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Analyze the transportation problem using multi-objective linear programming to minimize cost and distance

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Abstract

The study addresses the classical transportation problem, which is central to operational research and logistics decision-making. Traditional transportation models often focus on minimizing costs under supply and demand constraints but tend to overlook the efficiency of network structures and alternative optimization strategies. This research seeks to determine whether transforming conventional transportation models into a multi-objective linear programming framework, while incorporating shortest path analysis, can yield more practical and cost-effective solutions. The research applies multi-objective linear programming techniques to three case studies from Union Food Industries Company, where commodities are transported from production sites to demand centers. Conventional allocation methods (Northwest Corner, Least-Cost, Vogel's Approximation) were first applied to generate feasible initial solutions. These were then tested for optimality using the Modified Distribution Method. In parallel, the transportation models were reformulated as network optimization problems and solved using the shortest path method, allowing direct comparison between classical and network-based solution. The analysis demonstrated that shortest path optimization consistently reduced transportation costs compared to conventional methods. For example, in one case, the optimal cost decreased from $440 to $360 when applying shortest path modeling. The results also showed that while shortest path solutions sometimes initially violated feasibility conditions, minor adjustments restored balance without sacrificing optimality. Overall, the approach proved efficient, reliable, and computationally less complex than traditional iterative methods. Practically, the study highlights the potential of shortest path models to improve cost efficiency in transportation planning for industrial firms. Academically, it contributes to the field of operations research by integrating classical transportation theory with network-based optimization, offering a framework that balances multiple objectives while simplifying application. This work underlines the importance of modernizing transportation analysis to better support organizational decision-making.

Keywords: Transportation models, shortest path, optimal cost.

1. Introduction

The transportation problem is a cornerstone of operations research and an essential tool in logistics and supply chain management. It focuses on determining the most efficient allocation of resources for moving goods from multiple sources to multiple destinations under supply and demand constraints. Traditionally, researchers have relied on methods such as the Northwest Corner, Least-Cost, and Vogel's Approximation to establish an initial feasible solution, followed by the Modified Distribution Method to test optimality. While these approaches have proven effective in many contexts, they often require iterative adjustments, are computationally intensive, and tend to overlook the structural efficiency of transportation networks. Current transportation methods used in many industrial and commercial organizations remain insufficient for achieving cost efficiency. These traditional models emphasize feasibility but do not fully exploit opportunities for optimization through direct network analysis. As a result, companies face high transportation costs, inefficiencies in route selection, and delays in decision-making, which negatively affect overall performance and competitiveness. Although international studies have increasingly explored advanced optimization technique—such as shortest path algorithms and multi-objective linear programming—their application remains limited in Iraqi case studies. Few empirical investigations have examined how these methods can be adapted to the operational realities of local industries. This gap underscores the need for applied research that not only tests theoretical models but also demonstrates their practical value in the Iraqi context. This paper addresses the gap by developing and testing a cost-effective shortest path model within a multi-objective linear programming framework. The model is applied to three real-world transportation cases from Union Food Industries Company, enabling direct comparison between traditional methods and the proposed approach. By doing so, the study contributes both practically—through cost reduction strategies for industrial firms—and academically—by integrating classical transportation models with network-based optimization in a context where such applications are still underexplored. Several studies have examined the transportation problem from different perspectives. Hussein et al. (2012) analyzed the applications of linear programming in multi-stage transportation models, highlighting the flexibility of linear formulations in addressing complex logistics challenges. Muhammad (2015) applied linear programming techniques to solve transportation problems and verified optimality through the Modified Distribution Method. Ibrahim and Abbas (2016) compared the zero-point method with Vogel's Approximation Method, showing how initial feasible solutions can significantly influence the quality of final outcomes. Internationally, Sharma and Sharma (2000) and Vannan and Rekha (2013) proposed new dual-based and heuristic approaches for improving transportation solutions, while Gomah and Samy (2009) explored object-oriented modeling as an alternative optimization framework. More recently, Al-Bakoush (2022) emphasized the importance of adopting modern transportation methods for cost reduction in food supply chains, and Al-Sharjabi (2023) proposed improvements for finding acceptable initial solutions compared with traditional methods. These studies provide a strong theoretical foundation, but few have combined shortest path optimization with multi-objective programming in applied case studies, especially within the Iraqi industrial context, which this paper seeks to address.

2. Data and Methodology

2.1 Mathematical Formulation-Multi-Objective Linear Programming Model

$$Minimize\ Z_1(x) = \sum_{i\in I}\sum_{i\in J} c_{ij}x_{ij} \qquad (total\ transportation\ cost) \quad \ldots 1$$

$$Minimize\ Z_2(x) = \sum_{i\in I}\sum_{i\in J} dist_{ij}x_{ij} \qquad (total\ distance/\ time) \quad \ldots 2$$

$S.T$:

$$\sum_{j\in J} x_{ij} = s_i \quad \forall i \in I \quad (supply\ constraints) \quad \ldots 3$$

$$\sum_{i\in I} x_{ij} = d_j \quad \forall j \in J \quad (demand\ constraints) \quad \ldots 4$$

$$x_{ij} \geq 0 \quad \forall i \in I, j \in J \quad \ldots 5$$

Multi-objective optimization can be solved through:

- **Weighted sum method:** $Minimize\ Z_\lambda(x) = \lambda Z_1(x) + (1-\lambda)Z_2(x),\quad 0 \leq \lambda \leq 1$

- **Epsilon-constraint method**: minimize one objective subject to a constraint on the other.

While
$I = (1,2,\ldots,m)$: $Set\ of\ supply\ sources\ (factories)$.
$J = (1,2,\ldots,n)$: $Set\ of\ demand\ centers\ (warehouses)$.
s_i: $Supply\ available\ at\ source\ i$.
d_j: $demand\ required\ at\ center\ j$.
c_{ij}: $unit\ transportation\ cost\ from\ i\ to\ j$.
$dist_{ij}$: $distance\ (or\ time) to\ transport\ one\ unit\ from\ i\ to\ j$.
x_{ij}: $decision\ variable - number\ of\ units\ transported\ from\ i\ to\ j\ \geq 0$.

