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A Comparative Performance Evaluation of Some Signed-Rank Based Control Charts via Monte Carlo Simulations

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Abstract

Nonparametric control charts offer a robust alternative to traditional parametric charts, particularly when the underlying process distribution is unknown or difficult to specify. This paper investigates several *signed-rank* (SR) based control charts, including Shewhart, *cumulative sum* (CUSUM), and *exponentially weighted moving average* (EWMA) charts, through a comprehensive comparative evaluation. The performance of the control charts is evaluated under multiple sample sizes and across various process distributions (normal, uniform, Laplace, and logistic). A practical simulation example is also provided to demonstrate each chart's ability to detect shifts in process location. The performance of the control charts evaluated through Monte Carlo simulations, with the *average run length* (ARL) as a performance measure. The study provides actionable insights for selecting the most effective nonparametric control chart under varying conditions, offering both methodological and practical contributions to the literature.

Keywords: Nonparametric charts, Average run length, Simulation technique, signed-rank statistic

1. Introduction

Statistical quality control (SQC) is defined as a method for checking production processes in order to control and improve the quality of products. This approach uses tools such as control charts, Process Capability analysis, and acceptance sampling to ensure that the production or service process is operating efficiently and that it meets established quality specifications (Montgomery, 2019).

A main tool of SQC is the control chart, which graphically represents a quality characteristic measured or calculated from a sample in relation to the sample number or time. The graphic consists of a *center line* (CL) that denotes the mean value of the quality characteristic associated with the in-control condition. The chart additionally displays two more horizontal lines, referred to as the upper *control limit* (UCL) and *lower control limit* (LCL). The traditional control chart is designed based on some assumptions. Consider $\bar{X}$ and R charts for illustration, which are used for detecting a shift respectively in the process mean and process variation. These charts assume that the distribution of quality characteristics, (say X) is normal. The control chart procedure can be viewed as a testing of the hypothesis problem with the null hypothesis, H_0: Process mean μ equal to a specified value μ_0 . Here μ is the process target and μ_0 be its specified value. If all sample points fall within the UCL and LCL then the process is stable, which means the null hypothesis (H_0) is supported and the process is said to be in-control. On the other hand, when any sample point falls outside the UCL or LCL, then there is a change in the process, which means $\mu \neq \mu_0$, and the process is said to be an out-of-control.

The current SQC literature contains various types of control charts. The most ever control chart is the well-known Shewhart control chart (Shewhart, 1931). It is a memory-less control chart, that is, its control statistic for the current sampling instance is based on the sample taken at that sampling instance only. Shewhart chart is highly efficient in detecting the large process shifts (that is, the shifts in the process parameter being monitored), however, it is relatively less efficient in detecting the small ones. The *cumulative sum* (CUSUM) chart (Page, 1954) and the *exponentially weighted moving average* (EWMA) chart (Roberts, 1959) are the alternatives to the Shewhart control chart which are more efficient than the Shewhart chart in detecting the small process shifts. However, those are less efficient than the Shewhart chart in detecting the large shifts. Generally, the CUSUM and EWMA charts are approximately equally effective in detecting a process small shift size. These are the memory-based control charts, that is, their control statistics for the current sampling instance are based on the samples taken at the current as well as the past sampling instances.

Many times, in practice, the probability distribution of the underlying process is unknown because of insufficient information. In such situations, monitoring the underlying process using a parametric control chart is generally misleading if the actual probability distribution of the process differs from the assumed one, and hence, the process must be monitored using a nonparametric control chart since the in-control statistical performance of a nonparametric control chart remains the same for any underlying continuous process distribution. For a nonparametric charts, the chart's design parameters as well as the run length properties are derived based on the distribution of the corresponding nonparametric statistics which are used (such us the sign or the *Wilcoxon signed-rank* (SR) statistics) with only a minimal knowledge of the form of the underlying distribution for the characteristic to be monitored. We refer to Chakraborti et al. (2001), Qiu and Li (2011), Qiu (2018), and Chakraborti and Graham (2019) for extensive overviews on a nonparametric control charts.

The literature contains a variety of a nonparametric control charts based on SR statistic. For example, Bakir and Reynolds (1979) investigated the CUSUM chart based on the SR statistic while Bakir (2004) proposed the Shewhart SR chart. Chakraborti and Eryilmaz (2007) proposed the Shewhart SR chart using runs for identifying small shifts in location parameters efficiently. Graham et al. (2011) proposed the EWMA SR chart. Abid et al. (2018) proposed the CUSUM SR chart utilizing a ranked set sampling procedure. Alevizakos et al. (2020) proposed the double generally weighted moving average SR chart. Alevizakos et al. (2021) suggested the triple EWMA SR chart. Perdikis et al. (2021) investigated the exact run length properties of the EWMA SR chart. Recently, Abbas et al. (2022) proposed the progressive SR chart for monitoring the process location. Perdikis et al. (2023) investigated the performance of the EWMA SR chart for finite horizon processes. Recently, Talordphop et al. (2023) constructed and evaluated an extended EWMA SR chart. Shanshool et al. (2024) proposed the combined Shewhart CUSUM charts based on SR statistic. This paper investigates several nonparametric SR-based control charts, including Shewhart, CUSUM, and EWMA charts, by providing a comprehensive comparative evaluation. Unlike prior studies, the present work examines chart performance under multiple sample sizes and across diverse process distributions, including normal, uniform, Laplace, and logistic distributions. In addition, a practical simulation example is included to illustrate each chart's ability to detect shifts in process location.

