مقارنة مُقدِّرات تباين ARCH في ظل توزيعات شرطية مختلفة: تطبيق على أسعار النفط العراقي

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ECONOMICS AND ADMINISTRATIVE STUDIES JOURNAL (EASJ)

EISSN: 3005-3625, PISSN: 2790-2560, DOI: http://doi.org/10.58564/EASJ/5.1.2026.20

Comparing ARCH Variance Estimators Under Different Conditional Distributions: An Application to Iraqi Oil Prices

مقارنة مُقدِّرات تباين ARCH في ظل توزيعات شرطية مختلفة: تطبيق على أسعار النفط العراقي

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تاريخ الاستلام: 2026/01/19   تاريخ القبول: 2026/02/08   تاريخ النشر: 2026/03/01

Received: 19/01/2026   Accepted: 08/02/2026   Published: 01/03/2026

المستخلص:

يُعد تحليل السلاسل الزمنية أحد أهم الأدوات الإحصائية في التنبؤ. وتُظهر الكثير من بيانات السلاسل الزمنية، وخاصة في القطاع المالي، تباينًا وعدم خطية. تتعامل نماذج التباين الشرطي التلقائي (ARCH) مع السلاسل الزمنية التي تعاني من عدم استقرار التباينات. عد دراسة مقدرات التباين لنماذج ARCH الخطوة الأولى في عملية بناء نموذج أكثر دقة للسلاسل الزمنية. تبدأ عملية التحليل بمرحلة تشخيص النموذج، تليها مرحلة التقدير ومرحلة فحص مدى الملاءمة، ثم استخدام النموذج المرشح في التنبؤ. تهدف هذه الورقة إلى مقارنة نموذج التنبؤ لنماذج ARCH مع أربعة أنواع من التوزيعات الإحصائية، وهي (التوزيع الطبيعي، وتوزيع t، وتوزيع t الملتوي، وتوزيع الخطأ العام)، وذلك للحصول على النموذج الأكثر دقة. تم إجراء المقارنة على بيانات حقيقية تمثل بيانات يومية عن سعر برميل النفط العراقي (بالدولار) لفترة من (2008-2012) مع 1256 قيمة عينة. أظهرت النتائج أن نموذج ARCH ذو الرتبة (2) وتوزيع t المعمم المنحرف هو أفضل نموذج تنبؤي.

الكلمات المفتاحية : نماذج ARCH، التنبؤ الإحصائي، التوزيع الطبيعي، توزيع t، توزيع t الملتوي، توزيع الخطأ العام، متوسط مربع الخطأ

Abstract:

Time series is one of the important statistical topics in the analysis of forecasting, which leads to identification and interpretation of their behavior, and the more accurate the description of the features of the process leads that to generates the time series with greater the possibility of building an appropriate model for explain behavior of that phenomenon. Research deals with the study the Variance estimators for (Autoregressive Conditional Heteroskedasticity (ARCH)) and time series models that aim to Variance modeling with process of building these models which depends on the analysis of the time series Analysis process begins with the diagnosis stage of the model followed by the stage of estimating and stage of examining extent of suitability and then using the candidate model in prediction. Aim of the research is to reach the best prediction model for ARCH models with four types of statistical distributions, namely (normal distribution, t distribution, Skewed t distribution and general error distribution). The comparison was made on real data (Iraqi oil prices in dollars). The results showed that the ARCH model (2) When a

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Skewed generalized t distribution is the best predictive model. The aim of the research Scientific addition from the research Theoretical and practical framework for the research results.

Keywords: ARCH Models, Statistical Forecasting, normal distribution, t distribution, Skewed t distribution ,general error distribution, mean square error

Introduction

The subject of studying time series is one of the important topics, especially in analyzing the behavior of different phenomena and their interpretation, by studying and analyzing their historical development over different time periods with the aim of predicting the future of these phenomena. Time series analysis has been applied in many fields such as economic and financial fields, as the financial data and some economic data are mostly characterized by their volatility that occurs most often in the high-frequency financial time series, which makes models of (Box and Junkies(BJ)) gives imprecise results by assume that the variance of the random boundary over time is constant, or what is known as the homoskedasticity assumption.

