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STATISTICAL EVALUATION OF THE FLEXURAL MOMENT CAPACITY OF BEAMS REINFORCED WITH FRP AND HYBRID FRP-STEEL REINFORCEMENT

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ABSTRACT

Fiber reinforced polymer (FRP) bars are more commonly used as alternatives to conventional steel reinforcement because of their non-corrosive characteristics, especially in aggressive environments. Therefore, a large number of researchers have concentrated on studies involving the mechanical behavior of the flexural strength of concrete beams FSCB reinforced by FRP with regard to using FRP bars. In this paper, the flexural moment capacity of concrete beams reinforced with FRP bars and a combination of FRP and steel reinforcement are investigated. The authors carry out a statistical analysis using a database of 144 beam specimens, including 72 reinforced with FRP bars and 72 with hybrid FRP-steel reinforcement. The experimental flexural strength results were compared to theoretically predicted values obtained according to ACI 440.1R-2015, CSA S807-2012, and a hybrid reinforcement equation, in which the three models were analyzed with respect to their prediction accuracy and reliability. Mean, coefficient of variation (COV), standard deviation (SD), Mean Absolute Percentage Error (MAPE), Root-Mean-Square Error (RMSE), Pearson's r, and Nash -Sutcliffe efficiency index are some statistical metrics which were used to better compare the predictive performance of all the equations. The COV of the developed hybrid equations was 15.27 % and SD was 0.16. Contrarily, model ACI 440.1R-2015 presented such COV of 17.5% and SD = 0.17, whereas model CSA S807-2012 showed a COV of 17.11% and an SD = 0.15. It led to a 12% improvement in the prediction of error compared with both ACI 440.1R-2015 and CSA S807-2012, suggesting better precision and uniformity in predicting the bending resistance of reinforced concrete beams. These results highlight the accuracy of the predictive hybrid model, especially in the estimation of the most important parameters such as elastic modulus, FRP, concrete compressive strength, and reinforcement ratios.

KEYWORDS

Flexural Strength, FRP, Hybrid Reinforcement, Design Equation, Statistical Analysis.

1. INTRODUCTION

Corrosion of steel reinforcement is one of the common problems faced by the construction industry, which results in structural failure such as cracks, spalling and debonding from concrete (Kadhima and Zinkaaha, 2024; Zinkaah et al., 2019; Alhawat and Ashour, 2019; and Al-Turaihi and Al-Katib,.24). That said, there is a growing need for alternatives to traditional steel reinforcement that are more durable. Laminate (nonmetallic) composite reinforcement has recently been widely applied in new construction, and especially in special-purpose buildings and structures (Karpiuk et al., 2020; Yoon et al., 2011; Kadhim and Hassan, 2024; Khalid et al., 2025). reinforcement ratios. New materials used as reinforcement in concrete have been proposed due to the corrosion of steel caused by extreme environments (Abed et al., 2021;Karpiuk et al., 2020 and AlShadidi et al., 2016). FRPs, which consist of high-strength fibers and a binder, are an effective structural applications solution (Abdullah and Hassan, 2021; Kara & Ashour, 2012). FRP bars have been a challenging material for the use in reinforced concrete structures, especially applied on harsh or corrosive environments, when came to action and started to be used only since 1980th (Abed et al., 2021). The most often utilized categories of FRP include polymers reinforced with aramid fibers (AFRP), glass fibers (GFRP), basalt fibers (BFRP), and carbon fibers (CFRP). Additionally, to their resistance to corrosion, FRPs are non-magnetic, have low densities, are inoffensive, environmentally friendly, and lightweight. These properties make FRP practical and economically viable (Elmessalami et al., 2019; Habeeb and Ashour, 2008; Ftnan and Makki, 2022). However, despite these advantages, a sudden and brittle failure mode can result from FRP bars having a low elastic modulus and exhibiting linear elastic behavior up to failure. To satisfy serviceability requirements, concrete elements reinforced with FRP bars should be designed with over-reinforcement (Sam and Swamy, 2005). FRP exhibits greater deflections and larger crack widths due to its lower elastic modulus than steel. As a result, their design is primarily based on serviceability limit states (Cai et al., 2017). Hybrid reinforced members provide an important solution. The controlled cover thickness can be improved to become a good corrosion-resistance by placing the FRP bar closer to the surface and adding steel bars in the inner layer, which are subjected to tensile stress. This is intended to control bending response and limiting crack width. A number of guidelines, such as CSA S807-2012 (CAN/CSA-S806-12, 2012); ACI 440.1R06 (ACI-440. 1R-06, 2006); JSCE 1997 (JSCE, 1997), as well as ACI 440.1R-2015 (ACI-440. 1R-15, 2015) that are the basic concepts for application of FRP bars. Sudden and catastrophic member failure results due to the rupture of the FRP reinforcement. However, there is typically a noticeable warning before failure occurs, characterized by extensive cracking and significant deflection caused by the FRP reinforcement's considerable elastic elongation before rupture (ACI-440.1R-15, 2015). According to ACI 440.1R-2015 (ACI-440.1R-15, 2015); (JSCE, 1997); ACI 440.1R-06 (ACI-440.1R-06, 2006), and CSA S807-2012 (CAN/CSA-S806-12, 2012), compression-controlled behavior is slightly preferred for flexural members reinforced with FRP bars. The codes JSCE 1997 ((JSCE), 1997); ACI 440.1R-06 (ACI-440.1R-06, 2006); ACI 440.1R-2015 (ACI-440.1R-15, 2015), and CSA S807-2012 (CAN/CSA-S806-12, 2012) did not include design methods for hybrid reinforced beams; therefore, the equations proposed by the researchers relied upon (El Refai et al., 2015).

The behavior of FRP-reinforced concrete beams under flexural loading has been the focus of extensive research (Chaallal and Benmokrane, 1996; Douglas, 2012; Nanni et al., 2014; Adam et al., 2015; Lapko and Urbański, 2015; Pawłowski and Szumigała, 2015; Tomlinson and Fam, 2015; Benmokrane and Masmoudi, 1996; Dong et al., 2019; Grace et al., 1999; Gravina and Smith, 2008). These explorations have improved design procedures and guidelines (Kassem et al., 2011), particularly concerning serviceability (Theriault and Benmokrane, 1998; Lau and Pam, 2010; Kabashi et al., 2020; Nguyen et al., 2020; Vu and Phan, 2021) They investigated the use of GFRP in reinforcing beams, while (Abed et al., 2021; Aiello and Ombres, 2002; Leung and Balendran, 2003 and Bischoff, 2005) used BFRP to study flexural behavior (Kassem et al., 2011) An experimental study investigating the serviceability and FSCB reinforced with different FRP bars, including carbon, glass, and aramid FRP, was presented. Some studies explore using hybrid reinforcement, combining FRP and steel bars, to potentially enhance structural performance (Aiello and Ombres, 2002; Leung and Balendran, 2003; Bischoff, 2005; Sam and Swamy, 2005; Bischoff and Scanlon, 2007; Qu et al., 2009; Lau and Pam, 2010; Yoon et al., 2011; El Refai et al., 2015; Ge et al., 2015; Kara et al., 2015; Pang et al., 2016; Xue et al., 2016; Qin et al., 2017; Araba and Ashour, 2018; Sun et al., 2019; Kabashi et al., 2020; Lu et al., 2020; Nguyen et al., 2020; Yang et al., 2020; Vu and Phan, 2021). The researchers studied normal and high-strength concrete, employing different loading methods such as one-point and two-point loads.