2. Nonparametric Control charts

In this section we will give a review of Shewhart SR, CUSUM SR, and EWMA SR charts.

Let X denote a quality characteristic. Suppose that the probability distribution of X is and symmetric and continuous. Let μ be the median and σ be the standard deviation of X. Suppose we have to monitor μ at its target value μ_0. Further, suppose that an occurrence of an assignable cause shifts μ from μ_0 to $\mu_0 + \delta\sigma, -\infty < \delta < \infty, \delta \neq 0$, but does not affect σ. Further, suppose that once μ shifts from μ_0 to $\mu_0 + \delta\sigma$, it remains there until the detection of this shift.

To monitor μ, a sample of size n on X is taken at each sampling instance of a control chart which occurs repeatedly after a determined period. Let x_{ij}, $i = 1, 2, \ldots$, $j = 1, 2, \ldots, n$, be the j^{th}observation in the i^{th} sample. For the i^{th} sample, the SR statistic is defined as

$$SR_i = \sum_{j=1}^{n} sign(x_{ij} - \mu_0)R_{ij}$$

where

$$sign(x) = \begin{cases} 1 & if\ \ x > 0 \\ 0 & if\ \ x = 0 \\ -1 & if\ \ x < 0 \end{cases}$$

and $R_{ij} = Rank(|x_{ij} - \mu_0|), i = 1, 2, \ldots, j = 1, 2, \ldots, n$.

i. Shewhart SR charts

At the i^{th} sampling instance, i =1, 2…, the upper one-sided Shewhart SR chart for process location, proposed by Bakir (2004), uses the control statistic SR_i. The chart declares an out-of-control signal at the i^{th} sampling instance, if $SR_i \geq a_{sr}$, where a_{sr} is the UCL of this chart, which is chosen such that the chart has the desired in-control statistical performance. Likewise, the lower one-sided Shewhart SR chart issues an out-of-control signal at the i^{th} sampling instance, if $-SR_i \geq a_{sr}$.

At the i^{th} sampling instance, the two-sided Shewhart SR chart for location proposed by Bakir (2004) uses the control statistic SR_i and declares an out-of-control signal if $SR_i \leq -a_{sr}$ or $SR_i \geq a_{sr}$

ii. CUSUM SR charts

At the i^{th} sampling instance, i =1, 2…, the upper one-sided CUSUM SR chart (Bakir and Reynolds,1979), define by the control statistic $C_i^+ = \max\{0, C_{i-1}^+ + SR_i - k\}$, where $C_0^+ = 0$ and k is a positive constant (named the references value). This chart declares an out-of-control signal at the i^{th} sampling instance, if $C_i^+ \geq h_{sr}$, where h_{sr} is the UCL of the chart. similarly, the lower one-sided CUSUM SR chart defined by the control statistic $C_i^- = \max\{0, C_{i-1}^- - SR_i - k\}$ at the i^{th} sampling instance, i =1, 2…, where $C_0^- = 0$, and declares an out-of-control signal if $C_i^- \geq h_{sr}$.

The two-sided CUSUM SR chart proposed by Bakir and Reynolds (1979) uses the control statistics $C_i = \max\{C_i^+, C_i^-\}$, where C_i^+ and C_i^- given by

$$C_i^+ = \max\{0, C_{i-1}^+ + SR_i - k\}$$
$$C_i^- = \max\{0, C_{i-1}^- - SR_i - k\}$$

with initial values $C_0^+ = C_0^- = 0$. The chart signals an out-of-control at the i^{th} sampling instance, if $C_i \geq h_{sr}$.

iii. The EWMA SR chart

The EWMA SR chart (Graham et al., 2011) designed to monitor the location shifts defined as

$$Z_i = \lambda SR_i + (1-\lambda)Z_{i-1}$$

The process is said to be an out-of-control if $Z_i \leq LCL$ or $Z_i \geq UCL$, where the steady-state LCL and UCL are, respectively given by

$$LCL = -L\sqrt{\frac{n(n+1)(2n+1)}{6}\left(\frac{\lambda}{2-\lambda}\right)}$$

and

$$UCL = L\sqrt{\frac{n(n+1)(2n+1)}{6}\left(\frac{\lambda}{2-\lambda}\right)}$$

where λ $(0 < \lambda \leq 1)$ is the smoothing constant, and L is the charting constant of the EWMA SR chart with $Z_0 = 0$. The values of λ and L are to be selected such that the chart has desired in-control statistical performance.