In financial time series, this condition often does not exist, as variance and volatility appear in different periods of the series. The models that deal with this type of variance are the ARCH models that provide the modeling of conditional variance or (heteroskedasticity) which was proposed by (Angle) in (1882) while In(1987)introduced(Bollerslev)(Generalized ARCH models(GARCH)). When studying this type of models, we must have an idea of the shape or form of the curve of the frequency distribution and the direction of stacking the repeats, in other words, the measurement of the kurtosis coefficient, so using normal distribution in these models leads to tapered kurtosis

The research included distributions in which the tails are heavier than the normal distribution (heavy tails), which in turn are appropriate with these models. The goal is to reach the best distribution that follows the ARCH models of the lower degrees, especially the first and second degree (ARCH (1), ARCH (2)) when it is followed Errors with normal and abnormal distributions.

1-Time Series Models

The time series is a set of observations arranged during a period of time, Time Series Model is the function that links the values of the time series with previous values and errors, it is usually used to predict the future values of the phenomenon studied based on what happened in the past. The temporal is stable or stationary if it has a stationary computational medium around which data is collected free from the influence of the trend such that [2,3]:-

$$\mu = E y_t \quad \dots (1)$$

If the time series is un stationary, then its probabilistic properties are affected by time due to the existence of a general trend.

EISSN: 3005-3625, PISSN: 2790-2560, DOI: doi.org/10.58564/EASJ/5.1.2026.20

The economic and financial time series are often considered un stationary because they generally move in a direction and fluctuation.

Price movement has various different characteristics, it may be either normal or very high, as well as The time that prices spend in their direction is sometimes lengthened and sometimes shortened, and the trend is identified by looking at the highest and lowest points in the movement of price , which makes it have several mediums around which the data fluctuates, so it must be transformed into stationary time series easy to analysed by take the following transformation[2,4]:-

$$x_t = \mathrm{Ln}(p_t) - Ln(p_{t-1}) \approx \frac{P_t - P_{t-1}}{P_{t-1}} = \mu + \varepsilon_t \quad \dots (2)$$

With

P_t *represent the time series*

x_t *represent* Return series

μ *represent average series returns*

by Subtracting (μ)we get a series that suffers only from volatility, which is the aim of this research

2- Autoregressive Conditional Heteroscedastic (ARCH).

This kind of time series models with degree ($m \geq 1$) can be expressed as the following form[1,7] :-

$$x_t = \mu + \varepsilon_t \quad \dots (3)$$

$\varepsilon_t = \sigma_t \delta_t$

$\delta_t \sim D_\vartheta(0,1)$

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \cdots + \alpha_m \varepsilon_{t-m}^2 \quad \dots (4)$$

With

($i = 1,2, \dots m$ *when* $\alpha_i \geq 0$ and $\alpha_0 > 0$)

(m *represent degree of the model*)

$\mathcal{D}_\vartheta$ *represent probability density function for regenerative errors*

ϑ *represent dstribution parameter*

The necessary and sufficient conditions for fully stationary ARCH (m) models are when they are [5,6]

$\alpha_0 > 0 , \mathrm{E}\varepsilon_t^2 < \infty , \mathrm{E}\varepsilon_t^4 < \infty$

With

$$\sum_{i=1}^{m} \alpha_i < 1$$

3-ARCH Conditional Probability Distributions

A number of standard distributions (mean of zero and unit variance) commonly used in these models have been assumed. Before mentioning the distributions Unconditional standard distribution must be converted to the conditional distribution with the heterogeneity variance (ε_t) by the following form [8,9]:-

$$z_t = \frac{\varepsilon_t}{\sigma_t} \quad \dots (5)$$

EISSN: 3005-3625, PISSN: 2790-2560, DOI: doi.org/10.58564/EASJ/5.1.2026.20

And the distribution

$$f(\mathcal{E}_t) = f(z_t)|J|$$

With

$$J = \frac{\partial \mathcal{E}_T}{\partial z_T} = \frac{1}{\sigma_t}$$

4-The Statistical Distributions

Many statistical distributions can be proposed in this field such that[10] :-

4-1 *Gaussian* distribution (GD)

