



مجلة جمعية الليزر العراقية Iraqi Laser Society Journal

I.L.S.J.



Issue-1

3rd Year

2026

Estimation Model of Intercept in Use a Preliminary Test in Linear Regression

Shayma G. Salman, Baydaa A. Kuhdair, Hadeel M. Rasheed, Rasha S. Fayadh

Department of technology Research of Laser & Optoelectronics, Center research of Industrial application and Materials technology, Ministry of high Education / Scientific Research commission, Baghdad, Iraq

Email: Shayma.g.Salman@src.edu.iq

Email: baydaabd7@gmail.com

Abstract

Simple or multiple linear regression models contain three elements, each of which is variables, constants, and residuals. Among the common and commonly used constants in this model is the constant term or the intercept parameter, which is symbolized by the symbol β_0 . Due to the importance of the intercept parameter, we study and identify this parameter by using the preliminary test estimator to estimate the intercept parameter based on the Bayesian estimate, and showing the amount of bias (Bias) and the mean square error (MSE) of the estimator of the initial test of the intercept parameter. Then compare it with the least squares (OLS) method to demonstrate the efficiency of estimating the initial test. The MATLAB program will be added to solve the equations.

Keywords: standard linear regression, constant term, initial test value, least squares method.



مجلة جمعية الليزر العراقية

Iraqi Laser Society Journal

I.L.S.J.



Issue-1

3rd Year

2026

تقدير الحد الثابت في نموذج الانحدار الخطي باستخدام مقدار الاختبار الاولي

شيماء غازي سلمان ، بيداء عبد الحسين ، هديل ماجد رشيد، رشا سعد فياض

قسم بحوث الليزر والكهربويات ، مركز بحوث التطبيقات الصناعية وتكنولوجيا المواد، هيئة البحث العلمي، بغداد ، العراق

Email: : Shayma.g.Salman@src.edu.iq

Email: baydaabd7@gmail.com

الخلاصة

ان نماذج الانحدار الخطي البسيط او المتعدد يحتوي على ثلاث عناصر كل منهما وهي المتغيرات والثوابت والبواقي، ومن الثوابت الشائعة والمتداولة في هذا النموذج الحد الثابت او معلمة التقاطع والتي يرمز لها بالرمز β_0 . ولاهمية معلمة التقاطع نقوم بدراسة والتعرف على هذه المعلمة من خلال استخدام مقدر الاختبار الاولي (estimator preliminary test) لتقدير معلمة التقاطع بالاعتماد على تقدير بيز (estimate Bayesian)، وبيان مقدار التحيز (Bias) ومتوسط مربعات الخطأ (MSE) لمقدر الاختبار الاولي لمعلمة التقاطع ومن ثم مقارنته مع مقدار طريقة المربعات الصغرى (OLS) لبيان كفاءة تقدير الاختبار الاولي. وسيتم توظيف برنامج الماتلاب (MATLAB) لحل المعادلات.

الكلمات المفتاحية: الانحدار الخطي القياسي ، الحد الثابت ، مقدار الاختبار الاولي، طريقة المربعات الصغرى.

1.Introduction:

The linear regression model is one of the important statistical methods in the analysis and prediction of most phenomena. The equation that expresses this simple or multiple linear regression model contains three elements: variables, constants, and residuals. Among the common constants used in this model is the constant term or the intercept parameter. There are other cases that do not in it, the fixed term is written. The origin point fulfills the regression equation, since fitting a non-zero segment may lead to generating an inefficient model (1 Johan 1976)

The principle of linear regression can applied to fit a predictive model to an observed data set of response values and explanatory variables.

There are many studies and researches that have dealt with the subject of the regression model and methods for estimating its parameters.