3. The Statistical Performance Measure

The effectiveness of the control charts can be evaluated using the *average run length* (ARL), which is the average number of samples that taken by the chart from the occurrence of a process shift until it gives an out-of-control. There are two methods for calculating the ARL. The first approach is based on the assumption that the process shift exists at the onset of the chart application. The ARL calculated with this method is named the *zero-state ARL* (ZSARL). The second approach supposes that the process is in-control at the onset of the chart application and a shift happens at some random time later. The ARL measured with this approach is called *steady-state ARL* (SSARL). These methods have been well discussed by Crosier (1986) and Godase and Mahadik (2024).

Let $ZSARL_{\delta}$ be the ZSARL and $SSARL_{\delta}$ be the SSARL of a SR chart when $\omega = \omega_0 + \delta\sigma,\ -\infty < \delta < \infty$. Since $SSARL_{\delta}$ is a more realistic approach than $ZSARL_{\delta}$, we used it to evaluate the statistical performance of the charts.

For comparison purpose, the three charts are sets in such a way that uses the same sample size n and have same $ZSARL_0$. Then, the $SSARL_{\delta}$ values of these charts were determined for different values of shift δ through 100,000 simulation runs using R programming language when the underlying process distributions normal, uniform, Laplace and logistic. The probability density functions of these distributions and their in-control parameters considered in this simulation study are as shown in Table 1.

Table 1: The PDFs and the in-control parameters of the normal, uniform, Laplace, and Logistic distributions used in the simulation study

Distribution	Probability density functions	In-control parameters
Normal	$\frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu)}{\sigma}\right)^2}, -\infty < x < \infty$	$\mu = 0, \sigma = 1$
Uniform	$\frac{1}{b-a}, a < x < b$	$a = -\sqrt{3}, b = \sqrt{3}$
Laplace	$\frac{1}{2\lambda} e^{-\frac{1}{\lambda}\lvert x-\mu\rvert},\ -\infty < x < \infty$	$\mu = 0, \lambda = \frac{1}{\sqrt{2}}$
Logistic	$\frac{e^{-\left(\frac{x-\mu}{\sigma}\right)}}{\sigma\left(1+e^{-\left(\frac{x-\mu}{\sigma}\right)}\right)^2}, -\infty < x < \infty$	$\mu = 0, \sigma = \frac{\sqrt{3}}{\pi}$

Table 2 presents the design parameters for the Shewhart SR, CUSUM SR, and EWMA SR control charts under different sample sizes and in-control ARL. For $n = 10$, The Shewhart SR chart use a control limit $a_{sr} = 52$, while the EWMA SR chart is designed with a smoothing parameter $\lambda = 0.25$ and $L = 2.71$. The CUSUM SR chart, in contrast, relies on a decision interval $h_{sr} = 84$ and a reference value $k = 10$. Similarly, for $n = 12$, The Shewhart SR chart use a control limit $a_{sr} = 72$, while the EWMA SR chart is designed with a smoothing parameter $\lambda = 0.25$ and $L = 2.869$. The CUSUM SR chart, set with $h_{sr} = 139$ and reference value $k = 11$.

Table 2: The design parameters for the Shewhart SR, CUSUM SR, and EWMA SR

Design parameters	$ZSATS_0 \approx 250$			$ZSATS_0 \approx 410$		
	Shewhart	CUSUM	EWMA	Shewhart	CUSUM	EWMA
	$n = 10$	$n = 10$	$n = 10$	$n = 20$	$n = 12$	$n = 12$
a_{sr}	52	---	--	72	---	---
λ	---	---	0.25	---	---	0.25
L	---	---	2.71	---	---	2.869
h_{sr}	---	84	---	---	139	---
k	---	10	---	---	11	---

These design parameters were selected to achieve a comparable in-control performance, with in-control ARL values approximating 250 and 410, ensuring a fair basis for comparison across the different chart types. After determining these parameters, the ARL values were computed for all three charts under multiple process distributions. This facilitates a robust comparative evaluation of their detection performance across symmetric distributions.

Table 3: The $SSARL_\delta$ values of Shewhart SR, CUSUM SR, and EWMA SR charts for normal and uniform distributions with ARL≈250 and n=10

δ	$Normal$			$Uniform$		
	Shewhart	CUSUM	EWMA	Shewhart	CUSUM	EWMA
0.00	255.54	254.90	255.17	253.02	251.87	250.98
0.10	200.46	75.80	79.45	222.35	167.21	175.81

0.15	153.53	38.29	39.86	189.96	119.21	125.02
0.20	115.89	22.76	23.87	157.15	84.50	90.77
0.25	85.70	15.19	15.61	123.30	61.90	69.12
0.30	63.62	11.28	11.10	100.15	46.80	51.99
0.35	48.50	8.80	8.67	81.41	36.35	41.05
0.40	35.88	7.31	6.94	63.24	29.51	33.20
0.50	21.88	5.51	5.05	40.17	20.52	22.94
1.00	3.48	2.95	2.57	5.44	7.88	8.31
1.50	1.44	2.32	2.16	1.31	5.30	5.40
2.00	1.07	2.03	2.07	1.00	4.24	4.25
2.50	1.00	2.00	2.05	1.00	3.74	3.64
3.00	1.00	2.00	2.05	1.00	3.39	3.30
Design parameters	$a_{sr} = 52$	$h_{sr} = 84$ $k = 10$	$L = 2.71$ $\lambda = 0.25$	$a_{sr} = 52$	$h_{sr} = 84$ $k = 10$	$L = 2.71$ $\lambda = 0.25$

