IRAQI STATISTICIANS JOURNAL

https://isj.edu.iq/index.php/isj

ISSN: 3007-1658 (Online)

Alpha Power Families of Distributions Article Review

Raya S.AL-Rassam, Hayfa A.AL-Jawad, Safwan N. Rashed, Heyam A.Hayawi

Department of Statistics and Informatics, College of Computer Science and Mathematics, University of Mosul, Mosul, Iraq

ARTICLE INFO

Article history:

Received	8 November 2025
Revised	8 November 2025
Accepted	1 January 2026
Available online	13 January 2026

Keywords:

Alpha power transform family,
Zubair alpha power family,
Alpha-Beta power transform family,
Gull alpha power family.

ABSTRACT

Generalizing probability distributions is a very important issue, as are many other practical issues statistics, All generalization methods have been performed by introducing additional parameters such as location, scale or shape. In the past few years, this branch of statistics has received considerable attention, and new classes of generalized distributions have been proposed. Here we provide a brief overview of this branch and also introduce several new families of distributions.

1. Introduction

Adding parameters to Known probability distributions is very common in statistical distribution theory. These additional parameters increase the flexibility of class of distribution functions, and is very useful for the purposes of analyzing complex data, For example, a parameter has been added to symmetric distributions which transform them into skewed distributions that are more flexible than the original probability distribution. The added parameters is called the skewness parameters. The parameter value determines the shape of the skewness, which may be positive or negative skewness. Other methods have been proposed, such as the exponentiation of cumulative distribution function, Marshal-kin family, McDonald family, Kumaraswamy family, Topp- Leone family and many others.

The purpose of extending of probability distributions is to explain behavior of lifetimes in various branches such as public health ,engineering processes, biological and medical researches, insurance, biology and many other fields. Some probability distributions are very limited in their properties and applications sides, For example exponential distribution can be model phenomenas with constant hazard rate only, while Rayleigh distribution can only able to model phenomenas with increasing hazard rates over time .However ,Weibull distribution is able to model increasing, decreasing, or constant hazard functions, but the distribution can't able of modeling with non-monotonic hazard functions.

In the last decade , a new family of distributions which is a power transform function of a cumulative distribution function(CDF) of continuous random variable. This family is capable of modeling n-monotonic hazards. The aim of this paper is to a give review of newest alpha power transformed families of distributions. This review helps researchers to cover spaces in applications of classical probability

Corresponding author E-mail address: rayasalim73@uomosul.edu.iq
https://doi.org/10.62933/aq5hgb56

distributions and to help researchers to work on generalized models.

This paper organized as , section two consist of alpha power transform(APT) family. Section 3 contain the Zubair-G family, section 4 contain Gull alpha power family, section 5 consist of the alpha-Beta power family and last section contains some concluding remarks.

2. Alpha power transform(APT) family

The (APT) family was first introduced by [1]. This family summarizes the addition of an added parameter to a distribution function of baseline distribution to give more flexibility to a give distribution. This family defined as:

$$F_{MC}(x;\alpha,\theta) = \begin{cases} \dfrac{\alpha^{F(x,\theta)}-1}{\alpha-1} & \alpha,\theta>0, x\in R \\ F(x,\theta) & \alpha\neq 1 \end{cases} \quad (1)$$

Where $F(x,\theta)$ is a baseline (CDF) .[2] introduced a new generalization of power distribution. This generalization based on (1) ,the distribution named Alpha -power (APP) distribution. This distribution is more flexible than power distribution that can be unimodal or right skewed. They studied some it's statistical properties. The parameters of distribution were estimated by maximum likelihood (ML) method. They fit the (APP) and some competitive models to one real data set and it is shown that (APP) distribution is a better fit than other models evidenced by some goodness of fit criteria [3] introduced alpha power inverse exponential (APIE) distribution,[4] introduced alpha power transformed Lindley, [5] gave a generalization of invers Lindley distribution using (1), alpha power. Fréchet distribution [6], alpha power transformed power Lindley [7],[8] and [9] introduced alpha power transformed inverse Weibull distribution , alpha power Weibull [10], alpha power Pareto [11] , alpha power Maxwell [12] , Alpha power Aradhana [13] , alpha power transformed extended power Lindley [14] , alpha power inverted exponentiated Weibull [15].