The probability density function can be[11,12]:-

$$f(\varepsilon_t) = \frac{1}{\sqrt{2\pi\sigma_t^2}} \exp\left(-\frac{1}{2}\frac{\varepsilon_t^2}{\sigma_t^2}\right) \quad \dots (6)$$

With mean equal zero and variance (σ_t^2)

4-2 Student's t distribution (STD)

The probability density function can be [13,14]:-

$$f(\varepsilon_t) = \frac{\Gamma\left(\frac{v+1}{2}\right)}{\sqrt{\pi(v-2)\sigma_t^2}\Gamma(v/2)}\left(1+\frac{\varepsilon_t^2}{\sigma_t^2(v-2)}\right)^{-\frac{(v+1)}{2}} \quad \dots (7)$$

With mean equal zero and variance (σ_t^2)

Such that ($2 < v < \infty$) and ($\Gamma(*)$) represent Gamma function

4-3 General Error Distribution (GED)

The distribution was proposed by (Nelson) in (1991) with the following probability density function[15]:-

$$f(\varepsilon_t) = \frac{v\, 2^{-(1+\frac{1}{v})}}{\lambda\, \Gamma(\frac{1}{v})\, \sqrt{\sigma_t^2}}\; e^{-\frac{1}{2}\left|\frac{\varepsilon_t}{\lambda\sigma_t}\right|^v} \quad \dots (8)$$

Such that

$$\lambda = \sqrt{\frac{2^{-\frac{2}{v}}\Gamma\left(\frac{1}{v}\right)}{\Gamma\left(\frac{3}{v}\right)}}$$

The parameter(v) determines the shape curve of the(GED) distribution

If (v <1) the distribution has Heavy tails than the normal distribution

If (v> 2) the distribution has a thinner tail than a normal distribution

If (v = 2) the distribution approaches the normal distribution

4-4 skewed Student's t distribution (SSTD)

The skewness of t-distribution proposed by (Hansen) in(1994) is given by the following probability density function [16]:-

$$f(\varepsilon_t) = b\,\frac{\Gamma\left(\frac{v+1}{2}\right)}{\sqrt{\pi(v-2)\sigma_t^2}\Gamma(v/2)}\left(1+\frac{\zeta^2}{(v-2)}\right)^{-\frac{(v+1)}{2}} \quad (9)$$

EISSN: 3005-3625, PISSN: 2790-2560, DOI: doi.org/10.58564/EASJ/5.1.2026.20

$$\zeta = \begin{cases} \frac{(bz_t+a)}{1-\lambda} & \text{if} \quad \varepsilon_t < -\frac{a}{b} \\ \frac{(bz_t+a)}{1+\lambda} & \text{if} \quad \varepsilon_t \geq -\frac{a}{b} \end{cases}$$

$$b = \sqrt{1 + 3\lambda^2 - a^2}$$

$$a = 4\lambda c \frac{v-2}{v-1}$$

$$c = \frac{\Gamma\left(\frac{v+1}{2}\right)}{\sqrt{\pi(v-2)\sigma_t^2}\Gamma\left(\frac{v}{2}\right)}$$

(λ) represent (skew) parameter with ($-1 < \lambda < 1$)

With (λ=0) means that distribution will be (STD)

5- Identification Stage

There are many identification tests for ARCH models such that(ARCH test) this test was proposed by (Engle) in (1982) to detect disturbances that follow the ARCH model it based on the Lagrange multiplier (LM), and the test statistic is based on the determination coefficient (R^2) of the regression model according to the following method

Let

$$\eta_t = \varepsilon_t^2 - \sigma_t^2$$

Substitutions in an equation (4) and getting

$$\varepsilon_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \eta_t \qquad \dots (10)$$

$$t = 1,2,..k$$

The above formula represents the Autoregressive Model of order (p) and

(k)represents the number of observations

and (η_t) is an unrelated series.

null (H_0) and alternative (H_1) hypothesis will be

$$H_0: \alpha_1 = \alpha_2 = \cdots . = \alpha_p = 0$$

$$H_1: \alpha_1 \neq \alpha_2 \neq \cdots \neq \alpha_p \neq 0$$

(P) is the number of variables in the autoregressive equation and represent the square of the remainder at the displacement (p). The test statistic ($k \times R^2$) for the adopted model is distributed parallel as chi-square distribution with a degree of freedom (p).

