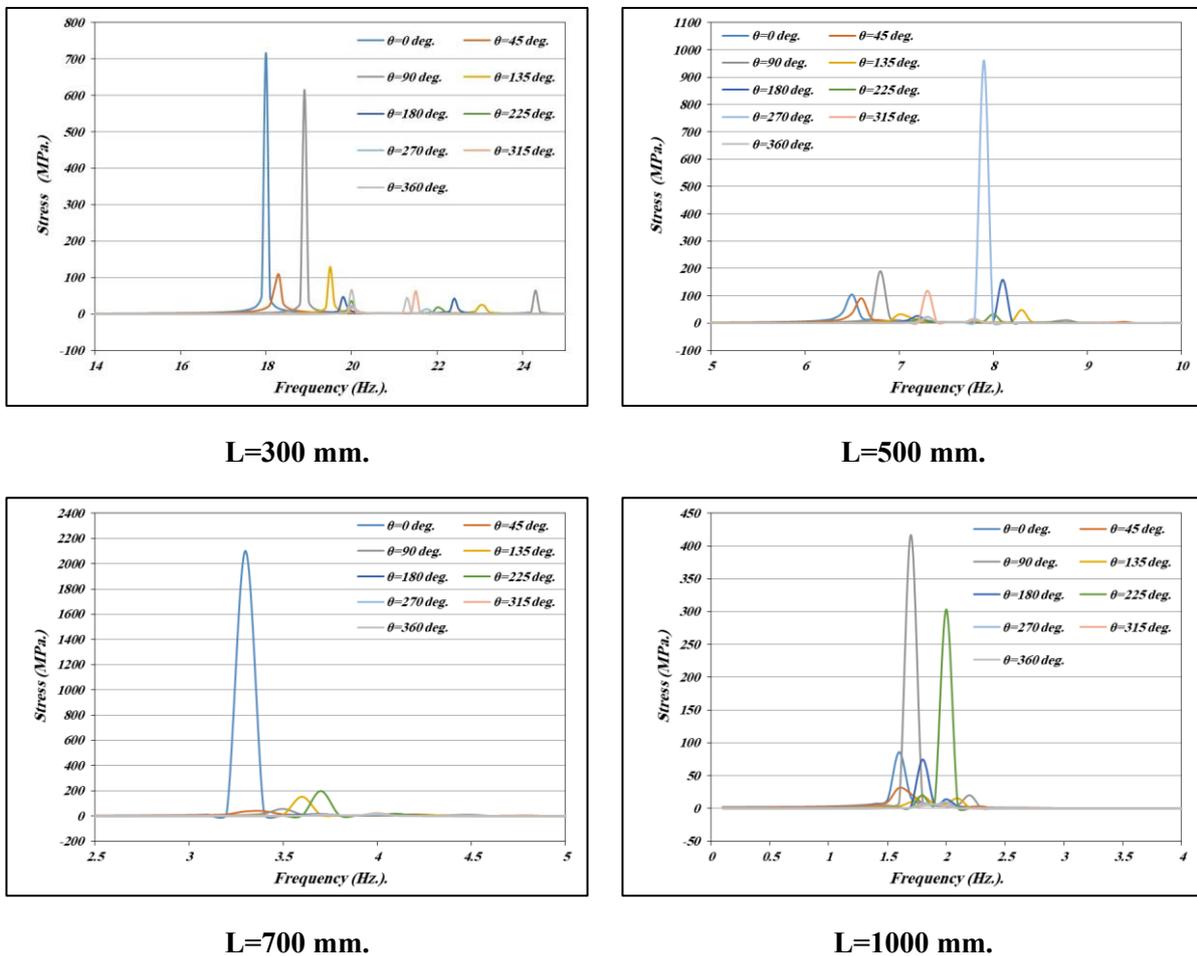
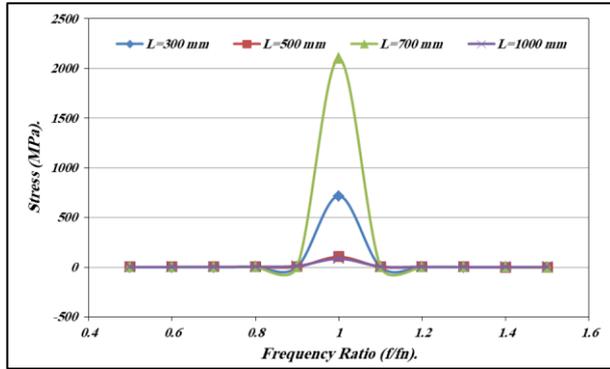
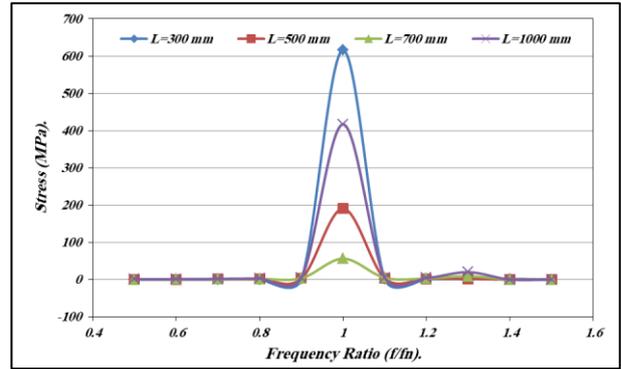