This paper provides important statistical considerations for the flexural behaviour of beams dominated by FRP bar reinforcement. It also investigates on the influence of using hybrid FRP-steel bars as flexural reinforcements through an extensive statistical study, and tries to develop a deep comprehension of how distinct reinforcing materials interact and work together in the structural adequacy/performance and robustness against different loading conditions of concrete beams. The analysis comprises statistical models ACI 440.1r 2015 (ACI-440. 1R-15, 2015); CSA S807-2012 (CAN/CSA-S806-12, 2012), and the hybrid reinforcement expression for predicting the flexural moment capacity of FRC members was reviewed and compared for appropriateness, accuracy, and safety margin. These models were subjected to statistical tests in order to validate their accuracy and the error of approximation. Key parameters affecting the flexural capacity of the beams, including compressive strength, elastic modulus, and FRP reinforcement ratio, were examined.

2. EXPERIMENTAL DATABASE

To investigate the flexural performance of FRP bar reinforced beam and hybrid FPR steel reinforcement under different loads and to assess the accuracy of the predicted equations, 144 specimens (72 beams using only FRP reinforcing bars and 72 with hybrid reinforcement) were used in bending mode. (Kassem et al., 2011; Theriault and Benmokrane, 1998; Lau and Pam, 2010; Kabashi et al., 2020; Nguyen et al., 2020; Vu and Phan, 2021; Abed et al., 2021; Aiello and Ombres, 2002; Leung and Balendran, 2003 and Bischoff, 2005) The tested beams were reinforced using GFRP, BFRP, and CFRP bars. At the same time, specimens in references (Aiello and Ombres, 2002; Leung and Balendran, 2003; Bischoff, 2005; Sam and Swamy, 2005; Bischoff and Scanlon, 2007; Qu et al., 2009; Lau and Pam, 2010; Yoon et al., 2011; El Refai et al., 2015; Ge et al., 2015; Kara et al., 2015; Pang et al., 2016; Xue et al., 2016; Qin et al., 2017; Araba and Ashour, 2018; Sun et al., 2019; Kabashi et al., 2020; Lu et al., 2020; Nguyen et al., 2020;Yang et al., 2020 and Vu and Phan, 2021) have been reinforced with hybrid FRP–steel bars as illustrated in Fig.1 .Table 1 presents the detailing of these beams. The width of the specimen B ranges between 130 mm and 280 mm, H denotes the overall height of the specimen, ranging from 180 mm to 380 mm, L represents the total length, and d corresponds to the effective depth. The symbol f'_c refers to the cylindrical compressive strength of concrete, and f_{fu} indicates the ultimate tensile strength of the FRP reinforcement. The modulus of elasticity of the FRP bars, denoted as ε_f, ranges from 36 GPa for BFRP to 155 GPa for CFRP. A_f represents the area of FRP and ($\rho_f = A_f/d \; x \; b$) is the ratio of longitudinal FRP bars, ranges between 0.21 – 2.285 %. A_s , ρ_s are area and ratio ($\rho_s = A_s/d \; x \; b$) of steel bars. f_y is the modulus of elasticity of steel bars, ranging from 324.4 MPa to 580 MPa.

Table 1. Flexural Design Parameters of Reinforced Beams Used in this Study.

Number of beams	Table database									
	FRP					Hybrid FRP-steel				
	Min	Max	M	SD	COV %	Min	Max	M	SD	COV %
B (mm)	130	280	173	41	23.7	150	280	181.50	34	18.73
H (mm)	180	380	252	54.3	21.52	170	400	270	55.54	20.51
L (mm)	1800	4600	2602	772.8	29,7	1800	4600	2665	712.43	26.7
d (mm)	160	360	226	55.6	24.53	150	380	250.8	55.5	22.1
f'_c *(MPa)*	28.1	116.88	46.7	19.2	41.17	28.1	75.9	41.8	9.64	23

Number of beams	Table database									
	FRP					Hybrid FRP-steel				
	Min	Max	M	SD	COV %	Min	Max	M	SD	COV %
f_y (MPa)	-	-	-	-	-	324.4	580	465.64	71.66	15.4
f_{fu} (MPa)	603	2068	1185.2	446.3	37.65	603	2068	1185.2	446.3	2.65
ε_f (GPa)	36	155	65.3	38.1	58.3	40.1	146.2	49.35	15.52	31.46
$A_s (mm^2)$	-	-	-	-	-	56.54	1205.76	250.2	200	79.71
$A_f (mm^2)$	56.55	1963.5	374.92	338	90.16	25.12	981.7	227.7	152.6	67
A_s / A_f	-	-	-	-	-	0.34	12	1.5	1.6	106.5
ρ_s %	-	-	-	-	-	0.085	2.91	0.55	0.37	66.33
ρ_f %	0.21	2.285	0.928	0.591	63.72	0.033	1.5	0.53	0.33	62.88

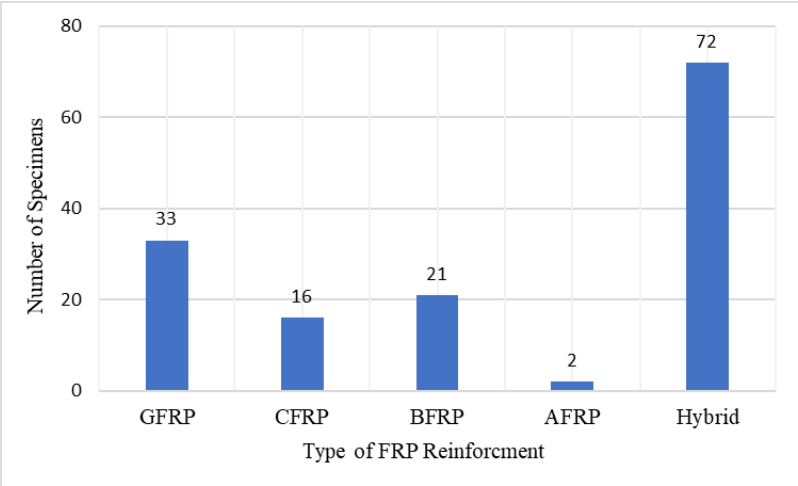


Fig.1. Distribution of the 144 Beam Specimens By FRP Reinforcement Type

3. ASSESSMENT OF THE FLEXURAL STRENGTH OF CONCRETE BEAMS

FRP is a practical substitute for traditional steel reinforcement because of its excellent corrosion resistance properties. Its use in structural buildings has become widespread. Some codes, such as the JSCE 1997 (JSCE, 1997) code, have permitted their use, while ACI 440.1R-2006 (ACI-440.1R-06, 2006), ACI 440.1R-2015 (ACI-440.1R-15, 2015), and CSA S807-2012 (CAN/CSA-S806-12, 2012) suggested equations to estimate flexural strength. According to ACI 440.1R-2015 (ACI-440.1R-15, 2015), the failure mode is influenced by the ratio of FRP reinforcement, and there are two modes of failure: compression and tension. The estimated failure modes can be identified by analyzing the reinforcement ratio ρ_f to balance ratio ρ_{fb} (Abdullah, 2020); the balance ratio can be calculated from Eq.1. (ACI-440.1R-15, 2015), where β_1 is the strength reduction factor calculated by Eq. 2.