6- Estimation Stage

After model identification stage the estimation stage comes, as the (ARCH)time series models are generally non-linear models by the parameters there are several methods of estimation and the most popular of these methods are conditional maximum likelihood (CML) which depend on the error distribution to be one of the following

6-1 Gaussian Maximum Likelihood (GML)

When the variable (Z_t) follows the standard normal distribution and ($Z_t = \frac{\varepsilon_t}{\sigma_t}$) then the natural logarithm function of the parameter vector will be:-

EISSN: 3005-3625, PISSN: 2790-2560, DOI: doi.org/10.58564/EASJ/5.1.2026.20

$$L(\theta) = \log \mathcal{L}(\theta) = \sum_{t=1+p}^{T} \mathcal{L}_t(\theta)$$

$$\log l(\theta) = -\frac{1}{2}\log 2\pi - \frac{1}{2}\log \sigma_t^2 - \frac{1}{2}\frac{\varepsilon_t^2}{\sigma_t^2} \quad \dots (11)$$

The partial derivative for logarithmic of conditional maximum likelihood, function will be:-

$$\frac{\partial \mathcal{L}(\theta)}{\partial \theta} = \sum_{t=1}^{n} \frac{1}{2\sigma_t^2}\frac{\partial \sigma_t^2}{\partial \theta}\left(\frac{\varepsilon_t^2}{\sigma_t^2} - 1\right) \quad \dots (12)$$

And the parameter vector

$$\theta = (\alpha_0 . \alpha_1 \dots \dots . . \alpha_m)$$

By equating equation (12) to zero, notice that the resulting equations are non-linear, so the estimators can be found by using numerical iterative metthods.

6-2 Student's t Maximum Likelihood (STML)

When the variable (Z_t) follows the Student's t distribution and ($Z_t = \frac{\varepsilon_t}{\sigma_t}$) then the natural logarithm function of the parameter vector

$$L(\theta) = \sum_{t=p+1}^{T} \left(\log\Gamma\left(\frac{v+2}{2}\right) - log\left[\Gamma\left(\frac{v}{2}\right)\right] - 0.5\left\{\left[log[\pi(v-2)] + \log(\sigma_t^2) + (1+v)log\left(1 + \frac{\varepsilon_t^2}{\sigma_t^2(v-2)}\right)\right]\right\}\right) \quad \dots (13)$$

6-3 General Error Maximum Likelihood (GEML)

When the variable (Z_t) follows the Student's t distribution and ($Z_t = \frac{\varepsilon_t}{\sigma_t}$) then the natural logarithm function of the parameter vector

$$L(\theta) = \sum_{t=p+1}^{T} \left(log(v/\lambda) - 0.5\left|\frac{\varepsilon_t}{\sigma_t \lambda}\right|^{v} - (1+v^{-1})log(2) - log\left(\Gamma\left(\frac{1}{2}\right) - 0.5log\,(\sigma_t^2)\right)\right) \quad \dots (14)$$

6-4 skewed Student's t Maximum Likelihood (SSTML)

When the variable (Z_t) follows the skewed Student's t distribution with location parameter (v) and skew parameter (λ) and($Z_t = \frac{\varepsilon_t}{\sigma_t}$) then the natural logarithm function of the parameter vector

EISSN: 3005-3625, PISSN: 2790-2560, DOI: doi.org/10.58564/EASJ/5.1.2026.20

$$L(\theta) = \sum_{t=p+1}^{T} \left(\log\Gamma\left(\frac{v+2}{2}\right) - log\left[\Gamma\left(\frac{v}{2}\right)\right] - 0.5\left\{\left[log[\pi(v-2)] + \log(\sigma_t^2) + (1+v)log\left(1+\frac{D^2}{\sigma_t^2(v-2)}\right)\right]\right\}\right) \quad \dots (15)$$