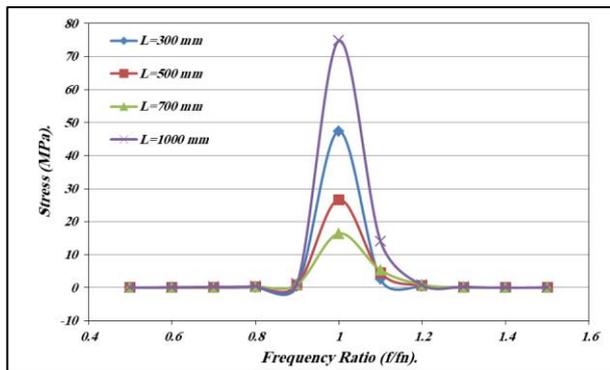
Fig.11. Stress of the Harmonic Vibration at the First Natural Frequency of Beam with Different Length and Different Twisting Angle.



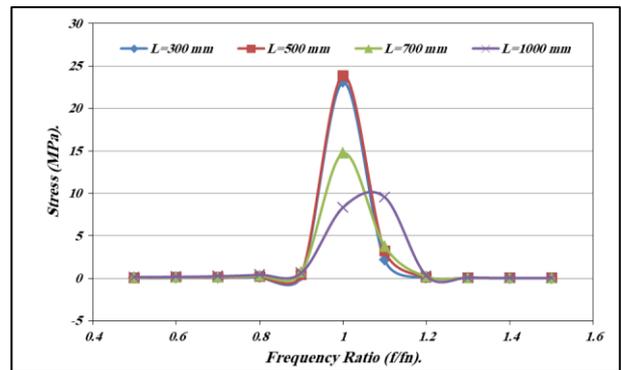
$\theta=0$ deg.



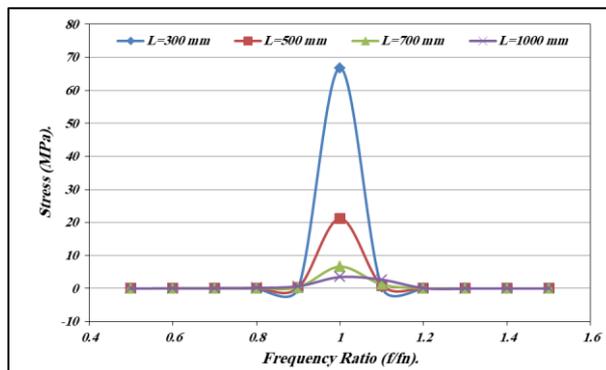
$\theta=90$ deg.



$\theta=180$ deg.