$$\rho_{fb} = \alpha_1\, \beta_1 \left(\frac{f`_c}{f_{fu}}\right)\left(\frac{E_f\, x\, \varepsilon_{cu}}{E_f \varepsilon_{cu} + f_{fu}}\right) \quad (1)$$

$$\beta_1 = 0.85 - 0.05\left(\frac{f`_c - 28}{7}\right) \geq\ 0.65 \quad (2)$$

Where, α_1 is the depth of equivalent rectangular stress block, ε_{cu} is the ultimate strain of concrete equal to 0.003.

The failure of concrete under compression occurs when the ratio of FRP is less than a balanced ratio $\rho_f < \rho_{fb}$, and the flexural moment can be estimated by Eq.3. In contrast, when the FRP ratio is overbalanced, tension failure will occur, and the bending moment capacity can be estimated by Eq.4. C represents the distance from the outermost compression fiber to the neutral axis, as described in Eq.5. f_f denotes the tensile strength of the FRP bars, which can be ascertained using Eq.6:

$$M_n = A_f \, x \, f_{fu} \, x \, (d - \frac{\beta_1 C}{2}) \quad (3)$$

$$M_n = \rho_f \, x \, f_f \, x \, b \, x \, d^2 \left(1 - 0.59 \, x \, \rho_f \, x \frac{f_f}{f`_c}\right) \quad (4)$$

$$C = \left(\frac{E_f \, x \, \varepsilon_{cu}}{E_f \varepsilon_{cu} + f_{fu}}\right) x \, d \quad (5)$$

$$f_f = (\sqrt{(\frac{1}{4}(E_F \, x \, \varepsilon_{cu})^2 + \frac{0.85 \, \beta_1 \, x \, f`_c}{\rho_f} \, x \, E_F \, x \, \varepsilon_{cu}} - 0.5 \, E_F \, \varepsilon_{cu} \le f_{fu} \quad (6)$$

For CSA S807-2012 (CAN/CSA-S806-12, 2012), the concrete flexural moment depends on the ratio of FRP reinforcement. The balance ratio can be computed from Eqs.1, 7, and 8. Where ε_{cu} is the maximum concrete strain is equal to 0.0035.

$$\alpha_1 = 0.85 - 0.0015 \, f`_c \quad (7)$$

$$\beta_1 = 0.97 - 0.0025 \, f`_c \quad (8)$$

When FRP bars achieve their ultimate tensile strength, $\rho_f < \rho_{fb}$, M_n can be calculated by Eq.9. Deposited if the FRP ratio is greater than the balanced ratio $\rho_f > \rho_{fb}$ the concrete will crash first before FRP reaches the ultimate tensile strength and the flexural strength estimate from Eq.10 and 11 (CAN/CSA-S806-12, 2012).

$$M_n = \rho_f \, f_{fu} \, b \, d^2 \left(1 - \frac{\rho_f \, f_{fu}}{2 \, \alpha_1 \, f`_c}\right) \quad (9)$$

$$M_n = \rho_f \, f_f \, b \, d^2 \left(1 - \frac{\rho_f \, f_f}{2 \, \alpha_1 \, f`_c}\right) \quad (10)$$

$$f_f = 0.5 \, E_f \, \varepsilon_{cu} \left[\left(1 + \frac{4 \, \alpha_1 \, \beta_1 \, f`_c}{\rho_f \, E_f \, \varepsilon_{cu}}\right)^{0.5} - 1\right] \quad (11)$$

For hybrid-RC beams, Numerous researchers have undertaken experiments to comprehend the behavior of hybrid reinforcement. Thresholds, and they have concluded that there are three different failure patterns. The balance ratio of steel and FRP $\rho_{s,b}$, $\rho_{f,b}$ that can be calculated by Eq.12 and Eq. 13 (Pang et al., 2016), the effective reinforcement for steel and FRP ($\rho_{eff,s}$ *and* $\rho_{eff,f}$) is estimated by Eq. 14, and Eq. 15.

$$\rho_{s,b} = 0.85 \; \beta_1 \left(\frac{f`_c}{f_y}\right) \left(\frac{E_s \varepsilon_{cu}}{E_s \varepsilon_{cu} + f_y}\right) \quad (12)$$

$$\rho_{f,b} = 0.85 \; \beta_1 \left(\frac{f`_c}{f_{fu}}\right) \left(\frac{E_f \varepsilon_{cu}}{E_f \varepsilon_{cu} + f_{fu}}\right) \quad (13)$$

$$\rho_{eff,s} = \rho_s + \frac{E_f}{E_s} \rho_f \quad (14)$$

$$\rho_{eff,f} = \rho_f + \frac{f_y}{f_{fu}} \rho_s \quad (15)$$

Compression failure will occur when the effective steel and FRP reinforcement ratio exceed the balance ratio, when ($\rho_{eff,s} > \rho_{s,b}$) and ($\rho_{eff,f} > \rho_{f,b}$). The flexural moment. In this case, is calculated using Eq.16. On the other hand, if the effective steel reinforcement ratio is less than the balance ratio ($\rho_{eff,s} < \rho_{s,b}$) and the FRP ratio exceeds the balance ratio ($\rho_{eff,f} > \rho_{f,b}$), the steel will reach to its yield limit before FRP ruptures, This failure is referred to an under-reinforced. The flexural moment is estimated by Eq. 16 and 17. In situation where of steel reinforcement ratio is below the critical ratio ($\rho_{eff,s} < \rho_{s,b}$), and FRP ratio is also less than the balance ratio ($\rho_{eff,f} < \rho_{f,b}$),tension failure will occur, and FRP bars will rupture first. the Bending moment (m_n) is calculated by Eq.18 and 19.

$$m_n = \left(A_S f_s + A_f f_f\right)\left(d - \frac{\beta_1 c}{2}\right) \quad (16)$$

$$f_f = \sqrt{\frac{\left(\frac{A_S f_y}{A_f} + E_f \varepsilon_{cu}\right)^2}{4} + E_f \varepsilon_{cu}\left(\frac{0.85\,\beta_1 f`_c}{\rho_f} - \frac{A_S f_y}{A_f}\right)} - 0.5\left(\frac{A_S f_y}{A_f} + E_f \varepsilon_{cu}\right) \leq f_{fu} \quad (17)$$

$$c_b = \left(\frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_{fu}}\right) \quad (18)$$

$$m_n = \left(A_S f_y + A_f f_{fu}\right)\left(d - \frac{\beta_1 c_b}{2}\right) \quad (19)$$

Where c_b is the distance from the extreme compression fiber to the neutral axis at the balanced strain condition.

