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The Impact of the Characteristics of Horizontal Wellbore and Hydraulic Fractures on the Reservoir Performance

Dheiaa Salah Mahdi

Oil and Gas Engineering Department, University of Warith Al-Anbiyaa, Kerbala 56001, Iraq.

Corresponding Author : Dheiaasalahmahdi@gmail.com

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Abstract

Hydraulic fracturing stimulation is a common practice in petroleum industry. The last two decades have witnessed great focusing on this process especially in North America and more or less attention in the European and Middle East oil fields. The process typically aims to increase the production recovery from oil and gas that can’t be produced by conventional techniques. It applies several hydraulic fractures along the horizontal wellbore that propagate to different zones in the formation. The success of fracturing process is controlled by several parameters. Fracture length, height and conductivity are the three parameters that influence these successes. Proppant Number, defined as the fracture volume to the reservoir volume, was proposed to judge the fracturing process performance. Proppant Number might be expressed by fracture conductivity and the ratio of the fracture length to reservoir width or the horizontal penetration. A new Proppant Number for multiple hydraulic fractures intersecting horizontal wellbore is presented in this paper. This number is determined from the volume of specific fracture and the volume of the drainage area surrounding the fracture. The new Proppant Number and the vertical penetration ratio, the ratio of the fracture height to the formation height, are two critical parameters affecting the productivity index of fractured formation. A new model for the productivity index of finite reservoirs having multiple hydraulic fractures has been introduced. This model reflects the effects of the boundary dominated flow. This paper shows that the productivity index increases with the increasing of Proppant Number and penetration ratio. The productivity index increases significantly for the increasing of Proppant Number when the number of fractures is small and slightly for large number of fractures. The change in productivity index with the penetration ratio can be noticed in short fracture lengths for large number of fractures and long fractures for small number of fractures.

Introduction

Since the early applications of hydraulic fracturing process, the great improvement in the performance and productivity of vertical and horizontal wells have been considered the motivated force for the increasingly uses of this stimulation technique all over the world. The significant increase in the production rate and the reasonable increase in the ultimate reserve are the two remarkable results obtained by fracking process. Both of them, from the economic point of view, are crucial parameters in the reservoir management. Physically the efficiency of the fracking process depends on how many fractures are propagating the formation and the dimensions of these fractures (length, width, and height). In addition, it is affected by several other parameters such as the penetration ratios, fracture conductivity and the proppant number. Of course, reservoir rock and fluid properties are taken into account when it comes to the estimation of the fracturing process efficiency. Two penetration ratios are subjected to study in this paper.

The first one is the vertical penetration ratio of hydraulic fractures or the ratio of the fracture height to the formation height. The successful fracking process aims to create hydraulic fractures fully penetrate the formation i.e., fracture height is equal to the formation height. As the penetration ratio increases, the productivity index increases also. This fact can be explained by the increasing of the fracture surface area that allows for reservoir fluid to flow from the matrix to the wellbore. Raghavan et al. 1978 were investigated the effect of the height of vertical fractures on pressure behavior for both uniform flux fracture and infinite conductivity fracture. They stated that extra pressure drop is resulted for the cases where the fracture height is less than the formation height. Later several researchers have studied fractures penetration ratio in the vertical direction on pressure behavior, flow regimes, skin factor and productivity index.

The second penetration ratio refers to the propagation of the fracture wings in the vertical plane normal to the horizontal wellbore. This ratio is typically called horizontal penetration ratio or the ratio of the fracture length to the formation width. It is not significantly important as the vertical penetration, but it has some effects on the flow regimes and productivity index of hydraulically fractured formations. Gringarten and Ramey 1974 were the first who indicated the effect of the partial penetration of a single horizontal fracture on pressure distribution in the reservoir. Several literatures have been presented later having the same scoop.

Proppant number (Economides E. et al. 2006, Economides M. et al. 2007, Daal and Economides 2006) is defined as the ratio of the proppant permeability and proppant volume (V_P) product to the formation permeability and reservoir volume (V_r) product. Based on the unified fracture design UFD approach presented by them, the model for the proppant number was written as follows:

$$N_P = \frac{2\,V_P}{V_r}\frac{h}{h_f}\frac{k_f}{k} \qquad (1)$$

It can be seen that the proppant number is intentionally included the vertical penetration ratio. In addition, it can be inferred that the horizontal penetration ratio is also included in the volume ratio in the above model. Recalling the definition for the fracture dimensionless conductivity and using the horizontal penetration ratio, the proppant number can be written as:

$$N_P = 4I_x^2 C_{fD} \qquad (2)$$

Demarchos et al. 2004 a,b stated that the maximum dimensionless productivity index for hydraulically fractured formation can be obtained from specific fracture dimensionless conductivity for any proppant number. This significant value of fracture conductivity can be used to assign fracture dimensions. However, these dimensions (width and length) vary corresponding to the formation properties. Therefore, wide and short fractures are expected to propagate at high permeability reservoir while narrow and long fractures are expected to extend in the low permeability formations.

The successful fracking process might be judged by the productivity index. Regardless the type of formation and type of wellbore, the index is defined as the amount or volume of reservoir fluids that can be produced daily by one psi pressure drop at the sand face. For a horizontal well with multiple hydraulic fractures, the productivity index is influenced by several factors such as the number of fractures, the spacing between them and the fracture dimensions. Reservoir permeability and reservoir fluid properties have a great influence on the productivity index as well as the geometry of the drainage area. Several models have been introduced during the last two decades for the productivity index of fractured formations. For example, Cinco-Ley and Samaniego 1981 presented a model for productivity index of fractured wells as follows:

$$J_D = \frac{1}{ln\left(0.472\,\frac{r_e}{x_f}\right) + f} \qquad (3)$$

where f is constant, called pseudo skin function, mainly represents skin factor. It was given by different researchers, however, Valko and Economides 1998 presented it as:

$$f = ln\left(\frac{x_f}{r_w}\right) + s = \frac{1.65 - 0.328\ln(C_{fD}) + 0.116\left[ln(C_{fD})\right]^2}{1 + 0.18ln(C_{fD}) + 0.064\left[ln(C_{fD})\right]^2 + 0.005\left[ln(C_{fD})\right]^3} \qquad (4)$$

Raghavan and Joshi 1993 presented a method to evaluate the productivity of wells with hydraulic fractures. A general and rigorous model was provided in their study for the productivity index as a function of reservoir variables and numbers of radials of fractures. Larsen 1998, 2001 introduced several analytical models for the productivity of fractured and non-fractured deviated wells in commingled reservoirs. Fokker et al. 2005, Kazem at al. 2023 presented a novel approach to determine the productivity of complex wells. They stated that their model is applicable for the finite-conductivity wells, well interference, non-homogenous reservoirs, and hydraulically fractured formations. Medeiros et al. 2008 discussed the performance of the fractured horizontal wells in heterogeneous and tight gas formations. They documented in their study the production characteristics and flow regimes in which the long transient periods may govern the productivity.
Al-Rbeawi and Tiab 2013 introduced a new technique for estimating the pseudo-steady state productivity index of horizontal wells intersecting multiple hydraulic fractures. They used the instantaneous source solutions for the diffusivity equation to obtain seven analytical models for different source solutions: Four of them represent the effect of the formation height and fracture height (the vertical direction), while the other three represent the solution for the horizontal plane. For vertical hydraulic fractures, the four solutions of the vertical direction, representing the pseudo-skin factor, are almost neglected. They stated that three horizontal plane solutions are the main parameters that control the productivity index and inflow performance of the fractured formations. In their technique, the horizontal wells are acting in finite reservoirs where the pseudo-steady state flow is expected to develop. Reservoir geometry, reservoir properties, and fracture dimensions were considered in this technique

.

Model description

Let us consider a horizontal well, extending in an infinite acting reservoir, having circular drainage area and the hydraulic fractures are propagating transversely to the well as shown in Fig. 1. The following assumptions are necessary to derive the model:

1. The fractures are symmetrical in dimensions.
2. The distances (spacing) between fractures are equal.
3. Reservoir fluids are slightly compressible.
4. Single phase fluid flow.

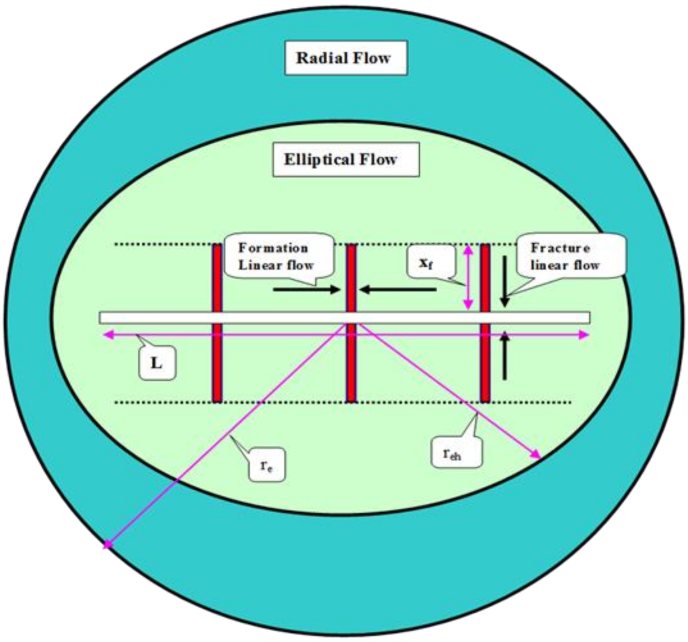

Figure 1: Horizontal well and hydraulic fractures in a circular reservoir.