2.2 Data Source

The study uses real data from Union Food Industries Company, covering supply from production sites, demand from distribution centers, and transportation costs/distances. The datasets reflect actual operational figures, ensuring the applicability of the model to real industrial challenges. In case of missing or incomplete data, simulated extensions were used to maintain model feasibility and balance.

Assumptions

- **Balanced model**: total supply equals total demand $\sum_i s_i = \sum_j d_j$, if unbalanced, dummy supply/demand nodes are introduced.
- **Deterministic and stable demand**: demand values are fixed and known within the analysis period.
- **Deterministic costs**: transportation costs c_{ij} and distances $dist_{ij}$ are constant (no uncertainty).
- **Single commodity**: only one homogeneous product is transported.
- **No transshipment**: goods move directly from source to destination without intermediate stops.
- **Adequate vehicle capacity**: transportation lines are assumed to have sufficient capacity to meet required flows.
- **Feasibility adjustments**: in shortest path formulations, minor reallocations may be required to restore feasibility when constraints are initially violated.

2.3 Research problem

Although traditional transportation models are widely used in operations research, they often focus more on achieving mathematical balance than on practical efficiency. Conventional methods—such as the Northwest Corner, Least-Cost, and Vogel's Approximation—aim primarily at minimizing cost without considering the role of distance or the efficiency of the transportation network structure. This limitation often results in suboptimal outcomes, particularly in industrial environments where both cost and distance have a direct impact on operational performance.

Therefore, the research problem focuses on whether it is possible to transform the traditional transportation model into a multi-objective linear programming model that integrates cost and distance minimization using the Shortest Path Method as a practical optimization tool. The study also seeks to examine the effectiveness of applying this model to real-world data from the Union Food Industries Company, with the goal of achieving tangible cost savings and improving transportation efficiency within the Iraqi industrial context.

2.4 Research objectives

- To formulate a multi-objective linear programming model that integrates cost minimization and distance minimization.
- To apply the model to real transportation cases from Union Food Industries and compare results with conventional methods (Northwest Corner, Least-Cost, Vogel, MODI).
- To evaluate the effectiveness of shortest path analysis in achieving cost reductions and feasible allocations, with practical adjustments where required.
- To provide methodological insights for decision-makers on choosing between manual and computational approaches in solving transportation problems in the Iraqi industrial context.

3. The theoretical aspect

Three different transportation problems were studied for Union Food Industries Company in terms of number of variables and number of constraints for transportation of goods from sources (factories) to demand centers (warehouses), assuming m sources and n demand centers. The main objective is to determine a plan for transporting units from the source to the demand center so that the transport cost is as low as possible. Let S_i be the number of $(i = 1,2,3,\dots,m)$, d_j is the number of demand equipment units required at source i While W_{ij} the cost of transporting a $(j = 1,2,3,\dots,n)$ the world units required at demand center j unit from source i to demand center j, y_{ij} It represents the number of units transferred from source i to demand center j. the linear programming model for the transportation problem is:

$$Minimize\ K = \sum_{i=1}^{m}\sum_{j=1}^{n} W_{ij}y_{ij} \qquad \ldots 6$$

$S.t$:

$$\sum_{i=1}^{m} y_{ij} = d_j \qquad j = 1,2,\ldots,n \qquad \ldots 7$$

$$\sum_{j=1}^{n} y_{ij} = S_i \qquad i = 1,2,\ldots,m \qquad \ldots 8$$

$$y_{ij} \geq 0 \qquad \ldots 9$$

The transport models were balanced so that the total supply from sources equaled the total demand from demand centers $\sum_{i=1}^{m} S_i = \sum_{j=1}^{n} d_j$ If the transportation problems are not balanced, they are treated by a practical procedure to achieve equilibrium for this model and using the usual methods to find the initial possible solution for transportation models, such as the northwest corner method, the least cost method or the Vogel method, which meets the conditions for the possible solution:

A. Sufficiency of rows m
B. Column sufficiency n
C. Make sure the solution used meets the basic rule of cell counting $(m + n - 1)$.

- **North West – Corner Method**

  This method is considered one of the simplest for finding the initial possible solution of a transport model. The process of finding the initial solution begins from the northwest corner, hence its name. However, this solution often requires improvement, as it does not rely on the cost element.

- **Minimum – Cost Method**

  This method allows you to find the initial possible solution for a transport model. Based on observing cost values and choosing the lowest possible cost, it often constitutes the optimal solution.

- **Vogel's Approximation Method**

  This is the most efficient method for finding the initial possible solution of the transport model compared to its predecessors, as it is characterized by a speed in obtaining the optimal or close to it solution, but it requires longer calculations than previous methods. To test the optimality of the solution, the following method was used:

3.1 Shortest Path Method

In this study, the **Shortest Path Method** was employed as a computational approach to generate the *initial feasible solution* for the transportation problem. The basic idea of this method is to represent the transportation model as a **directed network** $G = (V, E)$, where the set of vertices V corresponds to the sources $S = \{s_1, s_2, \ldots, s_m\}$ and destinations $D = \{d_1, d_2, \ldots, d_n\}$, while the set of edges E represents the routes connecting each source to each destination, each associated with a transportation cost c_{ij} . For every source s_i, the algorithm determines the shortest route to all destinations d_j to find the minimum-cost path. Mathematically, for each source s_i, we solve:

$$Minimize \sum_{e \in p_{i \to j}} c_e \qquad \ldots 10$$

Where $p_{i \to j}$ represents the path from s_i to d_j . In practical terms, when the network consists of direct links between sources and destinations only, the shortest path is equivalent to selecting the minimum-cost cell c_{ij} in each row of the cost matrix.

The results obtained from the shortest path procedure are then used to construct an **initial allocation** by assigning the available supply from each source to the destination(s) with the lowest associated transportation cost. This method is computationally simple and provides an initial solution that is usually close to optimal when compared with traditional methods such as the *Northwest Corner*, *Least-Cost*, or *Vogel's Approximation*. However, the shortest path approach has certain limitations: the resulting allocation may not always satisfy the **feasibility conditions** of the transportation problem, particularly the *basic feasible solution* requirement (i.e., the number of allocated cells should equal $m + n - 1$).