4. EVALUATION OF THE PREDICTED FLEXURAL STRENGTH EQUATIONS

4.1. Moment Prediction Equation

A total of 144 specimens were collected, consisting of 72 samples reinforced with FRP only and 72 samples reinforced with a hybrid FRP-steel reinforcement. These specimens were compiled from the 21 studies referenced previously to assess the theoretical moment anticipated by the equation and to compare it with the experimental findings. The main objective was to assess the accuracy of the equation in predicting the flexural moment capacity. The variables considered in this evaluation included the specimens' width, height, and length of beams, as well as the cylindrical compressive strength of concrete. Additionally, the yield and ultimate tensile strength of steel bars, as well as the ultimate tensile strength of the FRP bars, were also considered along with the modulus of elasticity for both materials. In addition, the steel and FRP material volumes as well as their reinforcement ratios were considered in this comparison. Three statistical relationships -Coefficient of Variation (COV), Mean (μ), and Standard Deviation (SD) were used to assess the accuracy, reliability, and consistency of the relationships. Table 2 shows the statistical results for the M_{exp}/M_{th} were obtained from ACI 440.1R-2015 (ACI-440.1R-15, 2015) and CSA S807-2012 (CAN/CSA-S806-12, 2012), a Hybrid reinforcement equation (El Refai et al., 2015). The hybrid reinforcement equations show a higher mean M value of 1.08 compared to ACI 440.1R-2015 (ACI-440.19R-15, 2015) 0.99 and CSA S807-2012 (CAN/CSA-S806-12, 2012) 0.92, indicating better overall performance in terms of the M_{exp}/M_t ratio. The standard deviations SD were 0.17, 0.15, and 0.16, respectively. The COV% for the hybrid method is significantly lower, 15.27%, compared to ACI 440.1R-2015 (ACI-440.1R-15, 2015), 17.5%, and CSA S807-2012 (CAN/CSA-S806-12, 2012), 17.11%, suggesting lower variability in the hybrid reinforcement results. The relationship M_{exp}/M_{th} moment capacities is presented in Fig. 2, where the experimental-to-theoretical moment capacity ratio closely follows the optimal line. The hybrid reinforcement equation exhibits higher accuracy when compared to ACI 440.1R-2015 (ACI-440.1R-15, 2015) and CSA S807-2012 (CAN/CSA-S806-12, 2012).In addition, the square Pearson's correlation coefficient (PSCC) was 0.947 for ACI 440.1R-2015 (ACI-440.1R-15, 2015), 0.958 for CSA S807-2012, and 0.948 for the hybrid reinforcement equation.

Table 2. The Ratio of Experimental to Theoretical Moment Capacity for Beams Within the Database.

M_{exp}/M_{th}	Min	Max	M	SD	COV %
ACI 440.1R-2015 (ACI-440.1R-15, 2015)	0.66	1.42	0.99	0.17	17.5
CSA S807-2012(CAN/CSA-S806-12, 2012)	0.56	1.31	0.92	0.15	17.11
Hybrid Equations	0.773	1.61	1.08	0.16	15.27

The performance criteria involve a variety of Formulas that are used to study the reliability and precision of the code equations. Namely, the Nash–Sutcliffe Efficiency Index (NSEI) expressed in Eq.24, Pearson's Correlation Coefficient (R) in Eq.20, Root-Mean-Square Error (RMSE) appearing in Eq.23, j Mean Absolute Percentage Error calculated by means of the formula represented by Eq.22 and a-index was defined as presented in Eq.25. Reasoning that error estimates could be directly utilized to differentiate between algorithms with close scores, especially when using only final models meeting minimum performance criteria, we not only computed NSEI and RMSE but also evaluated all alternatives based on MAPE. were measured as the criteria to judge the reliability and precision of code equations. Furthermore, a new engineering index – the a20-index is proposed for reliability assessment of these equations. The RMSE is of specific interest because it shows the average model error between experimental and predicted data. The smaller the RMSE, the better is the estimation. R value is the amount of variance the equation explains; it tells you how much less variance there would be in any other range by using that specific equation. R values can go from -1 to 1, where near 1 indicates a strong positive relationship between the two variables and near -1 is stating that variable one causes variable two. The closer to zero the value, the weaker or no correlation. These performance measures constitute a reasonable criterion for overall prediction ability.

A value of the NSEI, a20-index and R close to 1 implies good accuracy in the code equation. However, in order to consider the precision of the equation acceptable MAE, MAPE and RMSE should be tended toward zero. Miscellaneous are used to evaluate the accuracy of different formulae for evaluating final flexure MC. The performance indices (PI's) have been defined on the following basis.

$$R = \frac{\sum_{i=1}^{N}(M_{exp}-\overline{M_{exp}})(M_t-\overline{M_t})}{\sqrt{\sum_{i=1}^{N}(M_{exp}-\overline{M_{exp}})^2\,(M_t-\overline{M_t})^2}} \quad (20)$$

$$MAE = \frac{\sum_{i=1}^{N}|(M_{exp}-M_t)|}{N} \quad (21)$$

$$MAPE = \frac{\sum_{i=1}^{N}\left|\frac{(M_{exp}-M_t)}{M_{exp}}\right|}{N} x\ 100 \quad (22)$$

$$RMSE = \sqrt{\frac{\sum_{i=1}^{N}(M_{exp}-M_t)^2}{N}} \quad (23)$$

$$NSEI = 1 - \frac{\sum_{i=1}^{N}(M_{exp}-M_t)^2}{\sum_{i=1}^{N}(M_{exp}-\overline{M_t})^2} \quad (24)$$

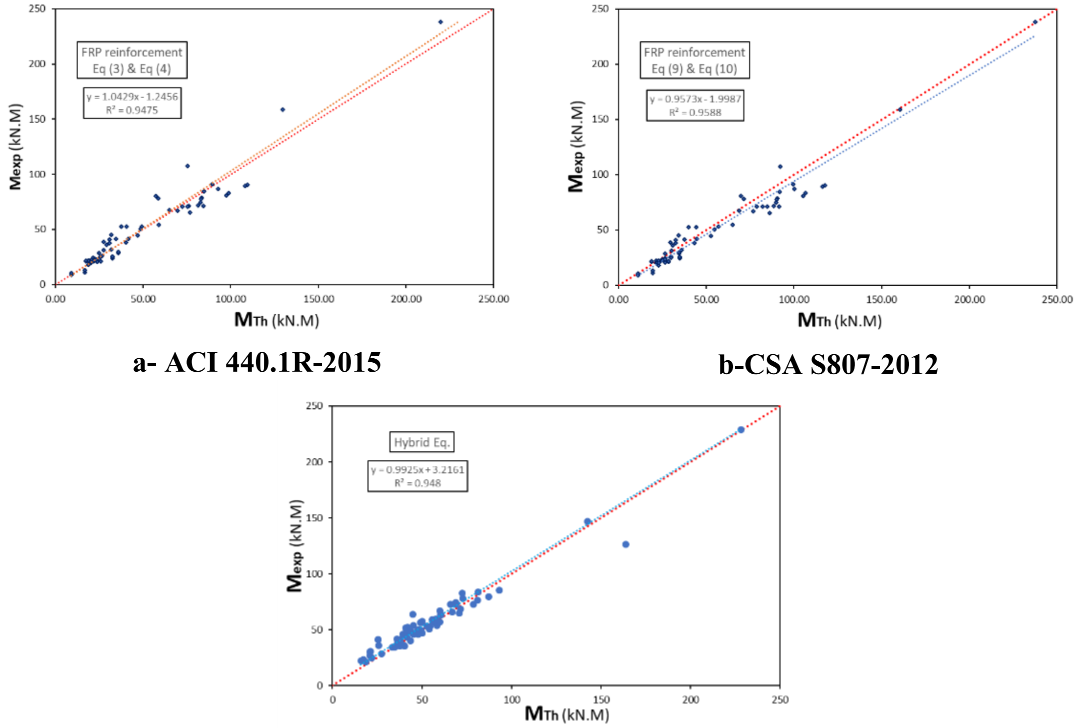
$$A20 = \frac{\sum_{i=1}^{N}(\frac{M_{exp}}{M_t})}{N} \quad (25)$$