3. Zubair G Family

[16] introduced new family of alpha power transformed named Zubair-G (Z-G) family of distributions. The (CDF) of this family defined as:

$$F_{2-G}(x;\alpha,\theta) = \frac{e^{\alpha F^2(x,\theta)}-1}{e^{\alpha}-1} \qquad \alpha,\theta>0 \ , \ X\in R \qquad (2)$$

The (Z-G) family applied on Weibull distribution named Z-Weibull (Z-W) distribution, The parameters of the distribution have been estimated by least squares ,(ML) , Maximum product spacing methods. This family has the ability for modeling data with monatomic and non-monatomic hazard rates. Two real data sets were fitted by (Z-W) model with other life models. It is shown that (Z-W) is better fit than another life distributions .[17] introduced Zubair-Lomax distribution , Extended Bur XII distribution[18][19] studied the flexibility of (Z-G) family based on baseline Dagum distribution, the distribution symbolized as (Z-D) distribution. Various properties of the distribution were studied. The (Z-D) distribution is positively skewed and has bathtub of different monatomic shapes and the survival function a monotonic decreasing function.[20] introduced Zubair-Rayleigh distribution, Zubair-inverse Lomax[21] , Zubair-Lomax [22] , Zubair-Exponential [23],Zubair-exponentiated Weibull [24], [25] introduced a new alpha power transform named new Zubair (NZ) family. The (CDF) of (NZ) family is :

$$F_{NZ}(x;\theta) = \frac{e^{F(x,\theta)}-1}{e-1} \quad x\in R \ , \theta>0 \qquad (3)$$

Which is a special case of (2). Also he gave the extended Zubair G (EZ-G) family whose (CDF) defined as:

$$F_{EZ-G}(x,\alpha,\beta,\theta) = \frac{e^{\alpha F^2(x,\theta)+\beta F(x,\theta)}-1}{e^{\alpha+\beta}-1} \qquad (4)$$

Where $,\beta,\theta>0\ , X\in R$, Another family proposed by [26] defined as:

$$F_{EX-APT}(x,\alpha,\theta) = \frac{\alpha^{F(x,\theta)}-e^{F(x,\theta)}}{\alpha-e} \quad \theta>0 \ ,\alpha\neq e \ ,\alpha>e \ ,X\in R \qquad (5)$$

This family named as extended alpha power transform (EX-APT) family. It is used to generalize Weibull distribution, All of these varieties of families defined in (2),(3) ,(4) ,(5) are useful in describing phenomenon resulting from real data sets.[27] introduced generalized alpha power transformation (GAPT) family.This family have left skewed , symmetric and reversed J shape densities , it is applied to Rayleigh baseline distribution. The (CDF) of proposed (GAPT) family defined as:

$$F_{GAPT}(x,\alpha,\beta)=\frac{\beta^{[F(x)]^{\alpha}}-1}{\beta-1} \qquad (6)$$

Where $\in R\,,\alpha,\beta>0\;\beta\neq 1$. When $=1$, GAPT family reduces to APT family, When $\alpha=1$ and $\beta=e$, GAPT family reduces to (NZ) family. [28] proposed a new extended alpha power (NE-APT) family whose (CDF) defined as:

$$F_{NE-APT}(x,\alpha,\delta,\theta) = \frac{\alpha^{F(x,\theta)}-[1-\overline{\delta}F(x,\theta)]}{\alpha-\delta} \qquad (7)$$

Where $X\in R\,,\alpha,\delta>0\,,\alpha>\delta$ and $\overline{\delta}=1-\delta$,[28] propose a weighted alpha power transform (WAPT) family whose (CDF) has the following form:

$$F_{APT}(x,\delta,\theta) = \frac{\delta^{F(x,\theta)}-1}{(\delta-1)(1+\delta-\delta^{F(x,\theta)})^2} \qquad (8)$$

Where $x\in R\,,\delta>0\,,\delta\neq 1$ [20] applied WAPT family to Weibull distribution.[31] introduced a new modified alpha power transform (MAPT) family and is used to generalize an exponential distribution. Its CDF defined as:

$$F_{MAPT}(x,\alpha,\delta,\theta) = \frac{\delta^{F^2(x,\theta)}\alpha^{F(x,\theta)}-1}{\delta\alpha-1} \qquad (9)$$

For $\in R,\theta>0\,,\alpha,\delta\geq 1\,,\alpha\delta\neq 1$, This family reduces to APT when $\delta=1$ .If $\delta=1\,,\alpha=e$;Itreduces to (NZ) family. The (MAPT) family applied to an exponential distribution.

4. Gull alpha power transform family of distributions

The Gull alpha power transform (GuAPT) family of distributions have been proposed by [32]. Its CDF defined as:

$$F_{GuAPT}(x,\delta,\theta)=\frac{\delta F(x,\theta)}{\delta^{F(x,\theta)}} \qquad \delta>0\,,\delta\neq 1\,,X\in R \qquad (10)$$

It is applied to Weibull distribution named (GuWeibull). The GuAPT family is suitable for monotonic and non-monotonic hazard rates,[32] found the parameter space of (GuAPT) family which makes the probability density function greater than zero.