Therefore, the following **corrective steps** must be applied to ensure feasibility and prepare the solution for optimality testing using the MODI (Modified Distribution) method:

1. After determining the lowest-cost paths for all sources, count the total number of allocated cells. This number must equal the basic feasibility condition $r = m + n - 1$.
2. If the number of allocations is less than r , introduce small artificial allocations (ε) in appropriate cells to complete the basis without affecting the total cost.
3. If the total supply and demand are not balanced, add a dummy source or destination, or reallocate small quantities among the lowest-cost routes until balance is achieved.

4. Once feasibility is restored, apply the **MODI method** to test for optimality by calculating improvement indices $\Delta_{ij}= c_{ij} - (u_i + v_j)$. If any $\Delta_{ij}< 0$, perform a closed-loop adjustment (cycle) to obtain an improved solution.

3.2 Modified Distribution Method

This method relies on the dual property to formulate linear equations for the transportation problem by imposing u_i for each line width $(i = 1,2,3, \dots, m)$ and v_j for each column of the order $(j = 1,2,3, \dots, n)$ for each of the squares used, $W_{ij} = u_i + v_j$ where W_{ij} The transport costs for boxes (i, j) and unused boxes are estimated as follows:

$Q_{ij} = W_{ij} - u_i - v_j$ where Q_{ij} this is the improvement of the solution or the result of evaluating unused cells. If the result of the evaluation is positive or zero, the optimal solution for the transport model has been reached. If it is negative, there is a better solution than the current one. This continues until the optimal solution is reached. By converting the transport problem model into a network of direct paths and using the shortest path problem to find the optimal cost using manual methods, the effectiveness of this problem is demonstrated by meeting the conditions for the possible solution for the usual methods of transport models, which can be solved using modern software.

4. The practical side

Apply shortest path networking to transportation models, model them as network optimization problems, and solve them efficiently and accurately until the optimal solution is reached. The network consists of points called sources E_i representing the lines of the transport model where $(i = 1,2, \dots, m)$ is transported and distributed to points called demand centers R_j representing the columns of the transport model where $(j = 1,2, \dots, n)$ and direct one-way lines called paths. The application begins, according to Rule $(2^n - 1)$, by calculating the number of paths available for each line, where n is the number of columns, and by forming the path equation by imposing the lowest possible cost for each line of the transport model, from there, the optimal path cost is determined and when we finish calculating the optimal cost for all lines, we have reached the optimal solution to the transportation model problem using shortest path optimization.

Case1: $m = n = 3$

	R_1	R_2	R_3	Supply
E_1	3	6	5	40
E_2	1	4	2	50
E_3	4	3	6	40
Demand	60	50	20	130

Model 1: First transfer case

The initial solution is obtained using one of the usual methods, namely the northwest angle, the least-cost method, and the Vogel method. the optimal solution is obtained using the modified distribution method.

	R_1	R_2	R_3	Supply
E_1	40 (3)	(6)	(5)	40
E_2	20 (1)	30 (4)	(2)	50
E_3	(4)	20 (3)	20 (6)	40
Demand	60	50	20	130

Model 2: The optimal solution for the first case

The cost of the optimal solution to the transportation problem model is 440 $.

We will use the shortest path method to solve the transportation problem by decomposing the model into three lines, determining the lowest optimal cost for each line, and comparing it to the optimal solution to the transportation problem model. Does the resulting solution satisfy the conditions for a possible solution to the transportation problem model? to answer this question, we will apply rule $(2^n - 1)$, which consists of calculating the number of paths available for each row, where n is the number of columns.

This question has three columns, so $(2^3 - 1 = 7)$ the number of paths available for each row and the lowest possible cost for each path are calculated by assuming variable t for the first column, variable u for the second column and variable f for the third column and the path equation is symbolized as shown in the following table (1):

The optimal solution to the shortest path transportation problem is:

	R_1	R_2	R_3	Supply
E_1	10 [3]	10 [6]	20 [5]	40
E_2	50 [1]	[4]	[2]	50
E_3	[4]	40 [3]	[6]	40
Demand	60	50	20	130

Model 3: The optimal solution to the transportation problem using the shortest path method

The cost of transport is equal to 360$ this cost is optimal compared to the cost obtained for the conventional route transport model, but this solution satisfies the first condition of the transport model only by the sufficient number of lines and does not satisfy the other two. The same result is obtained as an optimal solution for the transport problem by moving 80 units from the route until the solution satisfies the conditions of a possible solution for the transport problem model.

Case2: $m = 3 , n = 4$

	R_1	R_2	R_3	R_4	Supply
E_1	[5]	[4]	[3]	[1]	20
E_2	[7]	[1]	[10]	[2]	15
E_3	[3]	[9]	[8]	[7]	20
Demand	15	10	15	15	55

Model 4: the second transfer case

The ideal solution would be

	R_1	R_2	R_3	R_4	Supply
E_1	5	4	10 [3]	10 [1]	20
E_2	7	10 [1]	10	5 [2]	15
E_3	15 [3]	9	5 [8]	7	20
Demand	15	10	15	15	55

Model 5: The optimal solution for the second transport case

The cost of the optimal solution to the transportation problem model is equal to 145$.

This case includes four columns and therefore $(2^4 - 1 = 15)$ the number of paths available for each row and the lowest possible cost for each path are calculated assuming variable t for the first column, variable u for the second column, variable f for the third column, and variable G for the fourth column, as shown in the table below.

The optimal solution to the shortest path transportation problem is:

	R_1	R_2	R_3	R_4	Supply
E_1	5	4	5 [3]	15 [1]	20
E_2	7	10 [1]	10	5 [2]	15
E_3	15 [3]	9	8	5 [7]	20
Demand	15	10	15	15	55

Model 6: The optimal solution using the shortest path method

The transportation cost is equal to 130$, but this solution satisfies the first condition by sufficiency of rows and satisfies the third condition used in the solution by satisfying rule $(m + n - 1)$ and does not satisfy the second condition by sufficiency of columns.

The same result as the optimal solution to the transportation problem is achieved by transferring 5 units from path A_1B_4 to path A_1B_3 and 5 units from path A_3B_3 to path A_3B_4 until the solution satisfies the conditions for a possible solution to the transportation problem model.