Where M_{exp} is the experimental Flexural Moment capacity, M_t is the theoretical Flexural moment capacity, $\overline{M_{exp}}$, $\overline{M_t}$ are the mean of experimental and theoretical Flexural moment capacity, respectively. N is the number of samples.

Table 3 provides a detailed comparison of the performance indices for both reinforcement systems: ACI 440.1R-2015 (ACI-440.1R-15, 2015); CSA S807-2012 (CAN/CSA-S806-12, 2012), and the Hybrid Reinforcement method equation. The performance of each method is measured using a number of statistical metrics discussed earlier. As for prediction accuracy, the Hybrid reinforcement equation method has been proven to be more efficient independently. It exhibits the lowest MAE (5.76 kN·m) and the lowest MAPE (10.53%), meaning that its prediction is closest to the experimental value in terms of average values. Furthermore, the Hybrid reinforcement equation has the lowest RMSE (9.08 kN · m), because it can prevent large errors well. The A20 index of the Hybrid is also the maximum (1.09), which implies that a larger fraction of its predictions lies within 20% of experimental values compared with the other methods. CSA S807-2012 (CAN/CSA-S806-12, 2012) has the highest R value with the best fit for prediction/experimental values and NSEI of 0.947, which shows that have a good predicted efficiency too. Nonetheless, it has a larger MAE (7.65 kN. m) and (17.56%) than the Hybrid reinforcement equation, indicating poor accuracy in individual predictions. ACI 440.1R-2015 (ACI-440. 1R-15, 2015) performs reasonably well but is usually less accurate than the bars equation.

Table 3. The Statistical Analysis Results for the Tested Beam Equation.

Equation	R	MAE (kN.M)	MAPE %	RMSE (kN.M)	NSEI	A20
ACI 440.1R-2015 (ACI-440.1R-15, 2015)	0.9786	6.77	13.41	10.12	0.945	1.03
CSA S807-2012(CAN/CSA-S806-12, 2012)	0.9825	7.65	17.56	9.99	0.947	0.92
Hybrid equations	0.9787	5.76	10.53	9.08	0.942	1.09



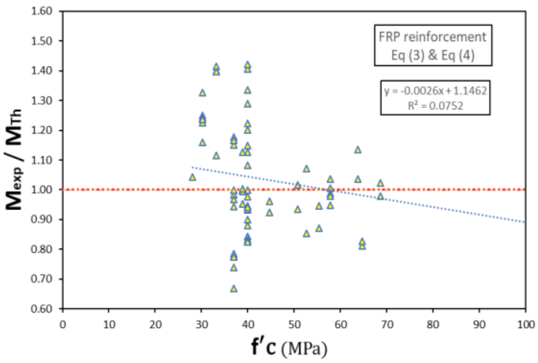
a- ACI 440.1R-2015 **b-CSA S807-2012**

c-Hybrid Eq

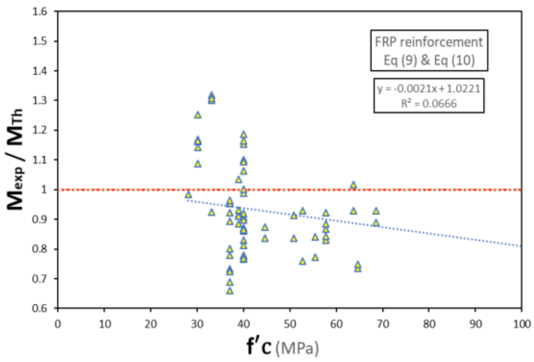
Fig. 2. The Relationship Between Experimental and Theoretical Flexural Moment Capacity is Predicted by a Different Equation.

4.2. Compressive Strength

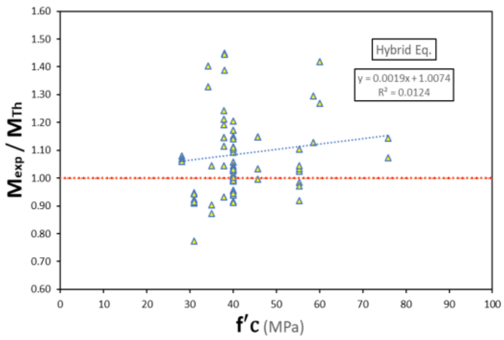
The compressive strength significantly influences the flexural strength of the beam, especially in the compression zone. Fig 3. compares ACI 440.1R-2015 (ACI-440.1R-15, 2015); CSA S807-2012 (CAN/CSA-S806-12, 2012), and the Hybrid reinforcement equation. The optimal linear trendline slope was -2.6E-3 for ACI 440.1R-2015 (ACI-440.1R-15, 2015); -2.1E-3 for CSA S807-2012 (CAN/CSA-S806-12, 2012), and 1.9E-3 for the hybrid reinforcement equation. Additionally, PSCC was 0.075 for ACI 440.1R-2015 (ACI-440.1R-15, 2015), 0.066 for CSA S807-2012 (CAN/CSA-S806-12, 2012), and 0.012 for the hybrid reinforcement equation. Concerning compressive strength, the hybrid reinforcement equation shows more safety and the largest consistency concerning ACI 440.1R-2015(ACI-440.1R-15, 2015) and CSAS807-12 (CAN/CSA-S806-12, 2012).



a- ACI 440.1R-2015



b- CSA S807-2012



c-Hybrid Eq

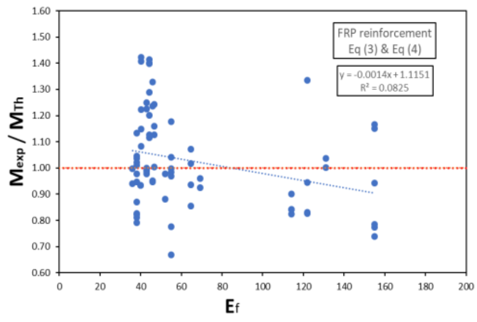
Fig. 3. The relationship between the experimental to theoretical ratio and the compressive strength.