Such that

$$D_t = \frac{(b\frac{\varepsilon_t}{\sigma_t}+a)}{(1-\lambda s)}$$

$$s = \begin{cases} 1 & z_t < -\frac{a}{b} \\ -1 & z_t \geq -\frac{a}{b} \end{cases}$$

7- Diagnostic checking Stage

In this step, estimated model is verified to be appropriate model for the data, and this done by examining the autocorrelation parameters of residual chain of the model by calculating the following formula:

$$\tilde{z}_t = \frac{\varepsilon_t}{\hat{\sigma}_t} \quad \dots (16)$$

This stage can be done by using one of the following tests :-

7-1 Ljung – Box test

The Ljung - Box test was used as a nonlinear statistic to test the effects on time series data. The statistic of the test proposed in (1978) by the researchers (Ljung & Box) and known according to the following formula:

$$\text{LB}(m) = n(n+2)\sum_{k=1}^{m} \frac{\rho_{\tilde{z}_t}^2(k)}{n-k} \quad \dots (17)$$

($m\ represent\ number\ of\ offsets$)

($n\ represent\ sample\ size$)

$\rho_{\tilde{z}_t}(k)\ can\ be\ found\ by\ using\ the\ following\ formula$

$$\rho_{\tilde{z}_t}(k) = \frac{\sum_{t=k+1}^{n}(\tilde{z}_t - \bar{\tilde{z}}_t)(\tilde{z}_{t-k} - \bar{\tilde{z}}_t)}{\sum_{t=1}^{n}(\tilde{z}_t - \bar{\tilde{z}}_t)^2} \quad \dots (18)$$

$$k = 1,2 \dots, n-1$$

with the following hypotheses

$H_0: \rho_1 = \rho_2 = \cdots . = \rho_m = 0$

$H_1: \rho_1 \neq \rho_2 \neq \cdots \neq \rho_m \neq 0$

The autocorrelation coefficients for the residuals that are excluded from the estimated model of the data are calculated. If the residues are independently and identically distributed (IID) then the distribution aligned to the test statistic ($LB(m)$)is chi-square of the degree of freedom (p)

$LB(m) \sim \chi^2(m-p)$

EISSN: 3005-3625, PISSN: 2790-2560, DOI: doi.org/10.58564/EASJ/5.1.2026.20

7- The test statistic ($LB(m)$)is compared with the tabular values of chi-square ($\chi^2(m-p)$) at the level of significance(α)

if ($LB(m) > \chi^2(m-p)$) this means rejecting of(H_0)and errors are not random

if ($LB(m) < \chi^2(m-p)$) this means un-rejection of (H_0) and the residual series is random.

2 Weighted Ljung – Box test

This test was founded in (2012) by the researchers (Fisher & Gallagher), which is a weighted formula for the Ljung - Box test and that the test statistic is given by the following formula:

$$F(m) = n\sum_{k=1}^{m}\left(\frac{\mathrm{m}-\mathrm{k}+(\mathrm{p}+1)}{\mathrm{m}}\right)\rho_{\tilde{z}_t}^2(k) \quad \dots(19)$$

That ($F(m)$) distributed Gamma with (α, β) with the following formula

$$\alpha = \frac{3}{4}\frac{(\mathrm{m}+\mathrm{p}+1)^2(\mathrm{m}-\mathrm{p})}{2\mathrm{m}^2+3\mathrm{m}+2\mathrm{mp}+2\mathrm{p}^2+3\mathrm{p}+1}$$

$$\beta = \frac{2}{3}\frac{2\mathrm{m}^2+3\mathrm{m}+2\mathrm{mp}+2\mathrm{p}^2+3\mathrm{p}+1}{m(m+p+1)}$$

8- Forecasting Stage

In the previous stages, the estimation and diagnosis of the time series to the ARCH model, where this model represented the best representation, and we have reached the last stage, which represents the main goal of this study, which it is the stage of predictions with the following general formula of the prediction process:-

$$\hat{h}_{t+\mathrm{L}}^2 = \alpha_0\frac{1-\left(\sum_{i=1}^{p}\hat{\alpha}_i\right)^L}{1-\left(\sum_{i=1}^{p}\hat{\alpha}_i\right)} + \left(\sum_{i=1}^{p}\hat{\alpha}_i\right)^{\mathrm{L}-1}\sigma_{\mathrm{t}}^2 \quad \dots(20)$$

The above equation is a general formula for computing the variance prediction for any(ARCH (p)) model.

