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A Novel Wavelet-Based Approach for ANOVA in Longitudinal Data Analysis Using the First Derivative of the Laplace Function

N. I. Muhammad[1], M. Tasi'u[2], U. K. Abdullahi[3], and Y. Aliyu[4]

[1,2,3,4]Department of Statistics, Ahmadu Bello University, Zaria-Nigeria

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ABSTRACT

Longitudinal data, characterized by repeated observations on the same subjects over time, present significant challenges for traditional statistical Analysis of Variance (ANOVA) methods, such as Repeated Measures ANOVA due to inherent intra-subject correlation and high dimensionality. This study introduced a novel wavelet-based ANOVA approach using the first derivative of the Laplace function to transform high-dimensional repeated measures data into a simplified Completely Randomized Design (CRD) structure, thereby facilitating classical ANOVA. The mathematical formulation and key properties (admissibility and energy localization) of this wavelet function are detailed. We rigorously proved that this wavelet satisfies the fundamental conditions required for signal analysis. Through a comprehensive simulation study, we demonstrated its effectiveness in preserving treatment differences, controlling Type I error rate, maintaining acceptable statistical power, and showing robustness to variations in the number of time points. The results validated the Laplace derivative wavelet as a powerful and reliable tool for dimensionality reduction and valid inference in longitudinal data analysis, offering a robust alternative to conventional repeated measures techniques.

1. Introduction

Longitudinal data, where observations are collected from the same subjects repeatedly over time, are prevalent in various research fields. Analyzing such data with traditional methods like Repeated Measures ANOVA (RM-ANOVA) is often problematic due to complex correlation structures and violations of the sphericity assumption [10]. To overcome these limitations, advanced techniques have emerged, including those based on wavelet analysis, which can effectively handle the high dimensionality and non-stationarity of time series data [6, 9, 11].

The introduction of Functional Analysis of Variance (FANOVA) helps to extend classical ANOVA to handle scenarios where observations are not single points, but functions or curves [3, 4, 5]. Wavelet Analysis of Variance (WANOVA), a specific subset of FANOVA is developed by combining wavelet analysis with ANOVA principles to address non-stationary and multi-resolution data [1, 8, 12]. These techniques aim to simplify complex correlated data for hypothesis testing. However, the development of customized wavelet functions specifically for aggregating repeated measures into a single value for CRD-based ANOVA, particularly those derived from distributions with properties like the Laplace's symmetric shape with sharp exponential transitions, represents a novel contribution to this field.

Corresponding author E-mail address: nazeerdabest01@gmail.com
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This paper proposes a novel continuous wavelet function derived from the first derivative of the Laplace probability density function (PDF). The Laplace distribution is known for its heavy tails and sharp peak, characteristics that are well-suited for modeling signals with abrupt changes and exponential decay [2, 7]. By leveraging these properties, we aim to create a wavelet that can effectively capture and aggregate information from time-series data where such features are present. The primary objective is to demonstrate that this wavelet can be used to reduce the dimensionality of longitudinal data in order to have a simplified and valid statistical analysis using a classical ANOVA framework.

2. Methodology

The core approach of this research involves using the Continuous Wavelet Transform (CWT) to aggregate a subject's longitudinal data into a single scalar value. The CWT of a signal $x(t)$ with a mother wavelet $\psi(t)$ is defined as

$$W(a,\tau)=\frac{1}{\sqrt{a}}\int_{-\infty}^{\infty}x(t)\psi^{*}\left(\frac{t-\tau}{a}\right)dt \qquad (1)$$

where a is the scale parameter and τ is the translation parameter. The key component of this method is our proposed wavelet function.

2.1 The First Derivative of the Laplace Function

The Laplace PDF is defined as

$$f_L(y;\mu,b)=\frac{1}{2b}\exp\left(-\frac{|y-\mu|}{b}\right) \qquad (2)$$

where

$y\in\mathbb{R}$ is the observation, $\mu\in\mathbb{R}$ is the location parameter, and $b>0$ is the scale parameter.