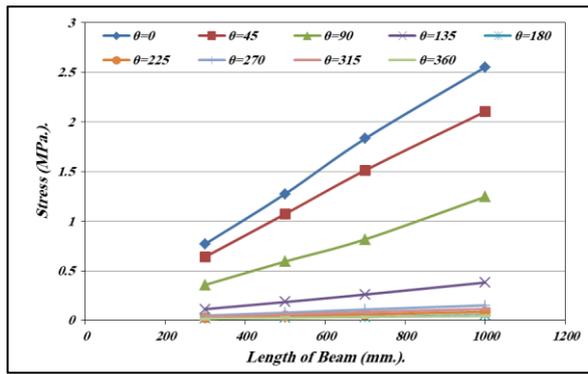


$\theta=270$ deg.

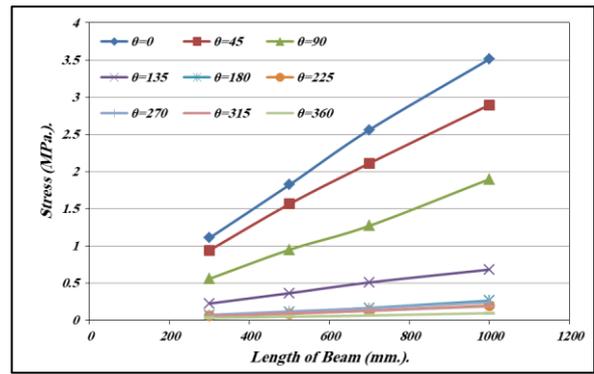


$\theta=360$ deg.

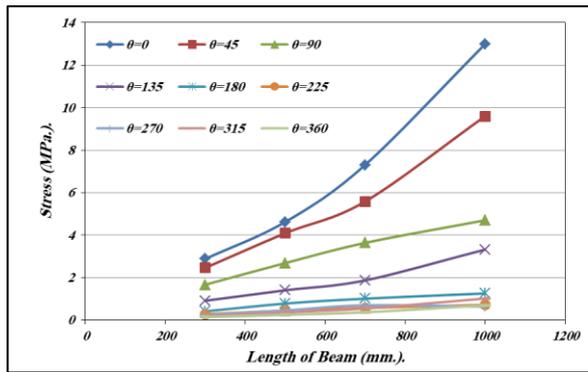
Fig. 12. Variation of Harmonic Stress Due to Increase Frequency Ratio Near the First Natural Frequency of Beam with Different Length and Different Twisting Angle.