9- Comparing Stage

Many formulas can be used for comparing forecasting values such that

$$TIC = \frac{\sqrt{\frac{1}{N}\sum_{t=1}^{N}(h_t^2-\sigma_t^2)^2}}{\sqrt{\frac{1}{N}\sum_{t=1}^{N}h_t^2}\sqrt{\frac{1}{N}\sum_{t=1}^{N}\sigma_t^2}} \quad \dots(21)$$

$$RMSE = \sqrt{\frac{1}{N}\sum_{t=1}^{N}(h_t^2-\sigma_t^2)^2} \quad \dots(22)$$

MSE=$\frac{1}{N}\sum_{t=1}^{N}(h_t^2-\sigma_t^2)^2$ ...(23)

With

(σ_t^2) *represent the true variance*

(h_t^2) *represent the estimated variance*

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10-Experimental Results

The experimental data represent Daily data on the price of a barrel of Iraqi oil (in dollars) for a period from (2008-2012) for a sample consisting of (1256) value.

the experimental results can be as :-

Table (1) the statistical indicators for daily data on the price of a barrel of Iraqi oil

Index	Statistical indicator	The value
1	Mean	89.41899
2	Median	92.3375
3	Maximum	138.8
4	Minimum	29.885

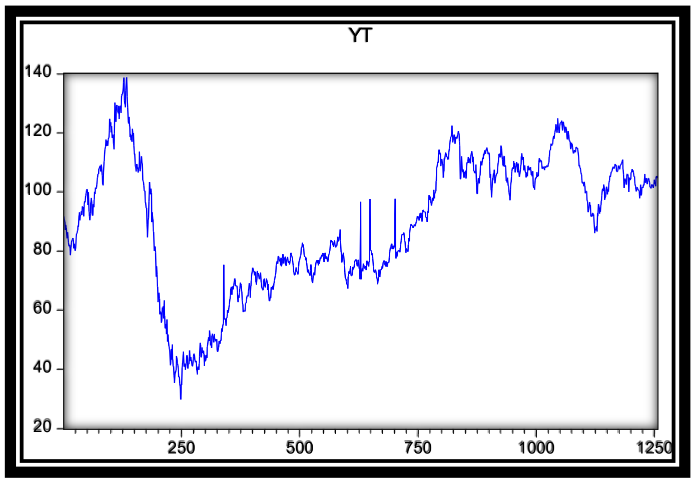

Figure (1) the daily data on the price of a barrel of Iraqi oil (in dollars) for a period from (2008-2012)

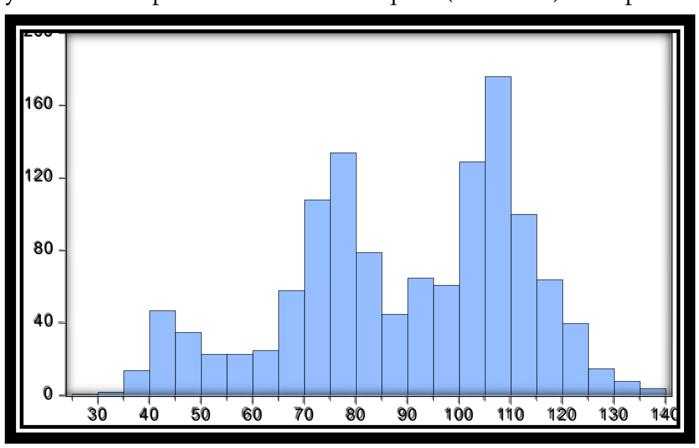

Figure (2) Frequency distribution of series errors(e_t)