4.3. Modulus of Elasticity (Young's Modulus)

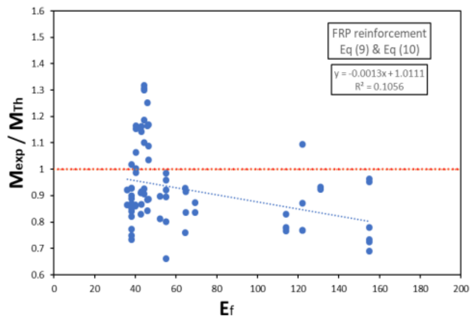
The modulus of elasticity of FRP bars significantly influences FSCB. Fig 4. Explain the relationship between M_{exp}/M_{th} and E_f.The optimal slope of the trendline for ACI 440.1R-2015(ACI-440.1R-15, 2015), CSA S807-2012(CAN/CSA-S806-12, 2012), and the hybrid reinforcement equation were -1.4E-3, -1.3E-3, and 0.8E3, respectively. Additionally, the PSCC was 0.082 in ACI 440.1R-2015 (ACI-440.1R-15, 2015), 0.1 for CSA S807-2012 (CAN/CSA-S806-12, 2012), and 0.005 for the hybrid reinforcement equation. The result demonstrates that the hybrid reinforcement equation exhibits greater consistency in safety than ACI 440.1R-2015(ACI-440.1R-15, 2015) and CSA S807-2012 (CAN/CSA-S806-12, 2012) because it takes into account the difference between Young's modulus of steel and FRP when calculating the effective modulus of elasticity.

4.4. The Ratio of the FRP Reinforcements.

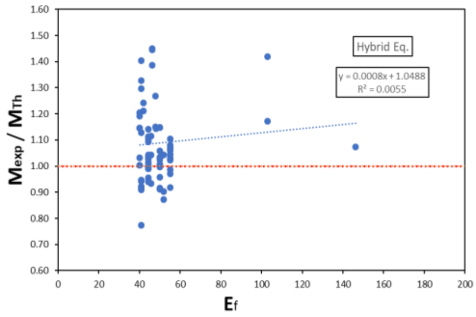
The flexural behavior of concrete is largely influenced by the main longitudinal reinforcement. Making it essential to consider the FRP reinforcement ratio. The PSCC values for ACI 440.1R-2015 (ACI-440.1R-15, 2015), CSA S807-2012 (CAN/CSA-S806-12, 2012), and the hybrid reinforcement equation were 0.051, 0.009, and 0.004, respectively. Additionally, the trendline slope was -6.9E-3 for ACI 440.1R-2015 (ACI-440.1R-15, 2015), -2.5E-3 for CSA S807-2012 (CAN/CSA-S806-12, 2012), and -3.3E-3 for the hybrid reinforcement equation. As shown in Fig. 5, it is evident that increasing the FRP reinforcement ratio enhances the accuracy of the hybrid reinforcement equation in predicting the flexural moment capacity.



a-ACI 440.1R-2015



b-CSA S807-2012

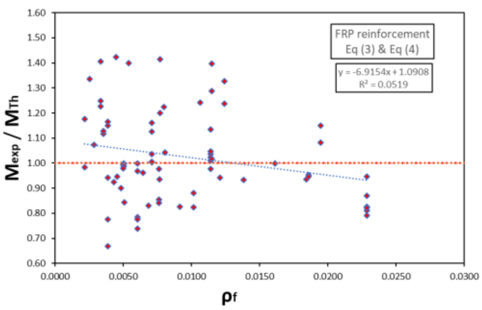


c-Hybrid Eq

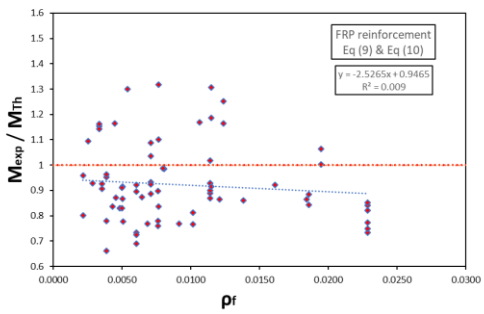
Fig. 4. The Relationship Between the Ratio of Experimental to Theoretical and the Modulus of Elasticity of FRP Bars.

5. CONCLUSIONS

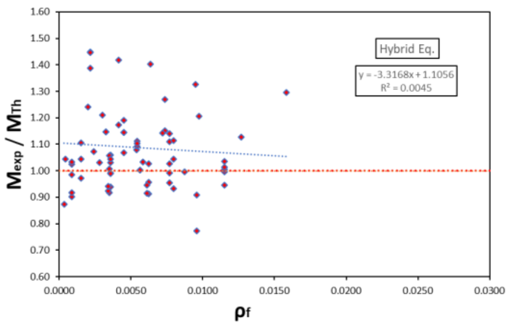
The statistical evaluation of FSCB reveals significant distinctions in the predictive performance of different design equations. The analysis robustly demonstrates that the hybrid reinforcement equation yields more accurate predictions when compared to ACI 440.1R-2015 and CSA S807-2012. This enhanced accuracy is quantitatively supported by a higher mean M value of 1.08 for the hybrid method, in contrast to 0.99 for ACI 440.1R-2015 and 0.92 for CSA S807-2012, indicating a superior agreement with experimental flexural moment capacity. While the hybrid reinforcement exhibits a lower Coefficient of Variation (COV) of 15.27%, suggesting lower data dispersion, a comprehensive analysis of key performance metrics, including a lower (MAE of 5.76 kN.m, a reduced MAPE of 10.53%, and a smaller RMSE of 9.08 kN.m, firmly establishes its superior predictive accuracy and minimization of error. Furthermore, the study highlights that the hybrid reinforcement equation demonstrates greater consistency in its predictions across variations in critical design parameters. Specifically, the model maintains a more stable predictive performance despite changes in concrete compressive strength, the elastic modulus of FRP bars, and the ratio of FRP reinforcement. This enhanced consistency is evident in the comparative analysis of trendline slopes. For instance, concerning compressive strength, the hybrid reinforcement equation exhibits a more gradual trendline slope (1.9E-3) compared to ACI 440.1R-2015 (-2.6E-3) and CSA S807-2012 (-2.1E-3), indicating a reduced sensitivity of the predictions to variations in concrete strength. Similarly, when considering the modulus of elasticity, the hybrid reinforcement equation's trendline slope (0.8E-3) demonstrates greater stability compared to ACI 440.1R-2015 (-1.4E-3) and CSA S807-2012 (-1.3E-3), further supporting its enhanced consistency. This enhanced consistency is essential for ensuring the reliability and reliability of structural designs, as it reduces the uncertainty associated with flexural capacity predictions. In contrast, other models exhibit increased sensitivity to these parameters, potentially leading to less reliable designs.



a- ACI 440.1R-2015



b- CSA S807-2012



c- ybrid Eq

Fig. 5. The Relationship Between the Experimental to Theoretical Ratio and the Ratio of the FRP Reinforcements.

