

Cauchy sequence on graph

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ABSTRACT

In this paper we study Cauchy sequence on graphs. Cauchy sequence introduced by the French mathematician Augustin Louis Cauchy. A Cauchy sequence is sequence which its terms become arbitrarily close to each other as it progresses. The amazing properties of Cauchy sequence can be used to understand the behavior of a system as time progresses. Cauchy and some Authors proved the converge of Cauchy sequence and some properties on it. In this paper we define Cauchy sequence and prove some properties on graph.

Key Words

Cauchy sequence on graph, Subsequence on graphs, set of verticies.

متابعة كوشي على البيانات

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الاختصاص العام mathematics

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الملخص

في هذا البحث سندرس متتابعة كوشي على البيانات. متتابعة كوشي قدمت من قبل الفرنسي أوغستيا لويس كوشي.

متابعة كوشي هي متتابعة التي حدودها تكون متقاربة من بعضها البعض اثناء تقدمه. الخواص المميزة لمتابعة كوشي يمكن استخدامها لفهم سلوك النظام كلما تقدم الوقت.

كوشي وبعض الباحثين برهنوا تقارب المتتابعة كوشي وبعض الخواص عليها.

في هذا البحث سنعرف متتابعة كوشي ونبرهن بعض الخواص على البيانات.

Introduction

Cauchy sequence is found by the French mathematician Augustin Louis Cauchy. Cauchy sequence and their properties used in fields like satellite design, manufacturing, construction, treatment plants, and so on. Some Authers study Sobolev spaces on graphs and the discrete analogue of Sobolev spaces of smooth functions [Mio05] and spectral graph theory, in [ChY95] and in [Chu97,chapter11] , " L_p – embeddability in graph metric" , see [BLMN04⁺] , [LMN02] and [Mat97] . [EmZ21] in 2021 ,study the L_q quasi normed space on graphs when $0 < q < 1$, and $f: V \rightarrow R$ "the discrete Sobolev seminorm on E " , is

$$\|f\| = \left(\sum_{(v \sim w) \in E} |f(v) - f(w)|^q \right)^{\frac{1}{q}},$$

For $0 < q < 1$. When the connected graph G the constant functions must remove to make $\|f\|_{E,p}$ a norm.

"The weighted Sobolev seminorm" as

$$\|f\| = \left(\sum_{v \sim w \in E} |f(v) - f(w)|^q \mu(v, w) \right)^{\frac{1}{q}},$$

For $0 < q < 1, \mu: E \rightarrow (0, 1)$.

In the beginning we define G a finite graph with the set of vertices and the set of edges , denoted by V and E respectively . we denote the degree of a vertex $v \in V$ by d_v .

We denote by $u \sim v$ to the edge connected between two vertices u and v .

Cauchy sequence on graph

Definition 1: we called $\{A_n\}$ a Cauchy sequence if given $\epsilon > 0$ there exists a natural $N \in \mathbb{N}$ such $\|A_n - A_m\| < \epsilon$ for all $n, m \geq N$. [And11]

To prove some theorems on Cauchy sequence we must define it on graphs.

Definition 2: we suppose that G a connected graph and V vertices set in G then We denote a sequence in R is a function $a_{v_i}: V \rightarrow R$ is called Cauchy sequence For all $v_i \in V, i = 1, \dots, \infty$, if for any given $\epsilon > 0$, there exist $w_n \in V$ such that

$$u_j, v_i \geq w_n \implies \|a_{u_j} - a_{v_i}\| < \epsilon.$$

Example

Suppose $\{A_{u_j}\}_{j=1}^{\infty}$, $u_j \in V$ a sequence converges to K . Let we have $\epsilon > 0$. so

there exist $w_n \in V, n = 1, \dots, \infty$ such that $\|a_{u_j} - K\| < \frac{\epsilon}{2}, \forall u_j \geq w_n$

Then if $u_j, v_i \geq w_n$, so $\|a_{u_j} - a_{v_i}\| = (\sum_{u_j, v_i \in V} |a_{u_j} - a_{v_i}|^q)^{\frac{1}{q}}, q \geq 1$

$$\begin{aligned} &= \left(\sum_{u_j, v_i \in V} |a_{u_j} - L + L - a_{v_i}|^q \right)^{\frac{1}{q}} \\ &\leq \left(\sum_{u_j \in V} |a_{u_j} - L|^q \right)^{\frac{1}{q}} + \left(\sum_{v_i \in V} |a_{v_i} - L|^q \right)^{\frac{1}{q}} \\ &\leq \|a_{u_j} - L\| + \|a_{v_i} - L\| \\ &\leq \frac{\epsilon}{2} + \frac{\epsilon}{2} \\ &= \epsilon \end{aligned}$$

Thus $\{a_{u_j}\}$ is Cauchy sequence.

In the following we prove that Cauchy sequence on graph is bounded.

Theorem 1

Let $a_{v_i}: V \rightarrow \mathbb{R}, i = 1, \dots, \infty$ Cauchy sequence when a set of vertices in G denote by V , then for $v_i \in V$ the sequence $\{A_{v_i}\}$ is bounded.