EISSN: 3005-3625, PISSN: 2790-2560, DOI: doi.org/10.58564/EASJ/5.1.2026.20

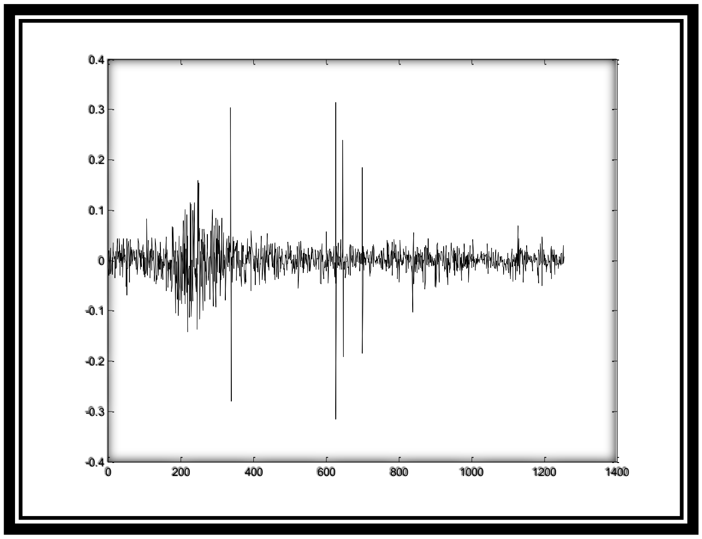

Figure (3) diffusion shape of the data

After finding the statistical indicators of the time series will started, diagnosis phase, where the data under study are diagnosed to make sure that data suffers from an ARCH problem. Therefore, the ARCH test was used with displacements (p = 1.5,10,15,20) plus to calculate the autocorrelation coefficients for the remainder that are important to know whether the remainders are related or not.

Table (2) shows the test statistics and the tabular (x^2) values. Where the table shows that ($\mathrm{LM(p)} > \mathrm{x}^2$) in all displacements, as this indicates the rejection of the null hypothesis which means (e_t) series suffers from the problem of heterogeneity in the variance.

Table (2) ARCH test with various displacement

displacement	$x^2(p)$	LM
1	3.8415	271.9
5	356.0	356.07
10	24.99	356.3
15	31.41	399.4
20	37.65	400.1

After the series was diagnosed, assessment stage and the suitability examination can be done, With table (3) to table(7) ,for ARCH(1) and ARCH(2) models with the proposed distributions , We can note that for ARCH(1) according (AIC) criteria the best model was (skewed Student's t-distribution) which gives the (6,-0.09)

According (Ljung – Box) and (Weighted Ljung – Box) tests the tests shows for all distributions errors series was uncorrelated .

And for (ARCH(2)) results show that

$$\hat{\alpha}_1 + \hat{\alpha}_2 = 1$$

For Gaussian distribution which means that un conditional variation un defined and errors of series is in perpetual flux. according (AIC) criteria the best model was for (skewed Student's t distribution), According (Ljung – Box) and (Weighted Ljung – Box) tests the tests shows for all distributions Except Gaussian distribution errors series was uncorrelated , according (AIC) criteria the best model was (ARCH(2)).

EISSN: 3005-3625, PISSN: 2790-2560, DOI: doi.org/10.58564/EASJ/5.1.2026.20

Table (3) ARCH(1) Estimators

Distribution	parameters	estimators	Stan error	t test	AIC
Gaussian	$\hat{\alpha}_0$	0.0007	0.000001	19.384	-5223
	$\hat{\alpha}_1$	0.4000	0.0625	6.398	
GED(1)	$\hat{\alpha}_0$	0.0005	0.0001	9.548	-5668
	$\hat{\alpha}_1$	0.363	0.1076	3.374	
Student's t(6)	$\hat{\alpha}_0$	0.0004	0.0000	15.371	-5672
	$\hat{\alpha}_1$	0.3336	0.0577	5.783	
SkewT(6,-0.09)	$\hat{\alpha}_0$	0.0004	0.0000	19.637	-5859
	$\hat{\alpha}_1$	0.2813	0.0379	7.416	