Pseudo-radial flow

Pseudo-radial flow regime is the dominant flow in the reservoir drainage area far from the vicinity of the wellbore when reservoir fluids flow in the horizontal XY plane radially toward the fractures area such as shown in Fig. 2. This flow is characterized by constant value (0.5) for the dimensionless pressure derivative curves on log-log plot of dimensionless pressure and dimensionless time. The pressure drop due to the pseudo-radial flow can be calculated by assuming the hydraulic fractures area represents the wellbore radius. Therefore, the governing equation for this flow derived from Darcy law is:

$$\Delta P_{RF} = \frac{q\mu B}{2\pi kh} ln(2r_e/L) \quad (5)$$

Elliptical flow

Elliptical flow regime indicates elliptical flow toward the fracture such as shown in Fig.3. This flow regime was described initially by Tiab 1994. It often occurs in the case of infinite conductivity fractures. However, it can be seen in a few cases of uniform flux fractures. This type of flow depends on the number of fractures and spacing between them. The governing equation for elliptical flow is:

$$\Delta P_{EF} = \frac{q\mu B}{2\pi kh}\left[ln\left(\frac{a+\sqrt{a^2-(L/2)^2}}{L/2}\right)+\left(\frac{I_{ani}h}{L} ln\left(\frac{I_{ani}h}{r_{we}(I_{ani}+1)}\right)\right)\right] \quad (6)$$

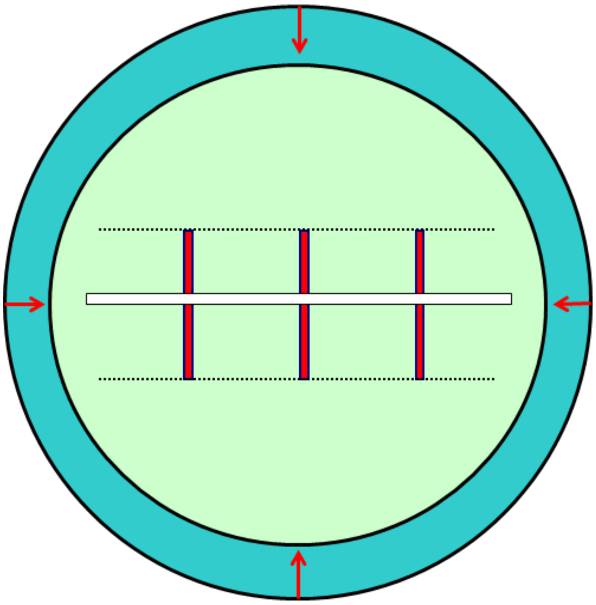
Figure 2: Pseudo- radial flow regime of hydraulic fractures.

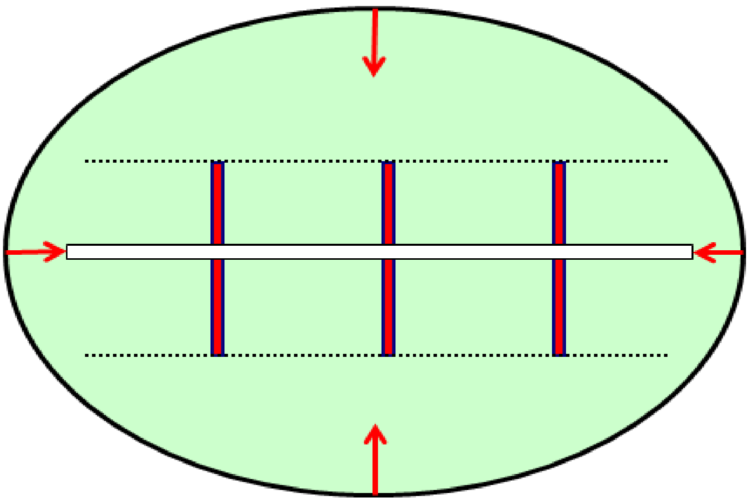
Figure 3: Elliptical flow regime for hydraulic fractures.

Formation linear flow

Reservoir fluids flow from the area around each fracture toward the fracture's vertical face in the vertical XZ plan as shown in Fig. 4. This flow takes place in the vicinity of the wellbore. It is one direction linear flow. The pressure drop caused by this flow can be written as:

$$\Delta P_{LF} = \frac{q\mu BLh}{2n^2 x_f h_f k} \qquad (7)$$

Fractures linear flow

This flow regime is observed inside the fractures where reservoir fluids flow linearly in one direction toward the wellbore as shown in Fig.4. In general, the pressure drop resulted from fractures linear flow is not significantly important at all times. The governing equation for the pressure drop of this flow regime can be approximated as:

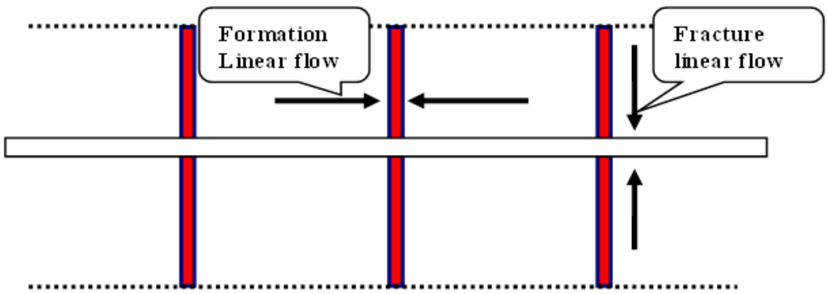

Figure 4: Formation and fracture linear flow.

$$\Delta P_{FLF} = \frac{q\mu B x_f}{2n w_f k_f h_f} \qquad (8)$$

Boundary dominated flow

For finite reservoir, boundary dominated flow definitely develops at late time when the production pulse reaches the boundary. The time for this flow to be developed depends on the reservoir properties. The equation that describes the pressure drop due to the boundary dominated flow in fractured formation is:

$$\Delta P_{BDF} = \frac{q\mu B}{2\pi k\left(\frac{1}{2}ln\left(\frac{4A}{\gamma C_{Af} r_f^2}\right)\right)} \quad (9)$$

$$r_f^2 = \frac{2Lx_f}{\pi} \quad (10)$$

Pressure drop due to skin factor

Skin factor has remarkable impact on the pressure drop of hydraulically fractured horizontal wells. Two types of skin are considered in this case. The first type is the mechanical skin factor resulted from drilling, completion, and fracturing process. The second one is the chock flow skin factor or the skin factor due to the moving from wide drainage area toward narrow one as the fluid reaches the fractures area. The following model is proposed by Brown and Economides (1992):

$$s = s_m + s_C = s_m + \frac{kh}{k_f w_f}\left[ln\left(\frac{h}{2r_w}\right) - \frac{\pi}{2}\right] \quad (11)$$

The pressure drop due to skin factor is:

$$\Delta P_S = \frac{q\mu B}{4n\pi k h_f} s \quad (12)$$

3-Total pressure drop

The total pressure drop in the wellbore can be estimated as the sum of different pressure drops caused by different flow regimes in addition to the pressure drop caused by skin factor:

$$\Delta P_t = \Delta P_{RF} + \Delta P_{EF} + \Delta P_{LF} + \Delta P_{FLF} + \Delta P_S \quad (13)$$

$$\Delta P_t = \frac{q\mu B}{2\pi kh}\Bigg[ln(2r_e/L) + ln\left(\frac{a + \sqrt{a^2 - (L/2)^2}}{L/2}\right) + \left(\frac{I_{ani}h}{L}ln\left(\frac{I_{ani}h}{r_{we}(I_{ani}+1)}\right)\right) + \frac{\pi hL}{n^2 x_f h_f} + \frac{\pi x_f kh}{nw_f k_f h_f} + \frac{1}{2n}\left[s_m + \frac{kh}{k_f w_f}\left(ln\left(\frac{h}{2r_w}\right) - \frac{\pi}{2}\right)\right]\Bigg] \quad (14)$$

Assuming constant flow rate and $(L = 2r_{eh})$, Eq. (14) can be written using dimensionless pressure as:

$$P_D = ln(2r_{e_D}) + ln(2.058) + \left(I_{ani}h_D\, ln\left(\frac{I_{ani}h_D}{r_{weD}(I_{ani}+1)}\right)\right) + \frac{\pi h_D}{n^2 x_{fD} h_{fD}} + \frac{\pi h_D}{n C_{fD} h_{fD}} + \frac{1}{2n}\left[s_m + \frac{h_D}{x_{fD}C_{fD}}\left\{ln\left(\frac{h_D}{2r_{wfD}}\right) - \frac{\pi}{2}\right\}\right] \quad (15)$$

The application of Eq. (15) to calculate the dimensionless pressure drop at the wellbore (PwD) for different dimensionless parameters are shown in figs. (5), (6), (7), and (8). The impact of the dimensionless reservoir radius (reD) is seen in Fig. (5) wherein the pressure drop increases with the increase of this radius. While the pressure drop is significantly declined when the dimensionless fracture length is increased as it inferred from Fig. (6). Similar decline behavior for the pressure drop is observed when the anisotropy of the reservoir is increased as it depicted from Fig. (7) and when the dimensioned hydraulic fracture conductivity is increased also as it illustrated in Fig. (8). It is important to emphasize that the relationships between these parameters and the pressure drop are all follow non-linear patterns except the reservoir anisotropy that exfibits linear behavior with for the pressure drop.

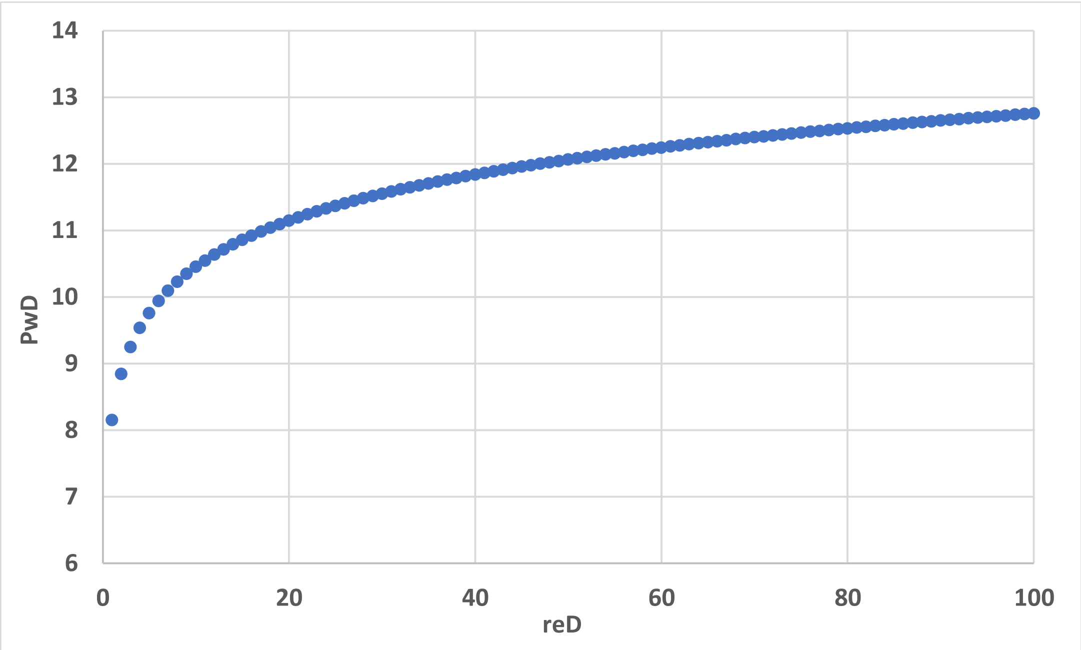

Figure 5: Pressure drop behavior with reservoir radius.