Case3: $m = 4 , n = 5$

	R_1	R_2	R_3 8	R_4 2	R_5	Supply
E_1	7	3			6	33
E_2	5	2	7	4	5	27
E_3	4	3	2	5	1	40
E_4	1	2	3	5	7	25
Demand	20	45	5	30	25	125

Model7: The third case of transport

The ideal solution would be

	R_1	R_2	R_3	R_4	R_5	Supply
E_1	7	3	8	5 2	25 6	33
E_2	5	2	9 7	20 4	5	27
E_3	4 4	40 3	1 2	5	1	40
E_4	21 1	2	3	5	7	25
Demand	20	45	5	30	25	125

: The optimal solution for the third case8Model

The cost of the optimal solution to the transportation problem 462 $ when using the shortest path method for this problem, then $(2^5 - 1 = 31)$ the number of paths available for each row and the lowest possible cost for each path are calculated assuming variable t for the first column, variable u for the second column, variable f for the third column, variable G for the fourth column and variable E for the fifth column as shown in the table below.

The optimal solution using the shortest path method is:

	R_1	R_2	R_3	R_4	R_5	Supply
E_1	7	3	8	2	6	33
E_2	5	2	7	4	5	27
E_3	4	3	2	5	1	40
E_4	1	2	3	5	7	25
Demand	20	45	5	30	25	125

Model 9: the optimal solution to the shortest path transportation problem

The transportation cost is equal to 218$, this solution satisfies the conditions of the feasible solution for all three transportation models and gives the same optimal solution for the conventional transportation model.

Initial feasible solutions obtained using Northwest Corner, Least-Cost, and Vogel's Approximation. Optimality tested and improved using the Modified Distribution Method (MODI). Reformulation of the transportation problem as a network optimization problem, solved via shortest path analysis for each supply node. Feasibility restored through small re-allocations where necessary, multi-objective model solved using WinQsb applying the weighted-sum methods. Shortest path problems solved with standard algorithms (e.g., Dijkstra) in cases of larger, more complex networks. Results compared to manual calculations to verify consistency and efficiency.

5. Discussion of Results

The results obtained from the three case studies demonstrate the comparative performance of traditional transportation methods versus shortest path optimization within the multi-objective framework.

Case1: The initial feasible solution obtained by conventional methods resulted in a transportation cost of $440 (Model 2). When reformulated using the shortest path approach, the cost decreased to $360 (Model 3), representing an approximate 18% reduction. However, the shortest path solution initially violated some feasibility conditions (specifically column sufficiency), requiring minor adjustments such as reallocating 80 units between routes. After adjustments, the solution satisfied feasibility while maintaining a lower cost. This highlights that shortest path optimization can outperform traditional allocation in terms of cost efficiency, though practical modifications may be required to ensure model balance.

Case2: In the second transportation problem, the optimal cost using conventional methods was $145 (Model 5). The shortest path solution reduced this further to $130 (Model 6), achieving a 10% reduction. In this case, the shortest path method satisfied the supply and demand conditions for rows but required adjustments for column sufficiency. By reallocating a total of 10 units across specific routes, feasibility was restored without loss of optimality. This confirms that shortest path solutions can yield lower costs while remaining practically adaptable.

Case3: The third case presented a larger and more complex network. The conventional optimal solution cost was $462 (Model 8). The shortest path method significantly reduced this to $218 (Model 9), nearly a 53% reduction. Unlike the earlier cases, the shortest path solution in this scenario satisfied all feasibility conditions without requiring adjustments. This indicates that as the network size and complexity increase, shortest path optimization becomes even more advantageous, providing both cost savings and full feasibility.

6. Conclusions

1. The study demonstrated that applying the shortest path approach in solving transportation problems yields more efficient results compared to traditional methods (Northwest Corner, Least-Cost, Vogel’s Approximation). Across the analyzed cases, transportation costs were reduced by approximately 10% to 53%.

2. Unlike conventional techniques that require multiple steps for generating initial feasible solutions and testing optimality, the shortest path model allows for a more direct and computationally less demanding path to optimal or near-optimal solutions.

3. Results showed that some shortest path solutions initially failed to satisfy all feasibility conditions (particularly supply–demand balance). However, these imbalances were corrected through minor reallocations without compromising optimality, confirming the method’s practical adaptability.

4. The effectiveness of the shortest path approach increased as the network size and complexity expanded. In the third case study, the method reduced transportation costs by more than half, highlighting its particular advantage in large-scale applications.

5. Integrating multi-objective linear programming with shortest path analysis provides a systematic framework that balances cost minimization and distance reduction, thus enhancing both the theoretical and practical value of the model.

6. From a practical standpoint, the proposed model offers Iraqi industrial companies a tool to achieve operational cost savings, accelerate decision-making, and strengthen competitiveness in the marketplace.

7. From an academic perspective, the research contributes to the field of operations research by bridging classical transportation models with network-based optimization within a multi-objective framework, paving the way for broader applications in diverse sectors.

8. The study recommends the adoption of specialized computational tools (e.g., WinQSB and similar software) to implement the model in real-world operations. Future research may extend this work to address multi-modal transportation systems and global supply chains.

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Annexes

Case1:

Table 1 Lowest cost for first class tracks

The path	**Path equation with the lowest possible cost**	**The sheriff is a man**
E_1R_1	$P.E = 3t\ , t = 40$	120
E_1R_2	$P.E = 6u\ , u = 50$	300
E_1R_3	$P.E = 5f$	Not available for the imposed value
$E_1R_2 + E_1R_1$	$P.E = 3t + 6u$	300
$E_1R_3 + E_1R_1$	$P.E = 3t + 5f\ \ , t = 20\ , f = 20$	160
$E_1R_3 + E_1R_2$	$P.E = 6u + 5f\ , u = 20\ , f = 20$	220
$E_1R_2 + E_1R_3 + E_1R_1$	$P.E = 3t + 6u + 5f, t = 0, u = 20, f = 20$	220

The optimal cost is the path $E_1R_3 + E_1R_2 = 220$ \$ for the first year.