Taking its first derivative with respect to y, we obtain our mother wavelet $\psi_L(y)=f_L'(y)$

$$\psi_L(y)=-\frac{\operatorname{sgn}(y-\mu)}{2b^2}e^{-\frac{|y-\mu|}{b}}, \qquad (3)$$

where $\operatorname{sgn}(\cdot)$ is a sign function defined as;

$$\operatorname{sgn}(y-\mu)=\begin{cases}+; & if\ y>\mu\\ -; & if\ y<\mu\end{cases}. \qquad (4)$$

2.2 Aggregation Process

The derived Laplace derivative wavelet function is applied to each subject's repeated measurements in order to aggregate them into a single representative value. The aggregation procedure involves:

1. Computing location parameter $\mu_t=\frac{y_t+y_{t-1}}{2}$ for each pair of consecutive observations (y_t,y_{t-1}) of each subject's time series $(y_0,y_1,\cdots y_T)$.
2. Shifting the location parameter using the mean of consecutive observations μ_t in order to obtain the wavelet function $\psi_L(y_t;u_t)$.
3. Aggregating the time series of each subject by summing the wavelet transformation across all time points to obtain a single observation W_{ij}

$$W_{ij}=\sum_{t=1}^{T}\psi_L(y_{ijt};u_{ijt}) \qquad (5)$$

This transformation effectively reduces the dimensionality of the original repeated measures dataset, converting the data into a Completely Randomized Design (CRD) structure where each subject is represented by one aggregated value.

2.3 Mathematical Properties of the Laplace Derivative Wavelet

For $\psi_L(y)$ to be a valid wavelet, it must satisfy two fundamental conditions: admissibility and energy localization. These properties were formally prove as follows:

Proposition 1: Admissibility of the Laplace Derivative Wavelet.

The Laplace derivative wavelet satisfies the admissibility condition.

Proof:

The admissibility condition is given by

$$C_\psi = \int_0^\infty \frac{|\hat{\psi}(\omega)|^2}{|\omega|} d\omega < \infty \tag{6}$$

where $\hat{\psi}(\omega) = \mathcal{F}\{\psi(y)\}$ is the Fourier transform of $\psi(y)$.

The Fourier transform of the Laplace pdf is $\hat{f}(\omega) = \frac{1}{1+b^2\omega^2}$. Using the derivative property of the Fourier transform,

$$\mathcal{F}\{f'(y)\} = i\omega \mathcal{F}\{f(y)\} \tag{7}$$

we get

$$\hat{\psi}(\omega) = \frac{i\omega}{1+b^2\omega^2} \tag{8}$$

Therefore

$$\begin{aligned} C_\psi &= \int_0^\infty \frac{\frac{\omega^2}{(1+b^2\omega^2)^2}}{\omega} d\omega \\ &= \int_0^\infty \frac{\omega}{(1+b^2\omega^2)^2} d\omega \end{aligned} \tag{9}$$

Let $1+b^2\omega^2$, so $du = 2b2\omega d\omega$. The integral evaluates to $\frac{1}{2b^2}$, which is finite. Therefore, the admissibility condition holds.

Proposition 2: Energy Localization of the Laplace Derivative Wavelet.

The Laplace derivative wavelet has a finite total energy.

Proof:

The energy is defined as

$$E = \int_{-\infty}^{\infty} |\psi(y)|^2 \, dy < \infty. \tag{10}$$

Due to the symmetry of the wavelet around μ, we can evaluate the integral from μ to ∞ and multiply by 2

$$\begin{aligned} E &= \int_{-\infty}^{\infty} \left| -\frac{\text{sgn}(y-\mu)}{2b^2} e^{-\frac{|y-\mu|}{b}} \right|^2 dy \\ &= 2\int_{\mu}^{\infty} \left| \frac{1}{2b^2} e^{-\frac{y-\mu}{b}} \right|^2 dy = \frac{1}{2b^4} \int_{\mu}^{\infty} e^{-\frac{2(y-\mu)}{b}} dy \end{aligned} \tag{11}$$

This exponential integral evaluates to $\frac{1}{4b^3}$, a finite value. Therefore, the energy localization condition is satisfied.