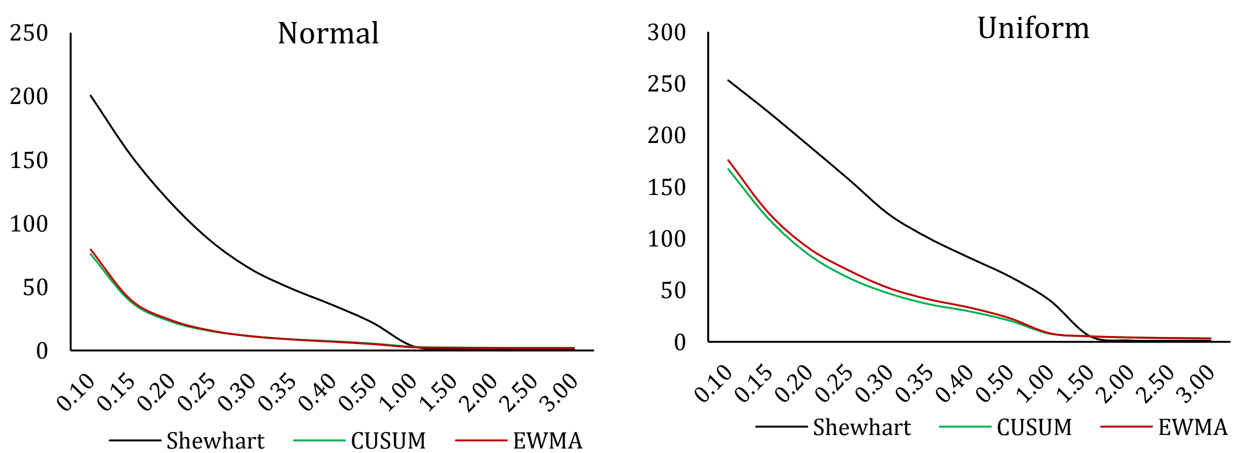

Figure 1: The $SSARL_{\delta}$ values for normal and uniform distributions with ARL≈250 and n=10

Table 4: The $SSARL_\delta$ values of Shewhart SR, CUSUM SR, and EWMA SR charts for Laplace and logistic distributions with ARL≈250 and n=10

δ	Laplace			Logistic		
	Shewhart	CUSUM	EWMA	Shewhart	CUSUM	EWMA
0.00	255.94	250.60	250.65	250.89	250.46	250.26
0.10	62.85	46.68	49.74	185.93	64.90	69.80
0.15	29.32	22.73	24.55	138.86	31.80	34.88
0.20	16.31	13.64	14.98	98.66	18.56	20.18
0.25	10.18	9.55	10.28	70.41	12.39	13.75
0.30	6.75	7.37	7.64	51.08	9.31	9.77
0.35	3.69	6.07	6.26	37.83	7.31	7.63
0.40	2.43	5.22	5.31	28.16	6.14	6.29
0.50	1.86	4.16	4.12	16.36	4.69	4.64
1.00	1.66	2.57	2.49	2.86	2.63	2.54
1.50	1.51	2.17	2.18	1.41	2.15	2.16
2.00	1.41	2.00	2.09	1.09	1.98	2.08
2.50	1.13	1.95	2.07	1.02	1.94	2.05
3.00	1.01	1.94	2.05	1.00	1.94	2.05
Design parameters	$a_{sr} = 52$	$h_{sr} = 84$	$L = 2.71$	$a_{sr} = 52$	$h_{sr} = 84$	$L = 2.71$
		$k = 10$	$\lambda = 0.25$		$k = 10$	$\lambda = 0.25$