The model is represented by the traditional estimators and the Bayes method estimator. The following is a presentation of some of these studies. In 2002, Al-Hasnawi and Al-Safawi presented a study on the issue of interpreting the parameters of a linear

regression model when it follows random errors (2 Al-Azzawi 2002). In 2004, Al-Qassab presented a study that included the process of estimating the parameters of the General Liner Model (GLM) and the apparently unrelated linear regression model (Sure) (3 Al-Qassab 2004), In (2007) Kevin. presented a study that included the use of prior distributions based on the probability function (4 Kevin p 2007). In 2009, Kazem presented the estimation of multiple regression model parameters based on two methods: OLS and goal programming (5 Kazem 2009). In (2016), Al-Obaidi used the Bayes estimator for the normal scaling parameter under the assumption of different initial distributions (6 Al-Obaidi 2016). In 2017 Ebru Turgal, and Beyza Doganay compared the results of models that include intercept and don't include intercepted term on hypothetic data sets. The approaches are demonstrated via both simulation studies. A simulation-based investigation is carried out under various scenarios, and the results obtained from this study (7 Ebru. T 2017). In 2019, Perron and Yamamoto pointed out the need to examine changes in model coefficients and error variance jointly, as they demonstrated that handling these two types of changes with a two-step testing approach can lead to large deviations in test size and loss of statistical power (8 Perron, P., & Yamamoto, Y. 2019). In 2018, Interest

also began to shift towards high-dimensional data, with Cowell finally proposing an effective method for estimating the breakpoint using regularization techniques without the need for extensive network research (9 Kaul, A., Jandhyala, 2018). In 2022, a unified framework for detecting discontinuities in high-dimensional regression models was introduced by Bai and Safikhani. This framework is characterized by theoretical consistency and its ability to accurately determine the number and location of discontinuity points in big data environments (10 Bai, Y., & Safikhani, A. 2022). These cases also include other expanded scenarios involving multiple equations, including compatibility with complex application environments (11 Schweikert, K. 2022).

As a result of the importance of the fixed term in econometric models, we will study and identify this parameter by using the preliminary test estimator to estimate the intercept parameter based on the Bayesian estimate, indicating the amount of bias (bias), and the mean square error. (MSE) for the initial test estimator of the intercept parameter and then compare it with the magnitude of the Ordinary Least Squares method (OLS) to demonstrate the efficiency of the initial test estimate of the initial test magnitude for the parameter β_0 .

2. Econometric Regression Model:

Simple linear regression includes three elements: variables, constants, and residuals. One of the important constants is the constant term, which we symbolize as β_0 . The constant term is considered an important element in economic measurement models, so it cannot be neglected and excluded for the following reasons (1 Johan 1976):

Excluding the fixed term in econometric models is due to a decrease in the accuracy of the estimates, which leads to an enlargement of the value of the statistic. Likewise, excluding this term affects the value of R^2 and makes it negative in some cases. Excluding it also affects and changes the values of the parameters of the explanatory variables.

If the fixed term is excluded from the model, this leads to a violation of some of the classical assumptions of regression analysis regarding the residuals that are assumed to have an expected value equal to zero.

The fixed term is important in the system of simultaneous equations, especially with regard to the diagnosis problem (12 M. Angeles 2014), as well as in other fields such as the dummy variable method.

If we assume that Y represents a dependent variable, that X represents the explanatory variables, and that U represents the error or disturbance term, then in this case the econometric model is:

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + U \quad \dots (1)$$

By rewriting the model using deviations from the averages ((X, X², Y), we obtain the following equation:

$$Y = \beta_1 X_1 + \beta_2 X_2 + U \quad \dots (2)$$

Whereas

$$X^2 = X_2 - X_2^-, \quad X = X_1 - X^-, \quad Y = Y - Y^-$$

The error term $U = u - \underline{u} \sim N(0, \sigma^2, X_1^2)$. (It is noted that the error term (U) suffers from the problem of heteroscedasticity. To get rid of this problem, we divide model (2) by the homogeneity amount $X_1 = \sqrt{x_1^2}$ so we get the equation:

$$y^* = \beta_1 + \beta_2 x^* + e \quad \dots (3)$$

Whereas

$$e = \frac{u}{x_1}, \quad x^* = \frac{x_2}{x_1}, \quad y^* = \frac{y}{x_1}$$

Equation (3) will have the following form

$$y^* = \beta_0 + \beta_1 x^* + e \quad \dots (4)$$

If we study the econometric model (4), where.

$$y_i = \beta_0 + \beta_1 x_i + e_i \quad i=1, 2, \dots, n \quad \dots (5)$$

And x_i are known and known quantities, e_i are independent random variables and are normally distributed with a mean of zero and an unknown variance, σ^2 The initial test value for the intercept parameter β_0 is known by the following formula:

$$\beta_0^* = \begin{cases} \underline{Y} & \text{if } |t| \leq R \\ R \hat{\beta}_0 & \text{if } |t| > R \end{cases} \quad \dots (6)$$