Table (4) ARCH(2) Estimators

Distribution	parameters	estimators	Stan error	t test	AIC
Gaussian	$\hat{\alpha}_0$	0.0004	0.0000	13.199	-5357
	$\hat{\alpha}_1$	0.4323	0.0646	6.681	
	$\hat{\alpha}_2$	0.5677	0.0740	7.687	
GED(1)	$\hat{\alpha}_0$	0.0004	0.0000	10.179	-5718
	$\hat{\alpha}_1$	0.3325	0.0753	4.419	
	$\hat{\alpha}_2$	0.3586	0.0779	4.601	
Student's t (6)	$\hat{\alpha}_0$	0.0003	0.0000	11.657	-5738
	$\hat{\alpha}_1$	0.2904	0.0560	5.183	
	$\hat{\alpha}_2$	0.309	0.0577	5.361	
Skew-t(6,-0.09)	$\hat{\alpha}_0$	0.0003	0.0000	14.868	-5927
	$\hat{\alpha}_1$	0.2440	0.0368	6.637	
	$\hat{\alpha}_2$	0.2617	0.0380	6.893	

above tables shows the ARCH (1) and ARCH (2) model, as well as the standard error and the t-statistic for each of the distributions used in this research, and we can compare all ARCH models using the Akaike criteria

AIC= -2log (maximized likelihood) +2 (number of parameters)

This is the best indicator that provides us with the number of parameters we used and it is preferred for the model that has least value for this criterion. We note from Table (3) that the AIC standard for the ARCH model (1) for t distribution is the most suitable for representing the data because it has the lowest value, which is

Table (5) (Ljung – Box) and (Weighted Ljung – Box) tests for ARCH(1) model

Distribution	Statistic	Sig-level
Ljung – Box with Gaussian	34.322	31.4*
WLB with Gaussian	17.108	17.35
Ljung – Box with GED(1)	33.405	31.4*
WLB with GED(1)	16.394	17.35
Ljung – Box with Student's t(6)	34.322	31.4*
WLB with Student's t(6)	15.808	17.35

EISSN: 3005-3625, PISSN: 2790-2560, DOI: doi.org/10.58564/EASJ/5.1.2026.20

Ljung – Box with Skew-t(6,-0.09)	32.576	31.4*
WLB with Skew-t(6,-0.09)l	15.765	17.35

Table (6) (Ljung – Box) and (Weighted Ljung – Box) tests for ARCH(2) model

Distribution	Statistic	Sig-level
Ljung – Box with GED(1)	31.985	31.4*
WLB with GED(1)	16.349	17.25
Ljung – Box with Student t (6)	31.68	31.4*
WLB with Student t (6)	16.108	17.25
Ljung – Box with Skew-t(6,-0.09)	31.592	31.4*
WLB with Skew-t(6,-0.09)	16.05	17.25

The prediction process was used on the estimated model according to the standard AIC which succeeded in the diagnostic process according to Weighted Ljung - Box which is ARCH (2) when the error follows the distributions GED (1) , Student's- t (6) , Skew- t(6,-0.09).

To evaluate the prediction process and choose the best predictive model, the accuracy criteria mentioned in the theoretical side were used, as the time series was divided into two halves, and the prediction within the time series was by half number of observations (682)

Table (7) shows that the best predictive model is ARCH (2) -SkewT. (6-, 0.09) because it has the lowest value out of all the criteria.

Table (7) accuracy criteria for ARCH(2) Model

Distribution	TIC	RMSE	MSE
ARCH(2)- Gaussian	0.9554	0.3248	0.1055
ARCH(2)-GED(1)	0.4834	0.0347	0.0012
ARCH(2)-$t_{(6)}$	0.2988	0.0278	0.0008
ARCH(2)-Skew $t_{(6-,0.09)}$	0.1110	0.0228	0.0005

11-Conclusions & suggestions

The analysis of time series data with diffrernt probability assmed for error term , we concluded:

1-The kurtosis of the data should treated to get more accurate forecasts.

2- It was found that the best predictive model for the price oil time series data is ARCH (2) when the error follows the Skew t-distribution.

3- Accuracy criteria shows that ARCH(2) forecasting best model is outperformed as compared with ARCH(1)

4- For future works , It is important to try use other distributions for estimation comparisons.

5-Use Weighted Ljung – Box criteria to estimate ARMA(2) model.

EISSN: 3005-3625, PISSN: 2790-2560, DOI: doi.org/10.58564/EASJ/5.1.2026.20

Funding

None

Acknowledgement

None

Conflicts of Interest

The author declares no conflict of interest.

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