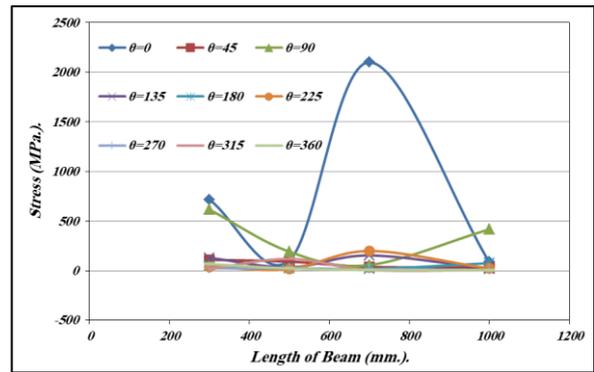
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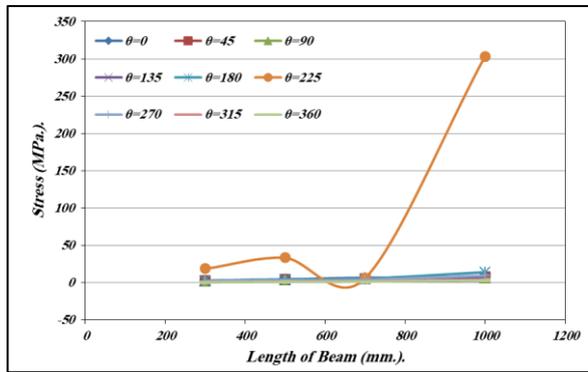
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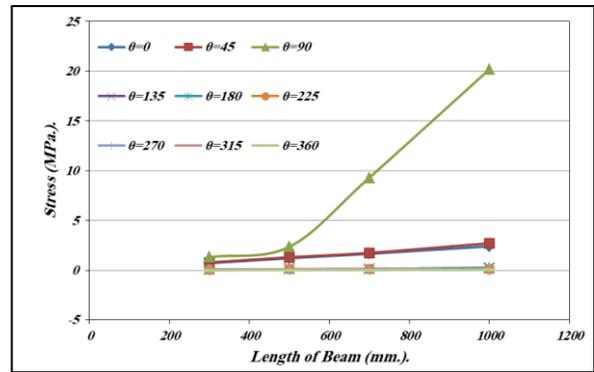
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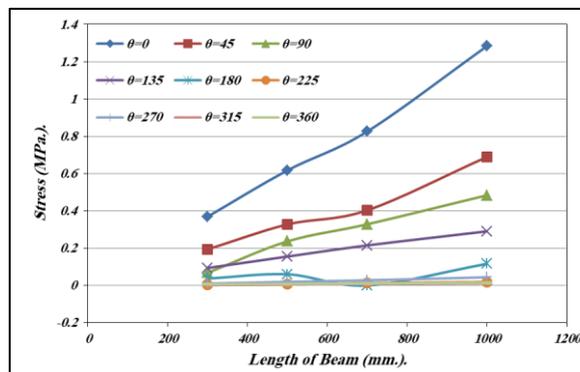
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f/fn=1.1.

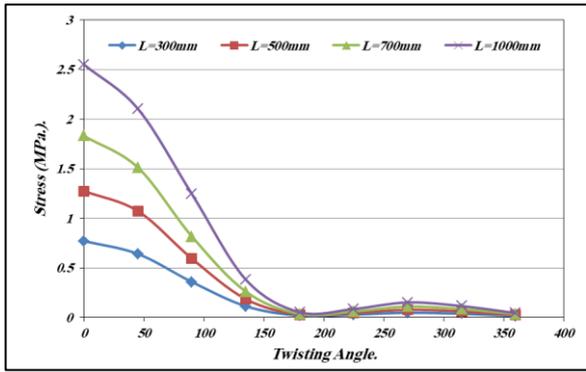


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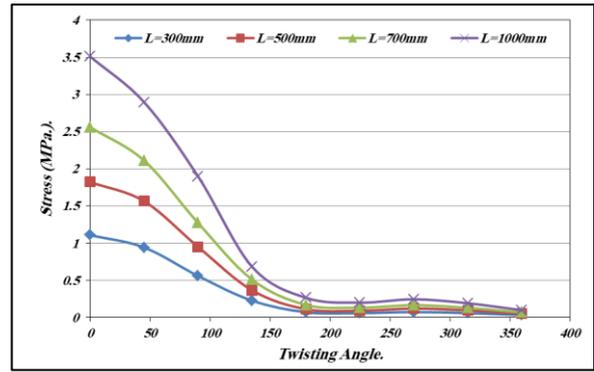


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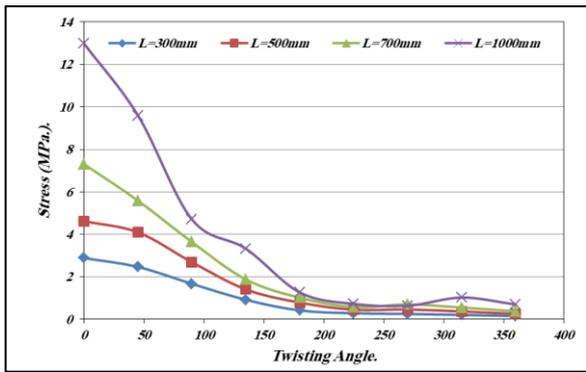
Fig.13.Variation of Harmonic Stress Due to Increase Length of Twisted Beam Near the First Natural Frequency with Different Frequency Ratio and Different Twisting Angle .