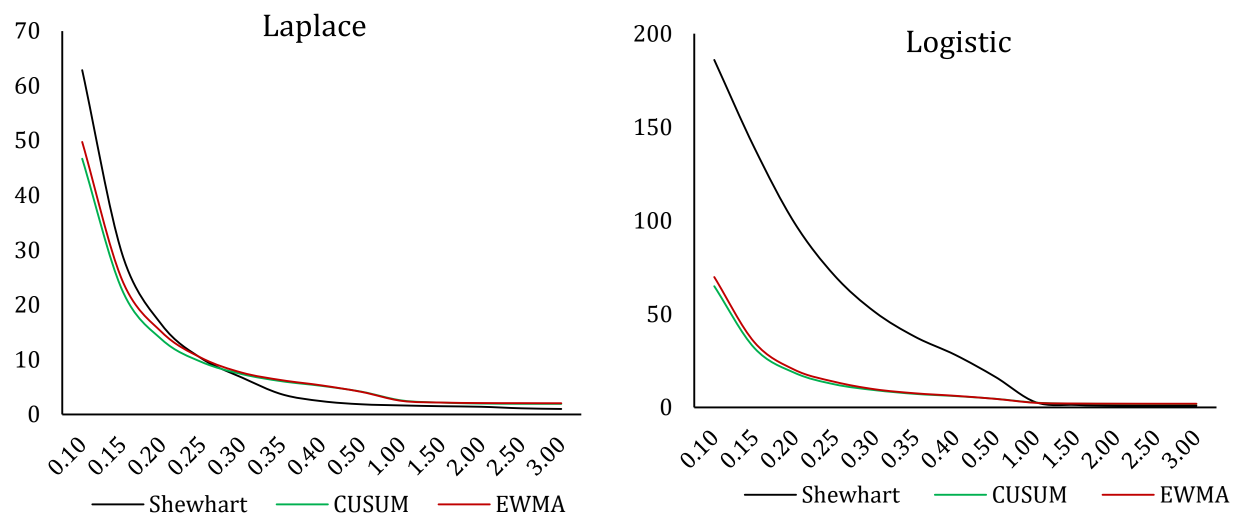

Figure 2: The $SSARL_\delta$ values for Laplace and logistic distributions with $ARL \approx 250$ and n=10

Table 5: The $SSARL_\delta$ values of Shewhart SR, CUSUM SR, and EWMA SR charts for normal and uniform distributions with ARL≈410 and n=12

δ	*Normal*			*Uniform*		
	Shewhart	CUSUM	EWMA	Shewhart	CUSUM	EWMA
0.00	410.32	408.64	405.87	409.98	409.32	405.97
0.10	292.23	70.99	93.60	336.86	209.46	102.77
0.15	207.98	32.40	42.85	266.58	129.21	49.08
0.20	145.85	19.00	24.10	203.74	82.34	27.13
0.25	103.00	12.93	15.39	159.51	57.72	17.35
0.30	73.33	9.79	10.67	119.08	41.66	12.26
0.35	52.90	7.77	8.25	90.44	32.19	9.50
0.40	38.84	6.57	6.56	67.61	25.41	7.50
0.50	21.23	5.10	4.79	39.40	17.95	5.44
1.00	2.81	2.86	2.47	4.00	7.60	2.60
1.50	1.24	2.59	2.10	1.11	5.33	2.06
2.00	1.02	2.51	2.00	1.00	4.37	1.98
2.50	1.00	2.49	1.98	1.00	3.85	1.98
3.00	1.00	2.49	1.98	1.00	3.52	1.98

Design parameters	$a_{sr} = 72$	$h_{sr} = 139$	$L = 2.869$	$a_{sr} = 72$	$h_{sr} = 139$	$L = 2.869$
		$k = 11$	$\lambda = 0.25$		$k = 11$	$\lambda = 0.25$

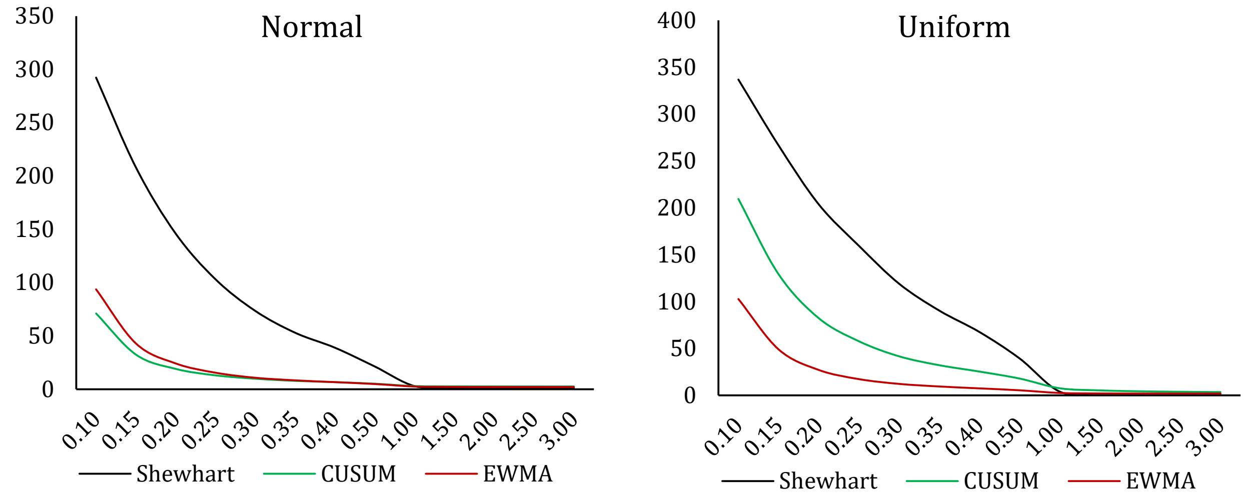

Figure 3: The $SSARL_\delta$ values for normal and uniform distributions with $ARL \approx 410$ and n=12

Table 6: The $SSARL_\delta$ values of Shewhart, CUSUM, and EWMA charts for Laplace and logistic distributions with ARL≈250 and n=10