The Ampadu Gull alpha power transform (AGuAPT) family proposed by [34] whose CDF defined as:

$$F_{AGuAPT}(x,\alpha,\theta)=\frac{(1+e^{-\alpha})F(x,\theta)}{1+e^{-\alpha F(x,\theta)}} \qquad (11)$$

Where $x,\alpha,\theta\in R$.

[35] introduced the Kumaraswamy-Gull alpha power transform (K-Gu) family where its CDF has the following form:

$$F_{K-Gu}(x,a,\delta,\beta,\theta) = 1-\left\{1-\left[\frac{aF(x,\theta)}{F_a(x,\theta)}\right]^{\delta}\right\}^{\beta} \qquad (12)$$

Where $x,\theta\in R,\delta,\beta>0,\alpha>1$. This family applied to exponential distribution.

5. Alpha-Beta power transformed family of distributions

The alpha-Beta power transformed (ABDT) family proposed by [36] where its CDF defined as:

$$F_{ABPT}(x,\beta,\alpha,\theta) = \frac{\beta^{F(x,\theta)}-\alpha^{F(x,\theta)}}{\beta-\alpha} \qquad (13)$$

Where $,\theta\in R\,,\alpha,\beta>0\,,\alpha\neq\beta=1$, They used a baseline exponential distribution where the hazard rate of (ABPTE) distribution can be decreasing, bathtub and unimodal.[37] gave a generalization of (ABPT) family named as exponented alpha-Beta power transform (EABPT) family where it's CDF defined as:

$$F_{EABPT}(x,\alpha,\beta,\delta,\theta) = \left(\frac{\beta^{F(x,\theta)}-\alpha^{F(x)}}{\beta-\alpha}\right)^{\delta} \qquad (14)$$

Where $\alpha,\beta,\delta>0$, $\alpha\neq\beta\neq 1$ when $\alpha=\delta=1$ it reduces to (APT) family.

6. Concluding Remarks

Researchers have relied on generalized distributions to model various phenomena , such as modeling life expectancy and device reliability, economic and financial data, Interest has focused on studying different families of probability distributions. The most recent alpha power transformation families , have better ability to model complex data than the established.

References

1. Mahdavi,A and Kundu,D,"A new method for generating distributions with an application to exponential distribution",Comman Stat. theory method,(2017),46(13):6547-6557.
2. Fathizadeh ,M. , Mahdavi , A. ,Jabbari,L.,"Alpha-power distribution", Journal of Biostatistics and epidemiology, 2018, 4(3): 129-135.
3. Unal,C.,Cakmakyapan,S.,and Ozel, G.,"Alpha power inverted exponential distribution: properties and application, Gazi university journal of science,2018,31(3): 954-965.
4. Dey,S. Ghash,I. and Kamar,D."Alpha- power transformed Lindley distribution: properties and associated inference with application to earthquake data", Annals of data science, 2013,6:623-650.
5. Dey,S.,Nassar , M.,and Kumar, D,"Alpha power transformed inverse Lindley distribution: A distribution with an upside-down bathtub-shaped hazard function", Journal of computational and applied mathematics, 2019,348:130-145.
6. Nasiru ,S., Mwita , P.N. and Ngsa, O.,"Alpha power transformed- Fréchet distribution, Applied mathematics and information sciences",2019,13(1);129-141.
7. Hassan ,A.,S.,Elgarhy,M., Mohamad,R.E., and Alrajhi ,S.,"On the alpha power transformed power Lindley distribution", Journal of probability and statistics,2019:1-13.
8. Ramadan,D.A. and Magdy ,W.A.,"On the alpha-power inverse Weibull distribution", International Journal of computer applications ,2018,181,(11),6-12.
9. Basheer ,A.M.,"Alpha power inverse Weibull distribution with reliability application, Journal of Taibuh university for science,2019,13(1):423-432.
10. Elbatal ,I; Ahmad ,Z.,Elgarhy,M., Almarashi,A.M.,"A new alpha power transformed family of distributions: properties and applications to the Weibull model", Journal of nonlinear sciences and applications,2019,12(1):1-20.
11. Ihtisham, S.,Khalil,A.,Manzor, S.,Khan,S.A.,Ali,A.,"Alpha power Pareto distribution: properties and applications ",Plos one,2019,14(6):1-15.
12. Erdogan,N.,Bagei,K.,Arslan,T.,Celik,H.E.,"Alpha power Maxwell distribution :properties and application", Journal of mathematical modeling,2021,9(4):585-595.
13. Muhiuddin,M.,Kaman,R.,"Alpha power transformed Aradhana distribution, its properties and applications", Indian journal of science and technology ,2021,14(30):2483-2493.
14. Eissa ,F. Y. and Sonar ,C.D. ,"Alpha power transformed extended power Lindley distribution ", Journal of statistical theory and applications", 2023,22(1):1-18.
15. Basheer ,A.M. ,"Alpha power inverted exponentiated Weibull distribution with data application", Journal of mathematics and statistics ,2024,20(1):63-73.
16. Ahmad,Z. ,"The Zubair-G family of distributions properties and applications",Analls of data science,2018:1-14.
17. Palov ,N.,Kyurkchiev,N.,Ilive,A.,Rahman ,A."A note on the Zubair -G family with baseline Lomax cumulative distribution function some applications", International journal of pure and applied mathematics, 2018,120(3):471-486.
18. Rahneva, O., Terzieva, T. and Golov, A.," Investigations on the Zubair family with baseline Ghosh Bourguignon's extended Burr XII cumulative sigmoid. Some applications", Newral parallel and scientific computation, 2019,27,(1); 11-22.
19. Uwaeme, O, R., Akpan, N. P.," The Zubair Dagum distribution", Asian journal of probability and statistics, 2020, 110(3); 25-35.
20. Swathi, k. S., Arana, C.H.V., Anjane yulu, G.V.S.R., "The Zubair Rayleigh distribution", International journal for scientific research and developement, 2020, 8(9):32-33.
21. Falgore, J. Y., The Zubair inverse Lomax distributions with applications", Asian Journal of probability and statistics 2020, 8(3): 1-14.
22. Bantan, R., Hassan, A. S., Elsehetry, M., "Zubair Lomax distribution: properties and estimation based on ranked set sampling ", Computers materials and continua, 2020, 65(3):2169-2187.
23. . Alsultan, R., "Zubair - exponentiated Weibull distribution with group acceptance Sampling schames and applications to bladder cancer", Advances and applications in Statistics, 2023, 90(2): 235-256.
24. Anabike, I. C., Igbokare, C. P., Onyekwere, C.K. Obulezi, O. J.," Inference on the parameters of Zubair Exponential distribution with application to Survival times of gulne a pigs", Journal of advances in mathematics and computer Science, 2023,38(7):12-35.
25. Ahmad z., Hamedani, G. G., Butt, N. S.,: Recent developments in distributions theory: A brief survey and some near generalized Classes