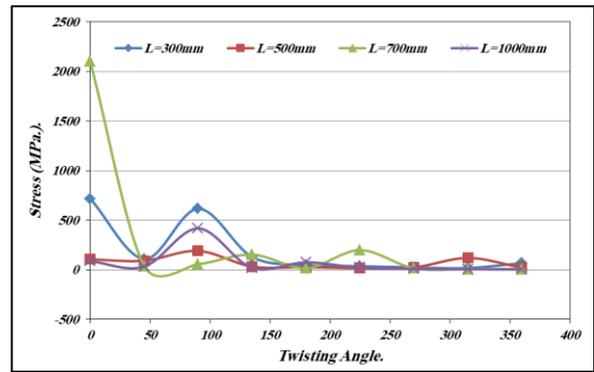
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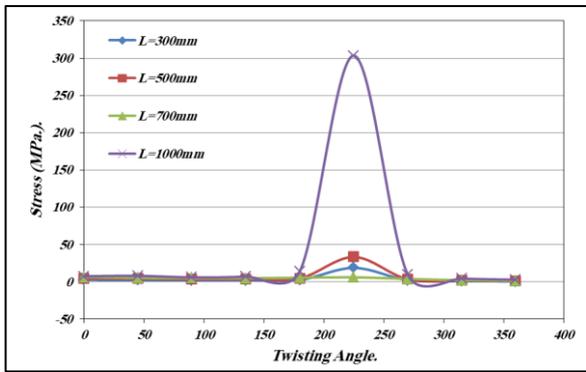
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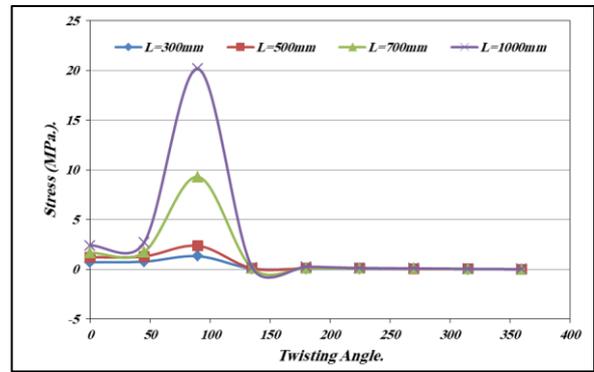
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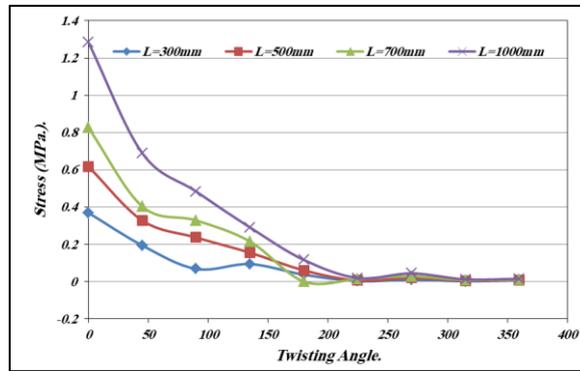
f/fn=1.0.



f/fn=1.1.



f/fn=1.3.



f/fn=1.5.

Fig.14.Variation of Harmonic Stress Due to Increase Twisting Angle Near the First Natural Frequency of Beam with Different Length and Different Frequency Ratio.

3.5. Damping Ratio (ξ):

One of the most important parameters in Harmonic analysis is the damping ratio of the beam, Fig.15 illustrates the changing of damping ratio as a result to change of twisting angle for various length of cantilever pre-twisted beam. When the length of pre-twisted beam increases, the damping ratio increases at any twisted angle. In other side, the damping ratio varies with increasing twisting angle at any length of pre-twisted beam. From the equation of damping ratio ($\xi = \frac{\omega_2 - \omega_1}{2\omega_n}$) (S. E. Sadiq et al., 2025) and procedure of calculation it, the natural frequency, profile of deflection response and maximum deflection of harmonic load are the main parameters effected on the damping ratio. Generally, the maximum damping ratio of pre-twisted beam with length (300, 500, 700 and 1000) mm finds if the twisting angle is (45, 135, 315 and 45) degree respectively.