Whereas

$$\hat{\beta}_0 = \underline{Y} - \hat{\beta}_1 \underline{x}$$

$$\hat{\beta}_1 = \frac{\sum_{i=1}^n x_i y_i - n \underline{x} \underline{y}}{\sum_{i=1}^n x_i^2 - n \underline{x}^2}$$

$$\underline{x} = \frac{\sum_{i=1}^n x_i}{n}, \quad \underline{y} = \frac{\sum_{i=1}^n y_i}{n}$$

$$t = \hat{\beta}_0 \left[\frac{(n-2) \sum_{i=1}^n (x_i - \underline{x})^2}{S_0^2} \right]^{\frac{1}{2}}$$

$$S_0^2 = \sum_{i=1}^n \left[(y_i - \underline{y}) - \hat{\beta}_1 (x_i - \underline{x}) \right]^2$$

R represents the critical value of a test at a significance level of α and a degree of freedom of n-2.

3.The amount of bias and the relative efficiency of the parameter:

In order to find the bias of the initial test estimator for the fixed term β_0^* , it is defined by the formula:

$$E(\beta_0^*) = E(\underline{y})p_r(|t| \leq R) + E(\hat{\beta}_0)P_r(|t| > R) \dots (7)$$

But

$$p_r(|t| \leq R) + P_r(|t| > R) = 1$$

$$E(\beta_0^*) = E(\underline{y})p_r(|t| \leq R) + E(\underline{y} - \hat{\beta}_1\underline{x}) [1 - p_r(|t| \leq R)]$$

After a series of simplifications, since $\hat{\beta}_0$ is an unbiased expression of the parameter β_0 , it is defined by the following formula:

$$E(\beta_0^*) = \beta_0 + \underline{x}E(\hat{\beta}_1)p_r(|t| \leq R)$$

$$Bias(\beta_0^*) = \underline{x}E(\hat{\beta}_1)p_r(|t| \leq R) \dots (8)$$

For the purpose of finding the mean square error of the initial test value of the parameter β_0^* , it is defined by the following formula:

$$MSE(\beta_0^*) = E(\beta_0^*)^2 - [E(\beta_0^*)]^2 + [Bias(\beta_0^*)]^2 \dots (9)$$

$$E(\beta_0^*)^2 = E(\underline{y})^2 p_r(|t| \leq R) + E(\hat{\beta}_0)^2 p_r(|t| > R) \dots (10)$$

But

$$\beta_0^* = \underline{y} - \hat{\beta}_1\underline{x} \quad , \quad \underline{y} = \beta_0^* + \hat{\beta}_1\underline{x}$$

By substituting and performing a set of operations, we arrive at the following formula:

$$E(\beta_0^{*2}) = E(\hat{\beta}_1^2) + 2\underline{x}E(\hat{\beta}_0\hat{\beta}_1)p_r(|t| \leq R) + \underline{x}^2E(\hat{\beta}_1^2)p_r(|t| \leq R) \dots (11)$$

To obtain MSE (β_0^*), we substitute equation (9) and it is:

$$MSE(\beta_0^*) = E(\hat{\beta}_1^2) + 2\underline{x}E(\hat{\beta}_0\hat{\beta}_1)p_r(|t| \leq R) + \underline{x}^2E(\hat{\beta}_1^2)p_r(|t| \leq R) - [\beta_0 + \underline{x}E(\hat{\beta}_0)p_r(|t| \leq R) + \underline{x}^2E(\hat{\beta}_1)p_r(|t| \leq R)]^2$$

Since $E(\hat{\beta}_0) = \beta_0$, $Var(\hat{\beta}_0) = \frac{\sigma^2}{n} + \frac{\sigma^2\underline{x}^2}{c^2}$

Where

$$Var(\hat{\beta}_1) = \frac{\sigma^2}{c^2}$$

$$C = E(\hat{\beta}_0^2) = Var(\hat{\beta}_0) + \hat{\beta}_0^2 \sum (x_i - \underline{x})^2$$

$$MSE(\beta_0^*) = \frac{\sigma^2}{n} + \frac{\sigma^2\underline{x}^2}{c^2} + 2\underline{x}E(\hat{\beta}_0\hat{\beta}_1)p_r(|t| \leq R) + \underline{x}^2E(\hat{\beta}_1^2)p_r(|t| \leq R) - 2\underline{x}\beta_0E(\hat{\beta}_1)p_r(|t| \leq R) \dots (12)$$

To calculate the value of the equation $E(\hat{\beta}_0^2)p_r(|t| \leq R)$, we assume that $\hat{\beta}_0 \sim N(\delta, 1)$, $Vx^2(n - 2)$.