δ	Laplace			Logistic		
	Shewhart	CUSUM	EWMA	Shewhart	CUSUM	EWMA
0.00	410.943	49.36	404.92	409.74	410.32	403.51
0.10	211.63	43.03	57.23	270.01	61.94	57.24
0.15	127.23	20.57	25.62	181.68	28.01	25.61
0.20	78.26	12.60	14.60	123.59	16.74	14.60
0.25	50.90	9.15	9.83	83.23	11.53	9.83
0.30	33.36	7.22	7.42	58.07	8.79	7.42
0.35	23.65	6.10	5.92	40.71	7.21	5.92

0.40	17.41	5.28	5.00	28.66	6.06	5.00
0.50	9.95	4.26	3.93	15.93	4.74	3.93
1.00	2.12	2.81	2.40	2.45	2.83	2.40
1.50	1.28	2.60	2.10	1.25	2.59	2.11
2.00	1.07	2.53	2.03	1.04	2.52	2.03
2.50	1.01	2.51	2.00	1.00	2.50	2.00
3.00	1.00	2.49	2.00	1.00	2.50	2.00
Design parameters	$a_{sr} = 72$	$h_{sr} = 139$	$L = 2.869$	$a_{sr} = 72$	$h_{sr} = 139$	$L = 2.869$
		$k = 11$	$\lambda = 0.25$		$k = 11$	$\lambda = 0.25$

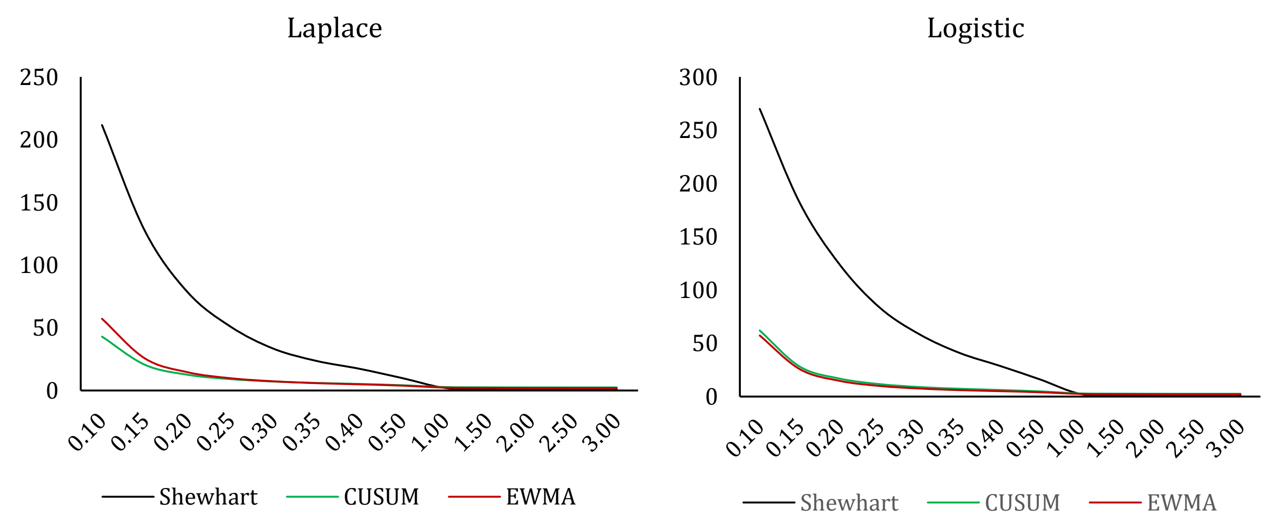

Figure 4: The $SSARL_{\delta}$ values for Laplace and logistic distributions with $ARL \approx 410$ and n=12

Tables 2–6 present the$SSARL_{\delta}$ values of the Shewhart SR, CUSUM SR, and EWMA SR charts across normal, uniform, Laplace, and logistic distributions for varying shift magnitudes δ and sample sizes n. The results demonstrate that for small shifts ($\delta \leq 0.25$), the Shewhart SR chart exhibits substantially higher $SSARL_{\delta}$ values than CUSUM and EWMA charts, indicating lower sensitivity in detecting minor changes, whereas CUSUM and EWMA charts show smaller values, reflecting superior detection capability for small to moderate shifts. As the shift magnitude increases ($\delta \geq 1.00$), all three charts converge toward unity, indicating similar efficiency in signaling large shifts. A similar conclusion is also observed when the sample size is $n = 12$, confirming the robustness of these findings across different subgroup sizes.

4. Case Study (Simulation example)

For the purpose of illustrating the computations of the control statistics of the Shewhart SR, CUSUM SR, and EWMA SR charts. Suppose the process distribution is $Normal\ (0,1)$ and in-control location parameter (that is, median) 65. Five samples each of size 12 were simulated from this distribution. Furthermore, additional 15 samples each of size 12 were simulated from normal distribution with the same parameter but the location parameter 65.50 to represent the location shift, the simulated data are shown in Table 7.