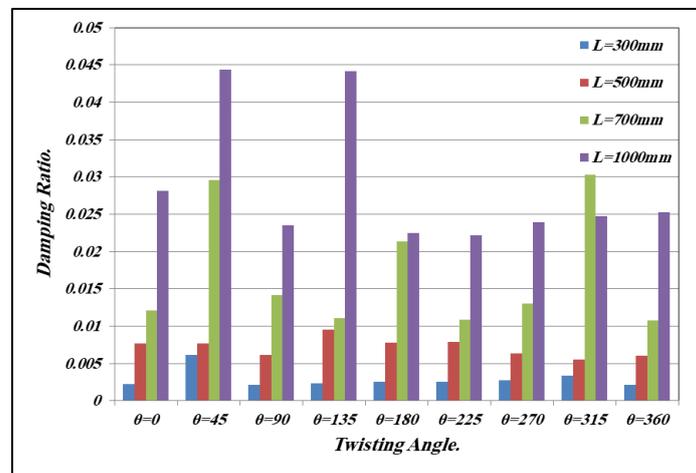


Fig.15.Variation of Damping Ratio Due to Length of Pre-twisted Beam and Twisting Angle Obtained by Harmonic Analysis.

4. CONCLUSION AND FUTURE WORK:

The first three natural frequencies of the pre-twist beam were calculated using the ANSYS software to study the effect of length of beam and twisting angle. Subsequently, the study examines how these variables affect the response of pre-twisted beams under harmonic forces, especially given the total deflection and equivalent stress. From the results that discussed in previous section, one can concluded the following points:

- The study shows a good agreement between the natural frequency values that are calculated by model and harmonic analysis. It can be observed that the natural frequency obtained by Harmonic analysis is smaller than that of experimental results. The max. error between them is (2.275 %) at (1000 mm) length and (360 o) twisting angle and the minimum error percentage between them is (-10.296 %) at (1000 mm) length and (225 o) twisting angle.
- In the first mode of transverse vibration in x-y plane, the natural frequency increases when

the twisting angle rises till 180 degrees. Then, the frequency is constant when the twisting angle is greater than 180 degrees. Also, in the second mode of transverse vibration in x-y plane (i.e. third mode), the maximum frequency is found when the twisting angle is 90 degree. While in the first mode of transverse vibration in x-z plane (i.e. second mode), the natural frequency reduces due to rise the twisting angle.

c- It can be noted that the influence of the length of beam on the deflection of harmonic response reduces if the twisting angle is greater than 180 degree. Also, the deflection of harmonic response rises when the frequency ratio is approximately unity.

d- The influence of the length of beam and twisting angle on the stress appears at the twisting angle more than 180 degree. Also, the stress rises linearly when the frequency ratio smaller than 0.9 or greater than 1.2. When the frequency ratio is less than 0.9 and greater than 1.3, the stress reduces when the twisting angle rises. While the stress troubles due to rise the twisting angle when the interval of frequency ratio is (0.9,1.3).

Finally, this work can extend to investigate the effect of twisting angle on the transient response using experimental and numerical simulation.

5. REFERENCES

Al-Ansari Luay S., Al-Hajjar Ali M. H., A. Husam Jawad, 2018. Calculating the natural frequency of cantilever tapered beam using classical Rayleigh, modified Rayleigh and finite element methods. *International Journal of Engineering & Technology* 7, 4866–4872.

Alansari, L., Jebur, M.A., Alansari, L.S., 2022. Simulation of Static Transverse Deflection for Non-Prismatic Beams. *NeuroQuantology* 20, 10728–10735. <https://doi.org/10.14704/nq.2022.20.10.NQ551038>

Alansari, L., Zainy, H.Z., Aljanabi, M., 2019. Calculating the natural frequency of hollow stepped cantilever beam.