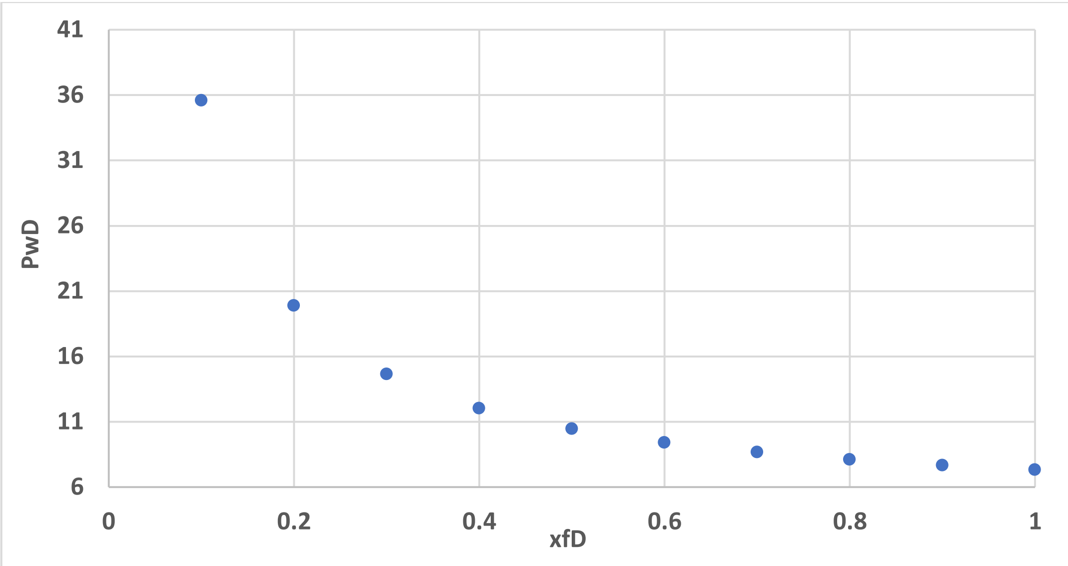

Figure 6: Pressure drop behavior with fracture length.

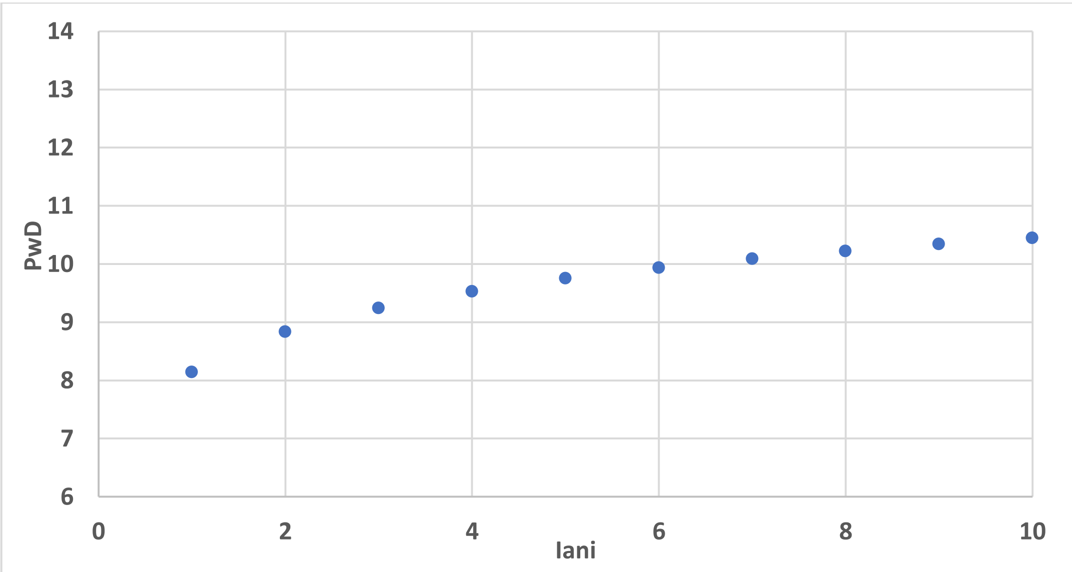

Figure 7: Pressurte drop behavior with reservoir anisotropy.

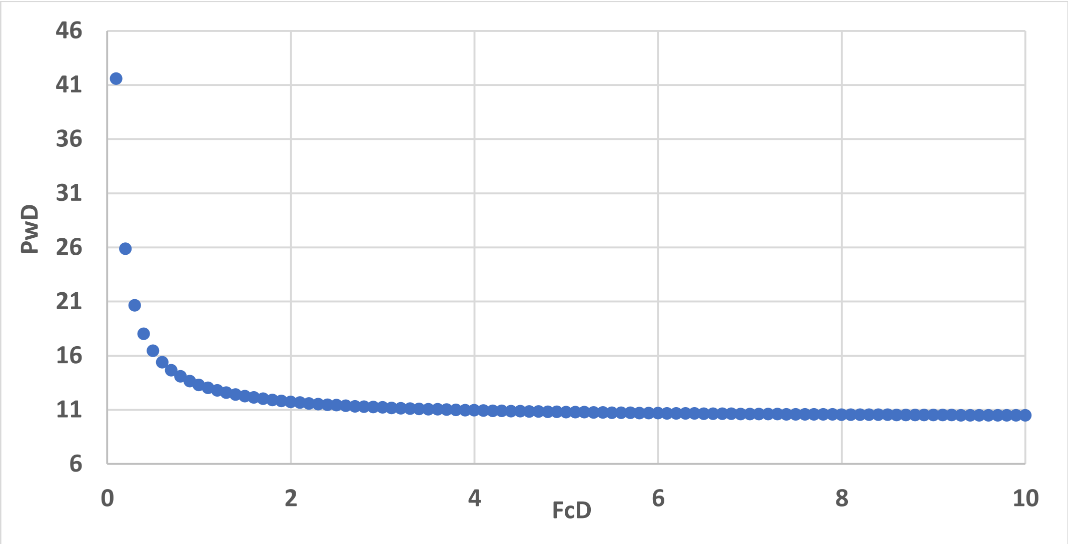

Figure 8: Pressurte drop behavior with fracture conductivity.

and it can also be written for the dimensionless flow rate assuming constant pressure drop as:

$$q_D = \frac{1}{\left[ln\left(2r_{e_D}\right) + ln(2.058) + \left(I_{ani}h_D\, ln\left(\frac{I_{ani}h_D}{r_{weD}(I_{ani}+1)}\right)\right) + \frac{\pi h_D}{n^2 x_{fD} h_{fD}} + \frac{\pi h_D}{nC_{fD}h_{fD}} + \frac{1}{2n}\left[s_m + \frac{h_D}{x_{fD}C_{fD}}\left\{ln\left(\frac{h_D}{2r_{wD}}\right) - \frac{\pi}{2}\right\}\right]\right]} \quad (16)$$

The pseudo-steady state productivity index of hydraulically fractured horizontal wells can be written in dimensionless form as:

$$J_D = \frac{1}{\left[ln\left(2r_{e_D}\right) + ln(2.058) + \left(I_{ani}h_D\, ln\left(\frac{I_{ani}h_D}{r_{weD}(I_{ani}+1)}\right)\right) + \frac{\pi h_D}{n^2 x_{fD} h_{fD}} + \frac{\pi h_D}{nC_{fD}h_{fD}} + \frac{1}{2n}\left[s_m + \frac{h_D}{x_{fD}C_{fD}}\left\{ln\left(\frac{h_D}{2r_{wD}}\right) - \frac{\pi}{2}\right\}\right]\right]} \quad (17)$$

In Field units, the productivity index is given by:

$$J = CJ_D \quad (18)$$

$$C = \frac{141.2\mu B}{kh} \quad (19)$$

Figs. (9), (10), (11), and (12) demonstrate the stabilized pseudo-steady state productivity index, in dimensionless form, for different dimensionless parameters. The productivity index is calculated using Eq. (17). Fig. (9) indicates a sharp decrease in the productivity index as the reservoir radius increases. However, figs. (10), (11), and (12) exhibit decreasing behavior for the productivity index when the fracture length, reservoir anisotropy, and the fracture conductivity are increased respectively.

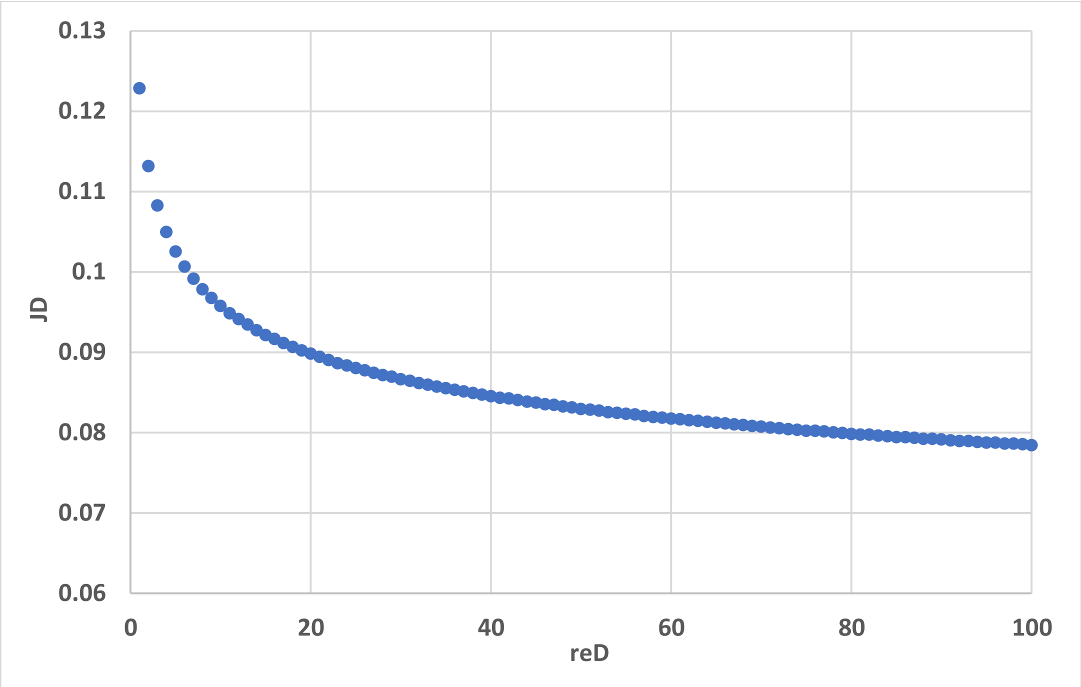

Figure 9: Productivity index behavior with reservoir radius.