Table 7: Simulated data

Sample No.	Observations											
	1	2	3	4	5	6	7	8	9	10	11	12
1	64.29	64.62	64.41	65.22	64.81	65.82	63.26	65.27	64.49	66.14	65.17	66.07
2	64.70	64.95	64.94	67.07	63.64	64.98	63.86	64.72	64.32	64.80	66.49	65.01
3	64.85	63.45	65.91	64.53	65.06	65.09	64.81	67.03	64.50	62.94	65.37	63.49
4	64.23	64.74	63.34	63.75	66.14	64.19	64.29	65.80	64.18	64.54	66.54	65.24
5	64.36	64.78	65.29	65.44	65.23	65.55	64.88	65.07	64.96	66.34	65.00	64.27
6	65.22	65.58	66.74	67.01	67.76	68.52	65.58	65.60	66.89	65.66	66.57	65.32
7	64.02	67.35	65.86	66.07	64.65	66.77	66.04	66.58	62.35	66.85	66.31	63.00
8	65.82	64.59	66.20	65.41	65.90	65.03	65.04	65.11	65.49	63.97	64.61	66.19
9	66.	64.	63.	63.	64.	64.	64.	64.	64.	66.	65.	64.

	30	33	15	42	62	83	64	70	85	10	90	83
10	65.16	65.40	65.02	64.91	66.70	64.46	63.92	66.87	65.50	64.94	65.60	66.08
11	65.10	66.84	64.39	64.08	66.39	65.77	67.00	64.25	65.16	65.26	64.68	64.30
12	65.54	65.86	65.74	65.90	63.75	65.49	65.42	64.03	66.62	64.93	65.26	65.90
13	64.98	66.37	64.68	66.04	66.39	67.10	65.07	67.45	65.06	66.95	65.63	66.35
14	66.49	68.72	65.94	66.45	65.73	67.38	65.51	65.33	64.76	65.69	66.19	66.68
15	67.51	67.36	66.43	65.10	65.31	67.36	63.82	66.25	65.76	66.09	64.31	65.75
16	65.27	64.93	66.82	65.98	65.63	65.68	64.97	65.20	64.84	67.45	66.25	64.81
17	66.22	65.81	65.47	65.82	66.20	64.98	65.04	63.79	66.29	66.08	65.60	65.15
18	65.46	64.96	64.42	64.81	64.66	66.39	65.59	66.74	64.77	65.23	66.90	65.32
19	65.50	67.13	65.69	66.77	66.77	64.42	66.50	64.42	64.74	64.83	66.41	65.54
20	65.29	66.87	65.46	64.41	65.43	65.15	64.21	65.59	65.64	65.10	64.91	62.54

Now suppose, the Shewhart SR, CUSUM SR, and EWMA SR charts is expected to have $ZSARL_0 \approx 405$. This leads to choose the parameter a_{sr} of the Shewhart SR chart to be 72, and the parameters for CUSUM SR $k \; and \; h_{sr}$ respectively to be 11 and 119, while the parameters $\lambda \; and \; L$ of the EWMA SR chart to be respectively, 0.25, and 2.869.

Now, let us discuss the implementation of the Shewhart SR, see that we have the value of the SR_1 statistic is -2.00 in the 1^{st}sample, as that value falls within the control limits of the chart (-72,72), we conclude that the process is in-control at the 1^{st}sampling instance and take the second sample after t time . For the second sample, we have SR_2 =-30.00 which also falls in (-72,72). The process of taking samples and computing SR_i , and comparing it with the control limits is continued up to the 6^{th} sample. For this sample, we have $SR_6 = 78$, which is an out-of-control signal as it falls outside (-72, 72).

Moving to the CUSUM SR chart, we monitor the process using both an upper and a lower CUSUM statistic defined in section 2. At each sampling instance, both statistics are computed and compared with their *decision interval* values h_{sr}. For the first sample, the values of C_1^+ and C_1^- are both 0.00, which fall within the respective limits, and thus the process is considered in-control at the first sampling instance. This procedure of taking samples, calculating C_1^+ and C_1^-, and comparing them with h_{sr} is continued for each subsequent sample. When, at a given sampling point, either $C_1^+ > h_{sr}$ (indicating an upward shift) or $C_1^- < -h_{sr}$ (indicating a downward shift), an out-of-control signal is generated and appropriate action can be taken. At 12^{th} sample, we have $C_{12}^+ = 129$, which is an out-of-control signal as $C_i^+ \geq h_{sr}$

For EWMA SR chart, see that we have the value of the $Z_1 = -0.50$ in the 1^{st}sample, as that value falls within the control limits $UCL\setminus LCL = \mp 27.65$, we conclude that the process is in-control at the 1^{st}sampling instance and take the second sample after t time. For the second sample, we have Z_2 = -7.88 which also falls control limits $UCL\setminus LCL = \mp 27.65$. The process of taking samples, computing Z_i , and comparing it with the $UCL\setminus LCL$ is continued up to the 13^{th} sample. For this sample, we have $Z_{13} = 32$, which is an OC signal as $Z_i \geq UCL$. Table 5 shows the values of the Shewhart SR, CUSUM SR, and EWMA SR along with their decisions.

Table 5: The values of the Shewhart SR, CUSUM SR, and EWMA SR along with their decisions.