Al-Raheem, S.K., Zainy, H.Z., Almawash, A.D., Alansari, L.S., Mohammed Ali, S.W., 2024. Static deflection of pre-twisted beam subjected to transverse load. *Results in Engineering* 21. <https://doi.org/10.1016/j.rineng.2024.101953>

Al-Saffar, A., Alansari, L., Alkhatat, A., Al-Saffar, A.A., Diwan, A.A., Al-Ansari, L.S., 2020. Experimental and Artificial Neural Network modeling of Natural Frequency of Stepped Cantilever shaft, *Journal of Mechanical Engineering Research and Developments*.

Baziyar Hamzehkhani, M., Zare, A., Gholami, M., Gorji Azandariani, M., 2024. Analysis of bending vibrations of a three-layered pre-twisted sandwich beam with an exact dynamic

stiffness matrix. *Composites Part C: Open Access* 14, 100473. <https://doi.org/10.1016/J.JCOMC.2024.100473>

Chen, G., Zeng, X., Liu, X., Rui, X., 2020. Transfer matrix method for the free and forced vibration analyses of multi-step Timoshenko beams coupled with rigid bodies on springs. *Appl Math Model* 87, 152–170. <https://doi.org/10.1016/j.apm.2020.05.023>

Diwan, A.A., Al-Ansari, L.S., Al-Saffar, A.A., Al-Anssari, Q.S., 2022. Experimental and theoretical investigation of static deflection and natural frequency of stepped cantilever beam. *Australian Journal of Mechanical Engineering* 20, 303–315. <https://doi.org/10.1080/14484846.2019.1704494>

Farghaly, S.H., El-Sayed, T.A., 2016. Exact free vibration of multi-step Timoshenko beam system with several attachments. *Mech Syst Signal Process* 72–73, 525–546. <https://doi.org/10.1016/j.ymsp.2015.11.025>

Hashim, W.M., Alansari, L.S., Aljanabi, M., Raheem, H.M., 2022. Investigating Static Deflection of Non-Prismatic Axially Functionally Graded Beam. *Material Design & Processing Communications* 2022, 1–12. <https://doi.org/10.1155/2022/7436024>

Hassib, Md.A., Islam, Md.R., Karim, Md.R., Hasan, Md.S., Hossain, K.R., 2024. RESEARCH PROGRESS OF 4D PRINTING TECHNOLOGY. *Kufa Journal of Engineering* 15, 107–133. <https://doi.org/10.30572/2018/KJE/150307>

Hoskoti, L., Misra, A., Sucheendran, M.M., 2021. Modal analysis of a rotating twisted and tapered Rayleigh beam. *Archive of Applied Mechanics* 91, 2535–2567. <https://doi.org/10.1007/s00419-021-01902-8>

Hossain, Md.I., Khan, Md.S., Khan, I.K., Hossain, K.R., He, Y., Wang, X., 2024. TECHNOLOGY OF ADDITIVE MANUFACTURING: A COMPREHENSIVE REVIEW. *Kufa Journal of Engineering* 15, 108–146. <https://doi.org/10.30572/2018/kje/150108>

Hu, W., Xu, M., Zhang, F., Xiao, C., Deng, Z., 2022. Dynamic analysis on flexible hub-beam with step-variable cross-section. *Mech Syst Signal Process* 180, 109423. <https://doi.org/10.1016/j.ymsp.2022.109423>

Hu, Y., Zhao, Y., Wang, N., Chen, X., 2020. Dynamic analysis of varying speed rotating pretwisted structures using refined beam theories. *Int J Solids Struct* 185–186, 292–310. <https://doi.org/10.1016/J.IJSOLSTR.2019.08.008>