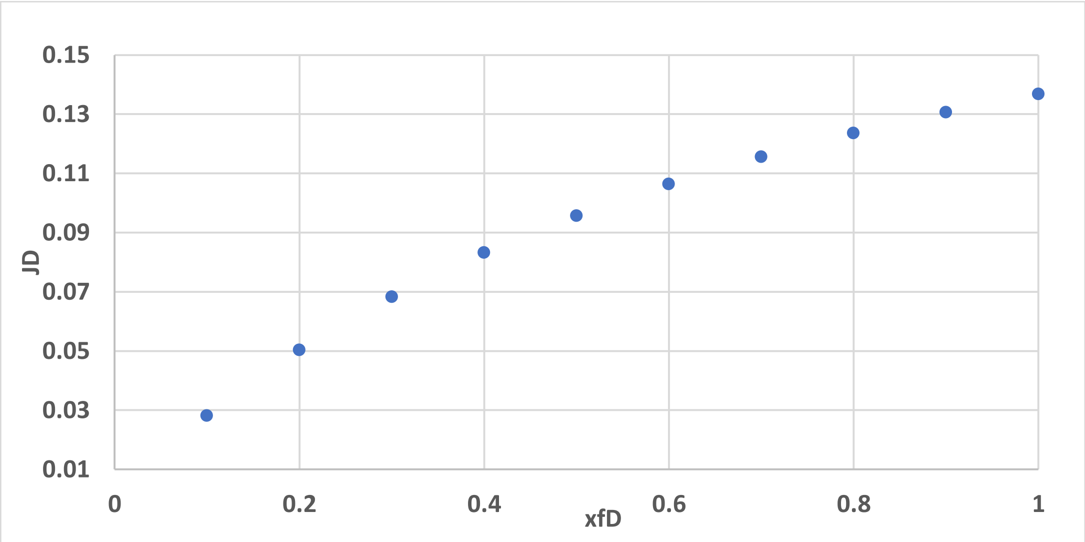

Figure 10: Productivity index behavior with fracture length.

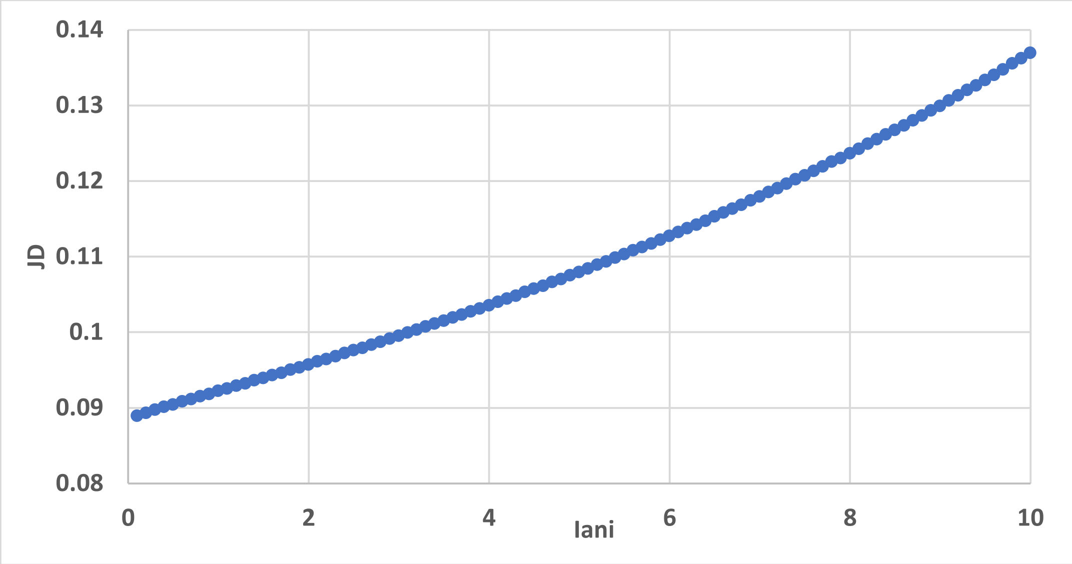

Figure 11: Productivity index behavior with reservoir anisotropy.

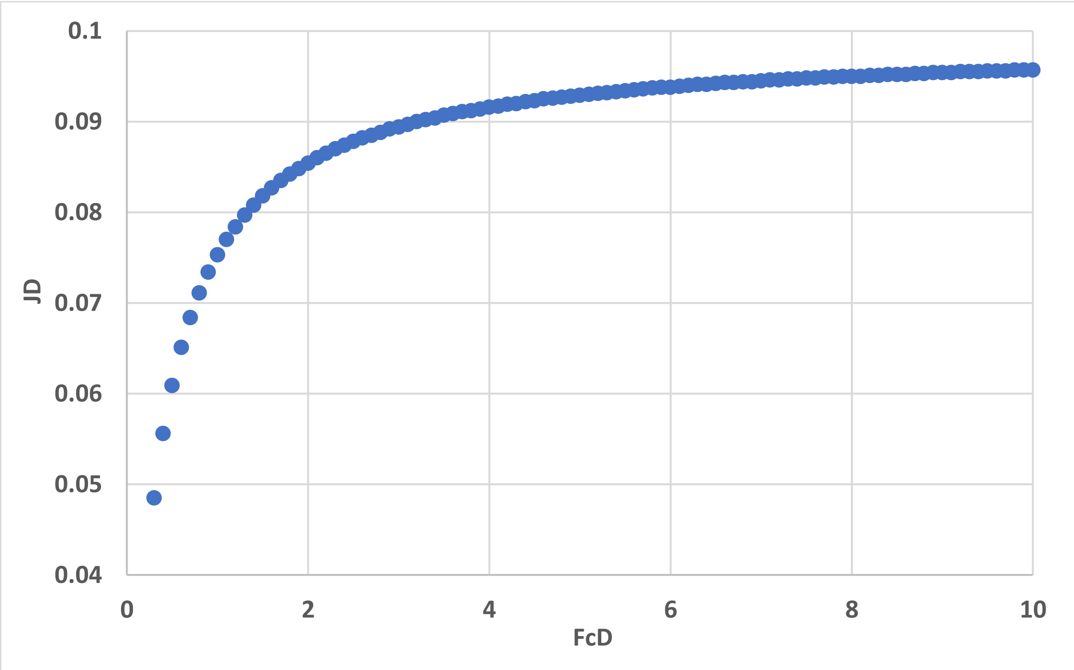

Figure 12: Productivity index behavior with fracture conductivity.

For the reservoirs having a rectangular drainage area rather than circular area as shown in Fig. 13, the dimensionless pseudo-steady state productivity index can be written as:

$$J_D = \frac{1}{\left[ln\sqrt{\left(2\pi x_{eD}y_{eD}/\gamma x_{fD}C_{Af}\right)}+ln\left(\frac{a+\sqrt{a^2-(L/2)^2}}{L/2}\right)+\left(I_{ani}h_D\,ln\left(\frac{I_{ani}h_D}{r_{weD}(I_{ani}+1)}\right)\right)+\frac{\pi h_D}{n^2x_{fD}h_{fD}}+\frac{\pi h_D}{nC_{fD}h_{fD}}+\frac{1}{2n}\left[s_m+\frac{h_D}{x_{fD}C_{fD}}\left\{ln\left(\frac{h_D}{2r_{wD}}\right)-\frac{\pi}{2}\right\}\right]\right]} \tag{20}$$

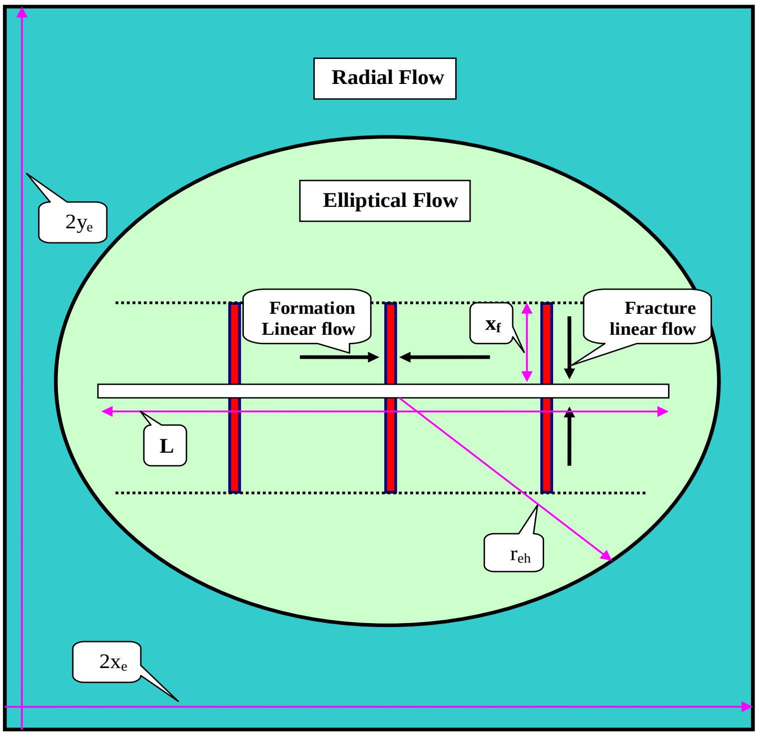

Figure 13: Horizontal well and hydraulic fractures in a rectangular reservoir.