Table 2 Lowest cost for second category tracks

The path	Path equation with the lowest possible cost	The sheriff is a man
E_2R_1	$P.E = t\ , t = 50$	50
E_2R_2	$P.E = 4u$	Not available for the imposed value
E_2R_3	$P.E = 2f$	Not available for the imposed value
$E_2R_2 + E_2R_1$	$P.E = t + 4u, t = 50\ , u = 0$	50
$E_2R_3 + E_2R_1$	$P.E = t + 2f\ \ , t = 50\ , f = 0$	50
$E_2R_3 + E_2R_2$	$P.E = 4u + 2f\ , u = 40\ , f = 20$	200
$E_2R_2 + E_2R_3 + E_2R_1$	$P.E = t + 4u + 2f, t = 50, u = 0, f = 0$	50

The optimal cost is the path $E_2R_1 = 50$ $ for the second year.

Table 3 The least expensive third-year courses

The path	Path equation with the lowest possible cost	The sheriff is a man
E_3R_1	$P.E = 4t\ , t = 40$	160
E_3R_2	$P.E = 3u, \quad u = 40$	120
E_3R_3	$P.E = 6f$	Not available for the imposed value
$E_3R_2 + E_3R_1$	$P.E = 4t + 3u, t = 0\ , u = 40$	120
$E_3R_3 + E_3R_1$	$P.E = 4t + 6f\ \ , t = 40\ , f = 0$	160

$E_3R_3 + E_3R_2$	$P.E = 3u + 6f\ , u = 40\ , f = 0$	120
$E_3R_2 + E_3R_3 + E_3R_1$	$P.E = 4t + 3u + 6f, t = 0, u = 40, f = 0$	120

The optimal cost is the path $E_3R_2 = 120\$$ for the third year.

Case2:

Table 4 Lowest cost for first-class tracks

The path	**Path equation with the lowest possible cost**	**The sheriff is a man**
E_1R_1	$P.E = 5t$	Not available for the imposed value
E_1R_2	$P.E = 4u$	Not available for the imposed value
E_1R_3	$P.E = 3f$	Not available for the imposed value
E_1R_4	$P.E = G$	Not available for the imposed value
$E_1R_1 + E_1R_2$	$P.E = 5t + 4u\ , t = 10, u = 10$	90
$E_1R_1 + E_1R_3$	$P.E = 5t + 3f\ \ , t = 5\ , f = 15$	70
$E_1R_1 + E_1R_4$	$P.E = 5t + G\ , t = 5\ , G = 15$	40
$E_1R_2 + E_1R_3$	$P.E = 4u + 3f\ , u = 5\ , f = 15$	65
$E_1R_2 + E_1R_4$	$P.E = 4u + G\ , u = 5\ , G = 15$	35
$E_1R_3 + E_1R_4$	$P.E = 3f + G\ , f = 5\ , G = 15$	30
$E_1R_1 + E_1R_2 + E_1R_3$	$P.E = 5t + 4u + 3f, t = 0, u = 5, f = 15$	65

$E_1R_1 + E_1R_2 + E_1R_4$	$P.E = 5t + 4u + G, t = 0, u = 5, G = 15$	35
$E_1R_1 + E_1R_3 + E_1R_4$	$P.E = 5t + 3f + G, t = 0, f = 5, G = 15$	30
$E_1R_2 + E_1R_3 + E_1R_4$	$P.E = 4u + 3f + G, u = 0, f = 5, G = 15$	30
$E_1R_1 + E_1R_2 + E_1R_3 + E_1R_4$	$P.E = 5t + 4u + 3f + G, t = 0, u = 0, f = 5, G = 15$	35

The optimal cost is the path $E_1R_3 + E_1R_4 = 30\$$ for the first year.

Table 5 Cheapest second-category tracks

The path	**Path equation with the lowest possible cost**	**The sheriff is a man**
E_2R_1	$P.E = 7t, \quad t = 15$	105
E_2R_2	$P.E = u$	Not available for the imposed value
E_2R_3	$P.E = 10f, \quad f = 15$	150
E_2R_4	$P.E = 2G, G = 15$	30
$E_2R_1 + E_2R_2$	$P.E = 7t + u, t = 5, u = 10$	45
$E_2R_1 + E_2R_3$	$P.E = 7t + 10f, t = 5, f = 10$	105
$E_2R_1 + E_2R_4$	$P.E = 7t + 2G, t = 0, G = 15$	30
$E_2R_2 + E_2R_3$	$P.E = u + 10f, u = 10, f = 5$	60
$E_2R_2 + E_2R_4$	$P.E = u + 2G, u = 10, G = 5$	20
$E_2R_3 + E_2R_4$	$P.E = 10f + 2G, f = 0, G = 15$	30

$E_2R_1 + E_2R_2 + E_2R_3$	$P.E = 7t + u + 10f, t = 5, u = 10, f = 0$	45
$E_2R_1 + E_2R_2 + E_2R_4$	$P.E = 7t + u + 2G, t = 0, u = 10, G = 5$	20
$E_2R_1 + E_2R_3 + E_2R_4$	$P.E = 7t + 10f + 2G, t = 0, f = 0, G = 15$	30
$E_2R_2 + E_2R_3 + E_2R_4$	$P.E = u + 10f + 2G, u = 10, f = 0, G = 5$	20
$E_2R_1 + E_2R_2 + E_2R_3 + E_2R_4$	$P.E = 7t + u + 10f + 2G, t = 0, u = 0, f = 0, G = 15$	30

The optimal cost is the path $E_2R_2 + E_2R_4 = 20\$$ for the second year.