The two random variables V and U are

$$\text{independent, as } t = \frac{u}{\sqrt{\frac{v}{n-2}}}$$

$$f(u, V) = f(u) \cdot F(V)$$

$$f(u, V) = \frac{e^{-\frac{(u-\delta)}{2}} V^{\frac{n}{2}-1} e^{-\frac{v}{2}}}{\sqrt{2\pi} 2^{\frac{n}{2}-1} \Gamma\left(\frac{n}{2} - 1\right)}$$

assuming that

$$E(u) = \frac{c\beta_1}{\sigma} = \delta, \quad \hat{\beta}_1 = \frac{\sigma}{c} u, \quad u = \frac{\hat{\beta}_1}{\sigma}$$

Ban notes.

$$E(\hat{\beta}_1)pr(|t| \leq R) = E\left[\left(\frac{\sigma}{c}u\right)^i pr(|t| \leq R)\right]$$

$$E(\hat{\beta}_1) = \int_u \int_v \left(\frac{\sigma}{c}u\right)^i \frac{e^{-\frac{(u-\delta)}{2}} V^{\frac{n}{2}-1} e^{-\frac{v}{2}}}{\sqrt{2\pi} 2^{\frac{n}{2}-1} \sqrt{\frac{n}{2}-1}} dudv$$

$$E(\hat{\beta}_1) = \left(\frac{\sigma}{c}\right)^i \int_{-\infty}^{\infty} \int_{-\infty}^{\frac{u^2}{v} \leq \frac{R^2}{n-2}} u^i \frac{e^{-\frac{(u-\delta)^2}{2}} V^{\frac{n}{2}-1} e^{-\frac{v}{2}}}{\sqrt{2\pi} 2^{\frac{n}{2}-1} \Gamma\left(\frac{n}{2} - 1\right)} dudv$$

Using the transformations where $W = \frac{u}{v}$,

$$dv = \frac{u^2}{w^2} dw, \text{ then.}$$

$$E(\hat{\beta}_1) = \left(\frac{\sigma}{c}\right)^i \int_{-\infty}^{\infty} \int_0^{\frac{R^2}{n-2}} U^i \frac{e^{-\frac{(u-\delta)^2}{2}} \left(\frac{u^2}{W}\right)^{\frac{n}{2}-2} \frac{u^2}{w^2}}{\sqrt{2\pi} 2^{\frac{n}{2}-1} \Gamma\left(\frac{n}{2} - 1\right)} dw du$$

..... (14)

Whereas $\Gamma(n/2-1)$ is the gamma function, while $\gamma(m,z)$ is the incomplete gamma function, which is known as:

$$\gamma(m, z) = \int_0^z t^{m-1} e^{-t} dt \quad \dots\dots (15)$$

$$E(\hat{\beta}_1) = \left(\frac{\sigma}{c}\right)^i \int_{-\infty}^{\infty} \frac{u^i e^{-\frac{(u-\delta)^2}{2}} v^{n-2}}{\sqrt{2\pi} 2^{\frac{n}{2}-1}} \int_0^{\frac{R^2}{n-2}} (W)^{\frac{-n}{2}} e^{-\frac{u^2}{2w}} dw du$$

..... (16)

Assuming that:

$$dw = \frac{u^2}{2t^2} dt, \quad w = \frac{u^2}{2t}, \quad t = \frac{u^2}{2w}$$

Using the method of transformations, making some simplifications, and taking advantage of the relationship (15), we obtain the following formula:

$$\int_0^{\frac{R^2}{n-2}} (W)^{\frac{-n}{2}} e^{-\frac{u^2}{2w}} dw = \frac{u^{-n+2}}{2^{\frac{-n}{2}+1}} \left[1 - \frac{\gamma\left(\frac{n}{2}-1, \frac{(n-2)u^2}{2R^2}\right)}{\Gamma\left(\frac{n}{2}-1\right)} \right] \dots\dots (17)$$