Effect of proppant number

The modified proppant number is calculated based on different reservoir configurations. For rectangular shape drainage area, the proppant number is:

$$N_P = nI_xI_y\,C_{fD} \tag{21}$$

Using the definition of the abovementioned proppant number, the productivity index for rectangular reservoirs can be written as:

$$J_D = \frac{1}{\left[\ln\left(\sqrt{\frac{2\pi x_{eD}y_{eD}}{\gamma x_{fD}C_{Af}}}\right)+\ln\left(\frac{a+\sqrt{a^2-\left(\frac{L}{2}\right)^2}}{\frac{L}{2}}\right)+\left\{I_{ani}\,h_D ln\left(\frac{I_{ani}h_D}{r_{weD}(I_{ani}+1)}\right)\right\}+\frac{\pi h_D}{n^2x_{fD}h_{fD}}+\frac{4\pi h_D x_{fD}^2}{y_{eD}x_{eD}N_P h_{fD}}+\frac{1}{2n}\left[s_m+\frac{4nh_D x_{fD}}{x_{eD}y_{eD}N_P}\left\{\ln\left(\frac{h_D}{2r_{wD}}\right)-\frac{\pi}{2}\right\}\right]\right]} \tag{22}$$

The following indications can be inferred from the relationship between the proppant number and the productivity index:

1-There two impacts for the proppant number on productivity index as shown in Figs. 14, 15, 16 and 17: The first impact, for short fracture length, is positive i.e. the productivity index increase as the fracture length increases. The second one, for long fracture length, is negative i.e. the productivity index decreases as the fracture length increases. Therefore, for each proppant number there is certain value for the fracture length where the productivity index reaches its maximum value. The reason for this behavior refers to the fact that the increasing of the fracture length leads to increase the area of the fracture's surface exposures to the reservoir fluid. This definitely means increasing the flow rate and hence the productivity index. At same point, the productivity index is no longer increases with the increasing of fracture length. This point indicates the maximum productivity index for certain proppant number. Beyond this point, the increasing of the fracture length leads to decreasing the productivity index due to the fact the great percentage of reservoir fluid flows only to the outermost fractures only. No more fluid flows to the inner fractures, therefore, flow rate decreases and productivity index decreases also.

2-This impact is observed for different numbers of fractures and different horizontal wellbore penetration ratios.

3- For the designed productivity index and reservoir configurations, the fracture dimensions can be determined based on the maximum productivity index for each proppant number.

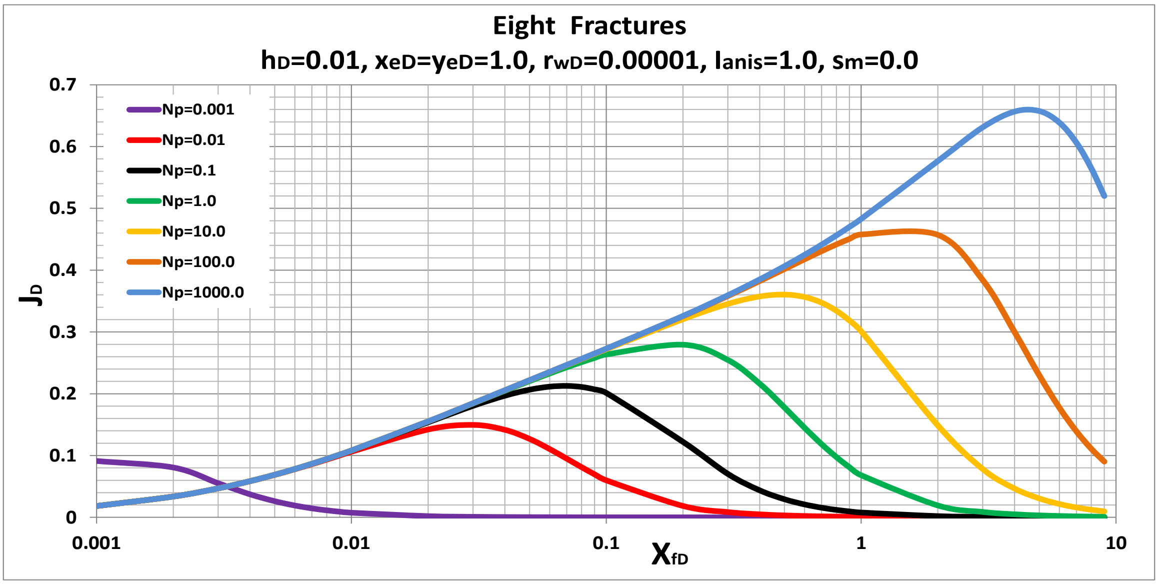

Figure 14: Productivity Index for Eight Fractures

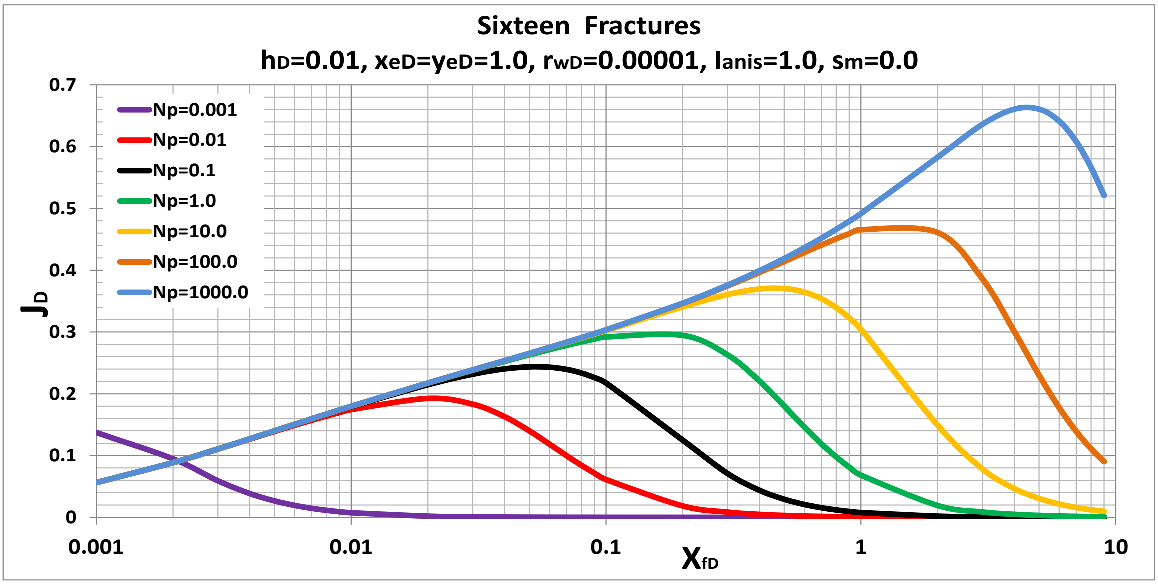

Figure 15: Productivity Index for Sixteen fractures.

Effect of vertical penetration ratio

Even though hydraulic fracturing process has been a common application in the petroleum industry during the last two decades, the final output of this process is significantly affected by several factors. The successful process has to produce maximum actual production from the total reserve in the formation. Fracture dimensions (half fracture length, fracture width, and fracture height) are of great importance in the performance as are the orientation of the fractures as well as the rock and fluid properties. Typically, it is preferred that the fracture height be equal to the formation height, where fully-penetrating fractures can be produced. Unfortunately, the fractures can not always penetrate totally the formation where partially penetrating fractures may be produced. Partially penetrating hydraulic fractures are undesirable stimulation process due to the possibility of reducing the expected production rate of the fractured formation. However, fully penetrating fractures in a reservoir with water and oil in contact may lead to an early or immediate water production. Therefore, partially penetrating fractures may be the only way to prevent the production of unwanted water.

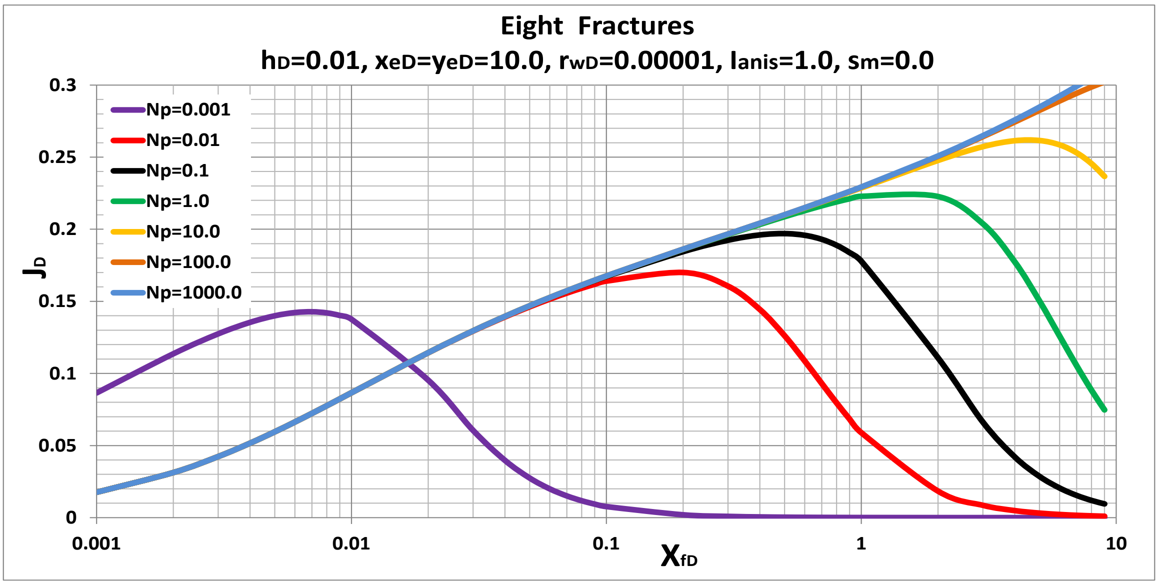

Figure 16: Productivity Index for Eight Fractures

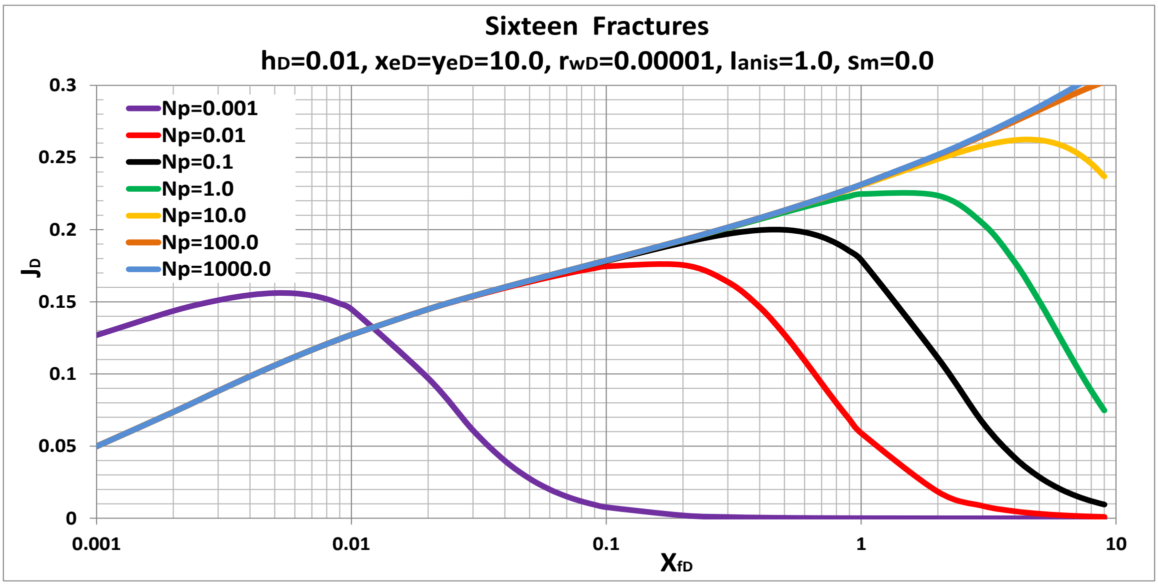

Figure 17: Productivity Index for Sixteen fractures.