Table 6 The least expensive third-year courses

The path	**Path equation with the lowest possible cost**	**The sheriff is a man**
E_3R_1	$P.E = 3t$	Not available for the imposed value
E_3R_2	$P.E = 9u$	Not available for the imposed value
E_3R_3	$P.E = 8f$	Not available for the imposed value
E_3R_4	$P.E = 7G$	Not available for the imposed value
$E_3R_1 + E_3R_2$	$P.E = 3t + 9u, t = 15, u = 5$	90
$E_3R_1 + E_3R_3$	$P.E = 3t + 8f, t = 15, f = 5$	85
$E_3R_1 + E_3R_4$	$P.E = 3t + 7G, t = 15, G = 5$	80
$E_3R_2 + E_3R_3$	$P.E = 9u + 8f, u = 5, f = 15$	165

$E_3R_2 + E_3R_4$	$P.E = 9u + 7G , u = 5 , G = 15$	150
$E_3R_3 + E_3R_4$	$P.E = 8f + 7G , f = 5 , G = 15$	145
$E_3R_1 + E_3R_2 + E_3R_3$	$P.E = 3t + 9u + 8f, t = 15, u = 0, f = 5$	85
$E_3R_1 + E_3R_2 + E_3R_4$	$P.E = 3t + 9u + 7G, t = 15, u = 0, G = 5$	80
$E_3R_1 + E_3R_3 + E_3R_4$	$P.E = 3t + 8f + 7G, t = 15, f = 0, G = 5$	80
$E_3R_2 + E_3R_3 + E_3R_4$	$P.E = 9u + 8f + 7G, u = 0, f = 5, G = 15$	145
$E_3R_1 + E_3R_2 + E_3R_3 + E_3R_4$	$P.E = 3t + 9u + 8f + 7G, t = 15, u = 0, f = 0 , G = 5$	80

The optimal cost is the path $E_3R_1 + E_3R_4 = 80$ $ for the third year.

Case3:

Table 7 The cheapest first-class tracks

The path	Path equation with the lowest possible cost	The sheriff is a man
E_1R_1	$P.E = 7t$	Not available for the imposed value
E_1R_2	$P.E = 3u, \quad u = 33$	99
E_1R_3	$P.E = 8f$	Not available for the imposed value
E_1R_4	$P.E = 2G$	Not available for the imposed value
E_1R_5	$P.E = 6E$	Not available for the imposed value

$E_1R_1 + E_1R_2$	$P.E = 7t + 3u , t = 0, u = 33$	99
$E_1R_1 + E_1R_3$	$P.E = 7t + 8f , t = 22 , f = 11$	242
$E_1R_1 + E_1R_4$	$P.E = 7t + 2G , t = 3 , G = 30$	81
$E_1R_1 + E_1R_5$	$P.E = 7t + 6E , u = 8 , E = 25$	206
$E_1R_2 + E_1R_3$	$P.E = 3u + 8f , u = 33 , f = 0$	99
$E_1R_2 + E_1R_4$	$P.E = 3u + 2G , u = 3 , G = 30$	69
$E_1R_2 + E_1R_5$	$P.E = 3u + 6E , u = 33 , E = 0$	99
$E_1R_3 + E_1R_4$	$P.E = 8f + 2G, f = 3, G = 30$	84
$E_1R_3 + E_1R_5$	$P.E = 8f + 6E$	Not available for the imposed value
$E_1R_4 + E_1R_5$	$P.E = 2G + 6E, G = 30, E = 3$	78
$E_1R_1 + E_1R_2 + E_1R_3$	$P.E = 7t + 3u + 8f, t = 0, u = 33, f = 0$	99
$E_1R_1 + E_1R_2 + E_1R_4$	$P.E = 7t + 3u + 2G, t = 0, u = 3, G = 30$	69
$E_1R_1 + E_1R_2 + E_1R_5$	$P.E = 7t + 3u + 6E, t = 0, u = 33, E = 0$	99
$E_1R_2 + E_1R_3 + E_1R_4$	$P.E = 7t + 8f + 2G, t = 3, f = 0, G = 30$	81
$E_1R_1 + E_1R_3 + E_1R_5$	$P.E = 7t + 8f + 6E, t = 8, f = 0, E = 25$	206
$E_1R_1 + E_1R_4 + E_1R_5$	$P.E = 7t + 2G + 6E, t = 0, G = 30, E = 3$	78
$E_1R_2 + E_1R_3 + E_1R_4$	$P.E = 3u + 8f + 2G, u = 3, f = 0, G = 30$	69
$E_1R_2 + E_1R_3 + E_1R_5$	$P.E = 3u + 8f + 6E, u = 33, f = 0, E = 0$	99

$E_1R_2 + E_1R_4 + E_1R_5$	$P.E = 3u + 2G + 6E, u = 3, G = 30, E = 0$	69
$E_1R_3 + E_1R_4 + E_1R_5$	$P.E = 8f + 2G + 6E, f = 0, G = 30, E = 3$	78
$E_1R_1 + E_1R_2 + E_1R_3 + E_1R_4$	$P.E = 7t + 3u + 8f + 2G, t = 0, u = 3, f = 0, G = 30$	69
$E_1R_1 + E_1R_2 + E_1R_3 + E_1R_5$	$P.E = 7t + 3u + 8f + 6E, t = 0, u = 33, f = 0, E = 0$	99
$E_1R_1 + E_1R_2 + E_1R_4 + E_1R_5$	$P.E = 7t + 3u + 2G + 6E, t = 0, u = 3, G = 30, E = 0$	69
$E_1R_1 + E_1R_3 + E_1R_4 + E_1R_5$	$P.E = 7t + 8f + 2G + 6E, t = 0, f = 0, G = 30, E = 3$	78
$E_1R_2 + E_1R_3 + E_1R_4 + E_1R_5$	$P.E = 3u + 8f + 2G + 6E, u = 3, f = 0, G = 30, E = 0$	69
$E_1R_1 + E_1R_2 + E_1R_3 + E_1R_4 + E_1R_5$	$P.E = 7t + 3u + 8f + 2G + 6E, t = 0, u = 3, f = 0, G = 30, E = 0$	69

The optimal cost is the path $E_1R_2 + E_1R_4 = 69\$$ for the first year.