of distributions", Pakistan journal of Statistics and operations research, 2019, 15(1) 87-110.

26. Ahmad, z., Ilyas, M., Hamedani, G. G., "The extended alpha power transformed family of distributions; properties and applications", Journal of data science, 2019, 17(4): 726-741.

27. El-Sherpieny, El-Sherpieny EI-S, A., and Hawas. H.K.,"Generalized alpha power transformation family of distributions with an application to exponential model", Journal of computational and theoretical nanoscience, 2021, 18:1-8.

28. Ahmad, Z., Mahmoudi, E., Hamedani, G. G.," A new extended alpha power transformed family of distributions; properties, characterization and an application to a data set in the insurance sciences", Communications for statistical applications and methods, 2021, 28(1) 1-19,

29. Alotaibi, R., Okash, H., Rezk, H., Nassar, M.,"A new weighted version of alpha power transformation method: properties and applications to COVID-19 and softus are reliability data ",Physica Scripta, 2021, 96: 215-221.

30. Alotaibi. R., Okash, H., Nassar, M., Elshahhat A., "A new modified alpha power trans formed Weibull distribution and its engineering applications", Computer modeling in engineering and sciences, 2023,135 (3): 2065-2089.

31. Hussein, M., Elsayed, H., Corderio, G. M.," A new family of continous distributions; properties and estimation", Symmetry, 2022, 14(276):1-18.

32. Ijaz, M., Asim, S. M., Farooq, A.M., Khan, S.A. Manzoor, S., "A Gull alpha power Weibull distribution with applications to real and simulated data", Plos One, 2020, 12:1-19.

33. Mukhlif, M. H. and Saieed, H, A, J.,"Study of Gull alpha power Weibull distribution with application", Iraqi journal of statistical sciences, 2024, 21(1):112-126.

34. Ampadu, C. B., "Gull alpha power of the Ampadu-type: properties and applications", Earthline journal of mathematical sciences, 2021, 6(1):187-207.

35. Kpangay, M., Odongo, L. O., Orwa, G.O.,"An extended Kumaraswamy-Gull alpha power exponential distribution, properties and applications to real data", International Journal of mathematics trends and technology, 2022. 60(8):72-104.

36. Semary. H. E., Hussain, Z., Hamdi, W, A. Aldahan, M.A,, Elbatal, I.," Alpha-Beta power family of distributions with applications to exponential distribution", Alexandria engineering journal, 2024, 100:15-31.

37. Semary, H. E., Thampi, A., Alghamadi, S. M., Nagarjuna, V. B. V.,"Generalized alpha Beta-Power family of distributions; properties and applications",Journal of radiation research and applied sciences, 2025, 18:1-11.