The productivity index of fractured formation is greatly affected by the vertical penetration ratio. This ratio is defined as the parentage of fracture height to the formation height. It can be seen in Figs. 18, 19, 20 and 21 that the productivity index has similar behavior for the penetration ratio regardless the number of fractures or the type of reservoir. The following points are observed:

1- For all cases, there is no remarkable difference in the productivity index as the penetration ratio decreases for both short $(x_{fD} < 0.001)$ and long $(x_{fD} > 1.0)$ fracture length. For short-length fractures, the reason for this conclusion might be referred to the non-sensible change in the surface area of fracture that allows for reservoir fluid to through fractures. While for long-length fractures, the reason is the flowing of reservoir fluid toward the outermost fractures only when the penetration ratio increases gradually to reach the formation height.

2- The significant impact of the penetration ratio on productivity index is seen in the moderate fracture length where the productivity index increases as the penetration ratio increase. The reason for this behavior can be explained due to the fact that belongs to the increasing of the surface area of flow given by the fractures when the penetration ratio increases. As a result, the flow rate increase and the productivity index increases also.

3- The productivity index is constant ($J_D = 0.7$) for steady-state flow regardless the number of fractures, fracture conductivity as shown in Figs. 14 and 15, the horizontal wellbore penetration ratio and the vertical penetration ratio when the fracture length is long. The constant productivity index corresponds to the beginning of the pseudo-radial flow in the infinite acting reservoirs. In addition, the point at which ($J_D = 0.7$) can be used to determine fracture dimensions and proppant number.

4- The productivity index increases significantly at high values of fracture length as shown in Fig. 18 and 19. The relationship between productivity index and fracture length is positive constant slope straight line. The slope is almost equal to (5) regardless the number of fractures, fracture conductivity, vertical penetration ratio and horizontal wellbore penetration ratio. The reason for this behavior might be understood as the proportional decreasing in the flow rate and pressure drop. The beginning of the straight line is an indication for the fracture length at which the boundary dominated flow is reached.

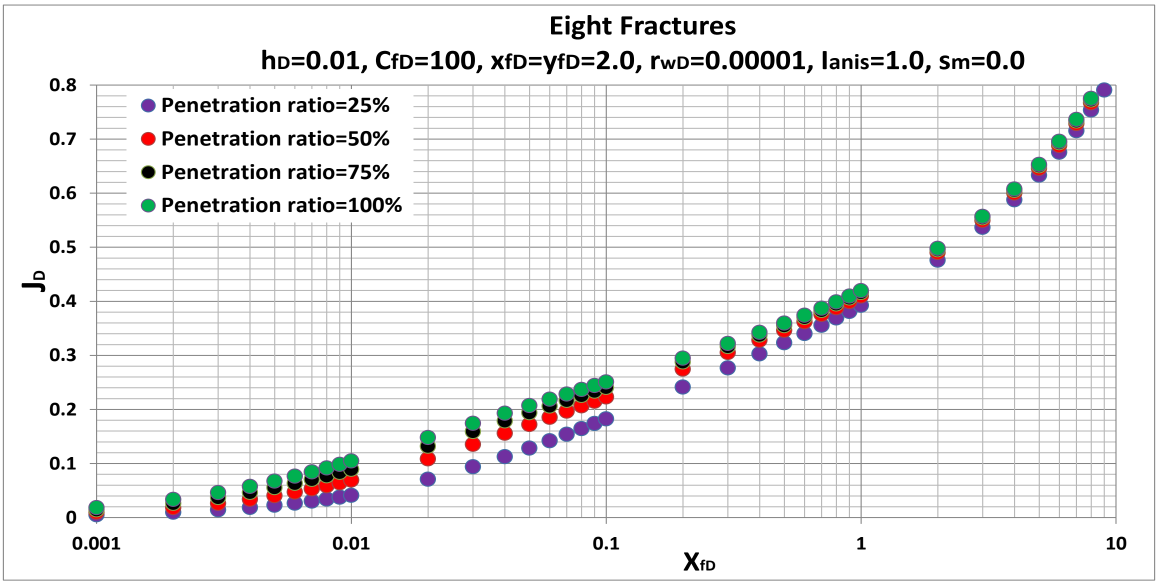

Figure 18: Pseudo-steady state Productivity Index for Eight Fractures

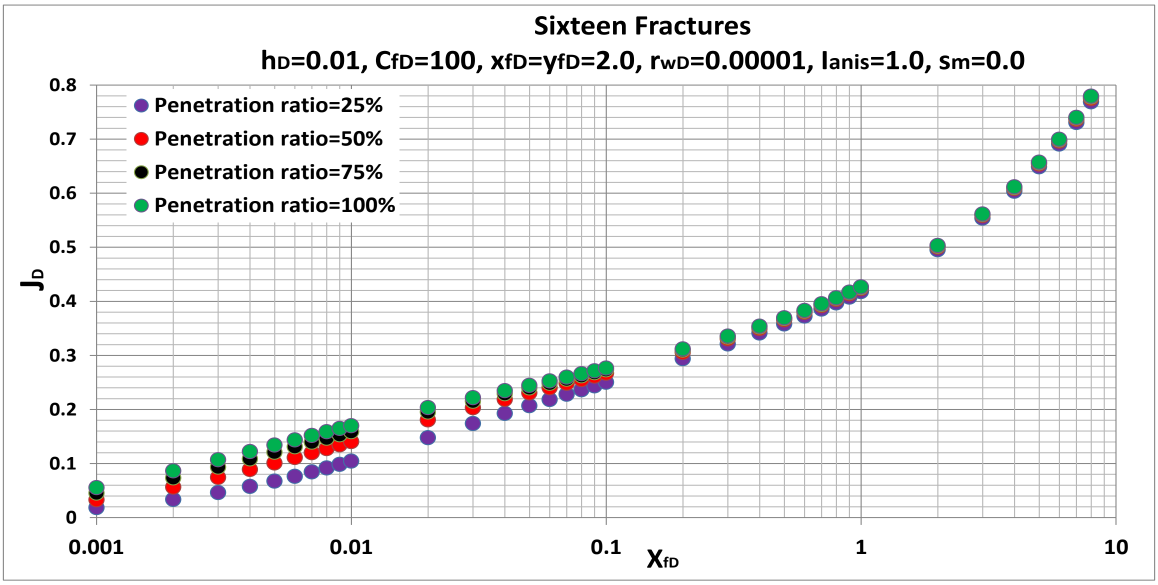

Figure 19: Pseudo-steady state Productivity Index for Sixteen fractures.