Table 8 The cheapest second-category slopes

The path	**Path equation with the lowest possible cost**	**The sheriff is a man**
E_2R_1	$P.E = 5t$	Not available for the imposed value
E_2R_2	$P.E = 2u, \quad u = 27$	54
E_2R_3	$P.E = 7f$	Not available for the imposed value
E_2R_4	$P.E = 4G , G = 27$	108
E_2R_5	$P.E = 5E$	Not available for

		the imposed value
$E_2R_1 + E_2R_2$	$P.E = 5t + 2u , t = 0, u = 27$	54
$E_2R_1 + E_2R_3$	$P.E = 5t + 7f$	Not available for the imposed value
$E_2R_1 + E_2R_4$	$P.E = 5t + 4G , t = 70 , G = 2$	108
$E_2R_1 + E_2R_5$	$P.E = 5t + 5E , t = 20 , E = 7$	135
$E_2R_2 + E_2R_3$	$P.E = 2u + 7f , u = 27 , f = 0$	54
$E_2R_2 + E_2R_4$	$P.E = 2u + 4G , u = 27 , G = 0$	54
$E_2R_2 + E_2R_5$	$P.E = 2u + 5E , u = 27 , E = 0$	54
$E_2R_3 + E_2R_4$	$P.E = 7f + 4G , f = 0 , G = 27$	108
$E_2R_3 + E_2R_5$	$P.E = 7f + 5E , f = 2 , G = 25$	139
$E_2R_4 + E_2R_5$	$P.E = 4G + 5E , G = 27 , E = 0$	108
$E_2R_1 + E_2R_2 + E_2R_3$	$P.E = 5t + 2u + 7f , t = 0, u = 27, f = 0$	54
$E_2R_1 + E_2R_2 + E_2R_4$	$P.E = 5t + 2u + 4G , t = 0, u = 27, G = 0$	54
$E_2R_1 + E_2R_2 + E_2R_5$	$P.E = 5t + 2u + 5E , t = 0, u = 27, E = 0$	54
$E_2R_1 + E_2R_3 + E_2R_4$	$P.E = 5t + 7f + 4G, t = 0, f = 0, G = 27$	108
$E_2R_1 + E_2R_3 + E_2R_5$	$P.E = 5t + 7f + 5E, t = 20, f = 0, E = 7$	135
$E_2R_1 + E_2R_4 + E_2R_5$	$P.E = 5t + 4G + 5E , t = 0, G = 27, E = 0$	108
$E_2R_2 + E_2R_3 + E_2R_4$	$P.E = 2u + 7f + 4G , u = 27, f = 0, G = 0$	54

$E_2R_2 + E_2R_3 + E_2R_5$	$P.E = 2u + 7f + 5E, u = 27, f = 0, E = 0$	54
$E_2R_2 + E_2R_4 + E_2R_5$	$P.E = 2u + 4G + 5E, u = 27, f = 0, E = 0$	54
$E_2R_3 + E_2R_4 + E_2R_5$	$P.E = 7f + 4G + 5E, f = 0, G = 27, E = 0$	108
$E_2R_1 + E_2R_2 + E_2R_3 + E_2R_4$	$P.E = 5t + 2u + 7f + 4G, t = 0, u = 27, f = 0, G = 0$	54
$E_2R_1 + E_2R_2 + E_2R_3 + E_2R_5$	$P.E = 5t + 2u + 7f + 5E, t = 0, u = 27, f = 0, E = 0$	54
$E_2R_1 + E_2R_2 + E_2R_4 + E_2R_5$	$P.E = 5t + 2u + 4G + 5E, t = 0, u = 27, G = 0, E = 0$	54
$E_2R_1 + E_2R_3 + E_2R_4 + E_2R_5$	$P.E = 5t + 7f + 4G + 5E, t = 0, f = 0, G = 27, E = 0$	108
$E_2R_2 + E_2R_3 + E_2R_4 + E_2R_5$	$P.E = 2u + 7f + 4G + 5E, u = 27, f = 0, G = 0, E = 0$	54
$E_2R_1 + E_2R_2 + E_2R_3 + E_2R_4 + E_2R_5$	$P.E = 5t + 2u + 7f + 4G + 5E, t = 0, u = 27, f = 0, G = 0, E = 0$	54

The optimal cost is the path $E_2R_2 = 54$ $ for the second year.

Table 9 The least expensive third-year tracks

The path	Path equation with the lowest possible cost	The sheriff is a man
E_3R_1	$P.E = 4t$	Not available for

		the imposed value
E_3R_2	$P.E = 3u, \quad u = 40$	120
E_3R_3	$P.E = 2f$	Not available for the imposed value
E_3R_4	$P.E = 5G$	Not available for the imposed value
E_3R_5	$P.E = E$	Not available for the imposed value
$E_3R_1 + E_3R_2$	$P.E = 4t + 3u , t = 0, u = 40$	120
$E_3R_1 + E_3R_3$	$P.E = 4t + 2f$	Not available for the imposed value
$E_3R_1 + E_3R_4$	$P.E = 4t + 5G , t = 20 , G = 20$	180
$E_3R_1 + E_3R_5$	$P.E = 4t + E , t = 15 , E = 25$	85
$E_3R_2 + E_3R_3$	$P.E = 3u + 2f , u = 35 , f = 5$	115
$E_3R_2 + E_3R_4$	$P.E = 3u + 5G , u = 40 , G = 0$	120
$E_3R_2 + E_3R_5$	$P.E = 3u + E , u = 15 , E = 25$	70
$E_3R_3 + E_3R_4$	$P.E = 2f + 5G$	Not available for the imposed value
$E_3R_3 + E_3R_5$	$P.E = 2f + E$	Not available for the imposed value
$E_3R_4 + E_3R_5$	$P.E = 5G + E , G = 15 , E = 25$	100
$E_3R_1 + E_3R_2 + E_3R_3$	$P.E = 4t + 3u + 2f , t = 0, u = 35, f = 5$	115
$E_3R_1 + E_3R_2 + E_3R_4$	$P.E = 4t + 3u + 5G , t = 0, u = 40, G = 0$	120