By substituting relationship (17) into equation (16), we obtain the following formula:

$$E(\hat{\beta}_1)pr(|t| \leq R) = \left(\frac{\sigma}{c}\right)^i \int_{-\infty}^{\infty} \frac{U^i e^{-\frac{(u-\delta)^2}{2}} v^{n-2}}{\sqrt{2\pi}} g(n, R, u) du$$

..... (18)

whereas:

$$g(n, R, u) = 1 - \frac{\gamma\left(\frac{n-1}{2}, \frac{(n-2)u^2}{2R^2}\right)}{\Gamma\left(\frac{n-1}{2}\right)} \dots\dots (19)$$

$$E(\hat{\beta}_1)pr(|t| \leq R) = \left(\frac{\sigma}{c}\right)^i E[U^i g(n, R, U)] \dots\dots (20)$$

Through equation (20), it is possible to find the amount of bias for the parameter, β_0^* which is:

$$Bias(\beta_0^*) = \underline{x} \frac{\sigma}{c} E[Ug(n, R, u)] \dots\dots (21)$$

The following amount must be found:

$$E(\hat{\beta}_0 \hat{\beta}_1)pr(|t| \leq R) = E\left[\left(\underline{y} - \hat{\beta}_1 \underline{x}\right) \hat{\beta}_1\right] pr(|t| \leq R) \dots\dots (22)$$

Since $y_i = \beta_0 + \beta_1 x_i + e_i$, and taking the averages of this equation, $\underline{y} = \beta_0 + \beta_1 \underline{x}$, and where $\underline{e} = 0$

Using these relationships in equation (22), we get:

$$E(\hat{\beta}_0 \hat{\beta}_1)pr(|t| \leq R) = (\beta_0 + \beta_1 \underline{x}) E(\hat{\beta}_1)pr(|t| \leq R) - \underline{x} E(\hat{\beta}_1^2)pr(|t| \leq R) \dots\dots (23)$$

Substituting all terms into the mean squared error equation and performing some operations, we get:

$$MSE(\beta_0^*) = \sigma^2 \left[\frac{1}{n} + \frac{x^2}{c^2} + \frac{2x^2 \delta}{c^2} E(ug(n, R, u)) - \frac{x^2}{c^2} E(u^2 g(n, R, u)) \right] \dots\dots (24)$$

To obtain the relative efficiency of the initial test estimator relative to the least square's method estimator

$$eff(\beta_0^*) = \frac{MSE(\hat{\beta}_0)}{MSE(\beta_0^*)}$$

$$eff(\beta_0^*) = \frac{\frac{1}{n} + \frac{x^2}{c^2}}{\frac{1}{n} + \frac{x^2}{c^2} + \frac{2x^2 \delta}{c^2} E(ug(n, R, u)) - \frac{x^2}{c^2} E(u^2 g(n, R, u))}$$

We can easily notice that the MSE approaches when R approaches zero, that is

$$\frac{MSE(\beta_0^*)}{\sigma^2} = \frac{MSE(\underline{y})}{\sigma^2} = \frac{1}{n} + \frac{x^2}{c^2}$$

While the MSE (β_0^*) approaches MSE ($\hat{\beta}_0$) when the value of R approaches infinity, that is.

$$\frac{MSE(\beta_0^*)}{\sigma^2} = \frac{MSE(\hat{\beta}_0)}{\sigma^2} = \frac{1}{n} + \frac{x^2 \delta^2}{c^2}$$

4. Practical aspect:

The MATLAB program was used to calculate the amount of bias for the initial test estimator, calculate the mean square error, and then calculate the relative efficiency of this estimator relative to the least squares method estimator and for several values of the constants on which these equations depend. The program was implemented for the following values: n = 5, 8, 15. As for the values of β_1 was as follows (0.5, 1, 2), and for the values adopted for the constant c it was (10, 30). The critical value

of the t-test was determined by a percentage of 0.05 and a degree of freedom of n-2, while the values of $\underline{\chi}^2$ that we adopted in the program It is (4,16), and the values of

the variable u were found based on the values of c, β_1 and populated enough data and the results in the table are as follows:

Table 1 showing the amount of bias and the relative efficiency of the estimator with respect to c, β_1, R, n at a significance level of $\alpha = 0.05$

N	$\hat{\beta}_1$		0.5		1		2	
	C	$\underline{\chi}^2$	4	16	4	16	4	16
5	10	RE	6.12	3.01	5.80	2.05	7.51	3.67
		Bias	0.75	1.26	0.51	1.03	0.53	0.83
	30	RE	7.25	2.34	6.13	2.01	7.82	3.25
		Bias	1.43	1.72	1.31	1.06	0.61	0.81
10	10	RE	4.03	1.21	4.75	2.82	4.32	3.95
		Bias	0.28	1.01	0.81	1.26	0.63	0.92
	30	RE	3.27	0.78	3.22	1.98	3.78	2.71
		Bias	1.62	2.01	1.75	1.93	1.17	1.64
18	10	RE	1.07	0.92	1.51	1.02	1.97	1.01
		Bias	1.23	1.94	1.30	1.97	0.93	1.92
	30	RE	1.02	0.61	1.53	1.09	1.73	1.46
		Bias	2.01	2.23	1.85	1.92	1.54	1.76

5. Conclusions:

The model give same conclusions which can get from table(1).

We note that the relative efficiency of the initial test estimator (β_0^*) decreases and decreases as the sample size n increases. This means that the amount of the initial test is a decreasing function of the sample size.

It is clear from the table that the amount of bias of the initial test estimator decreases as the value of β_1 increases, and that the amount of bias increases as the value of C increases.

We also notice that the relative efficiency of the initial test estimator increases as the value of β_1 increases. This indicates that the magnitude of the initial test is an increasing function with respect to the values of β_1 . As is clear from the table, the relative efficiency of the initial test estimator decreases as the value of C increases.

We recommend developing this research to be a more comprehensive and general research that takes into account many sample sizes and different values for the level of significance, in addition to using many values of β_1 .

References:

- [1] John Stewart, (1976), "Understanding Econometric", Hutchinson and Co.Ltd., London, pp (77-78).
- [2] Al-Azzawi, Dijla Ibrahim & Al-Khairi, Kawthar Muhammad, (2002), "A Comparative Study to Determine the Best Method for Estimating the Parameters of a Linear Regression Model When the Random Error Has a Multivariable (t) Distribution", Iraqi Journal of Statistical Sciences, College of Computer Science and Mathematics, University of Mosul, Issue 3/Volume2.
- [3] Al-Qassab, Osama Mohammed Jassim, (2004) "Using the Beta and Exponential Distributions in Estimating Parameters for a Linear Regression Model", PhD dissertation in Statistics, College of Administration and Economics, University of Baghdad
- [4] Kevin P. Murphy,(2007), "Conjugate Bayesian analysis of the Gaussian distribution", murphyk.
- [5] Kazim, Amouri Hadi & Muslim, Basim Shleibeh, (2009), "Advanced Economic Measurement: Theory and Application", Al-Taif Press, Baghdad
- [6] Al-Obaidi, Janan Abbas Nasser, (2016), "Bayes Estimator for the Measurement Parameter of the Normal Distribution under the Assumption of Different Initial Distributions", Journal of Economic and Administrative Sciences, Issue 92/Volume 22.
- [7] Ebru Turgal, and Beyza Doganay, "Include or Exclude a Constant Term in Regression Analysis", Conference: 3rd International

- Researchers, Statisticians and Young, Statisticians Congress (IRSYSC 2017).
- [8] Perron, P., & Yamamoto, Y. (2019). Pitfalls of Two-Step Testing for Changes in the Error Variance and Coefficients of a Linear Regression Mode
- [9] Kaul, A., Jandhyala, V., & Fotopoulos, S. (2018). An Efficient Two-Step Algorithm for High-Dimensional Change Point Regression Models
- [10] Bai, Y., & Safikhani, A. (2022). A Unified Framework for Change Point Detection in High-Dimensional Linear Models
- [11] Schweikert, K. (2022). Detecting Multiple Structural Breaks in Systems of Linear Regression Equations
- [12] M. Angeles Carnero "Estimation Statistics and Introduction to Econometrics" Chapter 7: The Multiple Regression Model Year 2014-15