- Khakalo, S., Niiranen, J., 2023. Structural buckling analysis of pre-twisted strips. *Eng Struct* 295, 116787. <https://doi.org/10.1016/J.ENGSTRUCT.2023.116787>
- Khalsan Al-Raheem, S., Alansari, L., n.d. Investigation of Static Deflection in Internal Stepped Cantilever Beam.
- Majid Jasim, Z., Jawad Abdulsamad, H., 2025. Investigation of Free Vibration Behavior for Composite Sandwich Beams with a Composite Honeycomb Core. *Kufa Journal of Engineering* 16, 177–199. <https://doi.org/10.30572/2018/KJE/160111>
- Migliaccio, G., Ruta, G., 2021. The influence of an initial twisting on tapered beams undergoing large displacements. *Meccanica* 56, 1831–1845. <https://doi.org/10.1007/s11012-021-01334-2>
- Ramaswamy, M., Stolarska, M.A., Stolarski, H.K., 2023. A modified buckling analysis of slender pretwisted bars. *Int J Solids Struct* 285, 112537. <https://doi.org/10.1016/J.IJSOLSTR.2023.112537>
- Sadiq, E.S., Zuhair Zainy, H., Mohammed Muneer, R., S. Al-Ansari, L., 2025. FREE VIBRATION ANALYSIS IN INNOVATIVE 3D PRINTING SANDWICH PANELS FOR AIRCRAFT STRUCTURE. *Kufa Journal of Engineering* 16, 265–282. <https://doi.org/10.30572/2018/KJE/160116>
- Sadiq, S.E., Abada, H.H., Al-Baidhani, H., Flayyih, M.A., Bakhy, S.H., Kazimierczuk, M.K., Jweeg, M.J., 2025. Theoretical Investigation of Forced Vibration of an Aircraft Sandwich Panel Structure Under Transient Load. *Mathematics* 13, 914. <https://doi.org/10.3390/math13060914>
- Sadiq, S.E., Bakhy, S.H., Jweeg, M.J., 2021. OPTIMUM VIBRATION CHARACTERISTICS FOR HONEY COMB SANDWICH PANEL USED IN AIRCRAFT STRUCTURE, *Journal of Engineering Science and Technology*.
- Sadiq, S.E., Jweeg, M.J., Bakhy, S.H., 2020. The Effects of Honeycomb Parameters on Transient Response of an Aircraft Sandwich Panel Structure, in: *IOP Conference Series: Materials Science and Engineering*. IOP Publishing Ltd. <https://doi.org/10.1088/1757-899X/928/2/022126>
- Shukur, Z.M., Hamad, R.F., Ali, Y.K., Al-Ansari, L.S., Al-Karaishi, M.H., 2023. Investigating the Effect of Applying Uniform Distributed Load on the Deflection of Simply Supported Axial - Functionally Graded Beam. *Journal of Aerospace Technology and Management* 15. <https://doi.org/10.1590/jatm.v15.1315>

Shukur, Z.M., Neamah, R.A., Abdulsamad, H.J., Al-Ansari, L.S., Wittayapiyanon, S., 2024. CALCULATING THE NATURAL FREQUENCY OF PRE-TWISTED BEAM. *Journal of Engineering and Sustainable Development* 28, 1–16. <https://doi.org/10.31272/jeasd.28.1.1>

Tang, J., Wang, G., Wu, G., Miao, Y., Han, C., Rui, X., 2022. Research on vibration characteristics of aircraft based on MSTMM, in: *1st International Conference on Mechanical System Dynamics (ICMSD 2022)*. Institution of Engineering and Technology, pp. 526–531. <https://doi.org/10.1049/icp.2022.1803>

Wadi, K.J., Yadeem, J.M., Mustafa khazaal, S., Al-Ansari, L.S., Abdulsamad, H.J., 2022. Static deflection calculation for axially FG cantilever beam under uniformly distributed and transverse tip loads. *Results in Engineering* 14, 100395. <https://doi.org/10.1016/j.rineng.2022.100395>

Zeng, J., Zhao, C., Ma, H., Wen, B., 2020. Dynamic modeling and coupling characteristics of rotating inclined beams with twisted-shape sections. *Frontiers of Mechanical Engineering* 15, 374–389. <https://doi.org/10.1007/s11465-019-0580-8>