6. REFRENCE

American Concrete Institute (ACI) 363R-10, A. (2010). Report on High-Strength Concrete (ACI 363R-10), ACI.

American Concrete Institute (ACI) 318R-19 (2019). Building Code Requirements for Structural Concrete ACI 318-19 and Commentary 318R–19, Farmington Hills, MI, USA, American Concrete Institute.

American Concrete Institute (ACI) Committee-440 (2006) Guide for the Design and Construction of Concrete Reinforced with FRP Bars.(ACI) -440.1R-06. Committee Detroit, Michigan, USA

Abdullah, M. M. (2020). Flexural Behavior of Normal and High Strength Continuous Beams Reinforced by GFRP Bars. AL-Mustansiriayah University. https://doi.org/10.31272/jeasd.25.1.2

Abed, F., Al-Mimar, M. & Ahmed, S. (2021). Performance of BFRP RC beams using high-strength concrete. Composites Part C: Open Access, 4, 100107. https://doi.org/10.1016/j.jcomc.2021.100107

Adam, M. A., Said, M., Mahmoud, A. A. & Shanour, A. S. (2015. Analytical and experimental flexural behavior of concrete beams reinforced with glass fiber reinforced polymers bars. Construction and building materials, 84, 354-366. https://doi.org/10.1016/j.conbuildmat.2015.03.057

Aiello, M. A. & Ombres, L. (2002). Structural performances of concrete beams with hybrid (fiber-reinforced polymer-steel) reinforcements. Journal of Composites for Construction, 6, 133-140. https://doi.org/10.1061/(ASCE)1090-0268(2002)6:2(133)

Al-turihi, A. A. A. & Al-katib, H. A. A. (2024). Behavior of Hybrid Reinforced Concrete Beams on Flexural Strength. Kufa Journal of Engineering, 15, 27-38. DOI: https://doi.org/10.30572/2018/KJE/150203

Al-hawat, M. & Ashour, A. Bond strength between corroded steel reinforcement and recycled aggregate concrete. Structures, (2019). Elsevier, 369-385. https://doi.org/10.1016/j.istruc.2019.02.001

Alshadidi, R. M., Hassan, H. F. & Mohammed, M. H. (2016). Shear behavior of high-strength concrete beams reinforced with GFRP bars and strengthened by CFRP sheets. Journal of Engineering and Sustainable Development, 20, 102-117.

Araba, A. M. & Ashour, A. F. (2018). Flexural performance of hybrid GFRP-Steel reinforced concrete continuous beams. Composites Part B: Engineering, 154, 321-336. https://doi.org/10.1016/j.compositesb.2018.08.077

Benmokrane & Masmoudi (1996). Flexural response of concrete beams reinforced with FRP reinforcing bars. Structural Journal, 93, 46-55. DOI: 10.14359/9839

Bischoff, P. H. (2005). Reevaluation of deflection prediction for concrete beams reinforced with steel and fiber-reinforced polymer bars. Journal of Structural Engineering, 131, 752-767. https://doi.org/10.1061/(ASCE)0733-9445(2005)131:5(752)

Bischoff, P. H. & Sacanlon, A. (2007). Effective moment of inertia for calculating deflections of concrete members containing steel reinforcement and fiber-reinforced polymer reinforcement. ACI Structural Journal, 104, 68.

Cai, J., Pan, J. & Zhou, X. (2017). Flexural behavior of basalt FRP reinforced ECC and concrete beams. Construction and Building Materials, 142, 423-430. https://doi.org/10.1016/j.conbuildmat.2017.03.087

Chaallal, O. & Benmokrane, B. (1996). Fiber-reinforced plastic rebars for concrete applications. Composites Part B: Engineering, 27, 245-252. https://doi.org/10.1016/1359-8368(95)00023-2

Dong, H.-L., Zhou, W. & Wang, Z. (2019). Flexural performance of concrete beams reinforced with FRP bars grouted in corrugated sleeves. Composite Structures, 215, 49-59. https://doi.org/10.1016/j.compstruct.2019.02.052

Douglas, D. (2012). An Investigation into the Flexural Behavior of GFRP Reinforced Concrete Beams [MSc thesis]. Canada Univ. Toronto.

El Refai, A., Abed, F. & AL-Rahmani, A. (2015). Structural performance and serviceability of concrete beams reinforced with hybrid (GFRP and steel) bars. Construction and Building Materials, 96, 518-529. https://doi.org/10.1016/j.conbuildmat.2015.08.063

EL-Messalami, N., EL Refai, A. & Abed, F. (2019). Fiber-reinforced polymer bars for compression reinforcement: A promising alternative to steel bars. Construction and Building Materials, 209, 725-737. https://doi.org/10.1016/j.conbuildmat.2019.03.105

Ftnan, A. H. & Makki, R. F. (2022). Behavior of Recycled Aggregate Concrete Beams Strengthened with FRP. Kufa Journal of Engineering, 13, 13-24. https://doi.org/10.30572/2018/KJE/130402

GE, W., Zhang, J., Cao, D. & TU, Y. (2015). Flexural behaviors of hybrid concrete beams reinforced with BFRP bars and steel bars. Construction and Building Materials, 87, 28-37.https://doi.org/10.1016/j.conbuildmat.2015.03.113

Grace, N. F., Sayed, G., Soliman, A. & Saleh, K. (1999). Strengthening reinforced concrete beams using fiber-reinforced polymer (FRP) laminates. ACI Structural Journal-American Concrete Institute, 96, 865-874.

Gravina, R. J. & Smith, S. T. (2008). Flexural behaviour of indeterminate concrete beams reinforced with FRP bars. Engineering Structures, 30, 2370-2380. https://doi.org/10.1016/j.engstruct.2007.12.019

Habeeb, M. & Ashour, A. F. (2008). Flexural behavior of continuous GFRP reinforced concrete beams. Journal of composites for construction, 12, 115-124. https://doi.org/10.1061/(ASCE)1090-0268(2008)12:2(115)

Japan Society of Civil Engineers (JSCE),(1997). Recommendation for Design and Construction of Concrete Structures using Continuous Fiber Reinforcing Materials. Concrete Engineering Series. Tokyo, Japan.