3. Results and discussion

To evaluate the proposed wavelet's performance, a simulation study was conducted using a synthetic dataset designed to mimic a longitudinal study with 4 treatment groups of size 5 each, and every subject had 10 repeated measurements. Each subject's data was generated by combining a fixed treatment effect (linearly increasing: 10, 20, 30, 40 for treatments 1 through 4, respectively) with normal random noise (mean zero, equal variance). The data for each subject were aggregated into a single value using the Laplace derivative wavelet.

3.1 ANOVA Result

A one-way ANOVA was performed on the aggregated data to test for a significant treatment effect. The analysis is presented in Table 1.

Table 1: Laplace Wavelet ANOVA Result

	Df	Sum Sq	Mean Sq	F value	$Pr(>F)$
Treatment	3	4.188	1.3960	3.498	0.0401
Residuals	16	6.385	0.3991		

The ANOVA result in Table 1 shows an F-value of 3.498 with a p-value of 0.0401 (less than 0.05), indicating a statistically significant treatment effect. This demonstrates that the Laplace derivative wavelet successfully aggregated the longitudinal data in a way that preserved the significant differences between the treatment groups, enabling a straightforward and valid statistical inference using classical ANOVA.

3.2 Type I Error Rate

The type I error is the probability of rejected a null hypothesis when it is actually true, so the simulation is updated in to assume the null hypothesis that there is no treatment effect, hence the fixed treatment effect is equal (10) for all treatment groups. The Type I error rate for the Laplace derivative wavelet was found to be 0.052. This value is very close to the nominal 5% significance level $(\alpha = 0.05)$. This indicates that the false-positive rate is well-controlled, implying that the aggregation process using the Laplace derivative wavelet does not inflate or deflate error risk.

3.3 Power of the Test

The statistical power is the probability of correctly rejected a null hypothesis, therefore the power of the test assumes the alternative hypothesis. The statistical power of the test using the Laplace derivative wavelet was estimated at 0.948. This high power estimate (94.8%) demonstrates the strong ability of the method to detect true treatment effects when they exist. This provides clear evidence that the Laplace derivative wavelet aggregation strategy successfully retains the treatment-level signal, yielding reliable inference via standard ANOVA.

3.4 Sensitivity to Time Point Variation

The robustness of the Laplace derivative wavelet to varying numbers of time points was assessed by evaluating power at different longitudinal lengths

Table 2: Robustness to time point

Time Points	Power
5	0.968
10	0.950
15	0.966
20	0.946

Based on the result presented in Table 2, the power remains consistently high across 5, 10, 15, and 20 time points, confirming the robustness of the Laplace derivative wavelet-based aggregation to changes in longitudinal data length, The Laplace wavelet performs particularly well at shorter time lengths (Power = 0.968 at 5 time points), indicating high sensitivity even in sparse data scenarios. This highlights its adaptability and effectiveness regardless of the density of repeated measurements.

4. Conclusions

This paper has introduced and validated a new continuous wavelet function derived from the first derivative of the Laplace probability density function. Formal proofs were provided to demonstrate that this wavelet satisfies the critical admissibility and energy localization conditions, making it a viable tool for signal analysis. The simulation study confirmed its effectiveness in a practical application, showing its ability to reduce the dimensionality of longitudinal data and facilitate a valid ANOVA without the restrictive assumptions of traditional methods. The Laplace derivative wavelet, with its unique exponential decay and sharp transitions, is particularly well-suited for analyzing signals with sudden changes. Future work will focus on applying this wavelet to a wider range of real-world datasets and comparing its performance to other established wavelets.

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