Proof

Since $\{A_{v_i}\}_{i=1}^{\infty}$ forms a Cauchy sequence for $\epsilon = 1$ there exists $w_n \in V, n = 1, \dots, \infty$ such that

$$\|A_{v_i} - A_{u_j}\| < 1, \forall u_j, v_i \geq w_n$$

In particular

Where $\|A_{v_i} - A_{u_j}\| = \left(\sum_{v_i, u_j \in V} |A_{v_i} - A_{u_j}|^p\right)^{\frac{1}{p}}$

$$\|A_{v_i} - A_{w_n}\| < 1, \forall u_j \geq w_n$$

Thus, if $\|A_{v_i}\| = \|A_{v_i} - A_{w_n} + A_{w_n}\| \leq \|A_{v_i} - A_{w_n}\| + \|A_{w_n}\|$

$$< 1 + \|A_{w_n}\|, \forall v_i \in V$$

Suppose $\mathcal{M} = \max\{|A_{n1}|, |A_{n2}|, \dots, |A_{nw-1}|, |A_{nw}|\}$.

So $|A_{v_i}| < \mathcal{M}$ for all $v_i \in V$

Hence $\{A_{v_i}\}$ is bounded.

Theorem2:

Suppose G be a connected graph then every convergent sequence is a Cauchy sequence on graph G .

Proof

Let $\{A_{v_i}\}_{i=1}^{\infty}$ be a sequence in V , and V the set of a vertices in G , $A_{v_i}: V \rightarrow \mathbb{R}$ which is converges to $x \in \mathbb{R}$

Suppose $\epsilon > 0$.

So there is a $w_n \in V, n = 1, \dots, \infty$, such that

$$\|A_{v_i} - x\| < \frac{\epsilon}{2}, \forall v_i \geq w_n \text{ for all } v_i, u_j \geq w_n,$$

$$\begin{aligned} \|A_{v_i} - A_{u_j}\| &= \left(\sum_{v_i, u_j \in V} |A_{v_i} - A_{u_j}|^p \right)^{\frac{1}{p}} \\ &= \left(\sum_{v_i, u_j \in V} |A_{v_i} - x + x - A_{u_j}|^p \right)^{\frac{1}{p}} \\ &\leq \left(\sum_{v_i \in V} |A_{v_i} - x|^p \right)^{\frac{1}{p}} + \left(\sum_{u_j \in V} |A_{u_j} - x|^p \right)^{\frac{1}{p}} \\ &\leq \|A_{v_i} - x\| + \|A_{u_j} - x\| \\ &< \frac{\epsilon}{2} + \frac{\epsilon}{2} \\ &= \epsilon. \end{aligned}$$

Then $\{A_{v_i}\}_{i=1}^{\infty}$ is a Cauchy sequence.

Theorem3

suppose G be a finite connected graph and V is a set of vertices in graph G .
 Cauchy sequence in G which has a convergent subsequence is convergent.

Proof

Suppose $\{A_{v_i}\}_{i=1}^{\infty}$ be a Cauchy sequence in V and $\{A_{v_{ik}}\}$ its subsequence which converges to $x \in \mathbb{R}$.

Then for any $\epsilon > 0$, there exist a w_1 and $w_2 \in V$ such that

$$\|A_{v_i} - A_{u_j}\| < \frac{\epsilon}{2} \text{ for all } v_i, u_j \geq w_1$$

And

$$\|A_{v_{ik}} - x\| < \frac{\epsilon}{2} \text{ for all } k \geq w_2,$$

Let $w_n = \max\{w_1, w_2\}$

If $v_k \geq w_n$, so for $v_{ik} \geq k$,

$$\begin{aligned} \|A_{v_k} - x\| &= \left(\sum_{v_k \in V} |A_{v_k} - x|^p \right)^{\frac{1}{p}}, p \geq 1 \\ &= \left(\sum_{v_k, v_{nk} \in V} |A_{v_k} - A_{v_{nk}} + A_{v_{nk}} - x|^p \right)^{\frac{1}{p}} \\ &\leq \left(\sum_{v_k, v_{nk}} |A_{v_k} - A_{v_{nk}}|^p \right)^{\frac{1}{p}} + \left(\sum_{v_{nk}} |A_{v_{nk}} - x|^p \right)^{\frac{1}{p}} \\ &\leq \|A_{v_k} - A_{v_{nk}}\| + \|A_{v_{nk}} - x\| \\ &\leq \frac{\epsilon}{2} + \frac{\epsilon}{2} \\ &\leq \frac{\epsilon}{2} \\ &< \epsilon \end{aligned}$$

So $A_{v_n} \rightarrow x$ as $n \rightarrow \infty$.

Conclusion

Cauchy sequence on graphs, Cauchy sequence is bounded, every cnvergent sequence is a Cauchy sequence, a Cauchy sequence in G which has a convergent subsequence is convergent was proven on graph when $q \geq 1$.

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