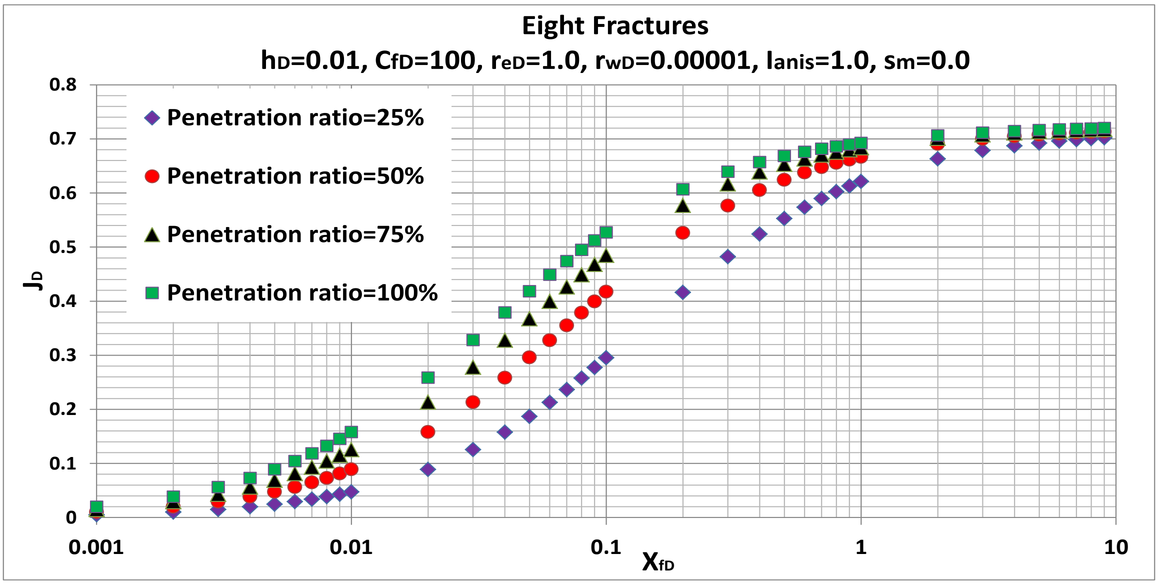

Figure 20: Productivity Index for Eight Fractures

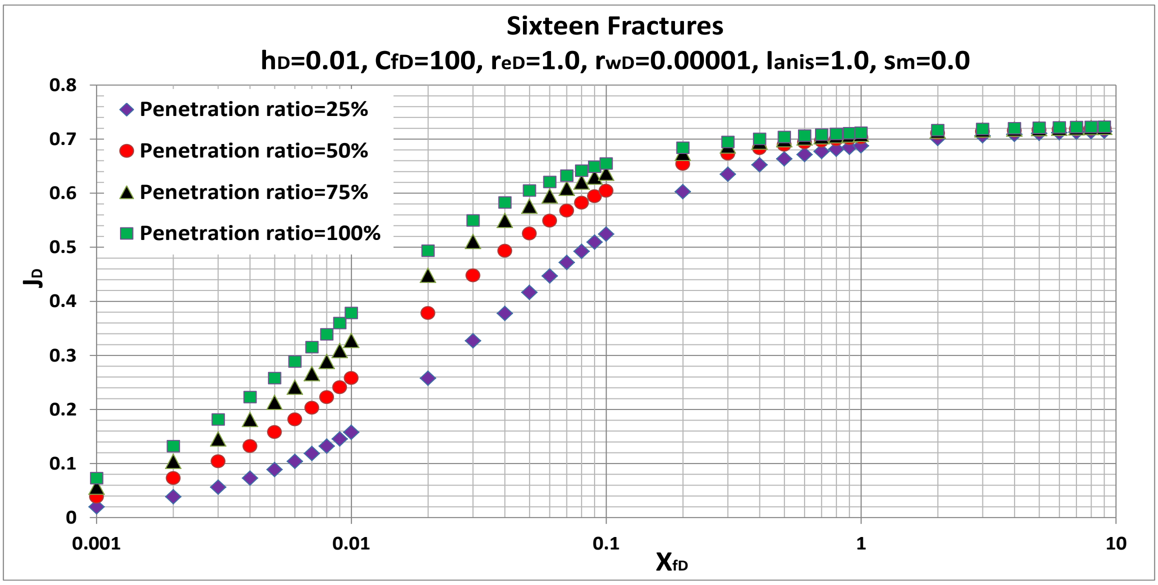

Figure 21: : Productivity Index for Sixteen fractures.

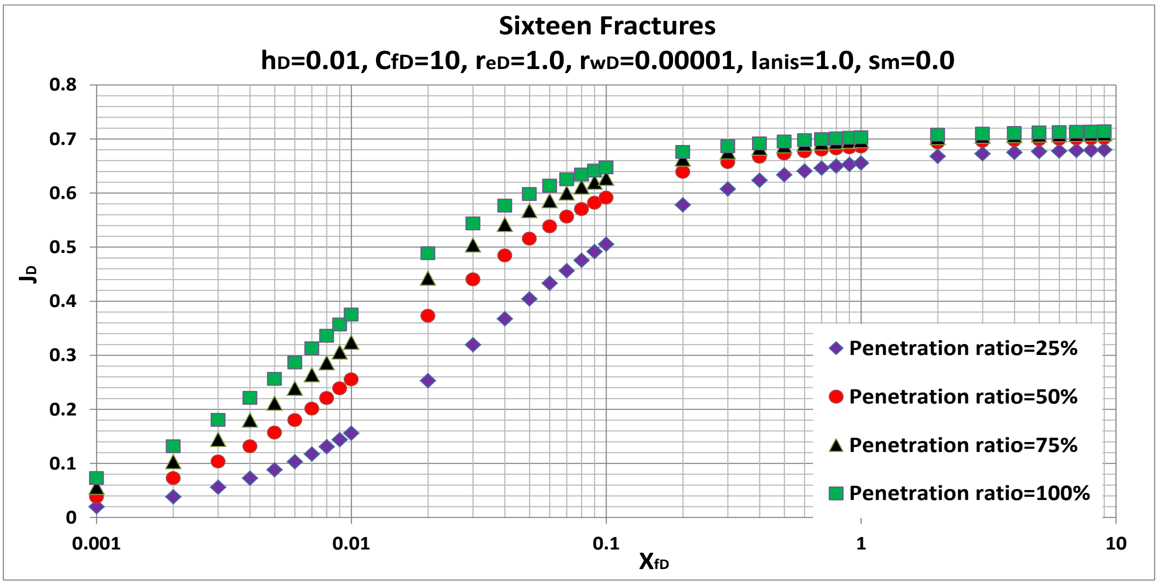

Figure 22: Productivity Index for Eight Fractures, CfD=10

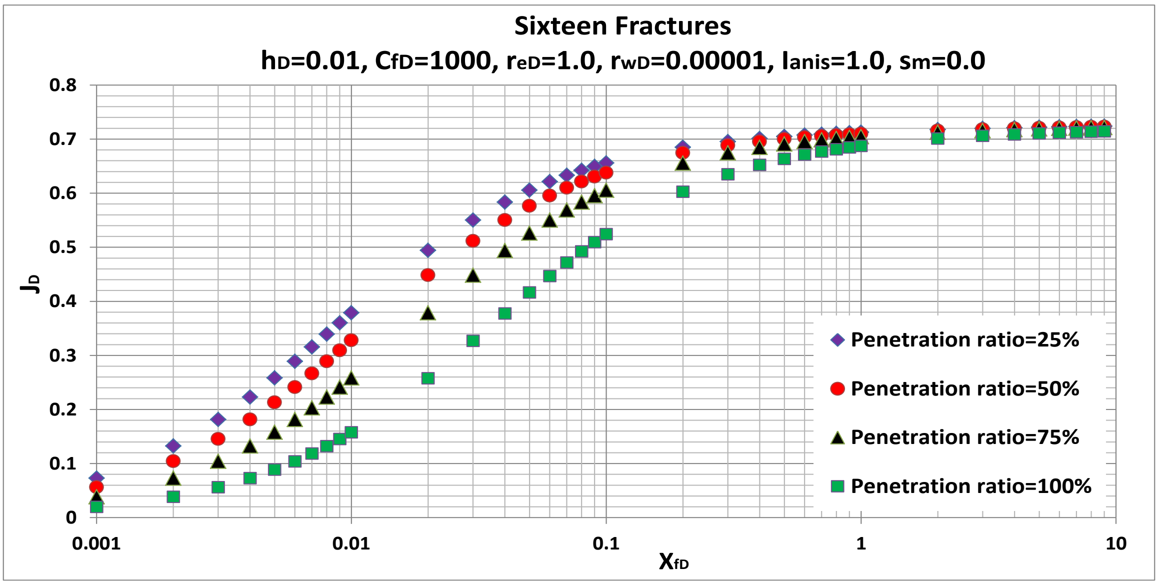

Figure 23: Productivity Index for Sixteen fractures, CfD=1000.

Effect of horizontal wellbore penetration ratio

Horizontal wellbore penetration ratio is defined as the ratio of the horizontal wellbore length to the reservoir boundary length parallel to the wellbore. The best scenario for this ratio is to be $\left(\frac{L}{2y_{eD}} = 1.0\right)$, i.e., the wellbore is fully penetrating the formation. This would increase the drainage area of the reservoir that can be undergone production. As this ratio decreases less than (1.0), the flow rate and the productivity index notably decrease as shown in Figs. 24 and 25. For short length fractures, there are no recognizable differences in the productivity index for different horizontal wellbore lengths. However, the differences are easily recognized for long length fractures. This behavior shows identical trend for different numbers of fractures and fracture conductivities.

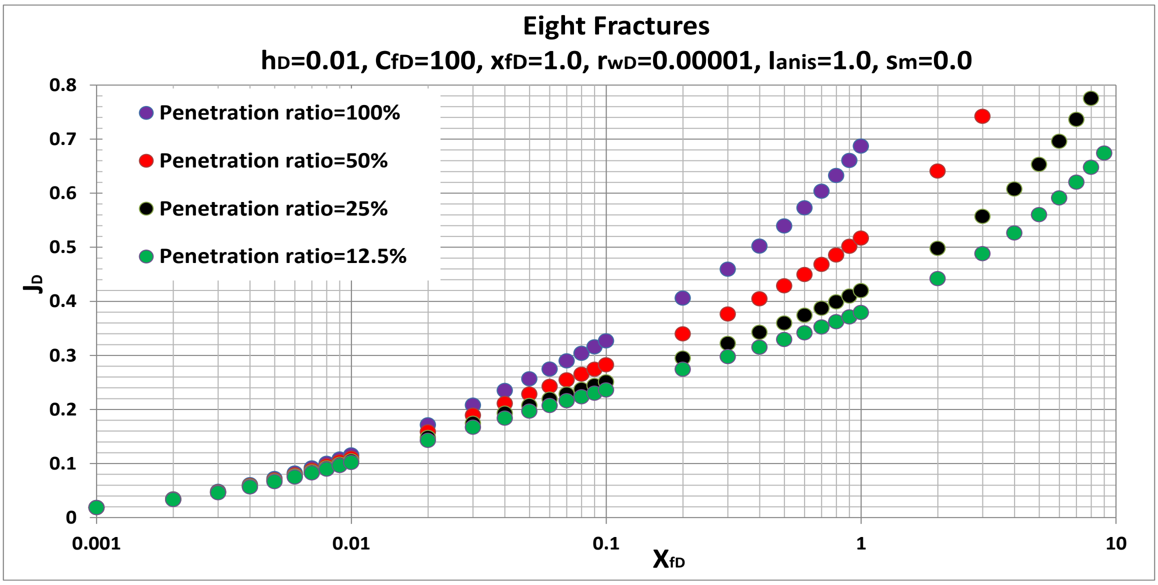

Figure 24: Productivity Index for Eight Fractures.