Kabashi, N., Avdyli, B., Krasniqi, E. & Kepuska, A. (2020). Comparative approach to flexural behavior of reinforced beams with GFRP, CFRP, and steel bars. Civil Engineering Journal, 6, 50-59. DOI: 10.28991/cej-2020-03091452

Kadhim, M. Q. & Hassan, H. F. (2024). Revisiting the Code Design Equations for Concrete Columns Reinforced with GFRP Bars. Journal of Engineering and Sustainable Development, 28, 664-674. https://doi.org/10.31272/jeasd.28.5.12

Kadhima, A. J. & Zinkaaha, O. H. (2024). Flexural behaviour of hybrid (FRP/steel) reinforced concrete beams: a review. Muthanna J. Eng. Technol, 12, 31-42. DOI:10.52113/3/eng/mjet/2024-12-01/31-42

Kara, I. F. & Ashour, A. F. (2012). Flexural performance of FRP reinforced concrete beams. Composite structures, 94, 1616-1625. https://doi.org/10.1016/j.compstruct.2011.12.012

Kara, I. F., Ashour, A. F. & Koroglu, M. A. (2015). Flexural behavior of hybrid FRP/steel reinforced concrete beams. Composite Structures, 129, 111-121. https://doi.org/10.1016/j.compstruct.2015.03.073

Karpiuk, V., Tselikova, A., Khudobych, A., Kostyuk, A. & Karpiuk, I. (2020). Study of strength, deformability property, and crack resistance of beams with BFRP. Восточно-Европейский журнал передовых технологий, 4, 42-53. DOI:10.15587/1729-4061.2020.209378

Kassem, C., Farghaly, A. S. & Benmokrane, B. (2011). Evaluation of flexural behavior and serviceability performance of concrete beams reinforced with FRP bars. Journal of composites for construction, 15, 682-695. https://doi.org/10.1061/(ASCE)CC.1943-5614.0000216

Khalid, N., Shallal, M., Naji H. & Hasson, A. (2025). An Experimental Investigation on Improving the Bond Behavior of Glass Fiber Reinforced Polymer Bars in Concrete. Kufa Journal of Engineering, 16, 81-95. DOI: https://doi.org/10.30572/2018/KJE/160205

Lapko, A. & Urbanski, M. (2015). Experimental and theoretical analysis of deflections of concrete beams reinforced with basalt rebar. Archives of Civil and Mechanical Engineering, 15, 223-230. https://doi.org/10.1016/j.acme.2014.03.008

Lau, D. & Pam, H. J. (2010). Experimental study of hybrid FRP reinforced concrete beams. Engineering Structures, 32, 3857-3865.https://doi.org/10.1016/j.engstruct.2010.08.028

Leung, H. & Balendran, R. (2003). Flexural behaviour of concrete beams internally reinforced with GFRP rods and steel rebars. Structural Survey, 21, 146-157.https://doi.org/10.1108/02630800310507159

Lu, C., Li, H., Xu, K., Xuan, G. & Abdullah, W.(2020) Experimental study of flexural behavior and serviceability of hybrid concrete beams reinforced by steel and G/BFRP bars. IOP Conference Series: Materials Science and Engineering, . IOP Publishing, 012007. https://doi.org/10.1088/1757-899X/770/1/012007

Nanni, A., De Luca, A. & Zadeh, H. (2014). Reinforced concrete with FRP bars. Reinforced Concrete with FRP Bars. CRC Press, London, United Kingdom. doi, 10, b16669.

Nguyen, P. D., Dang, V. H. & Vu, N. A. (2020). Performance of concrete beams reinforced with various ratios of hybrid GFRP/steel bars. Civil Engineering Journal, 6, 1652-1669.http://dx.doi.org/10.28.991/cej-2020-03091572

Pang, L., Qu, W., Zhu, P. & Xu, J. (2016). Design propositions for hybrid FRP-steel reinforced concrete beams. Journal of Composites for Construction, 20, 04015086. https://doi.org/10.1061/(ASCE)CC.1943-5614.0000654

Pawlowski, D. & SZUMIGAŁA, M.(2015). Flexural behaviour of full-scale basalt FRP RC beams–experimental and numerical studies. Procedia engineering, 108, 518-525. https://doi.org/10.1016/j.proeng.2015.06.114

Qin, R., Zhou, A. & Lau, D. (2017). Effect of reinforcement ratio on the flexural performance of hybrid FRP reinforced concrete beams. Composites Part B: Engineering, 108, 200-209. https://doi.org/10.1016/j.compositesb.2016.09.054

Qu, W., Zhang, X. & Huang, H. (2009). Flexural behavior of concrete beams reinforced with hybrid (GFRP and steel) bars. Journal of Composites for Construction, 13, 350-359. https://doi.org/10.1061/(ASCE)CC.1943-5614.0000035

Sam, A. R. M. & Swamy , R. N. (2005). Flexural behaviour of concrete beams reinforced with glass fibre reinforced polymer bars. Malaysian Journal of Civil Engineering, 17. DOI: https://doi.org/10.11113/mjce.v17.15672

Sun, Z., Fu, L., Feng, D.-C., Vatuloka, A. R., Wei, Y. & Wu, G. (2019). Experimental study on the flexural behavior of concrete beams reinforced with bundled hybrid steel/FRP bars.Engineering,Structures,197,109443.https://doi.org/10.1016/j.engstruct.2019.109443

Theriault, M. & Benmokrane, B. (1998). Effects of FRP reinforcement ratio and concrete strength on flexural behavior of concrete beams. Journal of composites for construction, 2, 7-16. doi/abs/10.1061/(ASCE)1090-0268(1998)2:1(7)

Tomlinson, D. & Fam, A. (2015). Performance of concrete beams reinforced with basalt FRP for flexure and shear. Journal of composites for construction, 19, 04014036. doi/abs/10.1061/(ASCE)CC.1943-5614.0000491

Vu, H. D. & Phan, D. N. (2021). Experimental and theoretical analysis of the cracking moment of concrete beams reinforced with hybrid fiber-reinforced polymer and steel rebars. Advances in Technology Innovation, 6, 222.doi.org/10.46604/aiti.2021.7330

Xue, W., Peng, F. & Zheng, Q. (2016). Design equations for the flexural capacity of concrete beams reinforced with glass fiber–reinforced polymer bars. Journal of Composites for Construction, 20, 04015069. doi/abs/10.1061/(ASCE)CC.1943-5614.0000630

Yang, Y., Sun, Z.-Y., Wu, G., Cao, D.-F. & Zhang, Z.-Q. (2020). Flexural capacity and design of hybrid FRP-steel-reinforced concrete beams. Advances in Structural Engineering, 23, 1290-1304. doi/abs/10.1177/1369433219894236

Yoon, Y.-S., Yang, J.-M., Min, K.-H. & Shin, H.-O. (2011). Flexural strength and deflection characteristics of high-strength concrete beams with hybrid FRP and steel bar reinforcement. Special Publication, 275, 1-22.

Zinkaah, O. H., Ashour, A. & Sheehan, T. (2019). Experimental tests of two-span continuous concrete deep beams reinforced with GFRP bars and strut-and-tie method evaluation. Composite Structures, 216, 112-126. https://doi.org/10.1016/j.compstruct.2019.02.078

American Concrete Institute ACI-440.1R-15 (2015). Guide for the design and construction of structural concrete reinforced with FRP bars. American Concrete Institute

Canadian Standareds Association CAN/CSA-S806-12 (2012). Design and Construction of Building Structures with Fiber Reinforced Polymers (CAN/CSA), Rexdale, ON, Canada, Canadian Standards Association.

Canadian Standareds Association CAN/CSA-S6-14 (2014). Canadian highway bridge design code. CAN/CSA-S6-14.

American Concrete Institute (ACI) 440.11-22, A. (2022). Building Code Requirements for Structural Concrete Reinforced with Glass Fiber-Reinforced Polymer (GFRP) Bars – Code and Commentary, American Concrete Institute.