$E_3R_1 + E_3R_2 + E_3R_5$	$P.E = 4t + 3u + E , t = 0, u = 15, E = 25$	70
$E_3R_1 + E_3R_3 + E_3R_4$	$P.E = 4t + 2f + 5G, t = 20, f = 5, \quad G = 15$	165
$E_3R_1 + E_3R_3 + E_3R_5$	$P.E = 5t + 7f + 5E, t = 20, f = 0, E = 7$	75
$E_3R_1 + E_3R_4 + E_3R_5$	$P.E = 4t + 5G + E, t = 15, G = 0, E = 25$	85
$E_3R_2 + E_3R_3 + E_3R_4$	$P.E = 5t + 4G + 5E , t = 0, G = 27, E = 0$	115
$E_3R_2 + E_3R_3 + E_3R_5$	$P.E = 2u + 7f + 4G , u = 27, f = 0, G = 0$	65
$E_3R_2 + E_3R_4 + E_3R_5$	$P.E = 2u + 7f + 5E , u = 27, f = 0, E = 0$	70
$E_3R_3 + E_3R_4 + E_3R_5$	$P.E = 2u + 4G + 5E , u = 27, f = 0, E = 0$	85
$E_3R_1 + E_3R_2 + E_3R_3 + E_3R_4$	$P.E = 7f + 4G + 5E , f = 0, G = 27, E = 0$	115
$E_3R_1 + E_3R_2 + E_3R_3 + E_3R_5$	$P.E = 5t + 2u + 7f + 4G , t = 0, u = 27, f = 0, G = 0$	65
$E_3R_1 + E_3R_2 + E_3R_4 + E_3R_5$	$P.E = 5t + 2u + 7f + 5E , t = 0, u = 27, f = 0, E = 0$	70
$E_3R_1 + E_3R_3 + E_3R_4 + E_3R_5$	$P.E = 5t + 2u + 4G + 5E , t = 0, u = 27, G = 0, E = 0$	75
$E_3R_2 + E_3R_3 + E_3R_4 + E_3R_5$	$P.E = 5t + 7f + 4G + 5E , t = 0, f = 0, G = 27, E = 0$	65

$E_3R_1 + E_3R_2 + E_3R_3 + E_3R_4 + E_3R_5$	$P.E = 2u + 7f + 4G + 5E$, $u = 27, f = 0, G = 0, E = 0$	65

The optimal cost is $E_3R_2 + E_3R_3 + E_3R_5 = 65$ $ for the third year.

Table 10 of the cheapest fourth-year tracks

The path	**Path equation with the lowest possible cost**	**The sheriff is a man**
E_4R_1	$P.E = t$	Not available for the imposed value
E_4R_2	$P.E = 2u, \quad u = 25$	50
E_4R_3	$P.E = 3f$	Not available for the imposed value
E_4R_4	$P.E = 5G$, $G = 25$	125
E_4R_5	$P.E = 7E$, $E = 25$	175
$E_4R_1 + E_4R_2$	$P.E = t + 2u$, $t = 20, u = 5$	30
$E_4R_1 + E_4R_3$	$P.E = t + 3f$, $t = 20, f = 5$	35
$E_4R_1 + E_4R_4$	$P.E = t + 5G$, $t = 20$, $G = 5$	45
$E_4R_1 + E_4R_5$	$P.E = t + 7E$, $t = 20$, $E = 5$	55
$E_4R_2 + E_4R_3$	$P.E = 2u + 3f$, $u = 25$, $f = 0$	50
$E_4R_2 + E_4R_4$	$P.E = 2u + 5G$, $u = 25$, $G = 0$	50
$E_4R_2 + E_4R_5$	$P.E = 2u + 7E$, $u = 25$, $E = 0$	50
$E_4R_3 + E_4R_4$	$P.E = 3f + 5G$, $f = 5, G = 20$	115
$E_4R_3 + E_4R_5$	$P.E = 3f + 7E$, $f = 5, E = 20$	155
$E_4R_4 + E_4R_5$	$P.E = 5G + 7E$, $G = 25$, $E = 0$	125

$E_4R_1 + E_4R_2 + E_4R_3$	$P.E = t + 2u + 3f\ , t = 20, u = 5, f = 0$	30
$E_4R_1 + E_4R_2 + E_4R_4$	$P.E = t + 2u + 5G\ , t = 20, u = 5, G = 0$	30
$E_4R_1 + E_4R_2 + E_4R_5$	$P.E = t + 2u + 7E\ , t = 20, u = 5, E = 0$	30
$E_4R_1 + E_4R_3 + E_4R_4$	$P.E = t + 3f + 5G, t = 20, f = 5, G = 0$	35
$E_4R_1 + E_4R_3 + E_4R_5$	$P.E = t + 3f + 7E, t = 20, f = 5, E = 0$	35
$E_4R_1 + E_4R_4 + E_4R_5$	$P.E = t + 5G + 7E, t = 20, G = 5, E = 0$	45
$E_4R_2 + E_4R_3 + E_4R_4$	$P.E = 2u + 3f + 5G\ , u = 25, f = 0, E = 0$	50
$E_4R_2 + E_4R_3 + E_4R_5$	$P.E = 2u + 3f + 7E\ , u = 25, f = 0, E = 0$	50
$E_4R_2 + E_4R_4 + E_4R_5$	$P.E = 2u + 5G + 7E\ , u = 25, G = 0, E = 0$	50
$E_4R_3 + E_4R_4 + E_4R_5$	$P.E = 2f + 5G + 7E\ , f = 5, G = 20, E = 0$	115
$E_4R_1 + E_4R_2 + E_4R_3 + E_4R_4$	$P.E = t + 2u + 3f + 5G\ , t = 20, u = 5, f = 0, G = 0$	30
$E_4R_1 + E_4R_2 + E_4R_3 + E_4R_5$	$P.E = t + 2u + 3f + 7E\ , t = 20, u = 5, f = 0, E = 0$	30
$E_4R_1 + E_4R_2 + E_4R_4 + E_4R_5$	$P.E = t + 2u + 5G + 7E\ , t = 20, u = 5, G = 0, E = 0$	30
$E_4R_1 + E_4R_3 + E_4R_4 + E_4R_5$	$P.E = t + 3f + 5G + 7E\ , t = 20, f = 5, G = 0, E = 0$	35

$E_4R_2 + E_4R_3 + E_4R_4 + E_4R_5$	$P.E = 2u + 3f + 5G + 7E$, $u = 25, f = 0, G = 0, E = 0$	50
$E_4R_1 + E_4R_2 + E_4R_3 + E_4R_4 + E_4R_5$	$P.E = t + 2u + 3f + 5G + 7E$, $t = 20, u = 5, f = 0, G = 0, E = 0$	30

The optimal cost is the path $E_4R_1 + E_4R_2 = 30$ \$ for the fourth year.