Sample No.	Control charts/ Decisions						
	SR_i	Decision	C_i^+	C_i^-	Decision	Z_i	Decision
1	-2.00	In-control	0.00	0.00	In-control	-0.50	In-control
2	-30.00	In-control	0.00	19.00	In-control	-7.88	In-control
3	-24.00	In-control	0.00	32.00	In-control	-11.91	In-control
4	-	In-control	0.00	45.00	In-control	-	In-control

	24.00					14.93	
5	13.00	In-control	2.00	21.00	In-control	-7.95	In-control
6	78.00	*Out-of-control*	69.00	0.00	In-control	13.54	In-control
7	---	---	84.00	0.00	In-control	16.65	In-control
8	---	---	112.00	0.00	In-control	22.24	In-control
9	---	---	77.00	13.00	In-control	10.68	In-control
10	---	---	101.00	0.00	In-control	16.76	In-control
11	---	---	106.00	0.00	In-control	16.57	In-control
12	---	---	129.00	0.00	*Out-of-control*	20.93	In-control
13	---	---	---		---	32.70	*Out-of-control*

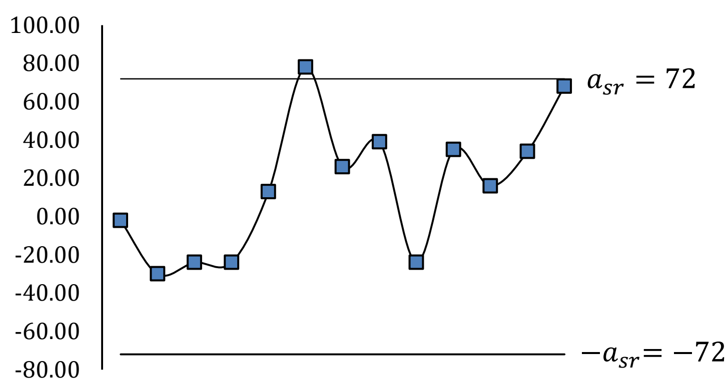

Figure 5: The Shewhart SR chart

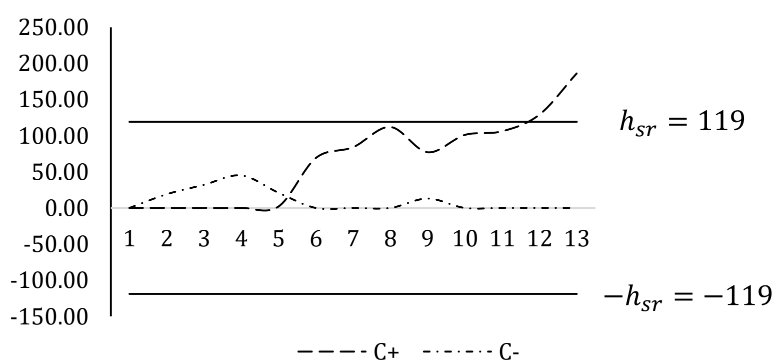

Figure 6: The CUSUM SR chart

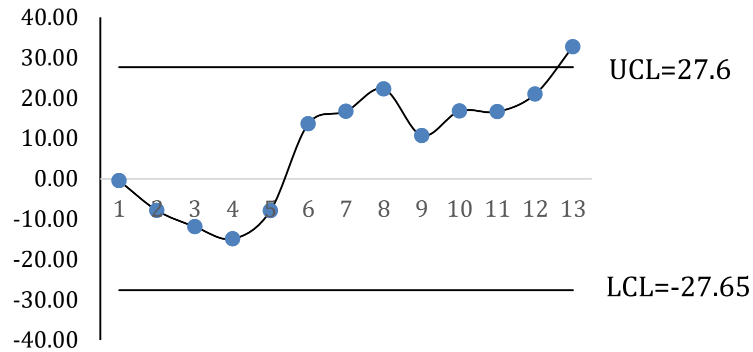

Figure 6: The EWMA SR chart

The results shows that the Shewhart SR chart signaled an out-of-control condition at sample No. 6, indicating its sensitivity to sudden and relatively larger shifts. In contrast, the CUSUM SR chart, did not indicate an out-of-control state until sample No.12, highlighting its strength in identifying moderate and sustained shifts rather than sudden ones. The EWMA SR chart, gives an out-of-control condition at sample 13, reflecting its effectiveness in detecting smaller shifts.

5. Conclusion

This study provided a comparative assessment of nonparametric control charts based on the SR statistic by evaluating their SSARL under various distributions and through a practical illustration of their decision-making behavior. The findings revealed that the Shewhart SR chart is more effective in detecting large process shifts but performs poorly for small ones, whereas the CUSUM SR and EWMA SR charts demonstrated superior sensitivity to small and moderate shifts.

From a practical standpoint, these results emphasize that in manufacturing and service industries where the underlying process distribution is unknown or difficult to specify, CUSUM SR and EWMA SR charts can provide more reliable early detection of process changes compared to the Shewhart SR chart. This highlights the value of nonparametric approaches as robust alternatives to conventional parametric charts, particularly when distributional assumptions are uncertain.

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