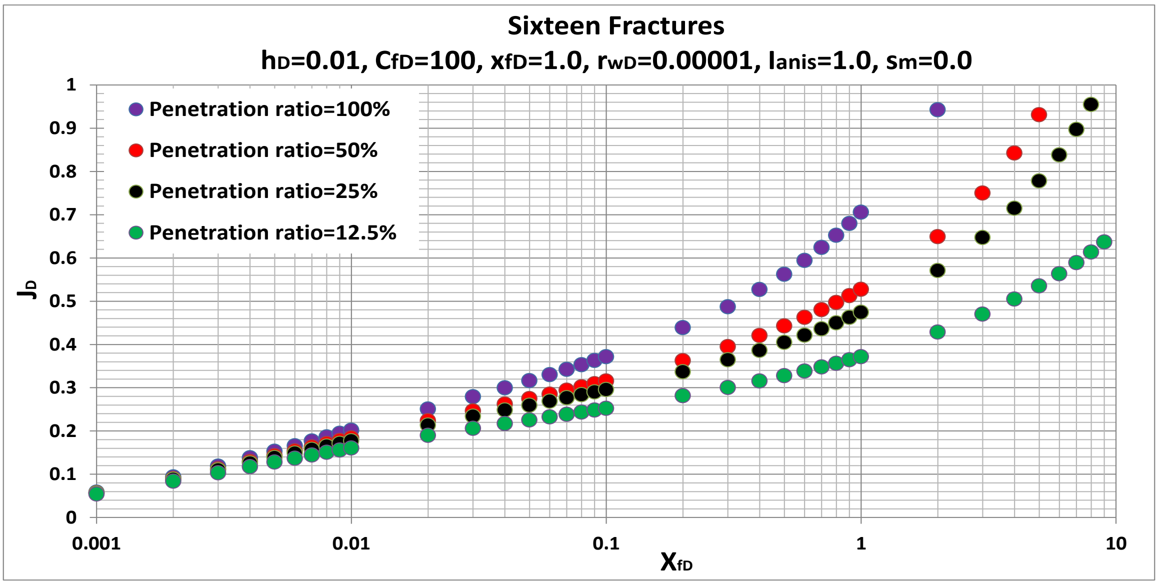

Figure 25: Productivity Index for Sixteen fractures.

Model application

The developed model has been examined for two field cases. The two cases have been taken from literatures (Li et al. 1996 and Guo and Yu 2008).

Case 1:

Pay zone thickness	46 ft
Reservoir permeability	1.3 md
Average reservoir pressure	2380 psi
Oil viscosity	3.5 cp
Horizontal wellbore length	939 ft
Wellbore radius	0.222 ft
Skin factor	4
Fracture length	
Fracture No. 1	836.4 ft (787.2ft*49.2ft)
Fracture No. 2	688.8 ft (590ft*98.4ft)
Fracture No. 3	705.2 ft (590ft*114.8ft)
Estimated fracture width	0.2 in
Fracture permeability	30000 md
Drainage Area	60 acres
Bottom hole flowing pressure	910 psi
Measured flow rate	41 STB/D
Formation volume factor	1.13res-bbl/STB

Due to the asymmetricity of fractures lengths, average fracture length has been used in the calculations $\left(x_{favg} = 371.73\ ft\right)$. The calculated parameters used in the calculations are given in Table-1.

Table 1: Calculated parameters used in case-1.

Parameter	Mathematical model	Result
r_e	$\sqrt{\frac{43560A}{\pi}}$	$912.3 ft$
a	$\frac{L}{2}\sqrt{0.5 + [0.25 + (2r_{eh}/L)^4]^{0.5}}$	974.6
r_{we}	$\sqrt{\frac{x_f h_f}{\pi}}$	$73.8 ft$
r_{eD}	r_e/L	0.9716
h_D	h/L	0.049
h_{fD}	h_f/L	0.049
x_{fD}	x_f/L	0.396
r_{weD}	r_{we}/L	0.0786
r_{wD}	r_w/L	0.000236
w_{fD}	w_f/L	0.00001775
C_{fD}	$\frac{k_f w_f}{k x_f}$	1.03465

Pseudo-radial flow is not expected to be developed. Therefore, elliptical flow, formation linear flow and fracture linear flow are the only flow regimes that expected to be developed. The dimensionless flow rate can be calculated as:

$$q_D = \frac{1}{[1.36 - 0.057 + 0.8813 + 1.0116 + 0.7279]} = 0.255$$

then, the calculated flow rate is:

$$q_{calculated} = \frac{q_D kh\Delta P_t}{141.2uB} = 40.12\ stb/day$$

The error can be estimated using the measured and calculated flow rate:

$$Error\% = \frac{41 - 40.12}{41} * 100 = 2\%$$

Case 2:

Pay zone thickness	39 ft
Reservoir permeability	7.5 md
Average reservoir pressure	2602 psi
Oil viscosity	4.8 cp
Horizontal wellbore length	1820 ft
Wellbore radius	0.19 ft
Skin factor	0
Fracture length	
Fracture No. 1	246 ft
Fracture No. 2	246 ft
Fracture No. 3	246 ft
Fracture No. 4	246 ft
Estimated fracture width	0.9 in
Fracture permeability	30000 md
Drainage Area	16 acres
Bottom hole flowing pressure	1279 psi
Measured flow rate	128 STB/D
Formation volume factor	1.084 res-bbl/STB

$$r_e = \sqrt{\frac{43560A}{\pi}} = 471.1ft$$

$$r_{we} = \sqrt{\frac{x_f h_f}{\pi}} = 39.1\ ft$$

Because $r_e < L/2$, the pseudo-radial flow regime close to the outer boundaries is not expected to occur. The configuration of the reservoir is rectangular (1820*383 ft). Therefore, elliptical flow, formation linear flow and fracture linear flow regimes only might be developed in the vicinity of the wellbore. Table-2 shows the calculated parameters that have been used in the calculations of case-2.

Table 2: Calculated parameters used in case-2.

Parameter	Mathematical model	Result
r_e	$\sqrt{\frac{43560A}{\pi}}$	$471.1ft$
r_{we}	$\sqrt{\frac{x_f h_f}{\pi}}$	$39.1ft$
h_D	h/L	0.0214
h_{fD}	h_f/L	0.0214
x_{fD}	x_f/L	0.0676
r_{weD}	r_{we}/L	0.0215
r_{wD}	r_w/L	0.0001
w_{fD}	w_f/L	0.0000412
C_{fD}	$\frac{k_f w_f}{k x_f}$	2.44

The dimensionless flow rate is:

$$q_D = \frac{1}{\left[\ln(2.058) + I_{ans} h_D \ln\left(\frac{I_{ans} h_D}{r_{weD}(Ians+1)}\right) + \frac{\pi h_D}{n^2 x_{fD} h_{fD}} + \frac{\pi h_D}{n C_{fD} h_{fD}} + \frac{1}{2n}\left[s_m + \frac{h_D}{x_{fD} C_{fD}}\left\{\ln\left(\frac{h_D}{2r_{wD}}\right) - \frac{\pi}{2}\right\}\right]\right]}$$

The dimensionless flow rate can be calculated as:

$$q_D = \frac{1}{[0.7217 - 0.015 + 2.904 + 0.322 + 0.05]} = 0.25$$

then, the calculated flow rate is:

$$q_{calculated} = \frac{q_D kh\Delta P_t}{141.2uB} = 132\ stb/day$$

The error can be estimated using the measured and calculated flow rate:

$$Error\% = \frac{132 - 128}{128} * 100 = 3\%$$

As a conclusion, the calculated flow rates by the proposed model showed good agreement with the measured flow rates for the two cases.

Conclusions

1- The modified proppant number and the two penetration ratios, the vertical and horizontal, have significant impacts on the productivity index of hydraulically fractured formations.

2- The impacts are identical regardless the number of fractures, fracture conductivity and reservoir configurations.

3- For certain proppant number, the productivity index has two different behaviors with fracture length. For short fracture length, the index increases with the increasing of fracture length. For long fracture length, the index decreases with the increasing of fracture length.

4- There is specific fracture length for each proppant number that gives maximum productivity index.

5- For infinite acting reservoir, the productivity index increases with the increasing of fracture length. At a certain fracture length, the index is no longer increased with fracture length. The maximum value of productivity index is (0.7). This value indicates the beginning of late radial flow.

6- For finite acting reservoir, the productivity index has constant slope straight line with long fracture length. The slope is almost (5). The beginning point of this line indicates the beginning of boundary dominated flow.

7- There are no significant impacts for the vertical penetration ratio on productivity index for both short and long fracture length. However, the remarkable impacts can be observed for moderate fracture length.

8- There is no significant impact for the horizontal wellbore penetration ratio on productivity index for short fracture length, but there is distinguished impact for long fracture length.

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Nomenclatures

A	Drainage area, acres.
a	Half the major axis of drainage ellipse and defined in Eq. (A-2), ft
B	Formation volume factor res-bbl/STB.
C_{Af}	Shape factor of fractured reservoir.
C_{fD}	Fracture conductivity
h	Reservoir height, ft.
h_f	Fracture height, ft.
I_{ani}	Anisotropy factor
k	Reservoir permeability, md.
k_f	Fracture permeability, md.
L	Horizontal wellbore length, ft.
N_P	Proppant number.
n	Number of fractures.
ΔP	Pressure drop, psi
q	Flow rate, STB/D.

r_e Reservoir radius, ft.

r_{eh} Hydraulic radius of drainage area, ft.

r_f Radius defined in Eq. (10), ft

r_w Wellbore radius, in.

r_{we} Radius defined in Eq. (A-3), ft

s skin factor.

x_f Fracture half length, ft.

x_e Reservoir width, ft

w_f Fracture width, in.

y_e Reservoir length, ft.

μ Viscosity, cp.

γ Constant

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Appendix-A: Symbols Definition

$$r_e = \sqrt{\frac{43560A}{\pi}} \qquad (A-1)$$

$$a = \frac{L}{2}\sqrt{0.5 + [0.25 + (2r_{eh}/L)^4]^{0.5}} \qquad (A-2)$$

$$r_{we} = \sqrt{\frac{x_f h_f}{\pi}} \qquad (A-3)$$

$$q_D = \frac{141.2q\mu B}{kh\Delta P_t} \qquad (A-4)$$

$$P_D = \frac{kh\Delta P_t}{141.2q\mu B} \qquad (A-5)$$

$$r_{eD} = \frac{r_e}{L} \qquad (A-6)$$

$$h_D = \frac{h}{L} \qquad (A-7)$$

$$h_{fD} = \frac{h_f}{L} \qquad (A-8)$$

$$x_{fD} = \frac{x_f}{L} \qquad (A-9)$$

$$r_{weD} = \frac{r_{we}}{L} \qquad (A-10) \qquad)$$

$$r_{wD} = \frac{r_w}{L} \qquad (A-11)$$

$$w_{fD} = w_f / L \qquad (A\text{ - }12)$$

$$C_{fD} = \frac{k_f w_f}{k x_f} \qquad (A-13)$$

$$x_{eD} = \frac{2x_e}{L} \qquad (A-14)$$

$$y_{eD} = \frac{2y_e}{L} \qquad (A-15)$$

$$I_x = \frac{x_f}{x_e} \qquad (A-16)$$

$$I_y = \frac{x_f}{y_e} \qquad (A-17)$$